

1 to 1 Million: Number Sense Progressions for K-5 NCTM 2018

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Free Resources on Progressions

Progressions for the Common Core State Standards in Mathematics (draft)

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29 May 2011

Free, online PDFs

https://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

http://commoncoretools.me/wp-content/uploads/2015/03/ccss_progression_nbp_k5_2015_03_16.pdf

K, Counting and Cardinality; K–5, Operations and Algebraic Thinking

Counting and Cardinality and Operations and Algebraic Thinking are the two domains that underlie Operations and Algebraic Thinking in Base Ten. It is telling how many in one group of multiplication, and division grow from very beginnings, this Progression is neither trivial nor obvious; these ideas that are interesting and engaging to

The Progression in Operations at the basic operations—the kinds of model and consequently the kinds to solve as well as their mathematics. Although most of the standards or involve whole numbers, the importance general because it describes representations that extend to other number algebra. For example, if the mass of mass of the rest of the solar system of the solar system as a whole is the example of additive reasoning, it does are whole numbers, fractions, decimals a property such as distributivity, hold that students will study in K–12, and

The generality of the concepts algebraic Thinking means that student designed to help them extend arithmetic the NF and NBT Progressions) and sions and equations in later grades. Addition and subtraction are the

Draft, 5/29/2011, comment at commoncoretools.wordpress.com

Grade 1

Students extend their work in three major and interrelated ways, by:

- Representing and solving a new type of problem situation (Compare);
- Representing and solving the subtypes for all unknowns in all three types;
- Using Level 2 and Level 3 methods to extend addition and subtraction problem solving beyond 10, to problems within 20. In particular, the OA progression in Grade 1 deals with adding two single-digit addends, and related subtractions.

Representing and solving a new type of problem situation (Compare) In a Compare situation, two quantities are compared to find "How many more" or "How many less." ^{K.CC.6, K.CC.7} One reason Compare problems are more advanced than the other two major types is that in Compare problems, one of the quantities (the difference) is not present in the situation physically, and must be conceptualized and constructed in a representation, by showing the "extra" that when added to the smaller unknown makes the total equal to the bigger unknown or by finding this quantity embedded within the bigger unknown.

The language of comparisons is also difficult. For example, "Julie has three more apples than Lucy" tells both that Julie has more apples and that the difference is three. Many students "hear" the part of the sentence about who has more, but do not initially hear the part about how many more; they need experience hearing and saying a separate sentence for each of the two parts in order to comprehend and say the one-sentence form. Another language issue is that the comparing sentence might be stated in either of two related ways, using "more" or "less." Students need considerable experience with "less" to differentiate it from "more"; some children think that "less" means "more." Finally, as well as the basic "How many more/less" question form, the comparing sentence might take an active, equalizing and counterfactual form (e.g., "How many more apples does Lucy need to have as many as Julie?") or might be stated in a static and factual way as a question about how many things are unmatched (e.g., "If there are 8 trucks and 5 drivers, how many trucks do not have a driver?"). Extensive experience with a variety of contexts is needed to master these linguistic and situational complexities. Matching with objects and with drawings, and labeling each quantity (e.g., J or Julie and L or Lucy) is helpful. Later in Grade 1, a tape diagram can be used. These comparing diagrams can continue to be used for multi-digit numbers, fractions, decimals, and variables, thus connecting understandings of these numbers in

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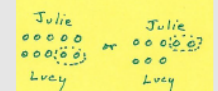
• Other Grade 1 problems within 20, such as $14 + 5$, are best viewed in the context of place value, i.e., associated with 1.NBT.4. See the NBT Progression.

• Compare problems build upon Kindergarten comparisons, in which students identified "Which is more?" or "Which is less?" without ascertaining the difference between the numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

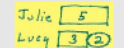
Representing the difference in a Compare problem



Compare problem solved by matching



Compare problem represented in tape diagram



Videos by Graham Fletcher

<https://gfletchy.com/progression-videos/>

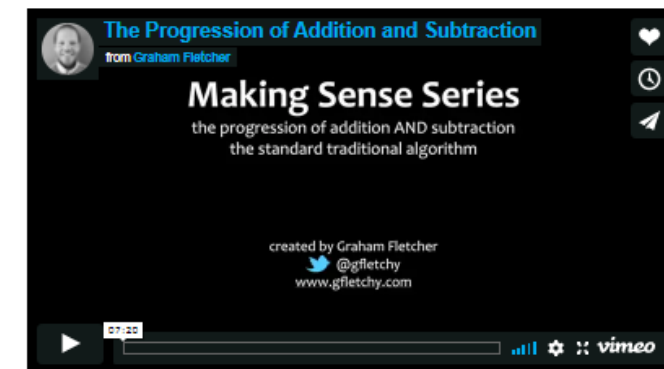


Progression Videos

Early Number & Counting



Addition & Subtraction

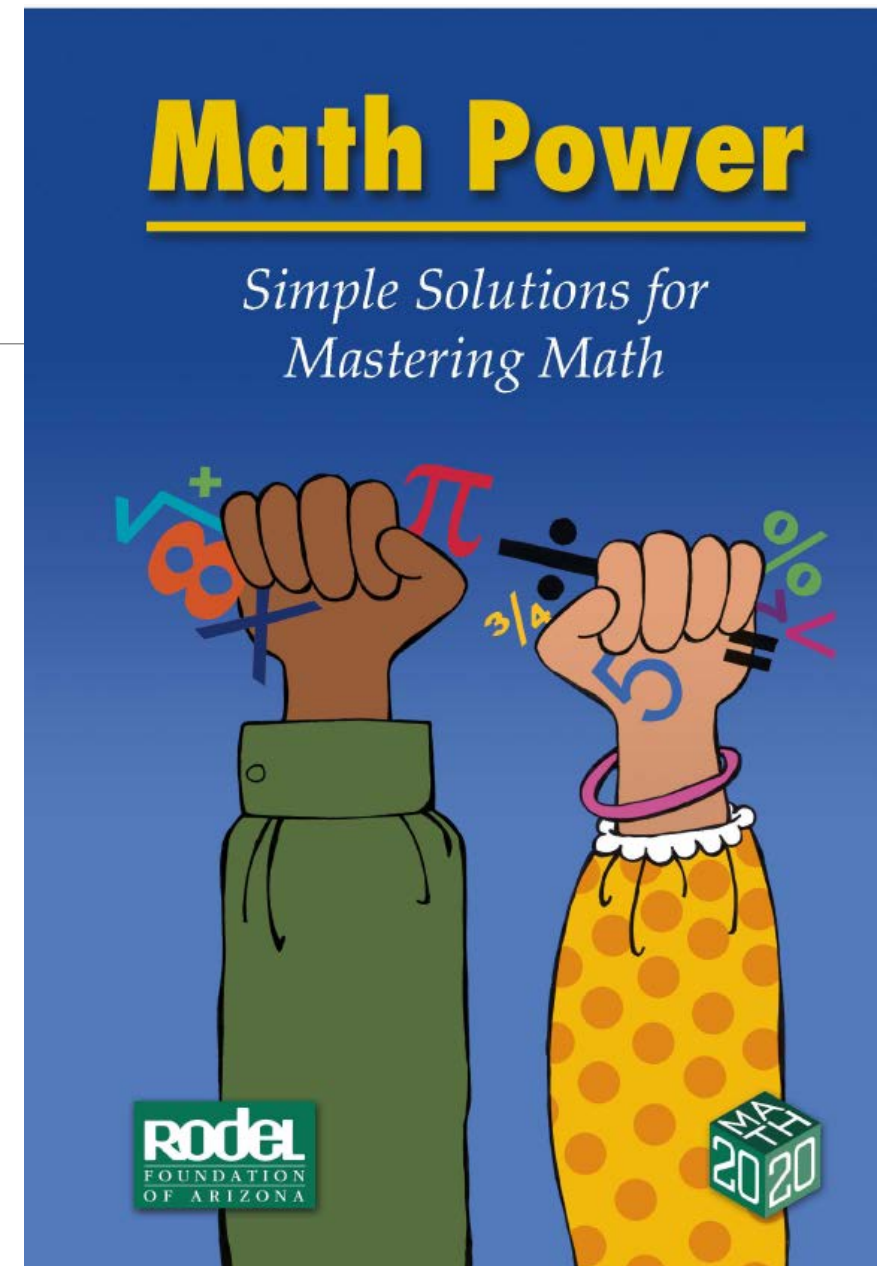


A Book from the Rodel Foundation of Arizona

Includes excellent graphics used in this presentation.

Free, online PDF

www.bit.ly/mathpowerbook



Math Tools from Conceptua Math

Excellent graphics used in this presentation came from these tools and their authors.

Free, online, and interactive tool

www.conceptuamath.com/math-tools/

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FRACTIONS

MULTIPLICATION & DIVISION

PLACE VALUE

Grade 3

Multiplication

Measurement Division

Commutative Property of Multiplication

Distributive Property

Equal Shares Division

Grade 4

Open Array Multiplication

Open Array Division

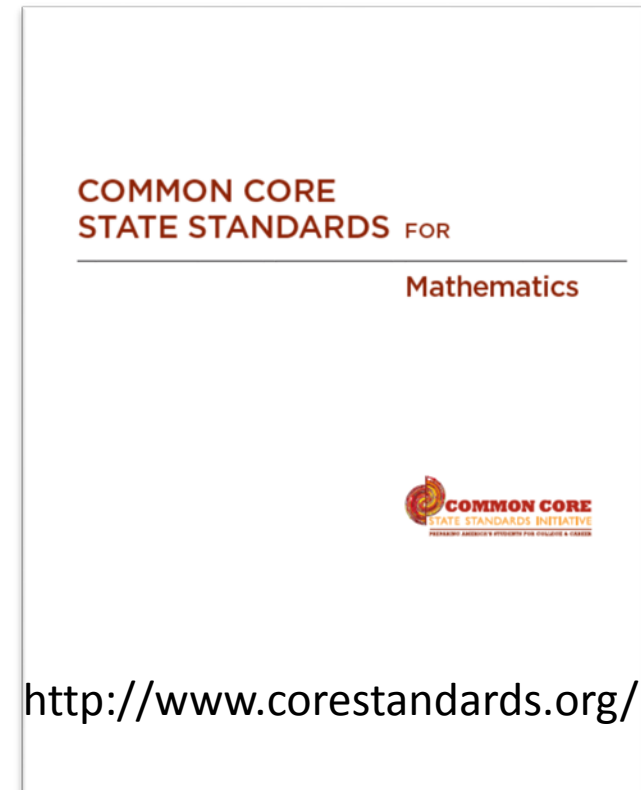
Grade 5

Multiplication

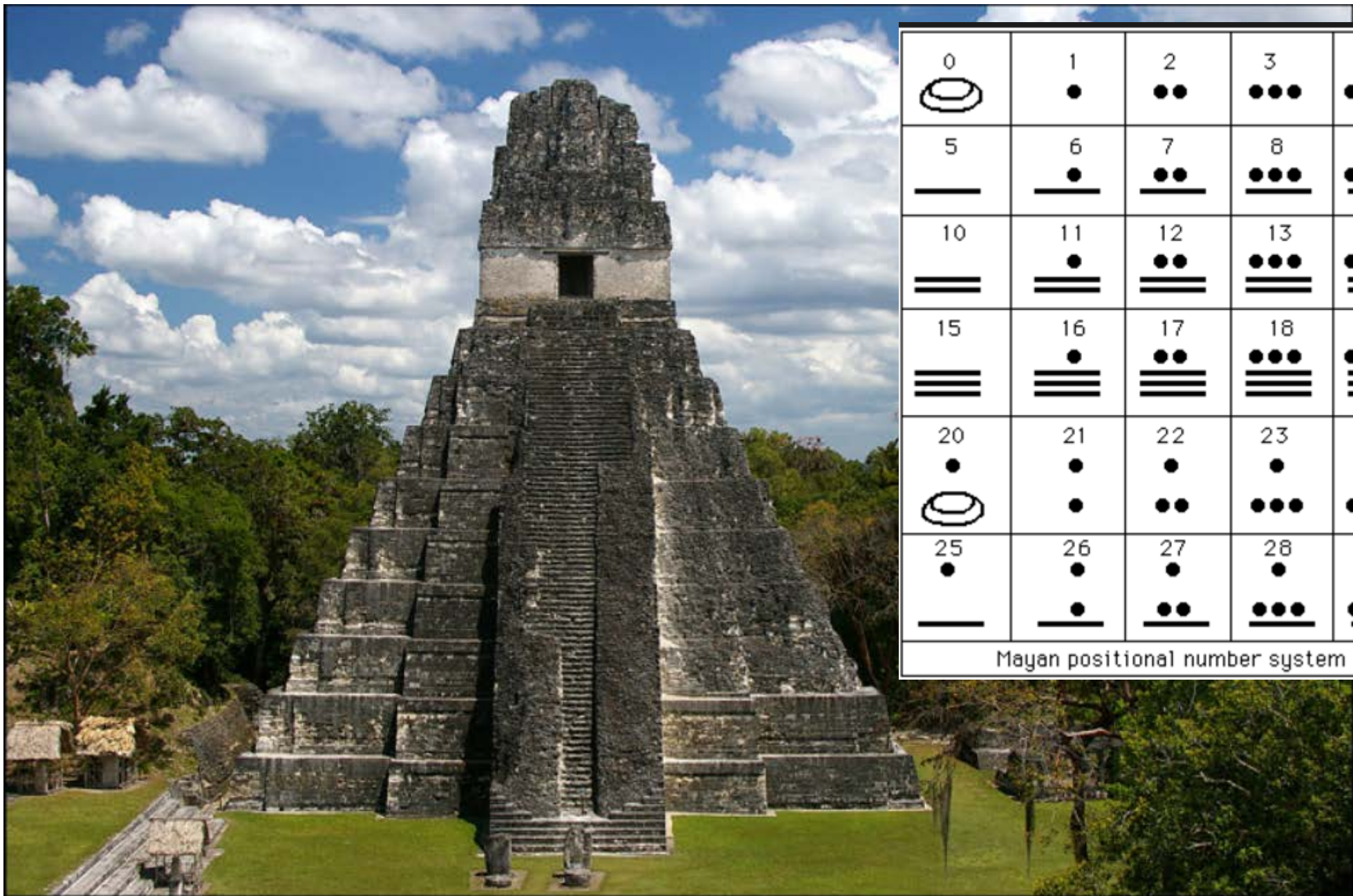
Division































Addition

Common Core State Standards for Math







| | | | | |
|---|---|---|---|---|
| 0  | 1  | 2  | 3  | 4  |
| 5  | 6  | 7  | 8  | 9  |
| 10  | 11  | 12  | 13  | 14  |
| 15  | 16  | 17  | 18  | 19  |
| 20  | 21  | 22  | 23  | 24  |
| 25  | 26  | 27  | 28  | 29  |
| Mayan positional number system | | | | |

Working in Base 4

Represent the value that is equivalent to 100.

What name would you give this amount?

Understanding Base 4


| | | | | |
|-------|------|------|-----|-----|
| | | | | |
| 256's | 64's | 16's | 4's | 1's |

Finding an Equivalent for 100

| | | | |
|--------|--------|-------|-------|
| 1 | 2 | 1 | 0 |
| 64 | 16 | 4 | 1 |
| 1 x 64 | 2 x 16 | 1 x 4 | 0 x 1 |

$$64 + 32 + 4 + 0 = 100$$

Finding an Equivalent for 100



| | | | |
|--------|--------|-------|-------|
| 1 | 2 | 1 | 0 |
| 64 | 16 | 4 | 1 |
| 1 x 64 | 2 x 16 | 1 x 4 | 0 x 1 |

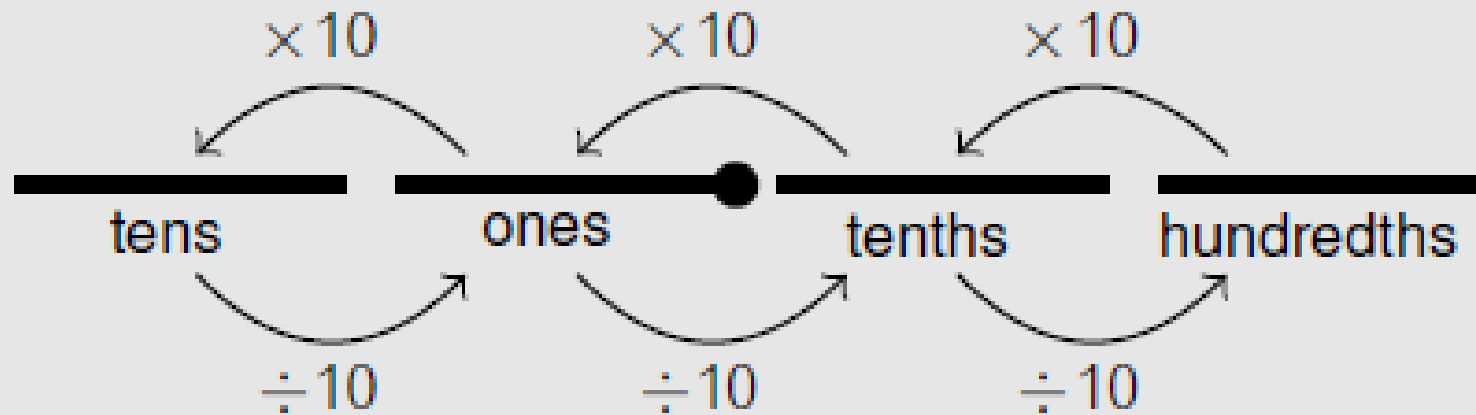
$$64 + 32 + 4 + 0 = 100$$

What Do We Call This?

