Connecting Quadratics

Through Completing the Square, Vertex Form, and Transformational Graphing
Parent Function

\[ f(x) = x^2 \]

Connecting Quadratics

SESSION GOALS

- Learn how multiple aspects of teaching quadratics are connected using the vertex form and transformational graphing.
- Graph quadratic functions using the vertex and symmetry.
- Connect the vertex form to transformations.
- Learn a concrete and pictorial model for completing the square.
- Leave with a deeper understanding of the quadratics connection!
What is your experience with teaching Quadratics?
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How are Quadratics typically taught?

Many traditional Algebra 1 textbooks and curriculum follow this (or a similar) sequence:

1. Teach Factoring of quadratic expressions
2. Briefly look at graphing & transformations
3. Focus on “Solving Quadratic Equations”
4. Teach Completing the Square & Quadratic Formula last

But is this the optimal sequence to gain a conceptual understanding of quadratics?
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How could Quadratics be taught?

What if we aimed for coherence of these ideas?

1. Hook students with real-world connections
2. Introduce graphing & transformations
3. Connect the different forms with key properties:
   a. Vertex Form: Completing the Square to reveal the vertex, then Solving for $x$- and $y$-intercepts.
   b. Intercept Form: Factoring to reveal the $x$-intercepts, then using different methods to find the vertex.
   c. Standard Form: Using the Quadratic Formula to find $x$-intercepts, then calculating the vertex coordinates.
4. Compare multiple quadratic functions & representations
Connecting **Quadratics**

**Hook students with real-world connections to Quadratics**

- A common situation in which we find parabolas (quadratics) are falling objects:
  - Throwing things in the air
  - Dropping things from a height

- We can construct quadratic equations in geometrical problems as well:
  - Area
  - Similar triangles
  - Right triangles (Pythagorean Theorem)

- Quadratics equations can also model pricing situations:
  - Maximizing profit
  - Minimizing expenses

- Many questions involving time, distance and speed use quadratic equations
Parent Function

\[ f(x) = x^2 \]

Connecting Quadratics

Falling Objects — “Will it Hit the Hoop?”

Dan Meyer created a Three-Act Task using Desmos to demonstrate this concept.

https://teacher.desmos.com/activitybuilder/custom/56e0b6af0133822106a0bed1

As a student, log in at:

https://student.desmos.com/

(My class code: X796Z)
Parent Function $f(x) = x^2$

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Falling Objects – “Will it Hit the Hoop?”

Shot #1 – Analyze

Drag the black points to transform the parabola and help you decide if the ball goes in the hoop or not.

Previously you predicted the ball goes out.

In  Out

Edit your response
Parent Function

\[ f(x) = x^2 \]

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Falling Objects – “Will it Hit the Hoop?”

Class Results

What conclusions can you draw from this graph?
(The green points represent your individual scores.)

Anthony
- It looks like the parabolas are a better prediction tool.

Bethany
- Parabolas are better to guess!

Charles
- My predictions weren’t good, but the parabolas helped
Connecting Quadratics

Graphing & Transformations of Quadratics

\[ f(x) = x^2 \] is the equation of the basic parabola, aka the Parent Function

Graphing & Transformations images from [www.coolmath.com](http://www.coolmath.com)
Can you graph without creating a table?

What points would come next?

Graphing & Transformations images from www.coolmath.com
Connecting Quadratics

Graphing & Transformations of Quadratics

F-BF.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative)...

Since using $k$ every time can be confusing, we can use different variables for each transformation:

- $f(x) + k \rightarrow f(x) + k$
- $k f(x) \rightarrow a f(x)$
- $f(kx) \rightarrow f(bx)$
- $f(x + k) \rightarrow f(x + h)$
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Graphing & Transformations of Quadratics

- $f(x) + k$
- $f(x + h)$
- $af(x)$
  - $-f(x)$ [$a < 0$]
  - $af(x)$ [$a > 1$]
  - $af(x)$ [$0 < a < 1$]

Graphing & Transformations images from www.coolmath.com
How do you transform the graph of a quadratic function using the parameters $a$, $h$, & $k$?

$$f(x) = a(x - h)^2 + k$$

- $a$: reflect horizontally (open up or down); stretch or compress vertically
- $h$: translate horizontally
- $k$: translate vertically
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Let's do a Quadratic sort!

• With your group, create a classifying (tree) map that quantifies different aspects of quadratics.

THINK ABOUT:
• What would you label each row?
• What are you most comfortable with as a teacher?
• What form is the most beneficial for students?
• Do we need to address all of the forms?
Parent Function

\[ f(x) = x^2 \]

Connecting Quadratics

Let's do a **Quadratic sort!**

- **Factored Form**
  
  \[ x^2 - 2x - 3 \]

- **Zero Product Property**
  
  \[ (x - 3)(x + 1) \]

- **Standard Form**
  
  \[ (x - 1)^2 - 4 \]

- **Vertex Form**
  
  \[ \text{vertex} \]

- **y-intercept**

- **Isolate variable**

- **x-intercepts**

- **Use quadratic formula**

- **Vertex**

- **Isolate variable**

- **x-intercepts**
# Let's do a Quadratic sort!

