Planting the seeds of Calculus in Algebra 2 and Precalculus

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Presentation materials: goo.gl/ZsR1x4

Prior to this investigation, students have learned to find a sum of a finite number of terms of a geometric sequence such as

$$S = 3 + 15 + 75 + \cdots + 29296875$$
$$-\ 5S = \quad 15 + 75 + \cdots + 29296875 + 146,484,375$$
$$-4S = 3 - 146,484,375$$

$$S = 36,620,593$$

In-class Experiment

Get a piece of paper. Call the area of the paper 1.

Step 1 Take your piece of paper and fold it in half. Cut into two pieces.

Step 2 Place one half on the desk in front of you. Write the area of the piece on your desk. Keep the other half in your hand.

Step 3 Repeat steps 1 and 2 with the piece of paper in your hand.

Repeat this process a few times.

Cut #	Area of Paper in hand	Area of each rectangle on your desk	Sum of the area of paper on desk (written as one #)

Head Scratcher(s)...

Remember...

The area of the original sheet was 1.

What if we continued this process forever...

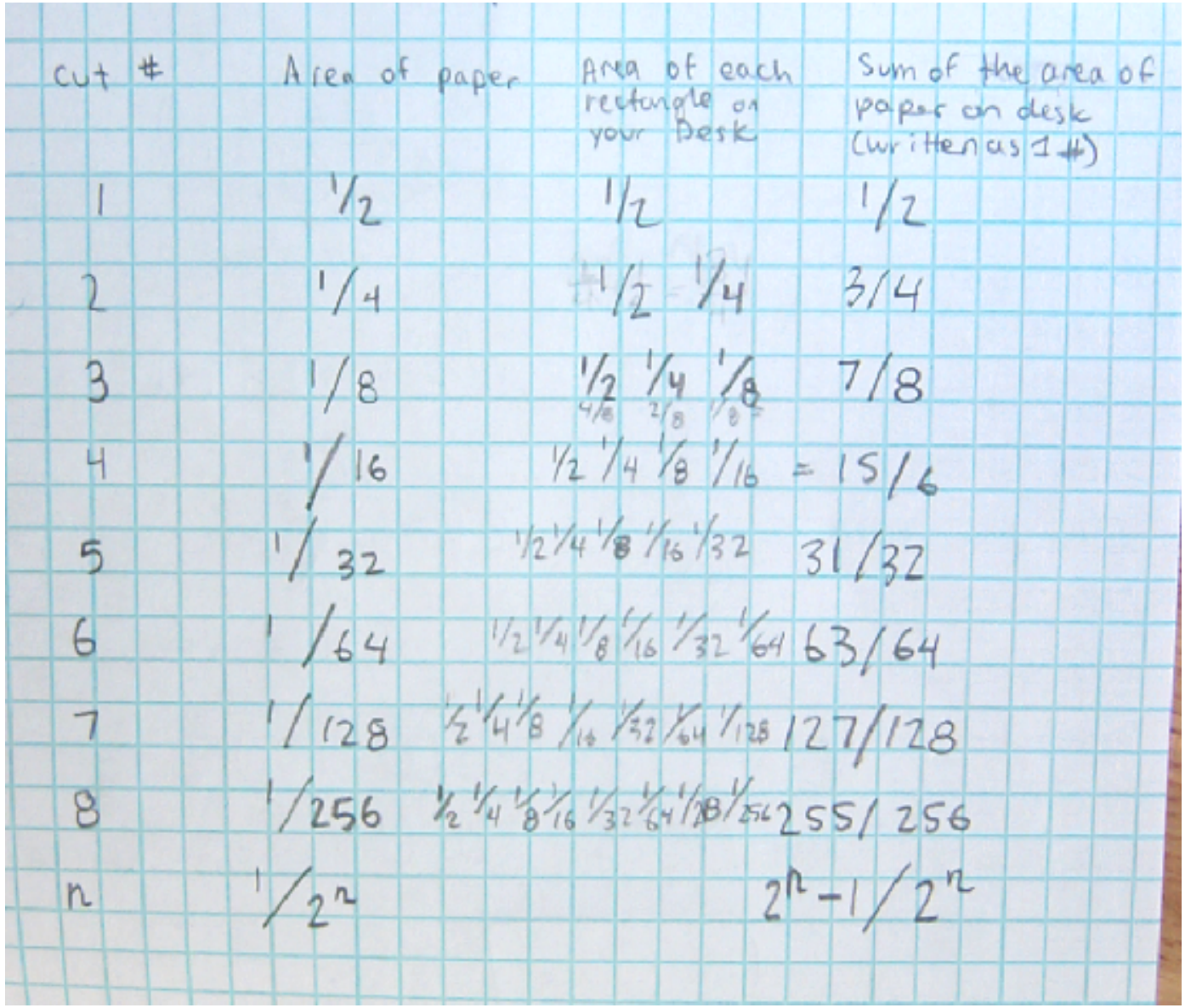
What would happen to the area of the rectangle in your hand?