1210

We have a GREAT number system!

Uniformity of the base-ten system



It works, really well!

1

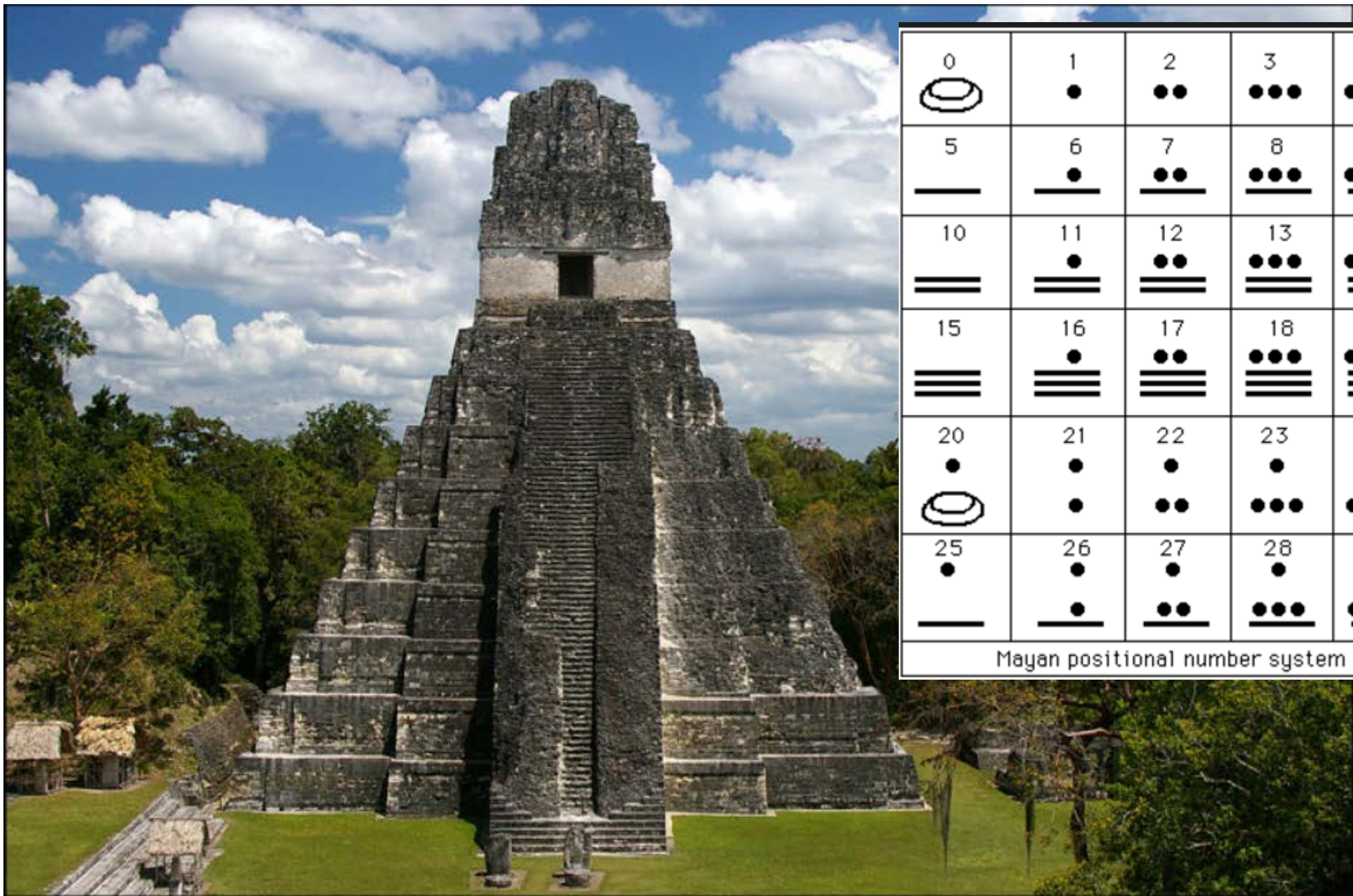
21































673

8,463

924,576

3,476,645



| | | | | |
|---|---|---|---|---|
| 0  | 1  | 2  | 3  | 4  |
| 5  | 6  | 7  | 8  | 9  |
| 10  | 11  | 12  | 13  | 14  |
| 15  | 16  | 17  | 18  | 19  |
| 20  | 21  | 22  | 23  | 24  |
| 25  | 26  | 27  | 28  | 29  |
| Mayan positional number system | | | | |

Counting and Cardinality

Recognizing and Building Quantities



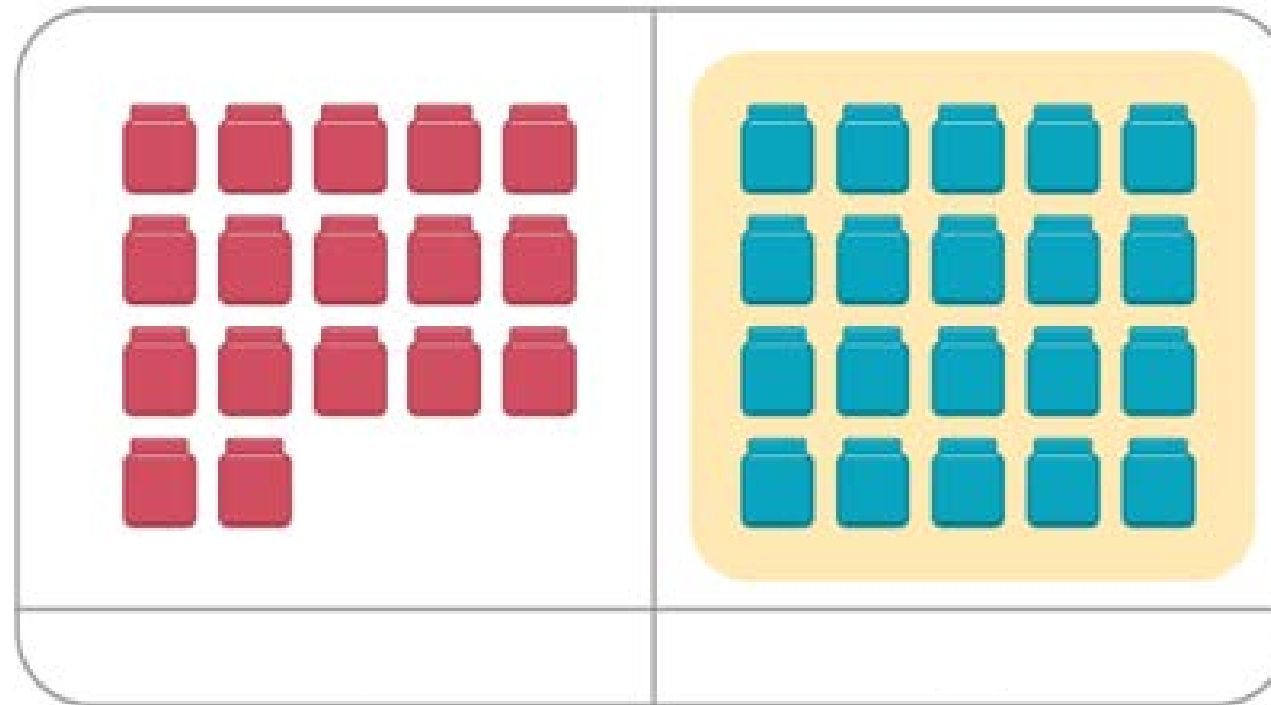
Recognizing and Building Quantities

Counting Collections

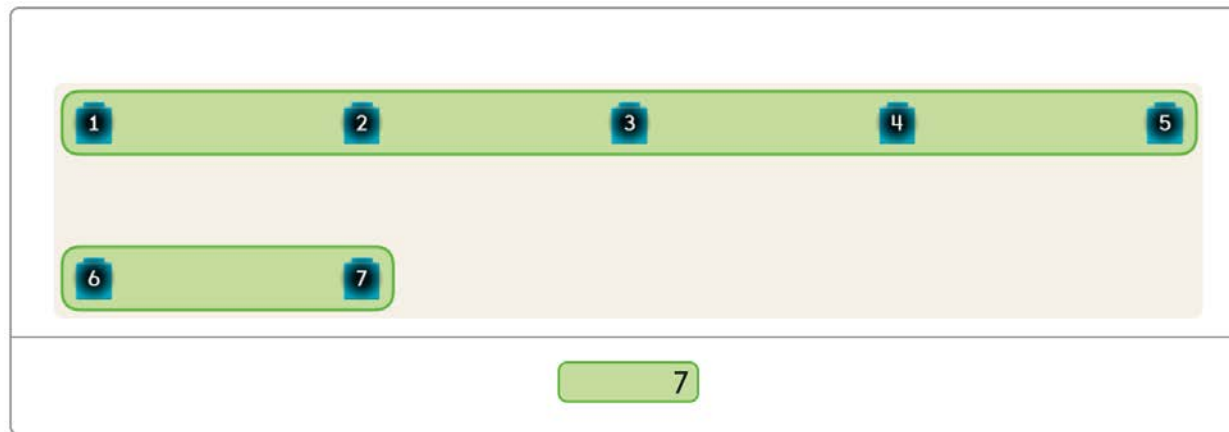
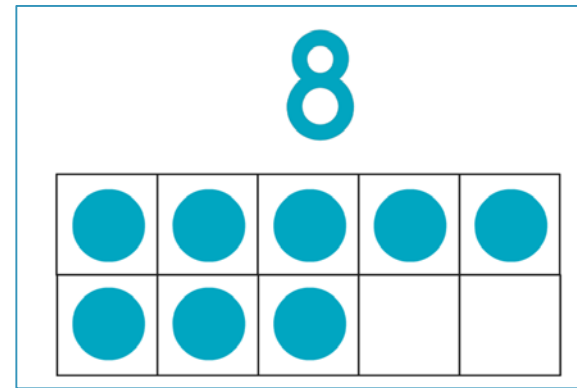
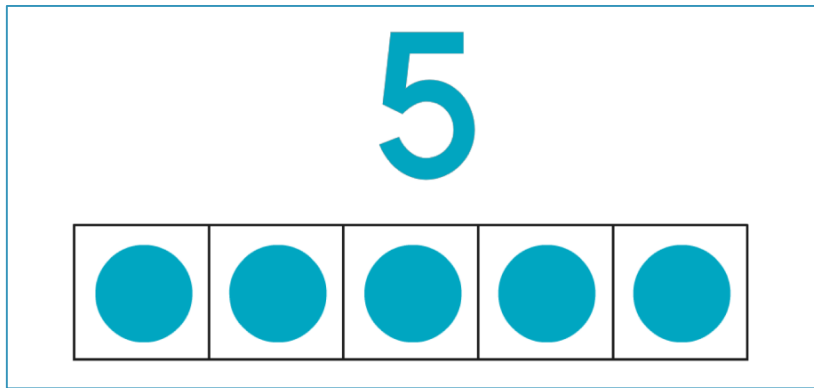


Comparing Numbers

Which has more?

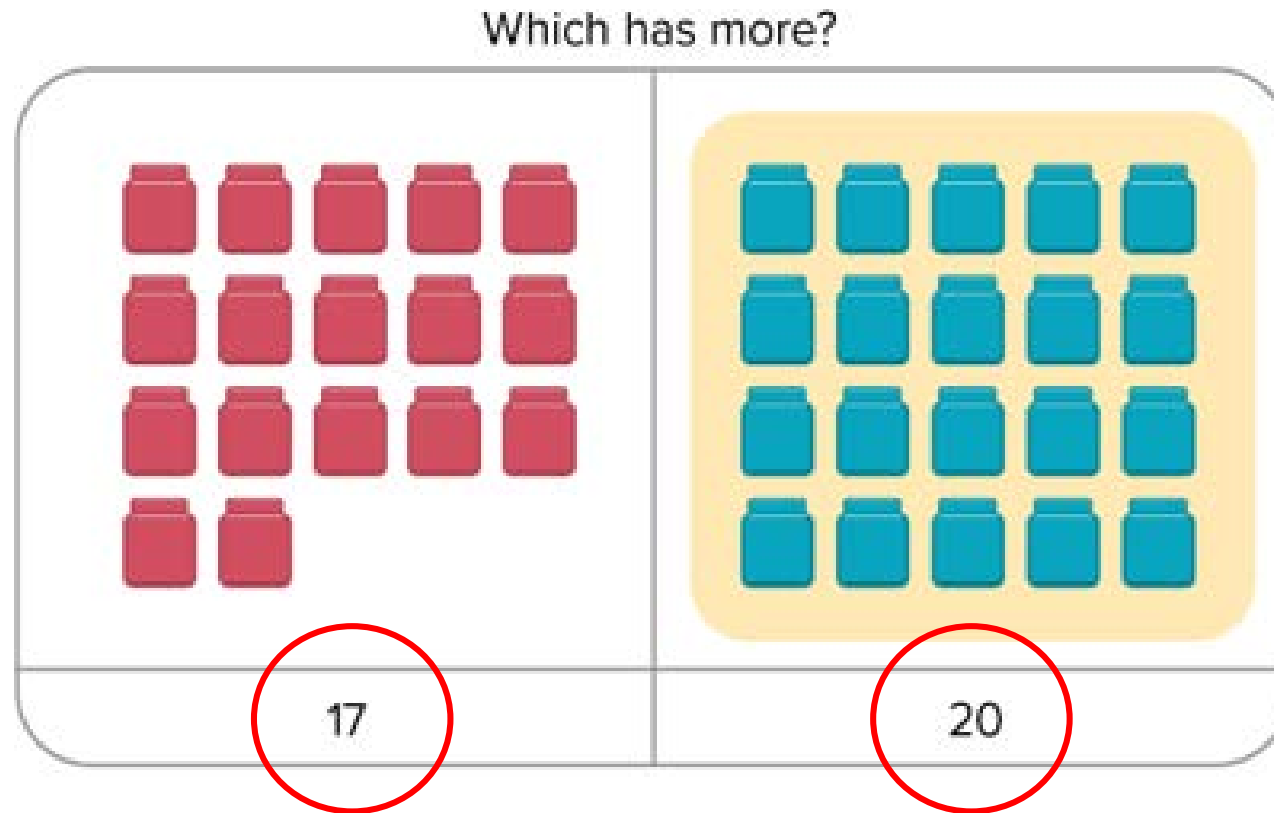


Number Names, Numerals, and Cardinality



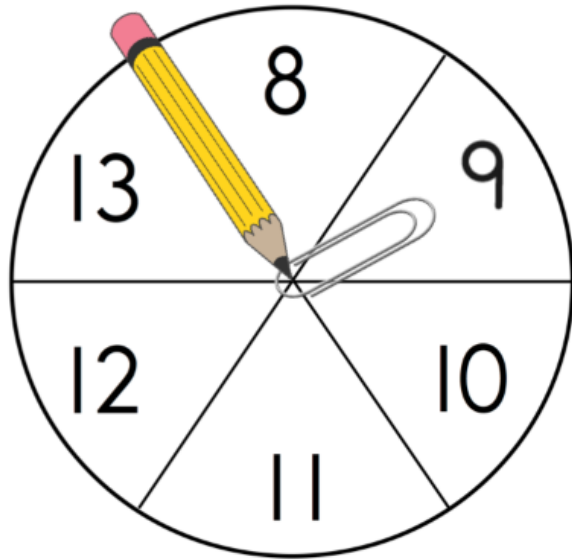
One-to-one
correspondence

Comparing Numbers



Writing Numbers

1. Spin

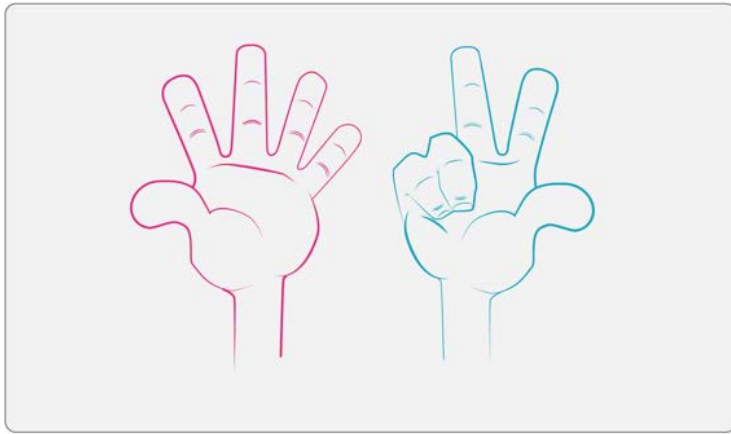


2. Record

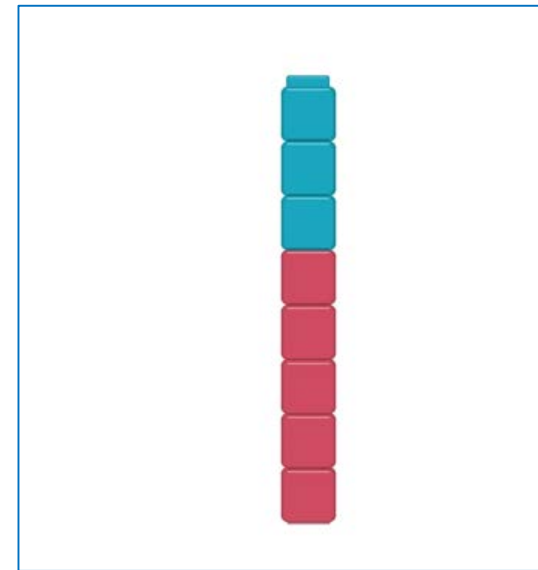
| | | | | | | | | |
|----|----|----|----|----|----|----|--|--|
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | | |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | | |
| 10 | 10 | 10 | 10 | 10 | 10 | 10 | | |
| 11 | 11 | 11 | 11 | 11 | 11 | 11 | | |
| 12 | 12 | 12 | 12 | 12 | 12 | 12 | | |
| 13 | 13 | 13 | 13 | 13 | 13 | 13 | | |

