Excerpts from *Curricular Coherence and Open Educational Resources*

NCTM has long recognized the importance of coherence in a well-developed mathematics curriculum. Broadly stated, coherence means that connections are made from one year to the next, from one idea to another, from one representation to another, and from one statement to many others that are implied by that statement. There is coherence pedagogically, logically, conceptually, in terms of learning science, and with the real world. …

The Curriculum Principle in *Principles and Standards for School Mathematics* (NCTM, 2000 p. 14–16) states, "A curriculum is more than a collection of activities." It further describes a well-articulated curriculum as both making clear the most important mathematics of the grade level, when concepts and skills are introduced and when they should be mastered, and how student conceptual understanding of big ideas develops across units and across multiple grade levels [NCTM 2016, 1, 2].

An excerpt from *Principles to Actions: Ensuring Mathematical Success for All*

An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world (NCTM 2014, 5, 70).

An excerpt from *Toward a Coherent Curriculum* (1995 ASCD Yearbook)

A "coherent" curriculum is one that holds together, that makes sense as a whole; and its parts, whatever they are, are unified and connected by that sense of the whole. The idea of coherence begins with a view of the curriculum as a broadly conceived concept—as *the* curriculum—that is about "something." It is not simply a collection of disparate parts or pieces that accumulate in student experiences and on transcripts. A coherent curriculum has a sense of the forest as well as the trees, a sense of unity and connectedness, of relevance and pertinence. Parts or pieces are connected or integrated in ways that are visible and explicit. There is a sense of a larger, compelling purpose, and actions are tied to that purpose (Beane 1995, 3).

An excerpt from Common Core State Standards for Mathematics

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are

> articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties). (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the “sequence of topics and performances” that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing “sequenced obstacles and challenges for students…absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise.” In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time [NGA Center and CCSSO 2010, 3–4].

An excerpt from *Designs for Science Literacy*

Types of Curriculum Coherence

In the context of this chapter, a coherent curriculum is one that focuses on the relatedness of particular knowledge and skills needed for science literacy, takes developmental considerations into account in deciding on the grade placement of specific learning goals in science, mathematics, and technology, and provides occasions for exploring thematic connections between science-related subjects and other fields. These three aspects of curriculum coherence—literacy goals, developmental sequence, and thematic connections—are discussed below. …

Thematic Connections

In principle, a coherent curriculum is one that makes conceptual sense at every level of instruction—from daily lesson plans to units, to courses, to the curriculum as a whole each year, to the curriculum over the years. Ideas and skills do not stand alone but are linked conceptually to other ideas and skills and appear in a variety of contexts.

This aspect of coherence, of course, is easier to achieve at the lesson-plan and unit levels than at higher ones. Indeed, the individual units making up a course of study are often very carefully organized, whereas the collection of units making up a course are largely independent of one another except for being in the same subject. Nevertheless, it is well within the reach of publishers to develop courses and even course sequences—blocks, in the language of earlier chapters of this book—in which the content is thoughtfully organized around a few pervasive themes, key ideas and skills are visited periodically (and with specific purpose) in different contexts, and their relationship is discussed explicitly. Crosscutting instructional themes can be used for creating such curriculum bridges. They can be concepts, applications, or historical episodes (Project 2061 AAAS 2000, 237, 240, 242).

From *Mathematical Musings*

What Does It Mean for a Curriculum to Be Coherent?

MARCH 29, 2017 / BILL MCCALLUM

Al Cuoco and I have been thinking about this question and have developed some ideas. I want to write about the first and most obvious one today, the principle of logical sequencing. I'll write about others in the weeks to come.

Remember the distinction between standards and curriculum. While standards might remain fixed—a mountain we aim to help our students climb—different curricula designed to achieve those standards might make different choices about how to get there. Whatever the choices, a coherent curriculum, focused on how to get students up the mountain, would make sense of the journey and single out key landmarks and stretches of trail—a long path through the woods, or a steep climb up a ridge.

By the same token, mathematics has its landscape. CCSSM pays attention to this landscape by laying out pathways, or progressions, that span across grade levels and between topics, so that a third grade teacher understands why she is teaching a particular topic, because it will help students with some other topic in the next grade and build on what they already know.

This leads us to the first property of a coherent curriculum: it makes clear a logical sequence of mathematical concepts.