What would happen to the area of the rectangle(s) on your desk?

Represent the areas on the desk symbolically.

What if we continued this process forever?
What would happen to the area of the rectangle in your hand?
What would happen to the area of the rectangle on your desk?

After cutting the paper n times,
what is the area of the rectangle in your hand?
what is the area of the paper on your desk?

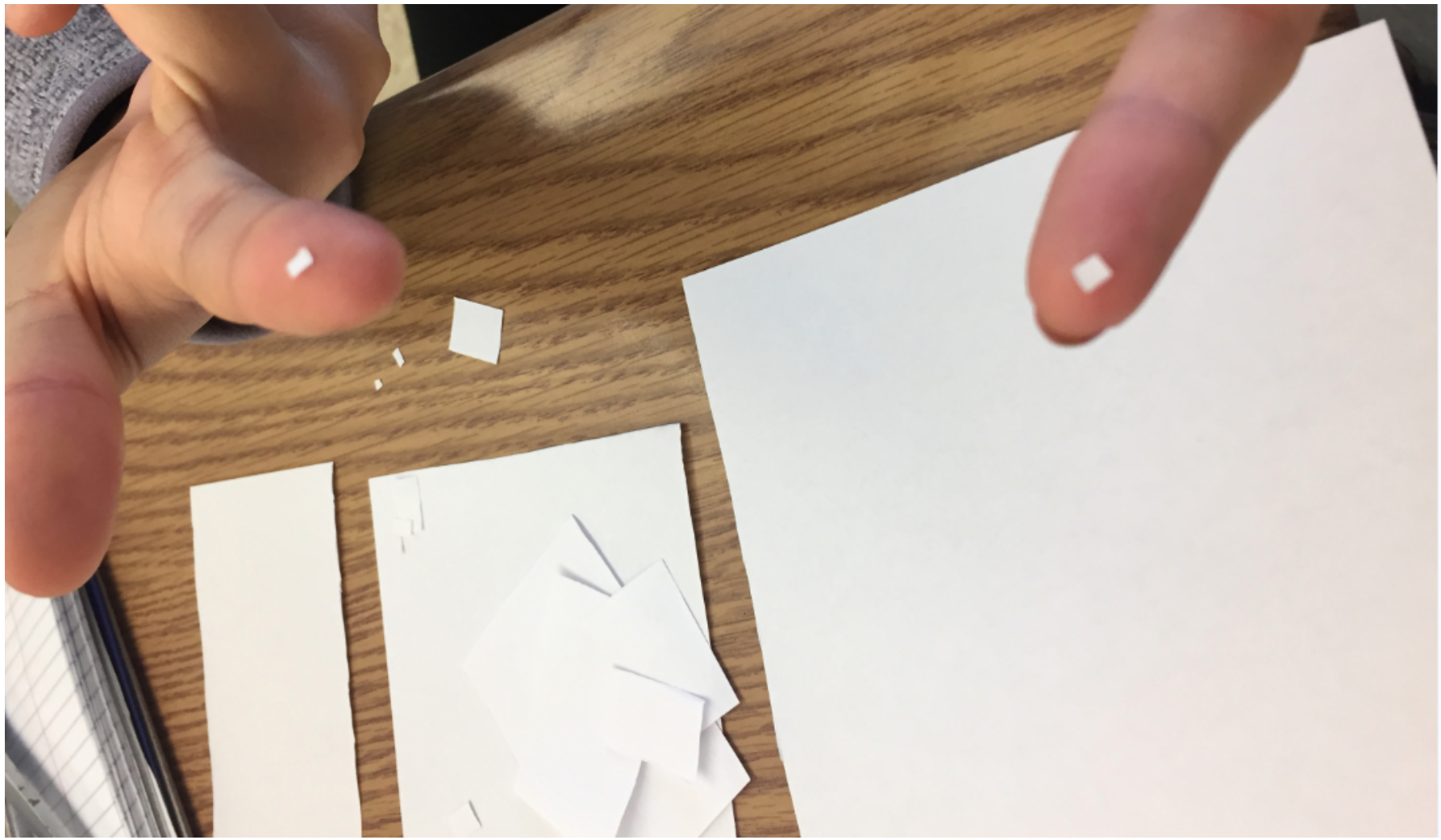


Cut #	Area of paper	Area of each rectangle on your Desk	Sum of the area of paper on desk (written as 1 #)
1	1/2	1/2	1/2
2	1/4	1/2 1/4	3/4
3	1/8	1/2 1/4 1/8	7/8