Compose Numbers

Ten Frames - Fingers

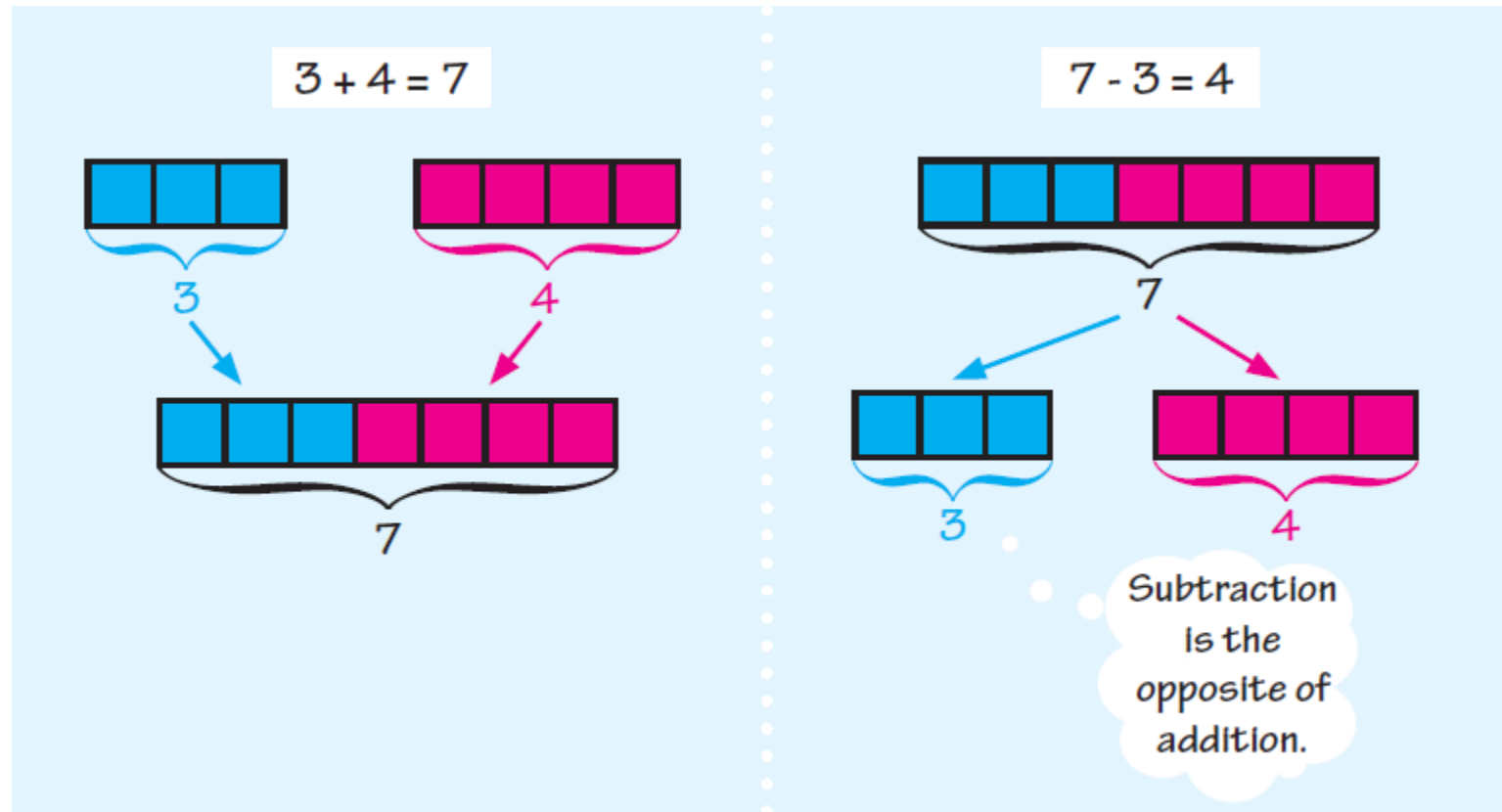


Linking Cubes



Hierarchical
inclusion

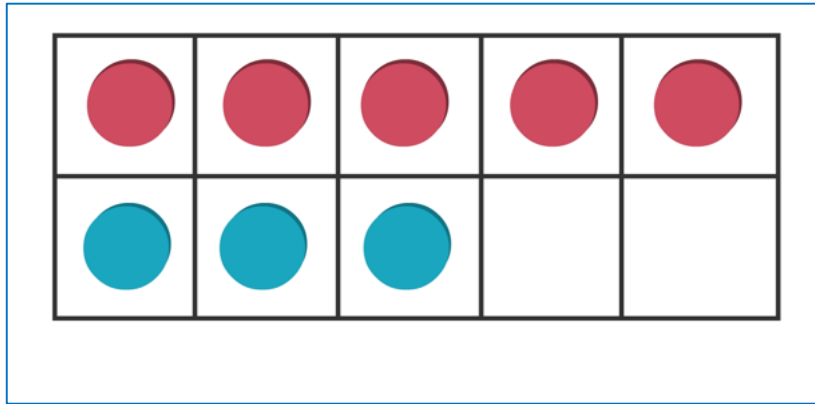
Composing and Decomposing Numbers



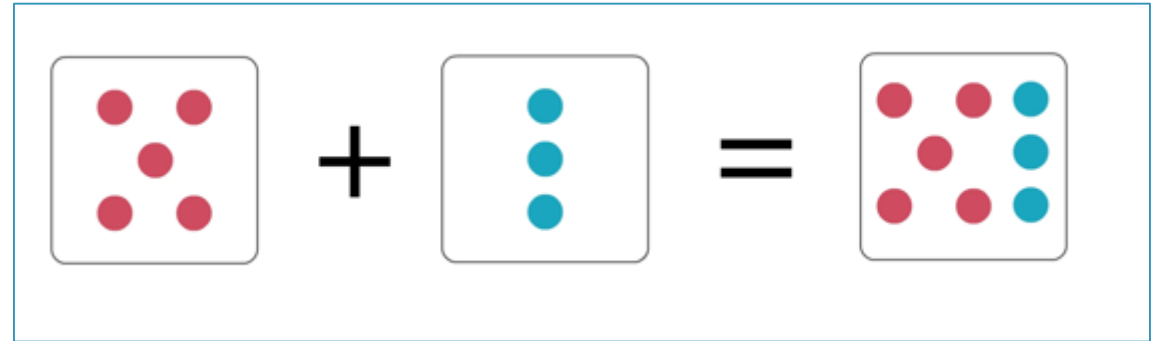
Number
Conservation

Seeing a Number

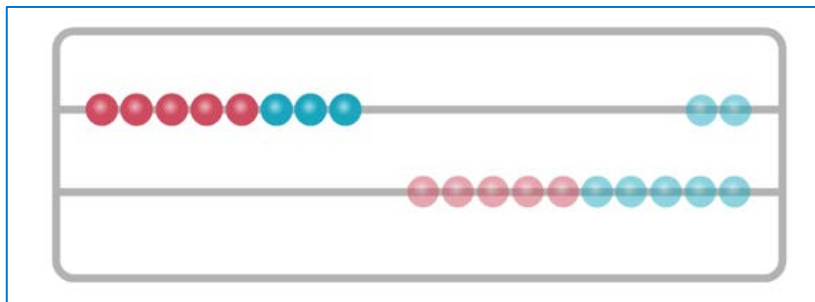
Ten Frames



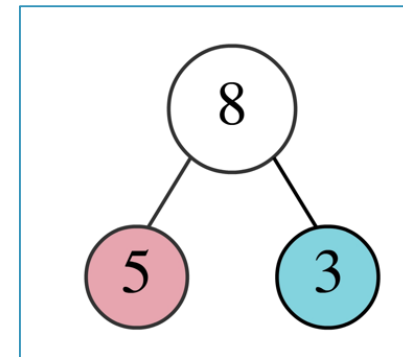
Dot Cards



Rekenrek



Number Bonds – Numerals!

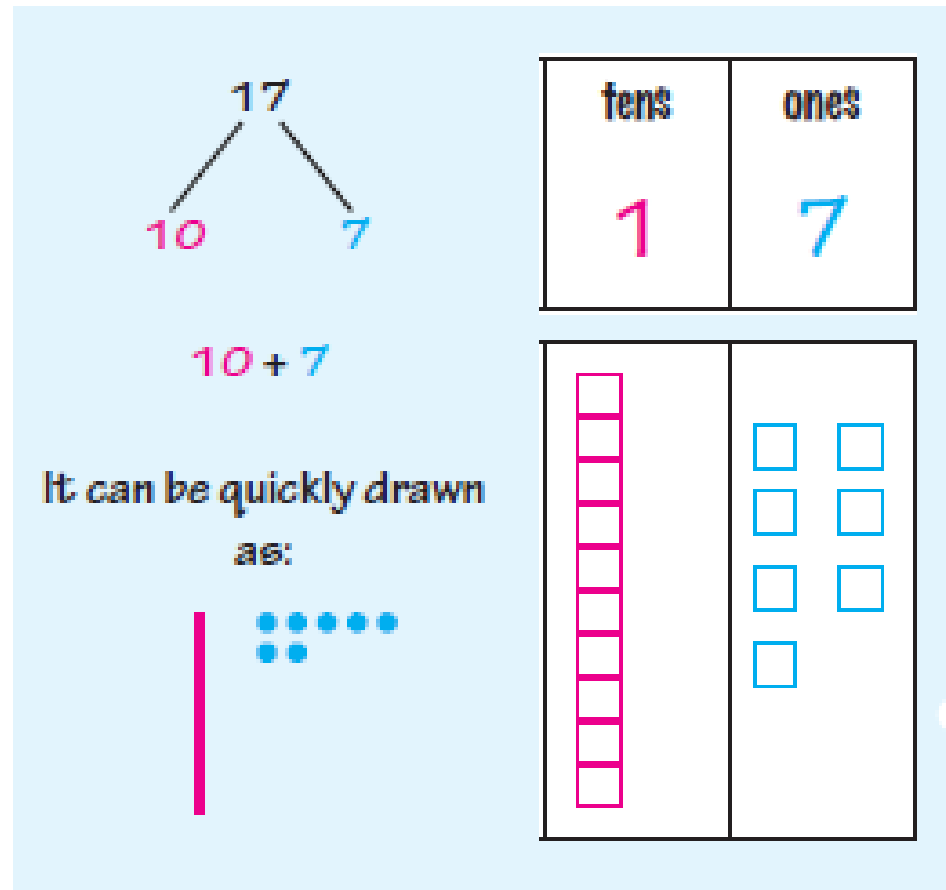


Numbers and Operations in Base 10

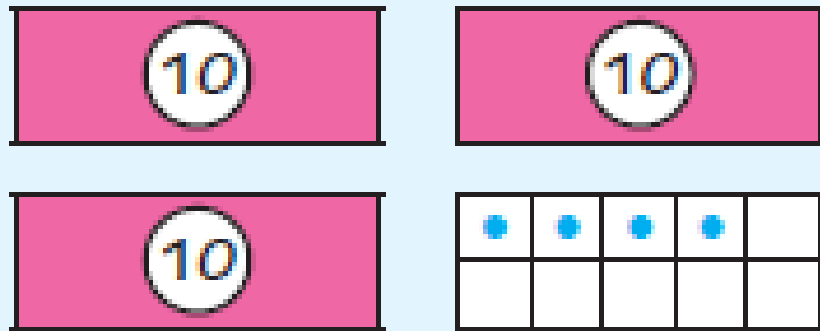
Kindergarten – It's all about 10 Units



First Grade – It's All About a Unit Called 10

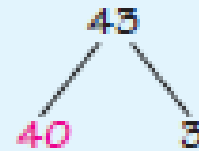


Ten Frames



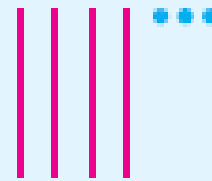
I see
3 tens and
4 ones.

Ten Frames, Ten Rods, and Visual Representations

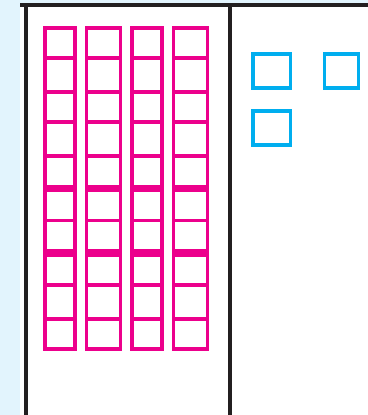


$$40 + 3$$

It can be quickly drawn
as:



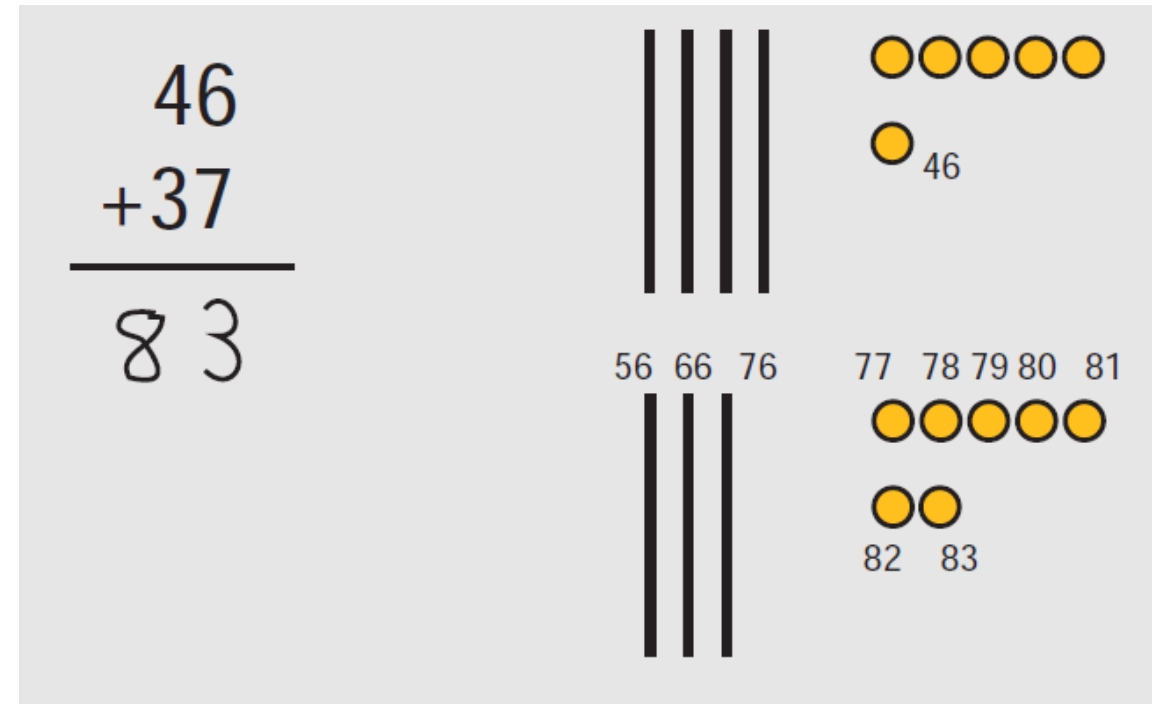
| tens | ones |
|------|------|
| 4 | 3 |



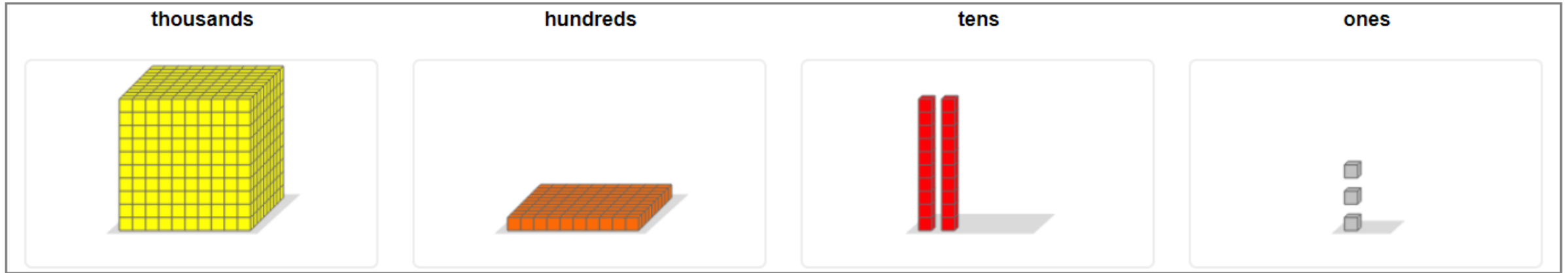
"forty-three"

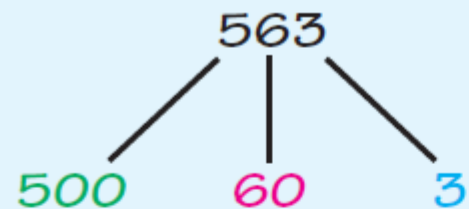
I have 4
full rods of 10 ones
which equals 4
groups of 10. I have 3
individual ones.

