Motivating an Intellectual Need (Purpose) for Proofs in Geometry

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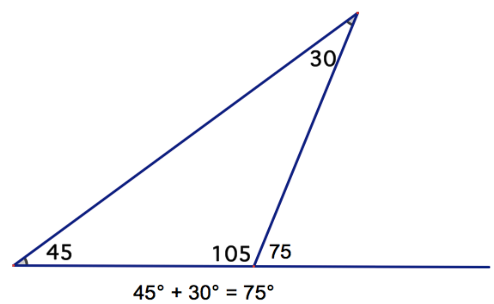
1. **Need for Certainty - to remove doubts about whether the claim is true**
   a. Prove universal claims, or statements about classes of objects ("all" or "any"), where it isn't possible to check every case.

      Example of a universal claim: "The opposite sides of a parallelogram are congruent" (statement about all parallelograms)

      Non-example: "If GEOM is a parallelogram, then $\overline{GM} \cong \overline{EO}$ and $\overline{GE} \cong \overline{MO}$" (statement about parallelogram GEOM)

   b. Involve students in a two-step proof process: 1) determine if the statement is true or false; 2) prove your answer. *Note*: This strategy only works if students are given multiple opportunities to prove both true and false statements.

      Example:
      - Show students a picture demonstrating a mathematical relationship (such as the exterior angle theorem shown here). Then ask students if this relationship is a coincidence, or will it *always* be true. After students make a claim about the validity of the statement, ask them to write a proof to support their claim.



Is this a coincidence?

2. **Need for Causality – to understand and explain *why* the claim is true**
   → Understanding *why* a claim is true can help motivate work beyond checking a few examples

   Examples:
   - After showing students the picture above, ask them to explain *why* the measure of the exterior angle equals the sum of the two remote interior angles.
   - After exploring the questions "do all quadrilaterals tessellate?" and "do all regular polygons tessellate?", ask students "Is there something special about quadrilaterals that will make it so that they will always tessellate?" (see back)

3. **Need for Communication – to persuade others that the claim is true**
   → If a proof is a convincing argument, then who should it be convincing to?

   - Once students write a rough draft of an argument, ask them to exchange papers and give each other feedback. Possible guiding questions: "What could be changed or added to make it more convincing? Is there anything that is unclear? Is there anything someone could say to poke a hole in the argument?"

Tessellation Tasks

The purpose of the tessellation tasks is to establish an intellectual need for proofs by asking students to think about how it's possible to know that a universal claim is *always* true (certainty) and *why* the statement is true (causality).

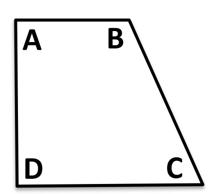
1. Do all quadrilaterals tessellate?
    a. Students explore the question using copies of different quadrilaterals (first convex, then concave quadrilaterals). Ask questions that focus on whether the student thinks the question is true for ALL quadrilaterals and *how* they know it's true.
    b. To help students become more systematic in their work, ask students to create "how to" directions to tessellate any quadrilateral.

2. Do all regular polygons tessellate?
    a. Start by talking through/exploring the cases that do work: equilateral triangles, squares, and hexagons. Then have students explore the cases that don't work: pentagons, septagons, and octagons.
    b. *Note:* I selected regular polygons for the follow up task because it fit within students' strategy to tessellate a shape by matching up the sides. Other types of polygons may be more appropriate depending on students' strategy for tessellating quadrilaterals.

3. Reflection/discussion about why all quadrilaterals, but not all regular polygons, tessellate.
    a. "Do you still think that all quadrilaterals tessellate? If yes, is there something special about quadrilaterals that make it so that they will always tessellate?"

4. Talk through proof for the claim, "All quadrilaterals tessellate". You could also just show a diagram and verbally talk through why the statement is true for all quadrilaterals.
    a. During the conversation, ask students to compare/contrast this explanation with their earlier answer that the quadrilateral statement was true because it worked for all of the examples they tried.

Prove: All quadrilaterals tessellate



<u>Proof:</u>

Assume the shape on the right is a quadrilateral where the angles add up to 360°. Take multiple copies of the quadrilateral and match the same side lengths together so that the opposite angles are touching. If you do this for all of the sides, then the angles A, B, C, D will all touch to form 360° without any gaps. This will always happen since the angles of quadrilaterals always add to 360°. Therefore, all these quadrilaterals tessellate.