The Rule of Four: Series Convergence Tests with a Rich Task and Multiple Representations

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Tinyurl with access to information
Man in the Mirror by Michael Jackson

Who is in the Room?

Outcomes

Teacher Hat

Student Hat

By the end of this workshop, you will be able to:

- Complete a task to represent series (and the corresponding sequence) in multiple ways.
- Discover the nth term test and the series convergence test “rules” for *p* series and geometric series.
- Identify teaching practices that support student achievement.

Why?

AP Calculus AB/BC
Schedule and Assignments
Chapter 8 Infinite Series

Date	Topic
	8.a Sequences of Real Numbers *Homework: 8.a Worksheet*
	8.b Infinite Series - Project *Homework: 8.b Homework Worksheet 1*
	8.b Infinite Series *Homework: 8.b Homework Worksheet 2*
	8.b Infinite Series *Homework: 8.c Video and Survey*
	8.c The Integral Test *Homework: 8.d Part 1 Video and Survey*
	8.d Limit Test *Homework: 8.d Part 2 Video and Survey*
	8.d Direct Comparison Test *Homework: 8.e Video and Survey*
	8.e Alternating Series and Error *Homework: 8.f Video and Survey*
	8.f Absolute Convergence and the Ratio Test *Homework: 8.g Video and Survey*
	Mixed Practice *Homework: Unit Study Sheet*
	Mixed Practice *Homework:*
	Mixed Practice *Homework:*
	Infinite Series Part 1 Test *Homework: None*

Why did that series converge?

How does that work?

What does it look like to go to infinity?

2012 – 2013: Taught Rules Only (Graphic Organizers and Flow Charts)

Tests for Series Convergence

	TEST	SERIES	CONVERGES	DIVERGES	COMMENTS
1.					
	n^{th} term test	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	This test cannot be used to show convergence
2.					
	Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\lvert r\rvert < 1$	$\lvert r\rvert \geq 1$	Sum: $S = \frac{a}{1-r}$
	p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	*Harmonic Series* when $p = 1$ **divergent** p-series $\sum_{n=1}^{\infty} \frac{1}{n}$
	Telescoping Series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	$\lim_{n\to\infty} b_n = L$		write out terms
	Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$0 \leq a_{n+1} \leq a_n$ and $\lim_{n\to\infty} a_n = 0$		
3.					
	Integral Test (f is continuous, positive, and decreasing)	$\sum_{n=1}^{\infty} a_n$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	
	Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} > 1$	Test is inconclusive if $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = 1$
	Ratio Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$	$\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$	Test is inconclusive if $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$
4.					
	Direct Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
	Limit Comparison ($a_n, b_n > 0$)	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	

2012 – 2013: Taught Rules Only
(Graphic Organizers and Flow Charts)

2013 – 2014: Investigation of Different Types of Series (Individual worksheets done station style)

2014 – 2015: Poster Project Version 1.0
(Posters with discussion but no formal work)

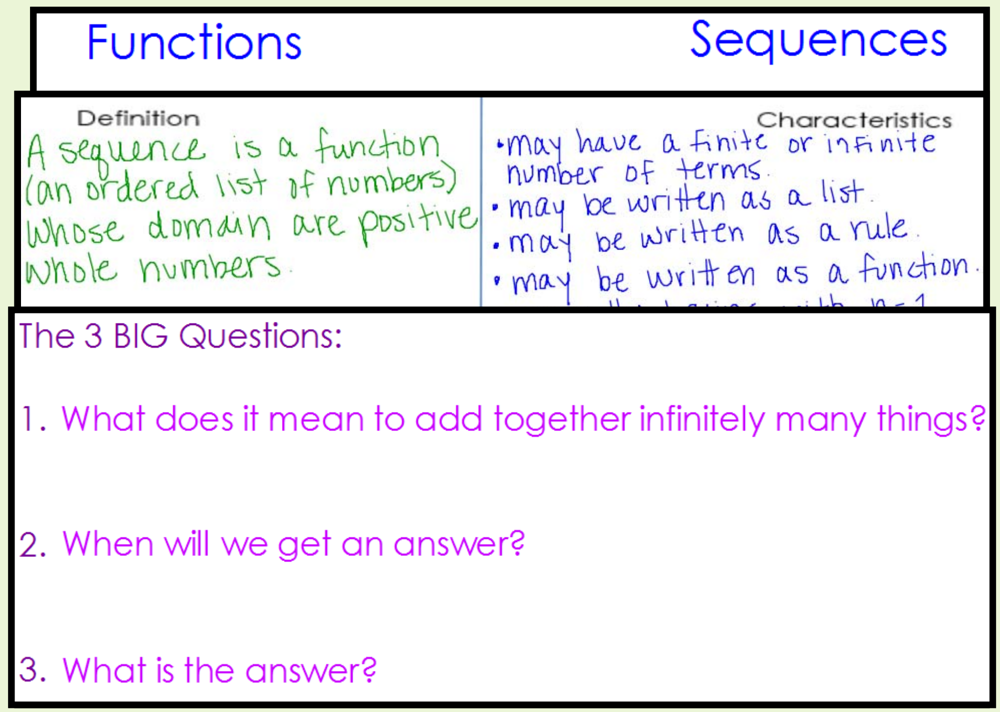
2015 – 2016: Poster Project Version 2.0
(Posters with opportunities to make generalizations and check for student understanding)