Grade 1 – Adding Within 100, Strategies around the Units of 10



Grade 2 – Fluent within 100, and Working with Hundreds



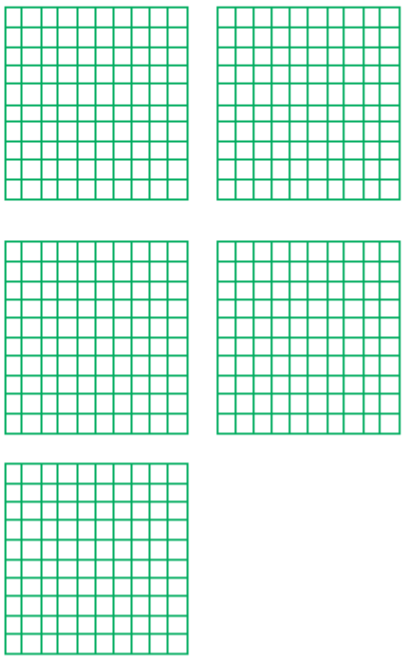
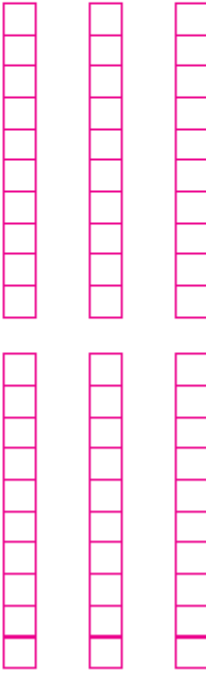



$$500 + 60 + 3$$

I have 5 full flats
of 100 ones
which equals 5
groups of 100.

“five hundred sixty-three”

| hundreds | tens | ones |
|----------|------|------|
| 5 | 6 | 3 |

| | | |
|--|--|---|
|  |  |  |
|--|--|---|

Example of Efficient Decomposition for a Subtraction Problem

The image illustrates three stages of solving the subtraction problem $425 - 278$ using base ten blocks and number lines.

Stage 1: Shows the initial setup with base ten blocks representing 425 (4 hundreds, 2 tens, 5 ones) and 278 (2 hundreds, 7 tens, 8 ones). A number line below shows the numbers 425 and 278.

Stage 2: Shows the first decomposition step. A pink line is drawn through the tens block of 425, and a blue line is drawn through the tens block of 278. A number line below shows the numbers 425 and 278, with the tens digit of 425 crossed out and the tens digit of 278 crossed out.

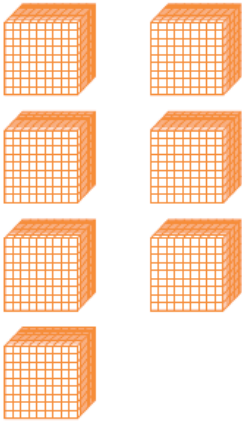
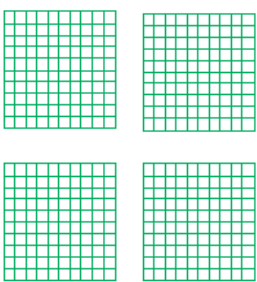
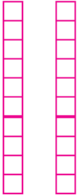
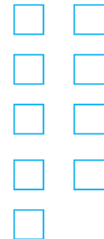
Stage 3: Shows the final result. The base ten blocks for 425 are now 3 hundreds, 1 ten, and 5 ones. The base ten blocks for 278 are 2 hundreds, 7 tens, and 8 ones. A number line below shows the numbers 425 and 278, with the tens digit of 425 crossed out and the tens digit of 278 crossed out. The final result is 147.

Grade 3 – Fluency with 1000, Working with Larger Numbers

The major focus is multiplication

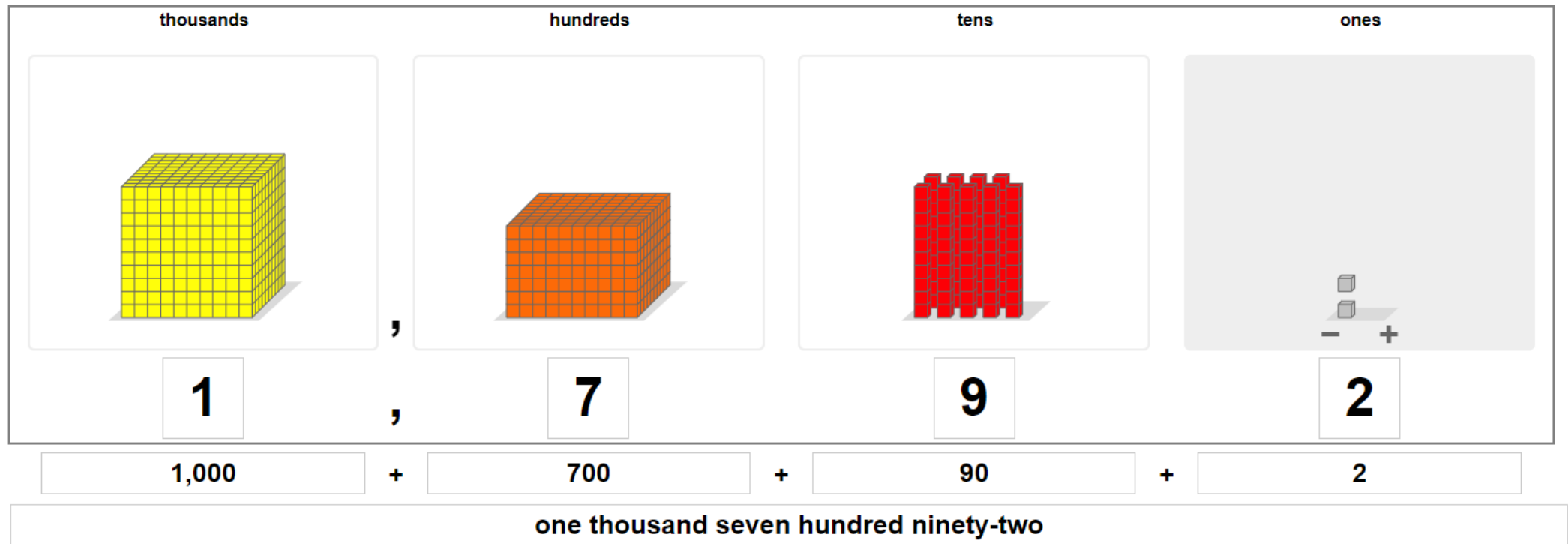
I have 7 full cubes of 1,000 ones which equals 7 groups of 1,000.

$$\begin{array}{c} 7,429 \\ \swarrow \quad \downarrow \quad \searrow \quad \swarrow \\ 7,000 \quad 400 \quad 20 \quad 9 \\ 7,000 + 400 + 20 + 9 \end{array}$$

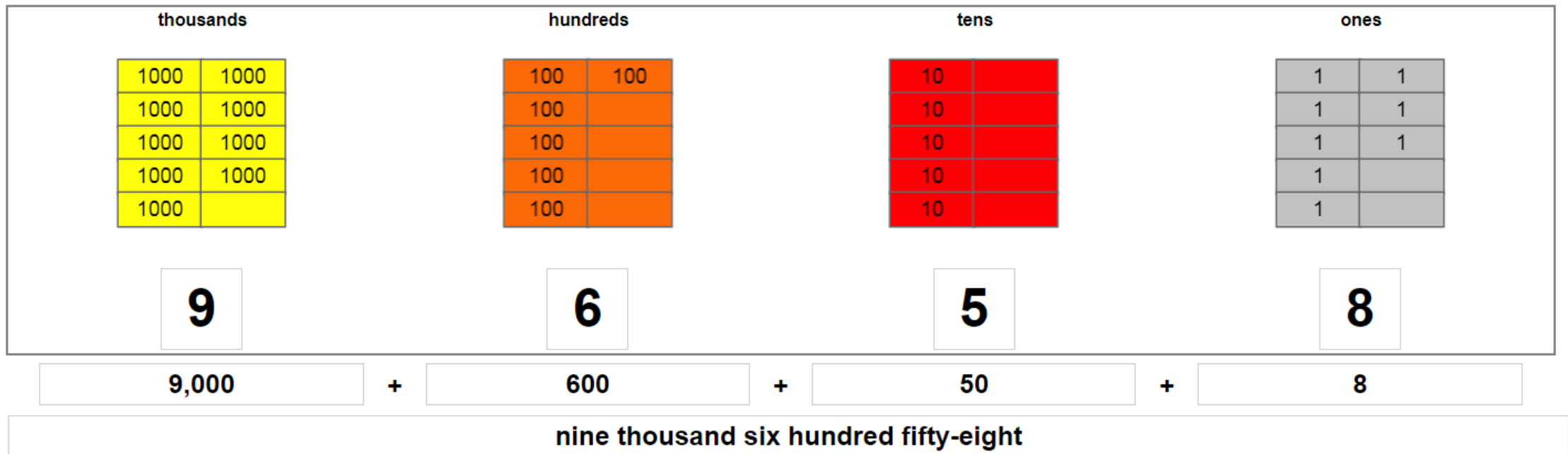
| thousands | hundreds | tens | ones |
|--|--|--|--|
| 7 | 4 | 2 | 9 |
|  |  |  |  |

“seven thousand four hundred twenty-nine”

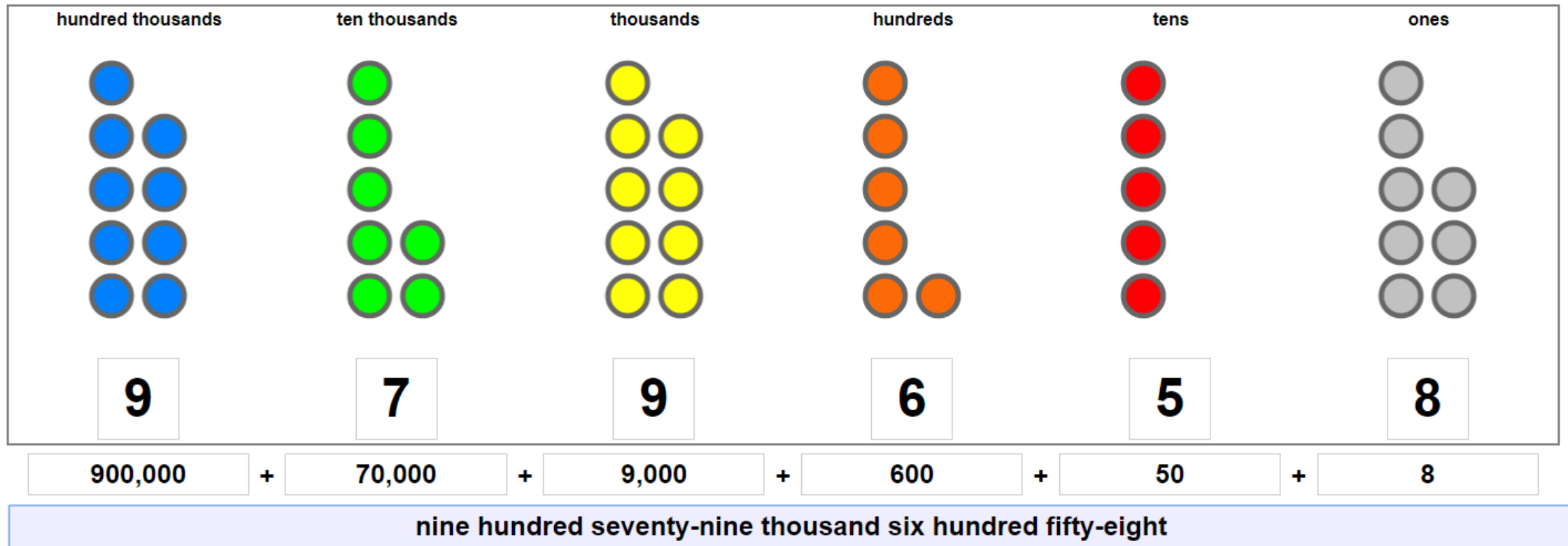
Grade 4 – Values to 1,000,000 with all Forms of Notation