## Quadratics

<table>
<thead>
<tr>
<th>Type of Equation</th>
<th>Standard Form</th>
<th>Factored Form</th>
<th>Vertex Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>$x^2 - 2x - 3$</td>
<td>$(x - 3)(x + 1)$</td>
<td>$(x - 1)^2 - 4$</td>
</tr>
<tr>
<td>Revealed Properties</td>
<td>y-intercept</td>
<td>x-intercepts</td>
<td>vertex</td>
</tr>
<tr>
<td>Graphs</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
<td><img src="image3.png" alt="Graph" /></td>
</tr>
<tr>
<td>Finding roots</td>
<td>Use quadratic formula</td>
<td>Zero Product Property</td>
<td>Isolate variable</td>
</tr>
</tbody>
</table>

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### Finding roots

- Use quadratic formula
- Zero Product Property
- Isolate variable
**Connecting Quadratics**

Connecting the different forms of **Quadratic** functions

<table>
<thead>
<tr>
<th>Quadratic Forms</th>
<th>Key Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td></td>
</tr>
<tr>
<td><strong>Vertex</strong> (aka Standard)</td>
<td></td>
</tr>
<tr>
<td><strong>Standard</strong> (aka General)</td>
<td></td>
</tr>
<tr>
<td><strong>Intercept</strong> (aka Factored)</td>
<td></td>
</tr>
</tbody>
</table>

- **Vertex (and Axis of Symmetry)**
- **y-intercept**
- **x-intercept(s)**

Parent Function

\[ f(x) = x^2 \]
Parent Function $f(x) = x^2$

Connecting Quadratics

The Standards concerning Quadratics: Completing the Square vs. Factoring

- **A-SSE.1** Interpret expression components in context
- **A-SSE.2** Use structure to rewrite
- **A-SSE.3a** Factor to reveal zeros
- **A-SSE.3b** Complete the square to reveal max or min
- **F-IF.4** Interpret graphed key features in context, sketch from verbal
- **F-IF.7a** Graph quadratics from equations, show key features
- **F-IF.9** Compare properties of two functions shown 2 ways
- **F-IF.8a** Use factoring & completing the square to show & interpret key features
- **A-REI.4a** Use completing the square to transform eqs and prove quadratic formula
- **A-REI.4b** Solve quadratic equations (all ways)

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Different methods for finding equivalent Quadratic forms

• So, hopefully it’s obvious by now that students need to be able to find both the vertex form and factored form of a quadratic…so how do we teach this in a way that students can grasp?

• **Factoring** is commonly taught using the **Area Model** (or Box Model), but **Completing the Square** is usually taught abstractly.

• However, using the **Area Model** to teach **Completing the Square** helps students grasp the conceptual understanding needed for this important method.
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Completing the Square using the Area Model (to find Vertex Form)

Start with a Quadratic with only a horizontal shift:

\[ f(x) = (x - 3)^2 \]

\[
\begin{array}{ccc}
  x & -3 \\
  x & x^2 & -3x \\
-3 & -3x & 9 \\
\end{array}
\]

\[ f(x) = x^2 + 3x + 9 \]
Now let’s take that same quadratic in Standard Form and change it back into Vertex Form:

\[ f(x) = x^2 - 6x + 9 \]

This is where Algebra Tiles really help!
Connecting Quadratics

Completing the Square using the Area Model (to find Vertex Form)

\[ f(x) = (x - 3)^2 \]

\[ f(x) = x^2 - 6x + 9 \]

Notice that with a Quadratic with only a horizontal shift, it’s quite easy to find both standard and vertex forms.
Connecting Quadratics

Completing the Square using the Area Model (to find Vertex Form)

But what about a Quadratic with both a horizontal and vertical shift?

\[ f(x) = x^2 - 2x - 3 \]

To make a “Zero Pair”

This is might take practice, but it’s worth it!
Completing the Square using the Area Model (to find \( x \)-intercepts)

\[
f(x) = (x - 1)^2 - 4
\]

\[
0 = (x - 1)^2 - 4
\]

\[
(x - 1)^2 - 4 = 0
\]

\[
(x - 1)^2 = 4
\]

\[
\sqrt{(x - 1)^2} = \sqrt{4}
\]

\[
(x - 1) = \pm 2
\]

\[
x = 1 \pm 2
\]

\[
x = 1 + 2 \quad \text{or} \quad x = 1 - 2
\]

\[
x = 3 \quad \text{or} \quad x = -1
\]
Connecting **Quadratics**

Comparing multiple **Quadratic** functions & representations

- Once students are fluent with converting between the different forms of Quadratics, they can compare multiple functions in multiple representations.

[Standards: F-IF.9, A-REI.11, and A-REI.7 (honors)]

Parent Function

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Did we meet our SESSION GOALS?

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Resources Provided:
Scan the following QR code or use the tinyurl to access all of the resources!

https://tinyurl.com/y7tnh56x