Consider, for example, the concepts of similarity and congruence. It is quite common in school curricula for similarity to be introduced before congruence. This comes out of an informal notion of similarity as meaning "same shape" and congruence as meaning "same shape and same size." However, the fact that the informal phrase for similarity is a part of the informal phrase for congruence is deceptive about the mathematical precedence of the concepts. For what does it mean for two shapes to be the same shape (that is, to be similar)? It means that you can scale one of them so that the resulting shape is both the same size and the same shape as the other (that is, congruent). Thus the concept of similarity depends on the concept of congruence, not the other way around. This suggests that the latter should be introduced first.

This is not to say you can never teach topics out of order; after all, it is a common narrative device to start a story at the end and then go back to the beginning, and it is reasonable to suppose that a corresponding pedagogical device might be useful in certain situations. But the curriculum should be designed so that the learner is made aware of the prolepsis. (Really, I just wrote this blog post so I could use that word.)

Although the progressions help identify the logical sequencing of topics, there is more work to do on that when you are writing curriculum. For example, the standards separate the domain of Number and Operations in Base Ten and the domain of Operations and Algebraic Thinking, in order to clearly identify these two important threads leading to algebra. But these two threads are logically interwoven, and it would not make sense to teach all the NBT standards in a grade level separately from all the OA standards.

In the next few blog posts, I will talk about three other aspects of coherent curriculum: the evolution from particulars to deeper structures, using deep structures to make connections between topics, and coherence of mathematical practice.

(McCallum 2017)

An excerpt from *K–8 Publishers' Criteria for the Common Core State Standards for Mathematics*

Coherence

Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject in which powerful knowledge results from reasoning with a small number of principles such as place value and properties of operations.[4] The Standards define progressions of learning that leverage these principles as they build knowledge over the grades.[5]

Coherence has to do with connections between topics. Vertical connections are crucial: these are the links from one grade to the next that allow students to progress in their mathematical education. For example, a kindergarten student might add two numbers using a "count all" strategy, but grade 1 students are expected to use "counting on" and more sophisticated strategies. It is critical to think across grades and examine the progressions in the standards to see how major content develops over time.

The Standards do not specify the progression of material within a single grade, but coherence across grades also depends on having careful, deliberate, and progressive development of ideas within each grade. Some examples of this can be seen in the *Progressions* documents.[6] For example, it would not make sense to address cluster 8.EE.B (understanding the connections between proportional relationships, lines, and linear equations) before addressing triangle similarity, as ideas of triangle similarity underlie the very definition of the slope of a line in the coordinate plane.

Connections at a single grade level can be used to improve focus, by closely linking secondary topics to the major work of the grade. For example, in grade 3, bar graphs are not "just another topic to cover." Rather, the standard about bar graphs asks students to use information presented in bar graphs to solve word problems using the four operations of arithmetic. Instead of allowing bar graphs to detract from the focus on arithmetic, the Standards are showing how bar graphs can be positioned in support of the major work of the grade. In this way coherence can support focus.

Materials cannot match the contours of the Standards by approaching each individual content standard as a separate event. Nor can materials align to the Standards by approaching each individual grade as a separate event. From the Appendix: "The standards were not so much assembled out of topics as woven out of progressions. Maintaining these progressions in the implementation of the standards will be important for helping all students learn mathematics at a higher level. … For example, the properties of operations, learned first for simple whole numbers, then in later grades extended to fractions, play a central role in understanding operations with negative numbers,

[3] See the Smarter/Balanced content specification and item development specifications, and the PARCC Model Content Framework and item development ITN. Complete information about the consortia can be found at www.smarterbalanced.org and www.parcconline.org.

[4] For some remarks by Phil Daro on this theme, see the excerpt at http://vimeo.com/achievethecore/darofocus, and/or the full video available at http://commoncoretools.me/2012/05/21/phil-daro-on-learning-mathematics-through-problem-solving/.

[5] For more information on progressions in the Standards, see http://ime.math.arizona.edu/progressions.