Building Towards Abstraction

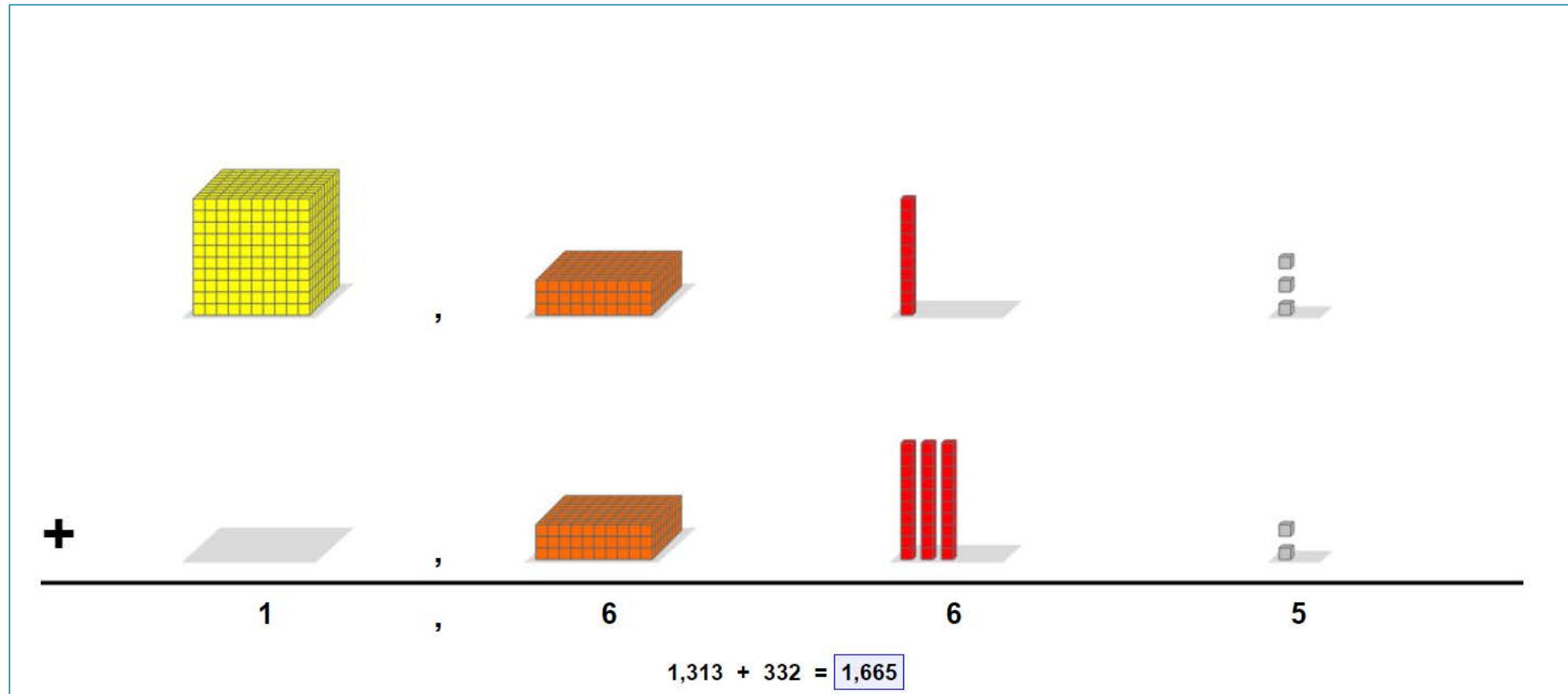


Heading Towards 1,000,000

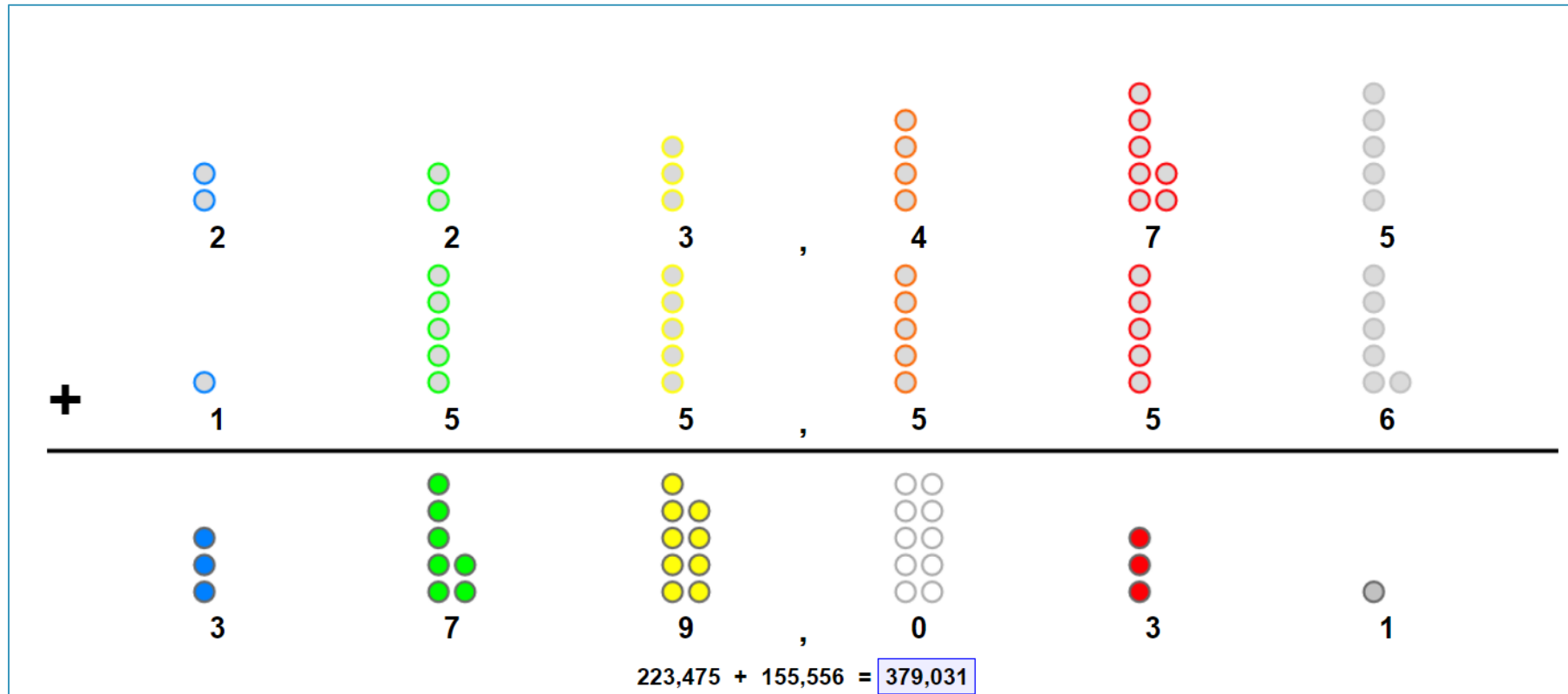


Nine hundred seventy-nine thousand six
hundred fifty-eight

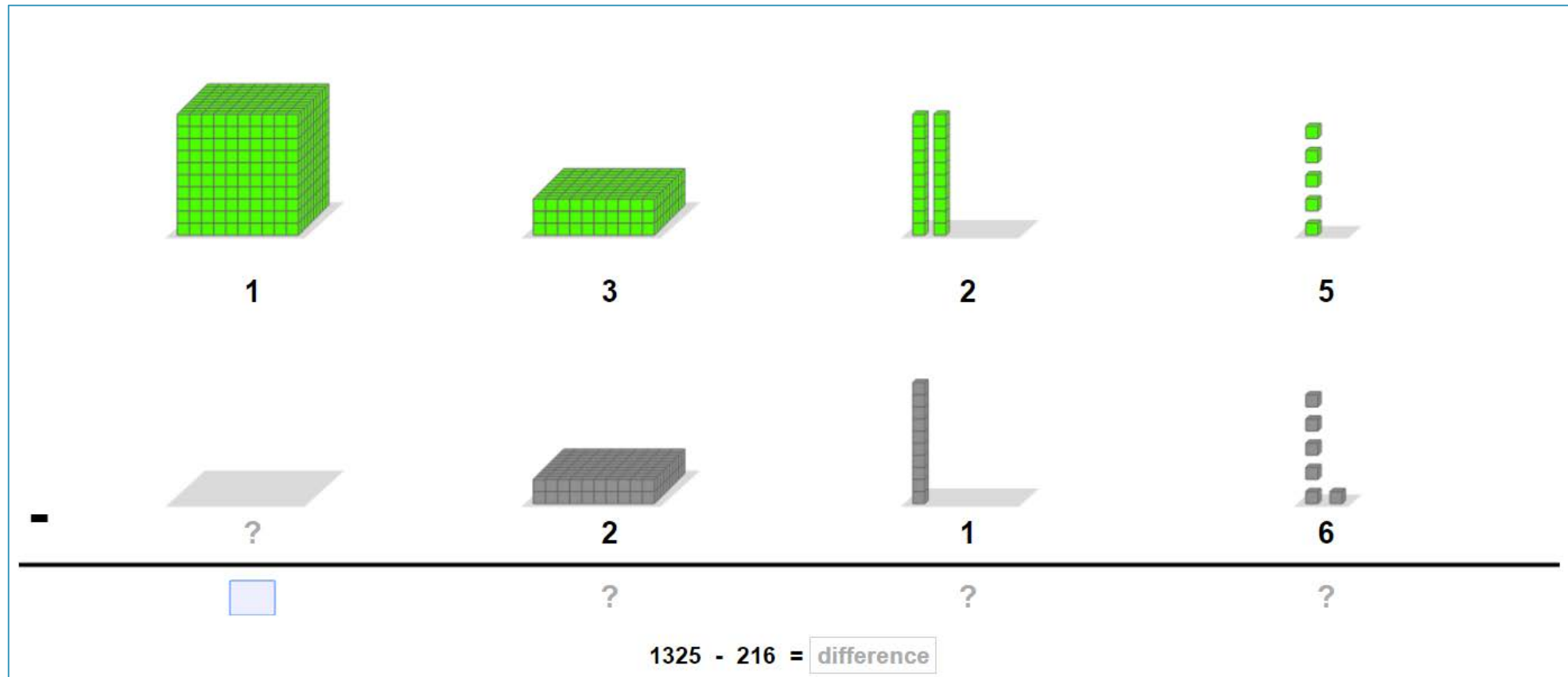
Place Value Addition



Place Value Addition with Large Numbers



Place Value Subtraction



The diagram illustrates the subtraction of 216 from 1325 using place value blocks. The minuend 1325 is represented by 1 thousand cube, 3 hundred flats, 2 tens rods, and 5 ones units. The subtrahend 216 is represented by 2 hundred flats, 1 ten rod, and 6 ones units. The subtraction is shown as a horizontal problem with a minus sign and a blank space for the difference. Below the blocks, the equation $1325 - 216 = \text{difference}$ is displayed.

| Thousands | Hundreds | Tens | Ones |
|-----------|----------|------|------|
| 1 | 3 | 2 | 5 |
| - | 2 | 1 | 6 |
| ? | | | |

$1325 - 216 = \text{difference}$

Place Value Subtraction with Large Numbers

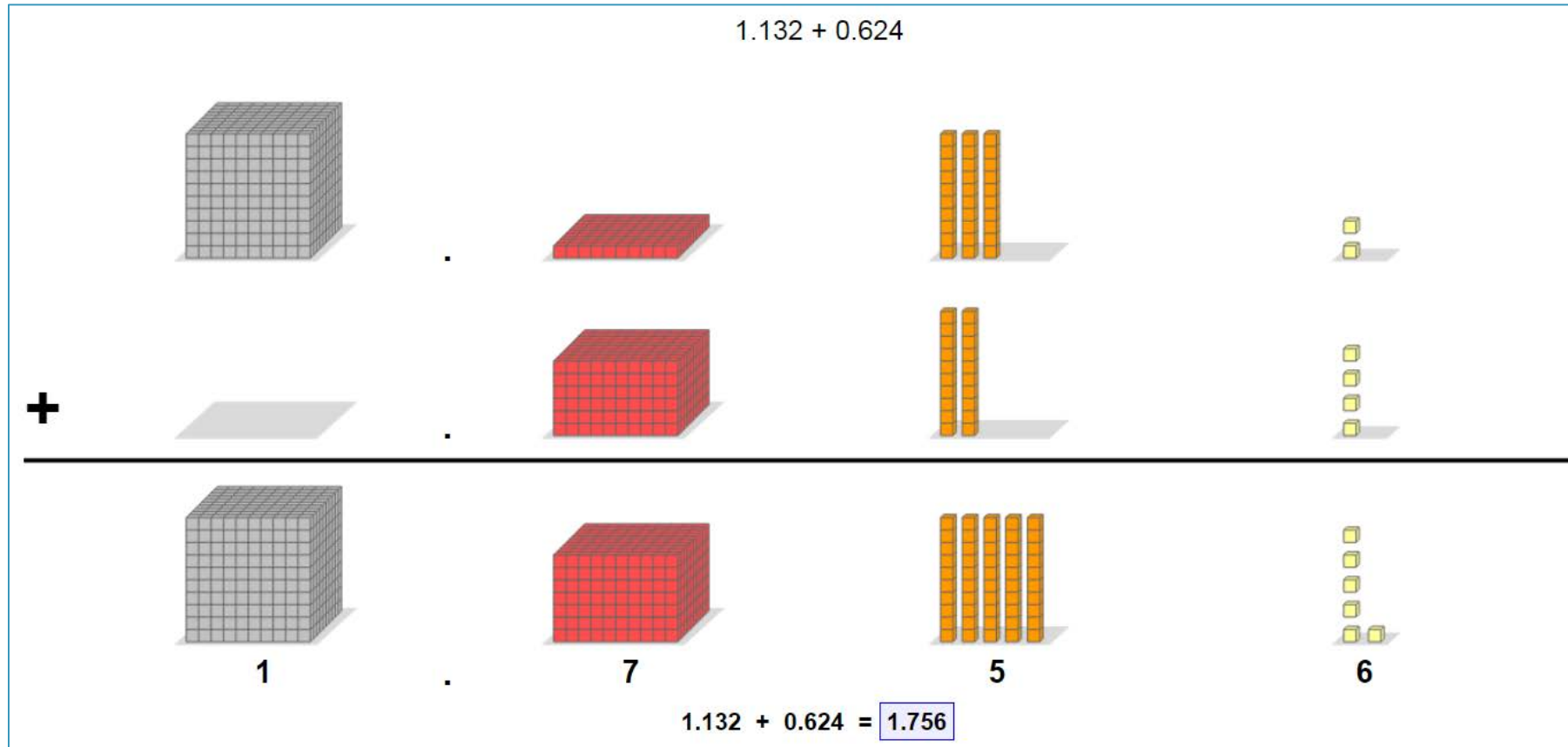
The diagram illustrates the subtraction of 94,134 from 415,325 using place value blocks. The blocks are arranged in columns corresponding to the place values: Hundred Thousands, Ten Thousands, Thousands, Hundreds, Tens, and Ones.

| Place Value | 415,325 (Top) | 94,134 (Bottom) |
|-------------------|---------------|-----------------|
| Hundred Thousands | 4 (4 blocks) | ? |
| Ten Thousands | 1 (1 block) | 9 (9 blocks) |
| Thousands | 5 (5 blocks) | 4 (4 blocks) |
| Hundreds | 3 (3 blocks) | 1 (1 block) |
| Tens | 2 (2 blocks) | 3 (3 blocks) |
| Ones | 5 (5 blocks) | 4 (4 blocks) |

A subtraction sign (-) is shown to the left of the top row. A horizontal line separates the top row from the bottom row. Below the line, the difference is shown in a box labeled "difference".

415,325 - 94,134 = difference

Grade 5 - Place Value Addition and Subtraction with Decimals



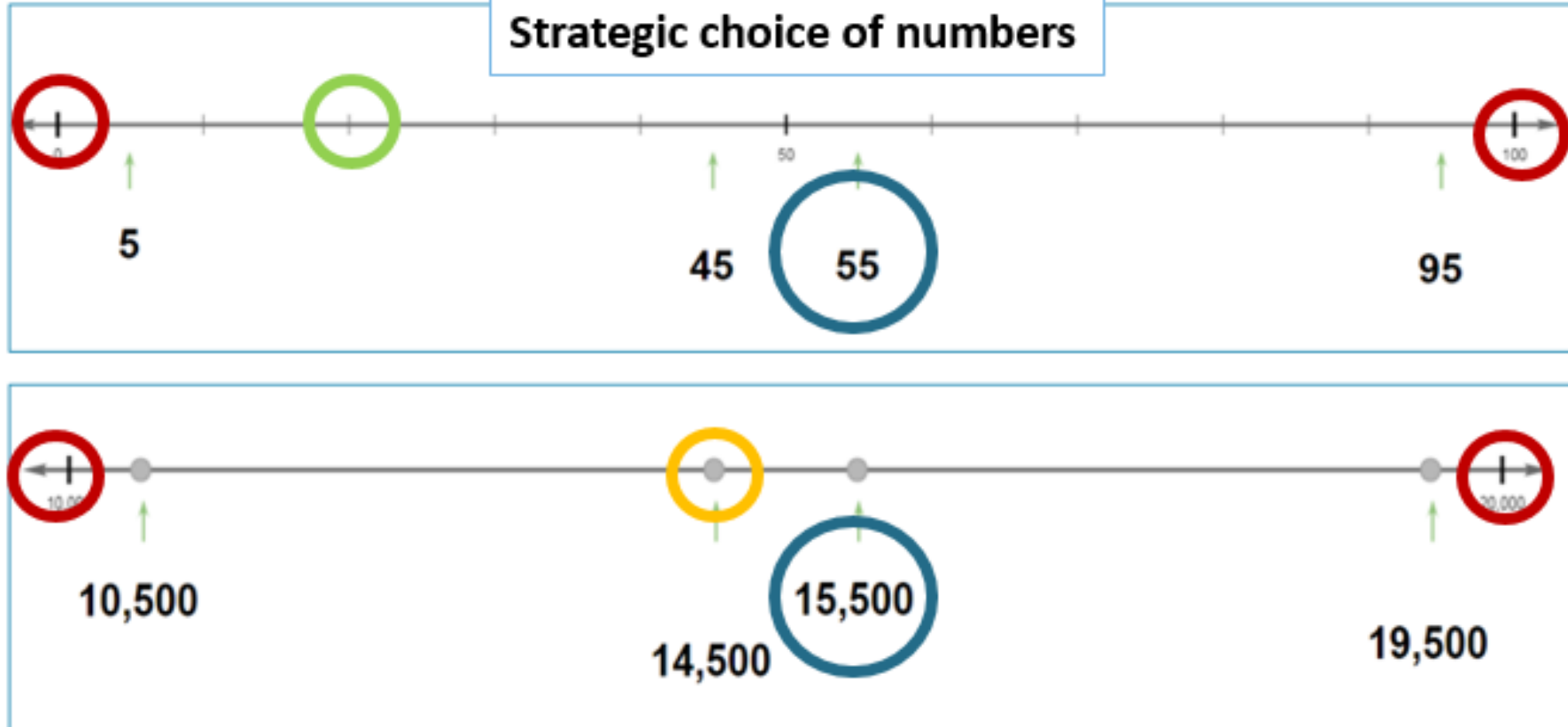
Grade 5 - Place Value Addition and Subtraction with Decimals

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|-------|---|---|---|---|---|---|---|--|--|--|--|--|---|--|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|------|---|------|------|------|------|------|------|------|------|------|---|-------|------|-------|---|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | <table border="1" style="background-color: #00FF00; border-collapse: collapse;"> <tr><td>1</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </table> | 1 | | 1 | | 1 | | | | | | | <table border="1" style="background-color: #00FF00; border-collapse: collapse;"> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </table> | | | | | | | | | | | | <table border="1" style="background-color: #00FF00; border-collapse: collapse;"> <tr><td>0.01</td><td></td></tr> <tr><td>0.01</td><td></td></tr> <tr><td>0.01</td><td></td></tr> <tr><td>0.01</td><td></td></tr> <tr><td>0.01</td><td></td></tr> </table> | 0.01 | | 0.01 | | 0.01 | | 0.01 | | 0.01 | | | <table border="1" style="background-color: #00FF00; border-collapse: collapse;"> <tr><td>0.001</td><td></td></tr> <tr><td>0.001</td><td></td></tr> <tr><td>0.001</td><td></td></tr> <tr><td>0.001</td><td></td></tr> <tr><td></td><td></td></tr> </table> | 0.001 | | 0.001 | | 0.001 | | 0.001 | | | | | | | | | |
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| - | <table border="1" style="background-color: #808080; border-collapse: collapse;"> <tr><td>1</td><td></td></tr> <tr><td>1</td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> <tr><td></td><td></td></tr> </table> | 1 | | 1 | | | | | | | | | | | <table border="1" style="background-color: #808080; border-collapse: collapse;"> <tr><td>0.1</td><td>0.1</td></tr> <tr><td>0.1</td><td>0.1</td></tr> <tr><td>0.1</td><td>0.1</td></tr> <tr><td>0.1</td><td>0.1</td></tr> <tr><td>0.1</td><td>0.1</td></tr> </table> | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | | <table border="1" style="background-color: #808080; border-collapse: collapse;"> <tr><td>0.01</td><td>0.01</td></tr> <tr><td>0.01</td><td>0.01</td></tr> <tr><td>0.01</td><td>0.01</td></tr> <tr><td>0.01</td><td>0.01</td></tr> <tr><td>0.01</td><td>0.01</td></tr> <tr><td>0.01</td><td>0.01</td></tr> </table> | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | | <table border="1" style="background-color: #808080; border-collapse: collapse;"> <tr><td>0.001</td><td>0.001</td></tr> <tr><td>0.001</td><td>0.001</td></tr> <tr><td>0.001</td><td>0.001</td></tr> <tr><td>0.001</td><td>0.001</td></tr> <tr><td>0.001</td><td>0.001</td></tr> <tr><td>0.001</td><td>0.001</td></tr> </table> | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
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| 0.1 | 0.1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 0.01 | 0.01 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
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| | 2 | . | 9 | | 9 | | 9 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | ? | . | ? | | ? | | ? | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

$3.054 - 2.999 =$

The Number Line

Start and end points
Magnitude of numbers
Tic marks vs discrete dots
Strategic choice of numbers



Number Line Considerations

Tic Marks: What is their purpose? How many? Labeled?