4	1/16	1/2 1/4 1/8 1/16	= 15/16
5	1/32	1/2 1/4 1/8 1/16 1/32	31/32
6	1/64	1/2 1/4 1/8 1/16 1/32 1/64	63/64
7	1/128	1/2 1/4 1/8 1/16 1/32 1/64 1/128	127/128
8	1/256	1/2 1/4 1/8 1/16 1/32 1/64 1/128 1/256	255/256
n	$1/2^n$		$2^n - 1/2^n$

Cut #	area Paper in hand	area of each rectangle on your desk	Sum of area of Paper on desk
1	1/2	1/2	1/2
2	1/4	1/2 1/4	3/4
3	1/8	1/2 1/4 1/8	7/8
4	1/16	1/2 1/4 1/8 1/16	15/16

$\frac{1}{2^n}$ = Paper in hand

$1-\frac{1}{2^n} = \frac{2^n-1}{2^n}$ = Sum of area



Follow Up...

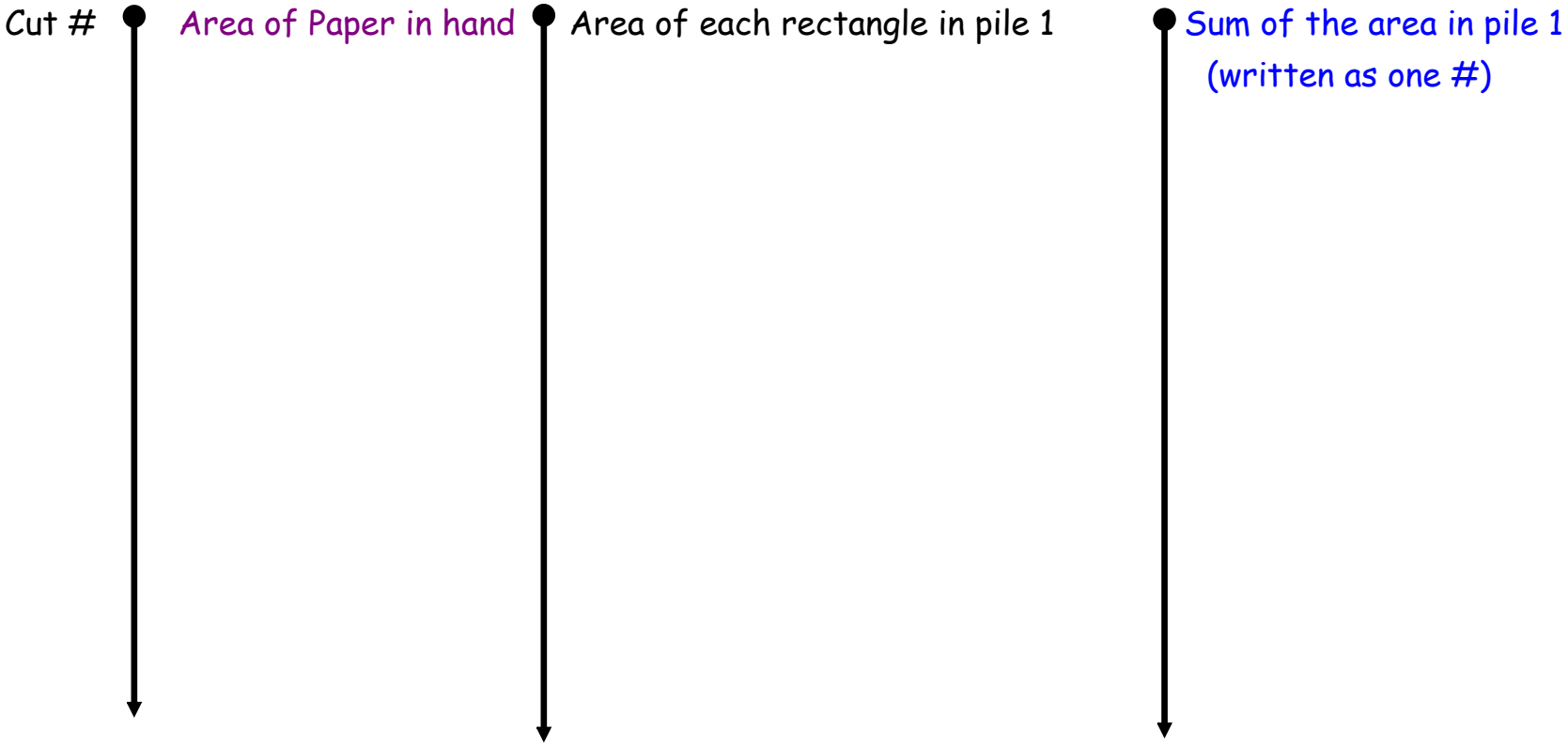
Get a piece of paper. Call the area of the paper 1.