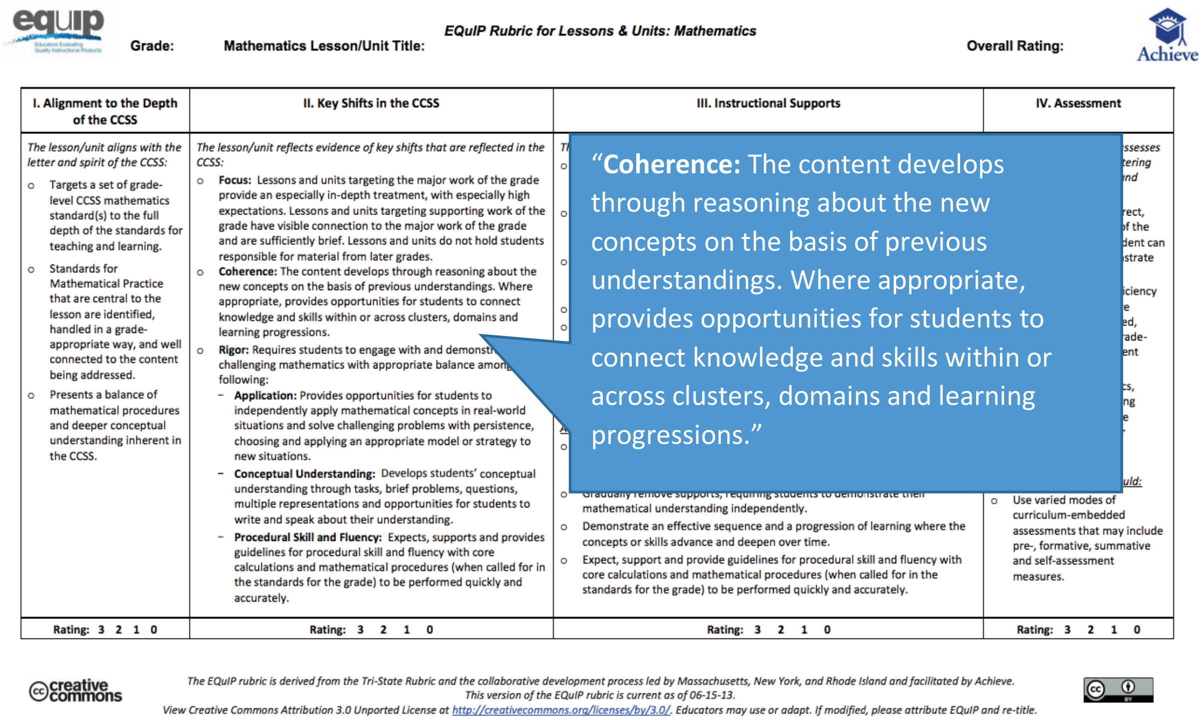
[6] http://ime.math.arizona.edu/progressions

expressions with letters and later still the study of polynomials. As the application of the properties is extended over the grades, an understanding of how the properties of operations work together should deepen and develop into one of the most fundamental insights into algebra. The natural distribution of prior knowledge in classrooms should not prompt abandoning instruction in grade level content, but should prompt explicit attention to connecting grade level content to content from prior learning. To do this, instruction should reflect the progressions on which the CCSSM are built."

"Fragmenting the Standards into individual standards, or individual bits of standards, … produces a sum of parts that is decidedly less than the whole" (Appendix). Breaking down standards poses a threat to the focus and coherence of the Standards. It is sometimes helpful or necessary to isolate a part of a compound standard for instruction or assessment, but not always, and not at the expense of the Standards as a whole. A drive to break the Standards down into 'microstandards' risks making the checklist mentality even worse than it is today. Microstandards would also make it easier for microtasks and microlessons to drive out extended tasks and deep learning. Finally, microstandards could allow for micromanagement: Picture teachers and students being held accountable for ever more discrete performances. If it is bad today when principals force teachers to write the standard of the day on the board, think of how it would be if every single standard turns into three, six, or a dozen or more microstandards. If the Standards are like a tree, then microstandards are like twigs. You can't build a tree out of twigs, but you can use twigs as kindling to burn down a tree.