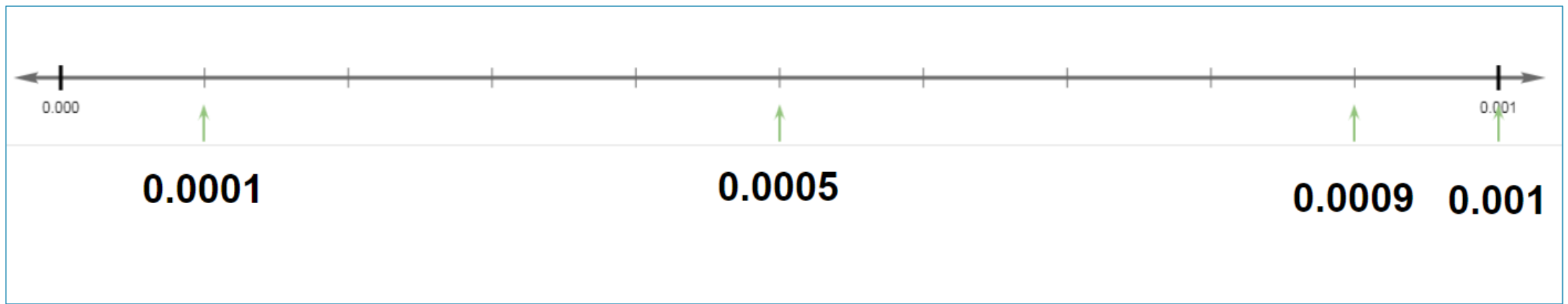
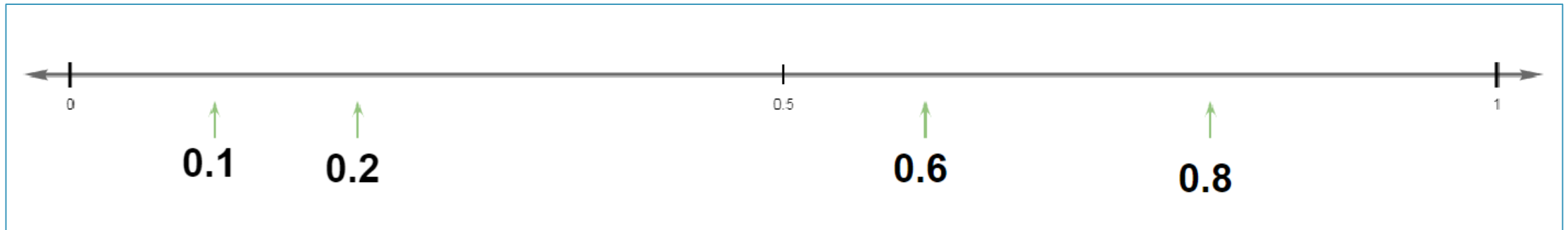
Dots: Consider discrete locations instead of tic marks.

Endpoints: Where does the number line segment start? End?

Magnitude: What place value are you teaching?

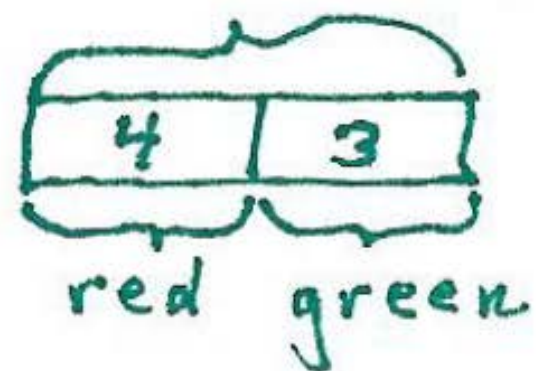
Numbers for Placement: What numbers? Why? How Many?

How Can You Apply this to Decimals?

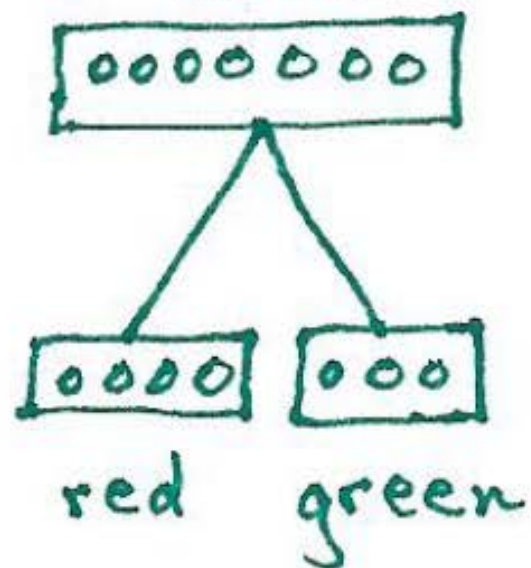


Operations and Algebraic Thinking

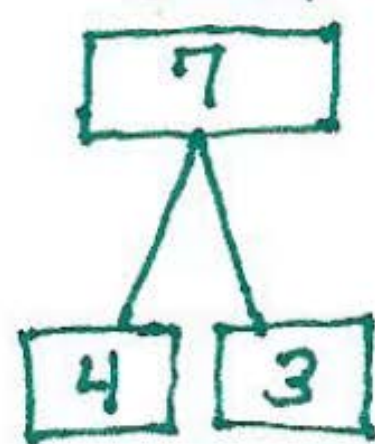
total is 7



total



total



Kindergarten

$5 + n$ pattern

$$6 = 5 + 1$$



$$7 = 5 + 2$$



$$8 = 5 + 3$$



$$9 = 5 + 4$$



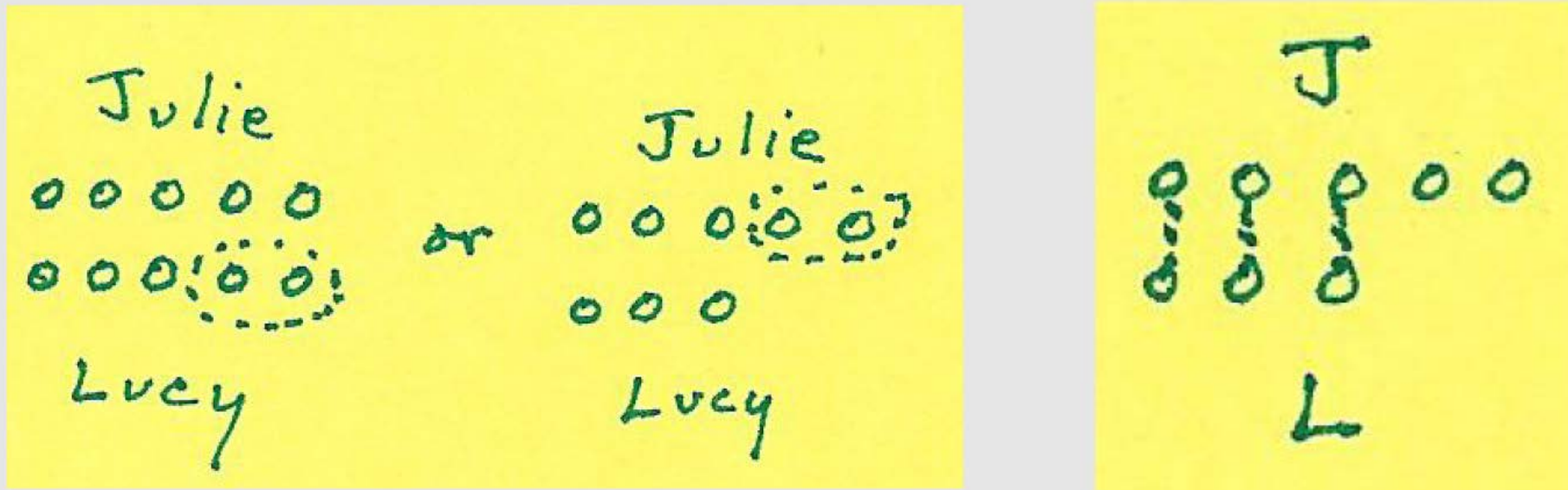
$$10 = 5 + 5$$



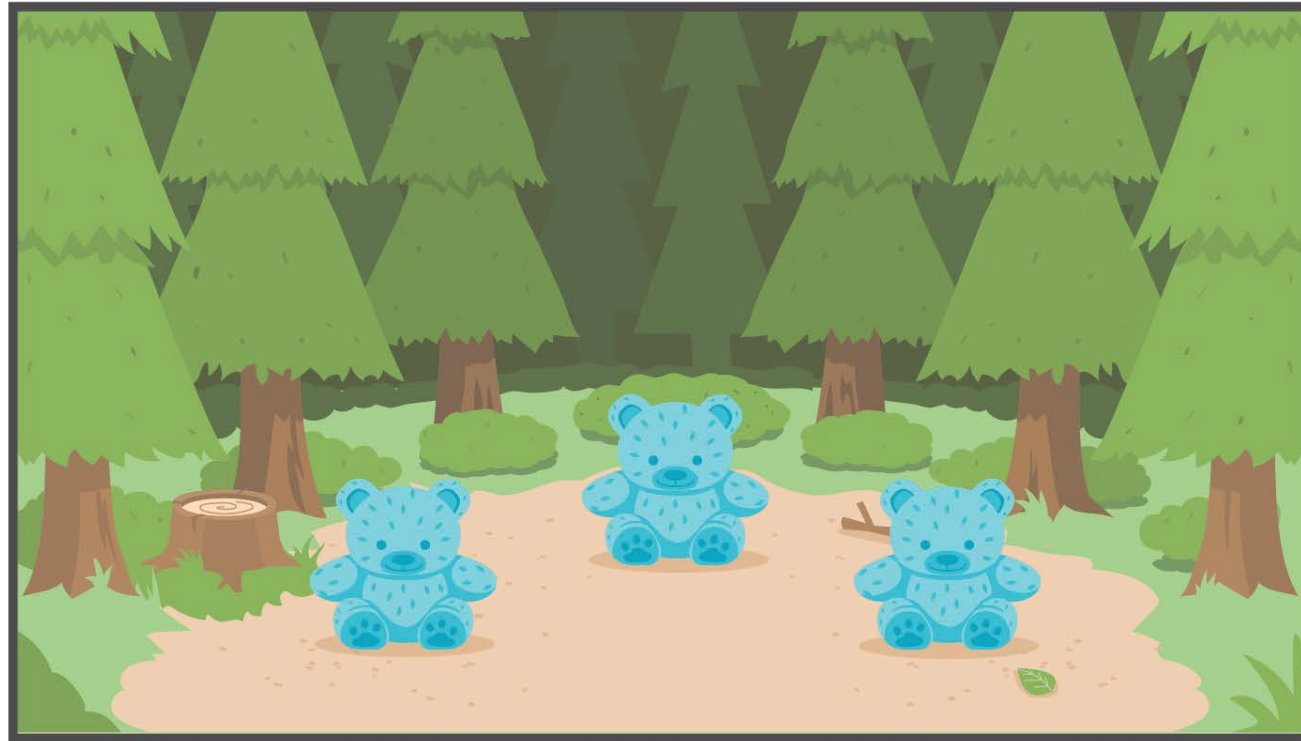
Composing and decomposing within 10

First Grade

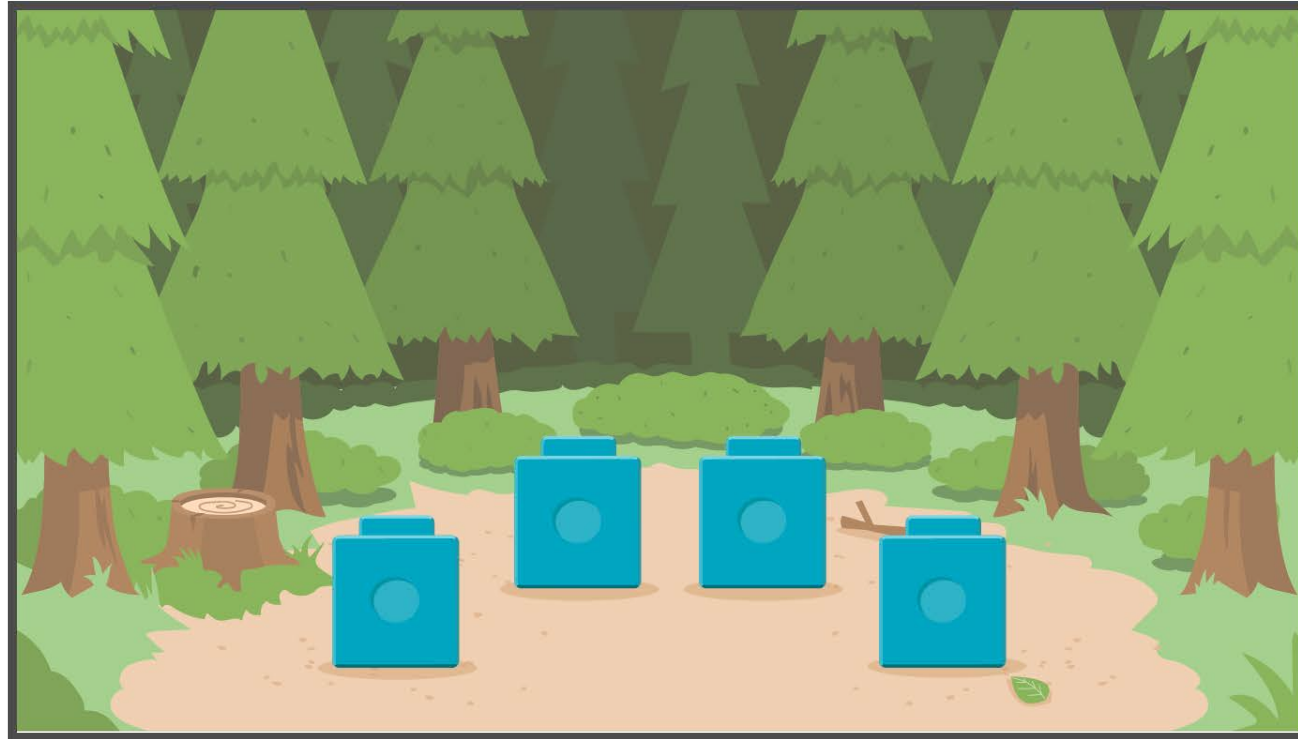
Representing the difference in a Compare problem



Three bears are sitting in the forest. Two more bears join them. How many bears are sitting in the forest now?



4 bears are playing in the forest. 2 bears leave.
How many bears are playing in the forest now?



| | Result Unknown | Change Unknown | Start Unknown |
|-----------|--|--|--|
| Add to | <p>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</p> $2 + 3 = ?$ | <p>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?</p> $2 + ? = 5$ | <p>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?</p> $? + 3 = 5$ |
| Take from | <p>Five apples were on the table. I ate two apples. How many apples are on the table now?</p> $5 - 2 = ?$ | <p>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?</p> $5 - ? = 3$ | <p>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?</p> $? - 2 = 3$ |

| Put Together/ Take Apart ² | Total Unknown | Addend Unknown | Both Addends Unknown ¹ |
|--|---|---|---|
| | Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$ | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$ |
| | | | |
| Compare ³ | Difference Unknown | Bigger Unknown | Smaller Unknown |
| | ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$ | (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$ | (Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$ |

Blair ate 5 red fish and 2 blue fish. How many more red fish did Blair eat than blue fish?