Step 1 Take your piece of paper and fold it into thirds. Cut it into three pieces.

Step 2 Place one piece in Pile 1, put one piece in Pile 2, keep one in your hand.

Step 3 Repeat steps 1 and 2 with the piece of paper in your hand.

Repeat this process a few times.



What is a closed form for

$$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots+\frac{1}{3^n}$$

Try to think of this two ways! Euclid's method and using the paper folding analogy.

9. An infinite number of mathematicians walk into a bar. The first orders a beer, the second orders half a beer, the third orders a quarter of a beer and so on.

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After the seventh order, the bartender pours two beers and says, "you fellas ought to know your limits."

Today is Alice's bday and she loves cake. In order to make the cake last a long time she eats
a 1/3 of the cake the first day and
1/3 of what was left each day after.

a. How much cake did she eat on the 10th day?

b. How much cake has she eaten after 10 days?

c. n days?

Her brother Harry has a birthday a few days after her and he too likes cake. But he eats 2/3 of the cake the first day and 2/3 of what is left cake each day after that. How do the answers change?


AH!-CHOO!

ACCHHHOOoooooo! Many of you have experienced, either directly or indirectly, the spread of influenza. The spread of a highly communicable disease through a confined area can be modeled by a mathematical function.

Class data collection: In order to model this we are going to simulate the spreading of a disease throughout our class. Don't worry no injections will be given. This disease will be called the 'Sulu-Clacerp Plague'. (*Pronounced Soo-Loo-Clay-Serp*) Everyone is a resident in the room with an assigned number so that we can keep track of who is infected and who is not. Record the data in the table below.

YOUR ARE RESIDENT NUMBER: _____

Day #	0													
People Infected	1													

Today is day zero, and one person is infected. BUT WHO! Let's keep track of who is healthy and who is infected.

Healthy Resident Numbers...not for long ☹

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

The Sulu-Clacerp disease has spread throughout our classroom and all are infected ☹

1.) Describe in words the rate at which the disease has spread. Offer an explanation as to why the disease spread in the manner in which it did.

2.) How will this impact the graph? Why does the curve behave as it does for the first half of the data collection? The second half? Think real life here…

3.) See how you did by considering a graph of the data. What do you notice about the graph? How is it similar to the concept of exponential growth discussed in class? How is it different?

4.) What are the equations of the asymptotes of the graph? What is the real life meaning behind these asymptotes?

5.) Over what interval is the spread of the disease increasing at an increasing rate? Over what interval is the spread of the disease increasing at a decreasing rate? How are these intervals represented in the graph?

6.) Approximate the instant when the spread of disease was the greatest. How is this represented in the graph? Why was the rate of the spread of disease greatest on that day?

7.) Approximate when the spread of the disease is the slowest. How is this represented in the graph? Why is the disease spreading at the lowest rate at the value you identified?

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