Background: Prior Knowledge

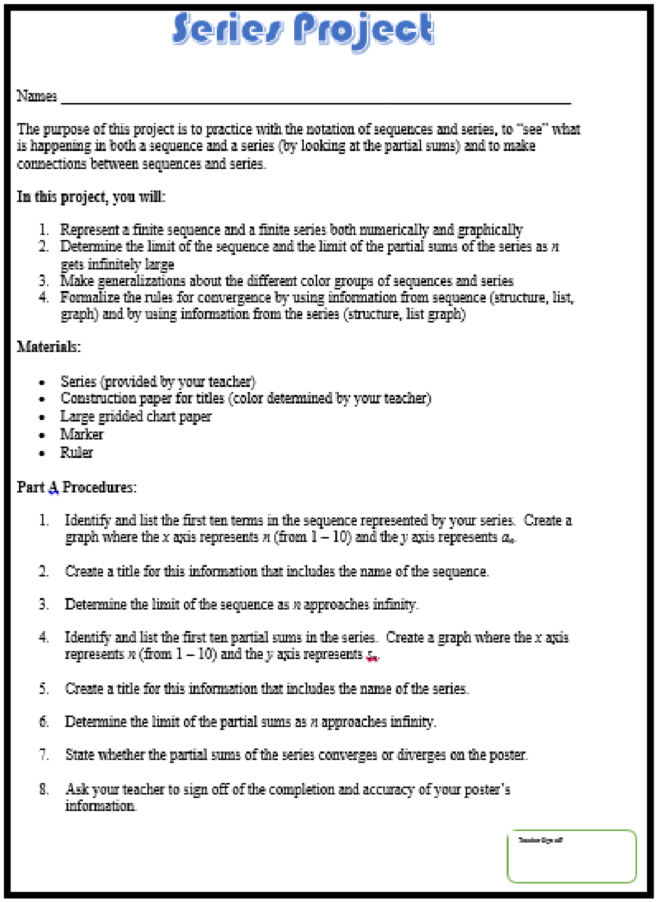
PreCalculus (and prior)

- Arithmetic and geometric sequences
- Explicit and recursive rules for sequences
- Arithmetic and geometric series
- Summation notation

Previous Calculus Lesson(s)



The Poster Task Project: The Rule of Four (Part A)



Series Project

Names ____

The purpose of this project is to practice with the notation of sequences and series, to "see" what is happening in both a sequence and a series (by looking at the partial sums) and to make connections between sequences and series.

In this project, you will:

1. Represent a finite sequence and a finite series both numerically and graphically
2. Determine the limit of the sequence and the limit of the partial sums of the series as n gets infinitely large
3. Make generalizations about the different color groups of sequences and series
4. Formalize the rules for convergence by using information from sequence (structure, list, graph) and by using information from the series (structure, list graph)

Materials:

- Series (provided by your teacher)
- Construction paper for titles (color determined by your teacher)
- Large gridded chart paper
- Marker
- Ruler

Part A Procedures:

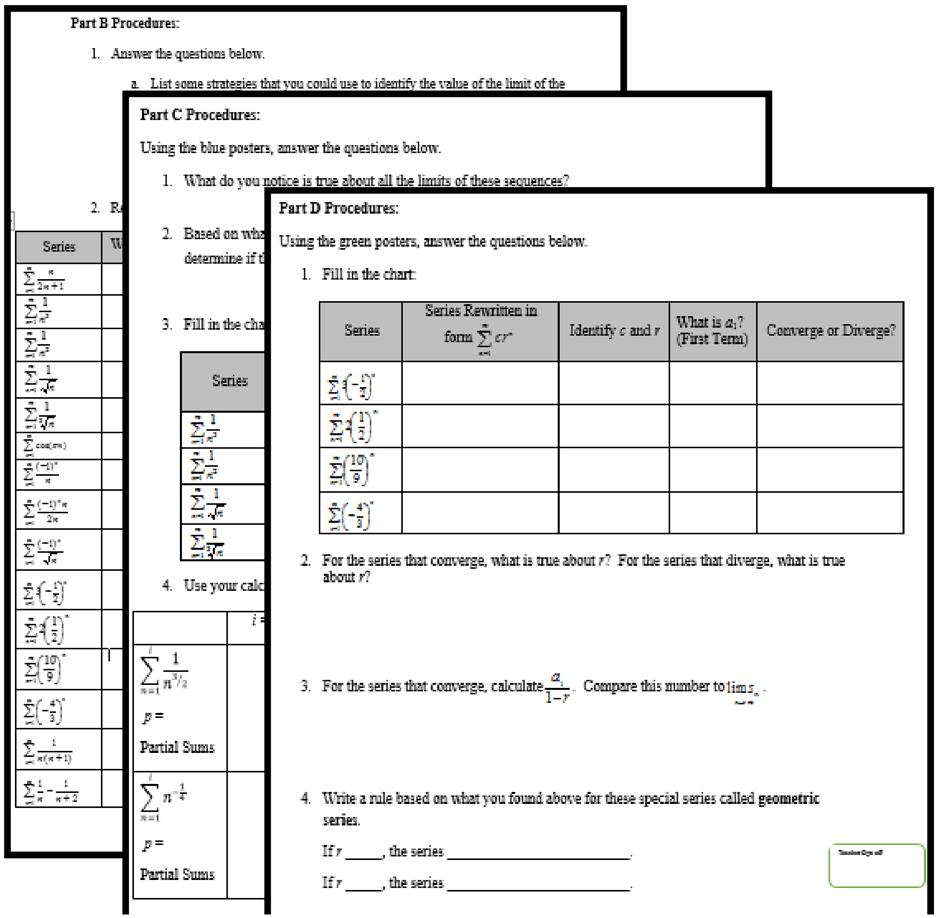
1. Identify and list the first ten terms in the sequence represented by your series. Create a graph where the x axis represents n (from 1 – 10) and the y axis represents a_n.
2. Create a title for this information that includes the name of the sequence.
3. Determine the limit of the sequence as n approaches infinity.
4. Identify and list the first ten partial sums in the series. Create a graph where the x axis represents n (from 1 – 10) and the y axis represents s_n.
5. Create a title for this information that includes the name of the series.
6. Determine the limit of the partial sums as n approaches infinity.
7. State whether the partial sums of the series converges or diverges on the poster.
8. Ask your teacher to sign off of the completion and accuracy of your poster's information.

Why is this part of the poster task project valuable?

Teacher Hat

Student Hat

The Poster Task Project: Convergence Testing Discovery (Parts B – D)



Part B Procedures:

1. Answer the questions below.
   a. List some strategies that you could use to identify the value of the limit of the

Part C Procedures:

Using the blue posters, answer the questions below.

1. What do you notice is true about all the limits of these sequences?

Part D Procedures:

Using the green posters, answer the questions below.

1. Fill in the chart:

Series	Series Rewritten in form $\sum_{n=1}^{\infty} cr^n$	Identify c and r	What is a_1? (First Term)	Converge or Diverge?
$\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n$				
$\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$				
$\sum_{n=1}^{\infty}\left(\frac{10}{9}\right)^n$				
$\sum_{n=1}^{\infty}\left(-\frac{4}{3}\right)^n$				

2. For the series that converge, what is true about r? For the series that diverge, what is true about r?

3. For the series that converge, calculate $\frac{a_1}{1-r}$. Compare this number to $\lim_{n\to\infty} s_n$.

4. Write a rule based on what you found above for these special series called **geometric** series.

   If r _____, the series ______________________.

   If r _____, the series ______________________.

Why is this part of the poster task project valuable?

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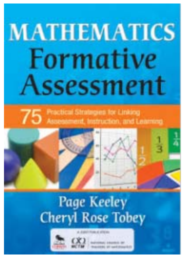
Formative Assessment



"Often it is hard to tell whether a particular technique or strategy serves an instructional, assessment, or learning purpose because they are so intertwined. Students are learning while at the same time, the teacher is gathering valuable information about their thinking that will inform instruction and provide opportunities for students to surface, examine and reflect on their learning."

(Mathematics Formative Assessment, Keely and Tobey, p 3)

MTPs & SMPs

Principles to Actions
Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

COMMON CORE
STATE STANDARDS INITIATIVE

Common Core State
Standards for Mathematical Practice
(Referred to as Math Practice Standards)

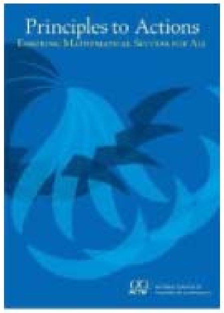
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Common Core Standards Initiative, 2012
http://www.corestandards.org

Conceptual Understanding to Procedural Fluency

"Effective teaching not only acknowledge the importance of both conceptual understanding and procedural fluency but also ensures that the learning of procedures is developed over time, on a strong foundation of understand the use of student generated strategies in solving problems."

(Principles to Action, p 46)



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