Put Together / Take Apart

9 dinner rolls are on the table, 6 are wheat and the rest are white. How many dinner rolls are white?



$$6 + ? = 9,$$

$$9 - 6 = ?$$

$$6 + 3 = 9$$

$$9 - 6 = 3$$

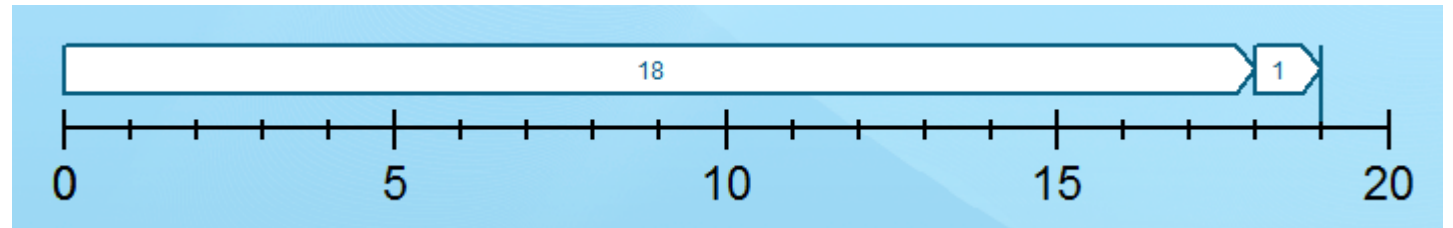
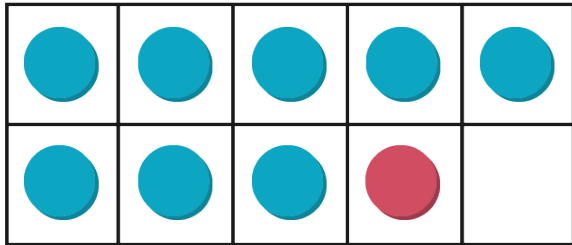
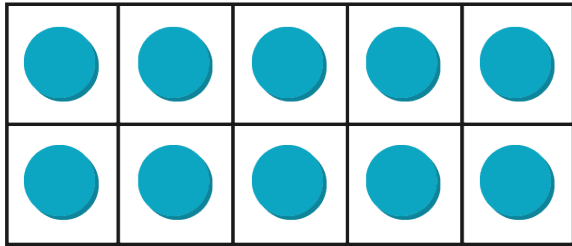
| | | | |
|---------------------------|--|---|---|
| Add To | Result Unknown <i>A</i> bunnies sat on the grass. <i>B</i> more bunnies hopped there. How many bunnies are on the grass now? $A + B = \square$ | Change Unknown <i>A</i> bunnies were sitting on the grass. Some more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies hopped over to the first <i>A</i> bunnies? $A + \square = C$ | Start Unknown Some bunnies were sitting on the grass. <i>B</i> more bunnies hopped there. Then there were <i>C</i> bunnies. How many bunnies were on the grass before? $\square + B = C$ |
| | Take From <i>C</i> apples were on the table. I ate <i>B</i> apples. How many apples are on the table now? $C - B = \square$ | Take From <i>C</i> apples were on the table. I ate some apples. Then there were <i>A</i> apples. How many apples did I eat? $C - \square = A$ | Take From Some apples were on the table. I ate <i>B</i> apples. Then there were <i>A</i> apples. How many apples were on the table before? $\square - B = A$ |
| Put Together / Take Apart | Total Unknown <i>A</i> red apples and <i>B</i> green apples are on the table. How many apples are on the table? $A + B = \square$ | Both Addends Unknown ¹ Grandma has <i>C</i> flowers. How many can she put in her red vase and how many in her blue vase? $C = \square + \square$ | Addend Unknown ² <i>C</i> apples are on the table. <i>A</i> are red and the rest are green. How many apples are green? $A + \square = C$ $C - A = \square$ |
| | Difference Unknown "How many more?" version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many more apples does Julie have than Lucy? "How many fewer?" version. Lucy has <i>A</i> apples. Julie has <i>C</i> apples. How many fewer apples does Lucy have than Julie? $A + \square = C$ $C - A = \square$ | Bigger Unknown "More" version suggests operation. Julie has <i>B</i> more apples than Lucy. Lucy has <i>A</i> apples. How many apples does Julie have? "Fewer" version suggests wrong operation. Lucy has <i>B</i> fewer apples than Julie. Lucy has <i>A</i> apples. How many apples does Julie have? $A + B = \square$ | Smaller Unknown "Fewer" version suggests operation. Lucy has <i>B</i> fewer apples than Julie. Julie has <i>C</i> apples. How many apples does Lucy have? "More" version suggests wrong operation. Julie has <i>B</i> more apples than Lucy. Julie has <i>C</i> apples. How many apples does Lucy have? $C - B = \square$ $\square + B = C$ |
| Compare | | | |

k

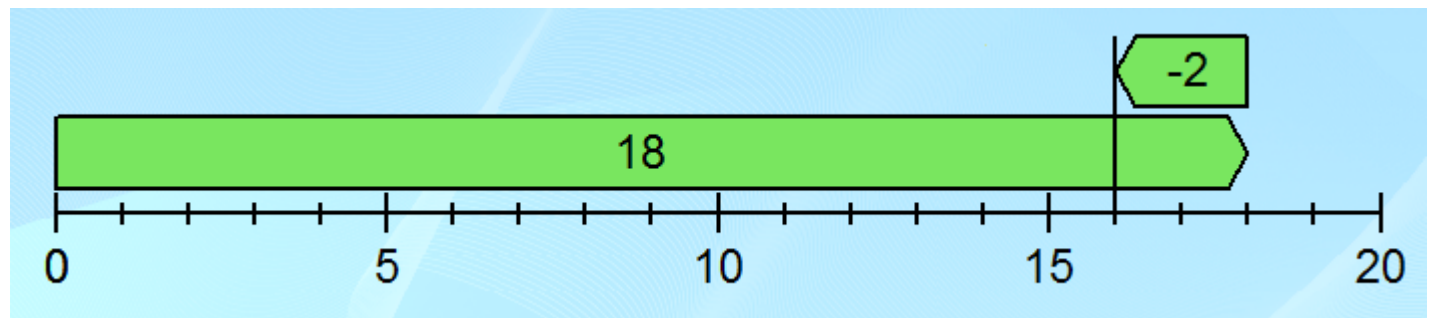
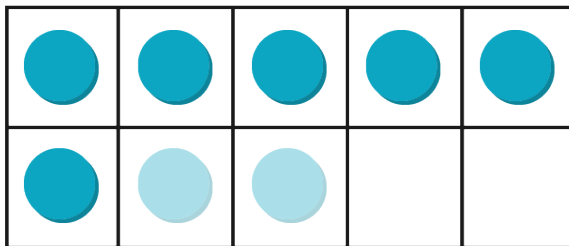
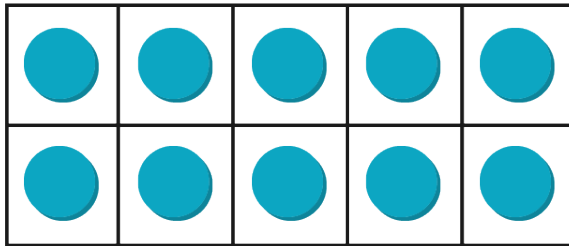
1

2

$$18 + 1 = ?$$

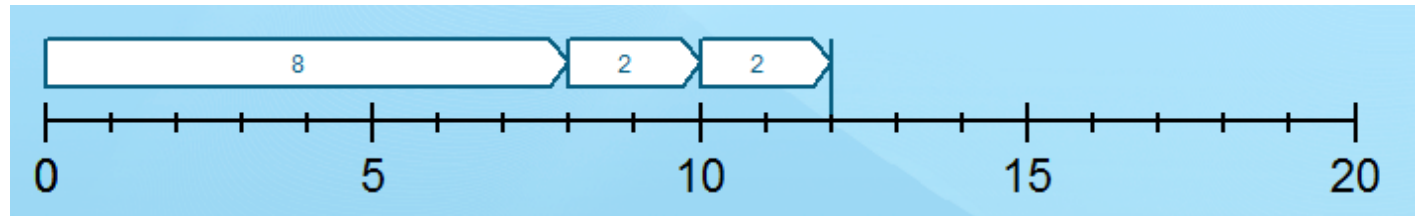
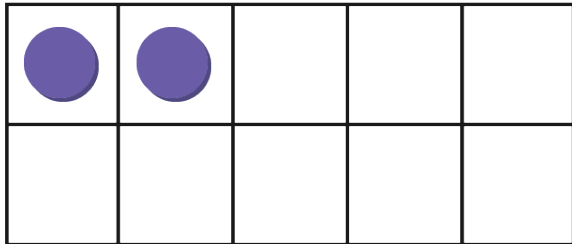
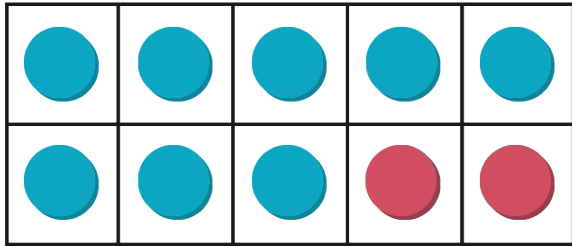


$$18 - 2 = ?$$



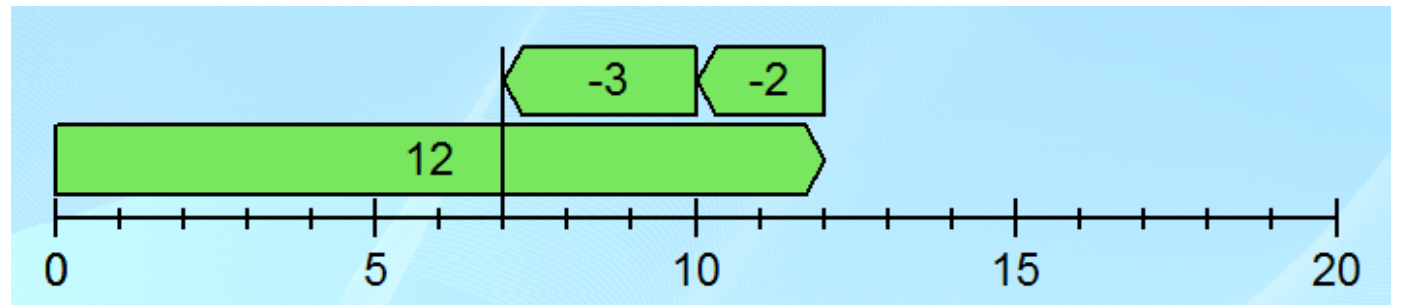
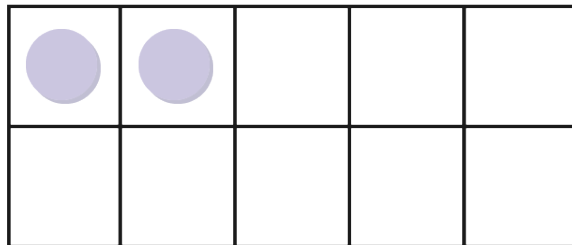
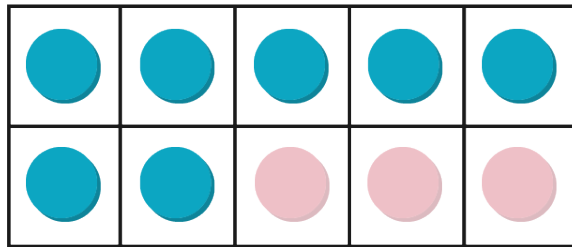
$8 + 4 =$

$$8 + 2 + 2 = ?$$



$$12 - 5 =$$

$$12 - 2 - 3 = ?$$



Grade 2 – Patterns in addition and subtraction to 100

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Grade 2 – Patterns in addition and subtraction to 100

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

**Repeated
Addition
and
Repeated
Subtraction**

**Multiplication
and Division**

| | Unknown Product | Group Size Unknown ("How many in each group?" Division) | Number of Groups Unknown ("How many groups?" Division) |
|---|---|---|--|
| | $3 \times 6 = ?$ | $3 \times ? = 18$, and $18 \div 3 = ?$ | $? \times 6 = 18$, and $18 \div 6 = ?$ |
| Equal Groups | <p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p> | <p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p> | <p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p> |
| Arrays, ⁴ Area ⁵ | <p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p> | <p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p> | <p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p> |
| Compare | <p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p> | <p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p> | <p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p> |
| General | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |

Multiplication

Equal Groups

Josie has 3 toy cars.
Each car has 4 wheels.
How many wheels are there in all?

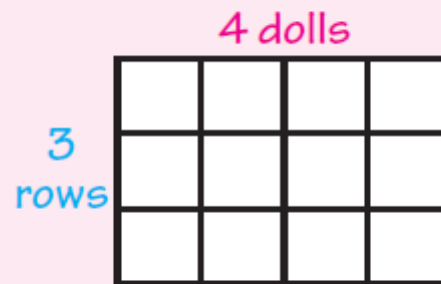


3 groups of 4

$$3 \times 4 = 12$$

Arrays of Objects

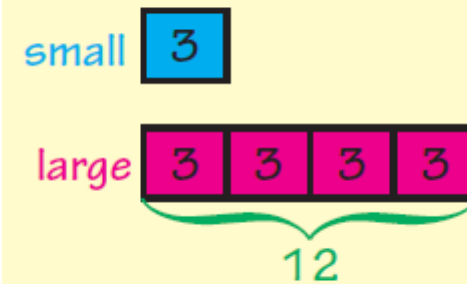
At the toy store,
Kaitlin notices 3 rows
of dolls with 4 dolls in
each row. How many
dolls are at the toy
store?



$$3 \times 4 = 12$$

Compare

A small balloon costs
\$3.00. The large
balloon costs 4 times
as much. How much
does the large balloon
cost?

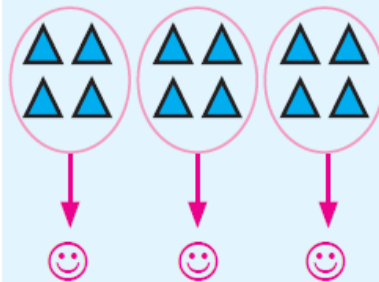


$$\$3 \times 4 = \$12$$

Division

Equal Groups

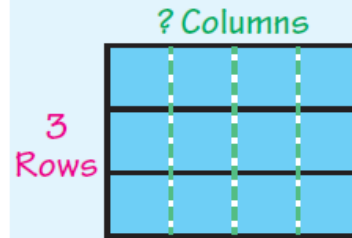
If 12 toys are divided equally among 3 brothers, how many toys will each brother receive?



$$12 \div 3 = 4$$

Arrays of Objects

If 12 apples are arranged into an array with 3 rows, how many apples will be in each row?

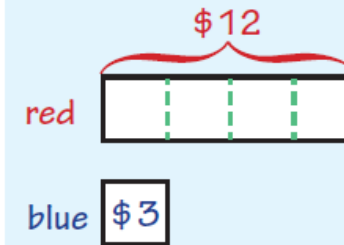


$$12 \div 3 = \square$$

$$12 \div 3 = 4$$

Compare

A red hat costs \$12 and a blue hat costs \$3. How many times more expensive is the red hat than the blue hat?

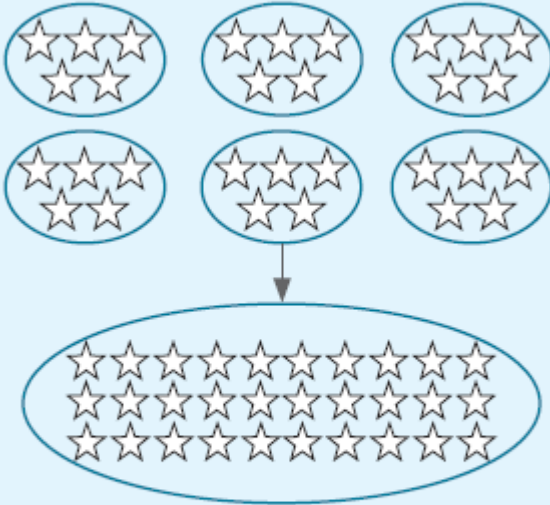


$$12 \div \square = 3$$

$$12 \div 4 = 3$$

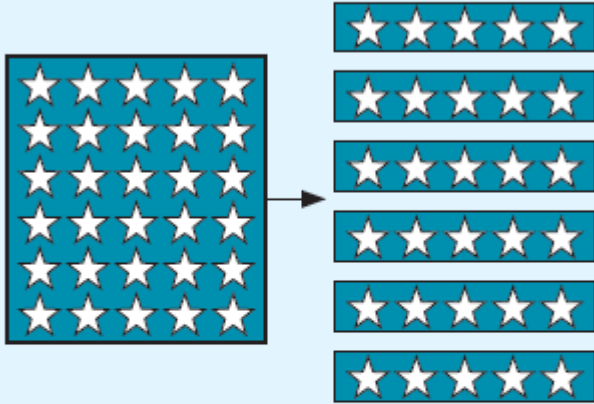
Grade 3 – Multiplication and Division

$6 \times 5 = 30$



6 groups of 5 are 30.