(NGA Center and CCSSO 2013, 4–5)

An excerpt from "EQuIP Rubric for Lessons and Units: Mathematics"



equip — Grade: — Mathematics Lesson/Unit Title: — *EQuIP Rubric for Lessons & Units: Mathematics* — Overall Rating: — Achieve

I. Alignment to the Depth of the CCSS	II. Key Shifts in the CCSS	III. Instructional Supports	IV. Assessment
The lesson/unit aligns with the letter and spirit of the CCSS: ○ Targets a set of grade-level CCSS mathematics standard(s) to the full depth of the standards for teaching and learning. ○ Standards for Mathematical Practice that are central to the lesson are identified, handled in a grade-appropriate way, and well connected to the content being addressed. ○ Presents a balance of mathematical procedures and deeper conceptual understanding inherent in the CCSS.	*The lesson/unit reflects evidence of key shifts that are reflected in the CCSS:* ○ **Focus:** Lessons and units targeting the major work of the grade provide an especially in-depth treatment, with especially high expectations. Lessons and units targeting supporting work of the grade have visible connection to the major work of the grade and are sufficiently brief. Lessons and units do not hold students responsible for material from later grades. ○ **Coherence:** The content develops through reasoning about the new concepts on the basis of previous understandings. Where appropriate, provides opportunities for students to connect knowledge and skills within or across clusters, domains and learning progressions. ○ **Rigor:** Requires students to engage with and demonstr… challenging mathematics with appropriate balance amon… following: – **Application:** Provides opportunities for students to independently apply mathematical concepts in real-world situations and solve challenging problems with persistence, choosing and applying an appropriate model or strategy to new situations. – **Conceptual Understanding:** Develops students' conceptual understanding through tasks, brief problems, questions, multiple representations and opportunities for students to write and speak about their understanding. – **Procedural Skill and Fluency:** Expects, supports and provides guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	… mathematical understanding independently. ○ Demonstrate an effective sequence and a progression of learning where the concepts or skills advance and deepen over time. ○ Expect, support and provide guidelines for procedural skill and fluency with core calculations and mathematical procedures (when called for in the standards for the grade) to be performed quickly and accurately.	… ○ Use varied modes of curriculum-embedded assessments that may include pre-, formative, summative and self-assessment measures.
Rating: 3 2 1 0	Rating: 3 2 1 0	Rating: 3 2 1 0	Rating: 3 2 1 0

The EQuIP rubric is derived from the Tri-State Rubric and the collaborative development process led by Massachusetts, New York, and Rhode Island and facilitated by Achieve.
This version of the EQuIP rubric is current as of 06-15-13.
View Creative Commons Attribution 3.0 Unported License at http://creativecommons.org/licenses/by/3.0/. Educators may use or adapt. If modified, please attribute EQuIP and re-title.

(EQuIP, Achieve 2013)

An excerpt from EdReports.org *Quality Instructional Materials Tool: Grades K–8 Mathematics*

March 2015

Rating Sheet 3: Coherence

- For 'Coherence' to attain a score of 'Meets Expectations,' material must earn at least 7 points.

CRITERION	INDICATORS	RATING	EVIDENCE
Coherence: Each grade's instructional materials are coherent and consistent with the Standards.	**1c.** Supporting content enhances focus and coherence simultaneously by engaging students in the major work of the grade.[10]	0 1 2*	
	1d. The amount of content designated for one grade level is viable for one school year in order to foster coherence between grades.	0 1 2	
Earned: _____ of 8 points	**1e.** Materials are consistent with the progressions in the Standards.[11]		
☐ **Meets expectations (7-8 points)**	i. Materials develop according to the grade-by-grade progressions in the Standards. If there is content from prior or future grades, that content is clearly identified and related to grade-level work.	0 1 2**	
	ii. Materials give all students extensive work with grade-level problems.		
☐ **Partially meets expectations (5-6 points)**	iii. Materials relate grade level concepts explicitly to prior knowledge from earlier grades.		
	1f. Materials foster coherence through connections at a single grade, where appropriate and required by the Standards.[12]		
☐ **Does not meet expectations (<5 points)**	i. Materials include learning objectives that are visibly shaped by CCSSM cluster headings.	0 1 2**	
	ii. Materials include problems and activities that serve to connect two or more clusters in a domain, or two or more domains in a grade, in cases where these connections are natural and important.		

**This Rating Sheet allows for reviewers to identify Indicators that 'Do Not Meet Expectations' (0); 'Partially Meet Expectations' (1); and 'Meet Expectations (2)'.*

*** For Indicators 1e and 1f, evidence of each sub-indicator must be provided although the reviewer will determine one score for all indicators.*

[10] Refer also to Criterion #3 (page 5) in the Publisher's Criteria.
[11] Refer also to Table 1 (page 9) in the Publisher's Criteria.
[12] Refer also to Criterion #6 (page 13) in the Publisher's Criteria.

(EdReports.org 2015)

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