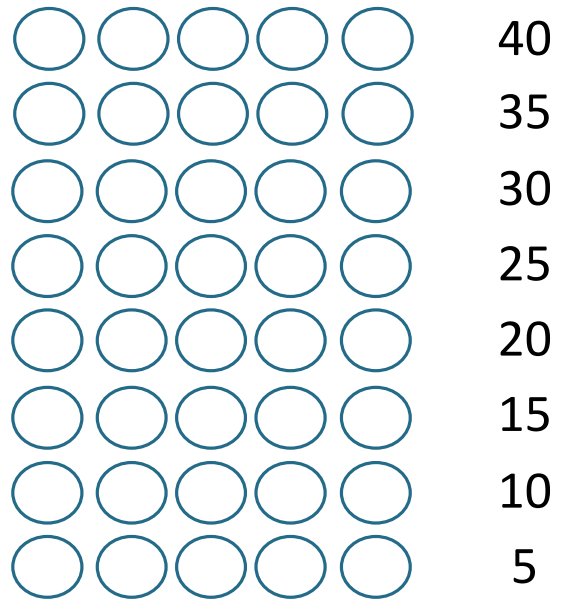
$30 \div 6 = 5$



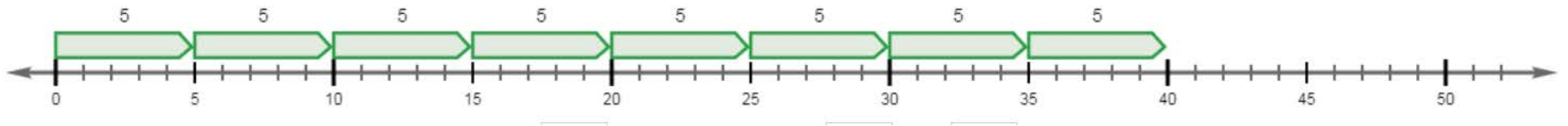
30 split into 6 equal groups is 5 in each group.

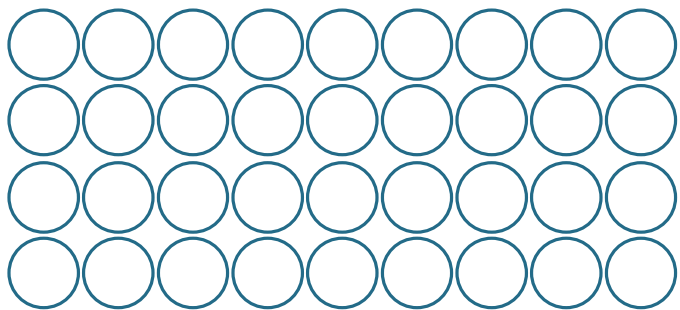
Division is the opposite of multiplication.

| | Unknown Product | Group Size Unknown ("How many in each group?" Division) | Number of Groups Unknown ("How many groups?" Division) |
|---|---|---|--|
| | $3 \times 6 = ?$ | $3 \times ? = 18$, and $18 \div 3 = ?$ | $? \times 6 = 18$, and $18 \div 6 = ?$ |
| Equal Groups | <p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p> | <p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p> | <p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p> |
| Arrays, ⁴ Area ⁵ | <p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p> | <p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p> | <p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p> |
| Compare | <p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p> | <p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p> | <p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p> |
| General | $a \times b = ?$ | $a \times ? = p$, and $p \div a = ?$ | $? \times b = p$, and $p \div b = ?$ |



$$8 \times 5$$

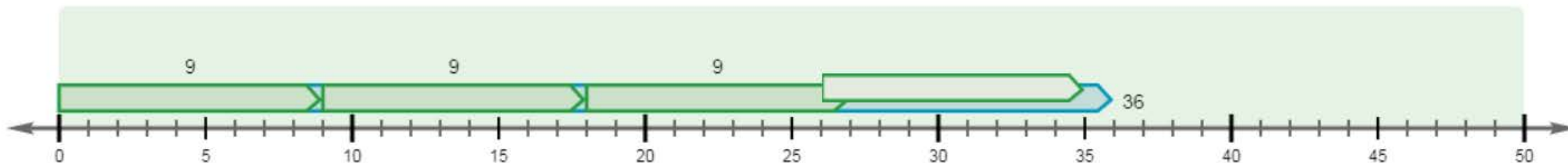
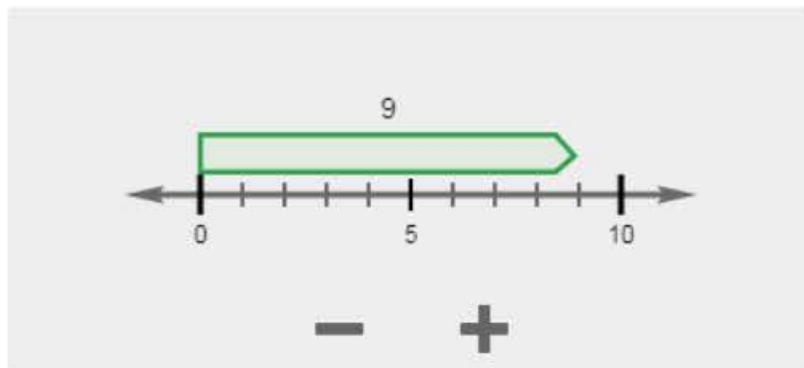




4

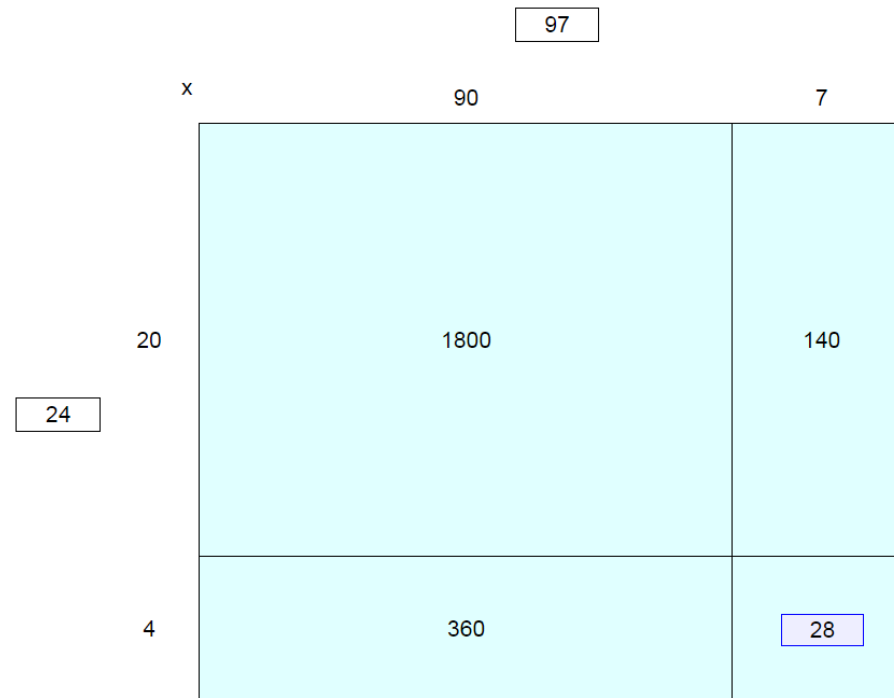
$$36 \div 9$$

9



Grade 4 – Multiplication, Larger Numbers

$$\begin{array}{r} 97 \\ \times 24 \\ \hline 28 \\ 360 \\ 140 \\ + 1800 \\ \hline \end{array}$$



$$\boxed{97} \times \boxed{24} = \boxed{\text{Product}}$$

Grade 4 – Division, Larger Numbers

$$\begin{array}{r}
 \begin{array}{r}
 98 \\
 10 \times 98 \\
 10 \times 98 \\
 10 \times 98 \\
 10 \times 98 \\
 + 1 \times 98 \\
 \hline
 41
 \end{array}
 \quad
 \begin{array}{r}
 41 \\
 \overline{) 4114} \\
 \underline{380} \\
 314 \\
 \underline{296} \\
 184 \\
 \underline{184} \\
 0
 \end{array}
 \end{array}$$

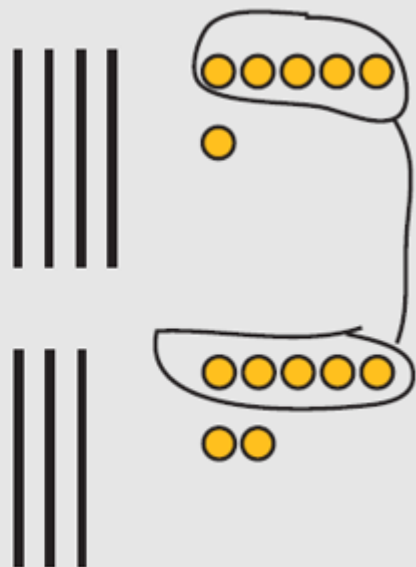
| | 4114 ÷ | 98 |
|----|--------|-----|
| 10 | | 980 |
| 10 | | 980 |
| 10 | | 980 |
| 10 | | 980 |
| 1 | | 98 |
| | | r96 |
| 41 | | |

$$4114 \div 98 = 41 \text{ r } 96$$

A Difficult 5th Grade Problem

$$8.4 \div 1.4 = ?$$

$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$



| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

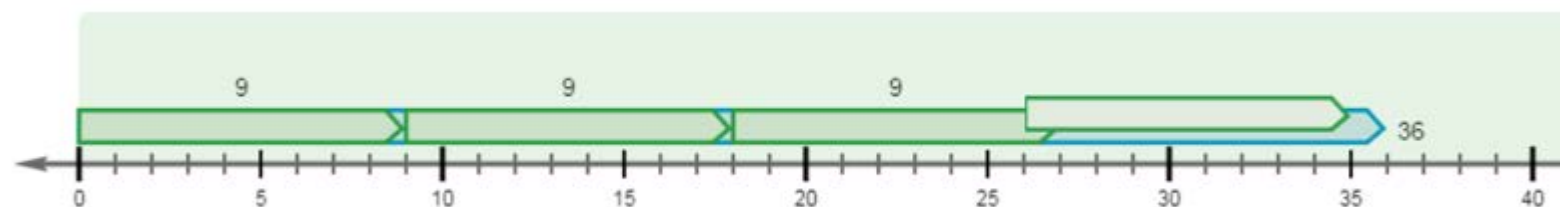
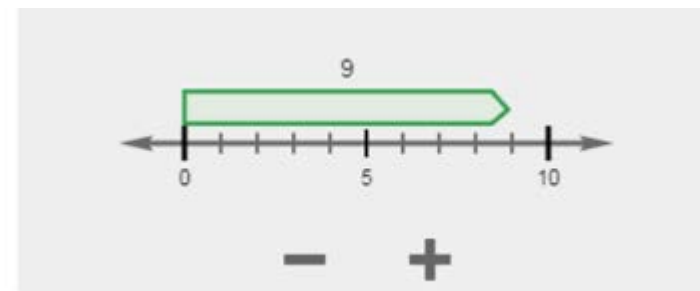
$6 \times 5 = 30$

6 groups of 5 are 30.

$30 \div 6 = 5$

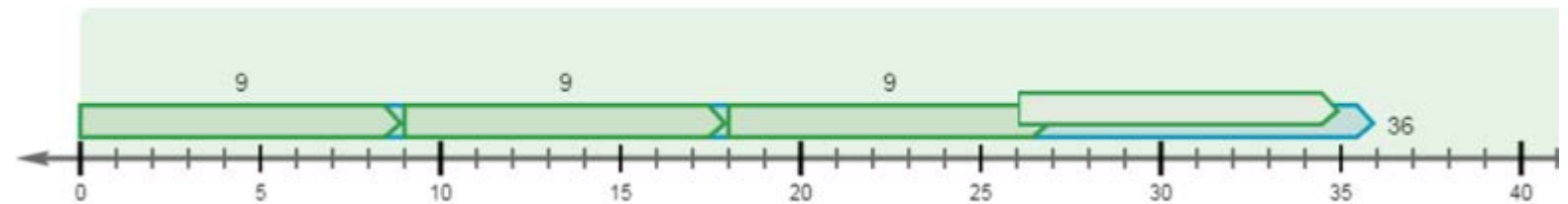
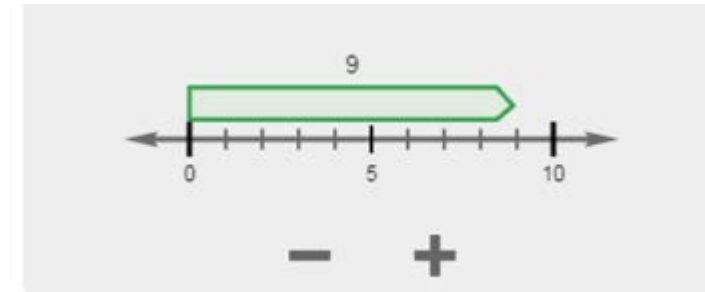
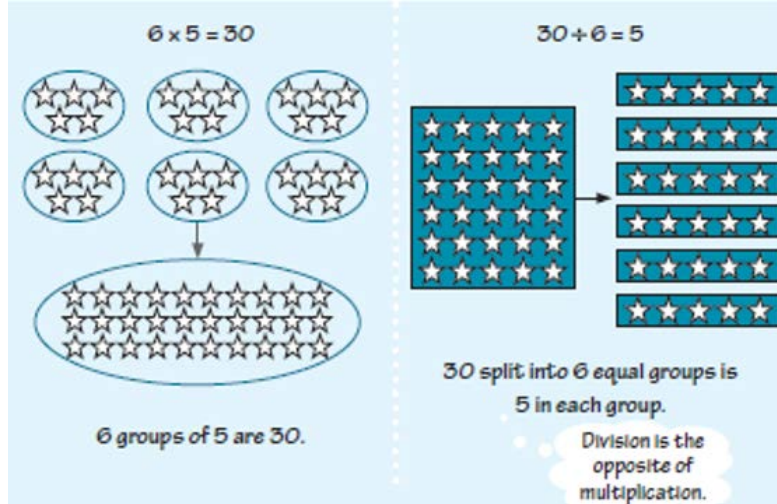
30 split into 6 equal groups is 5 in each group.

Division is the opposite of multiplication.

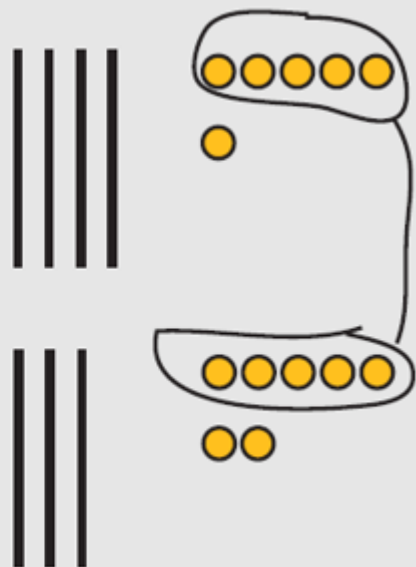


Addition is all about base ten

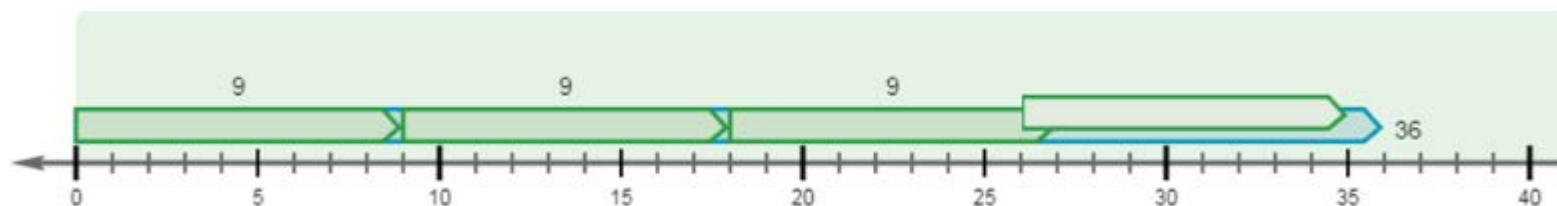
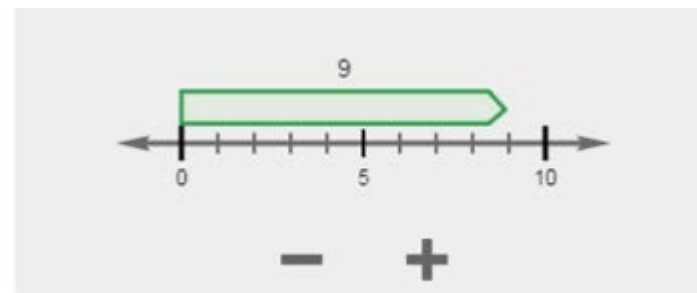
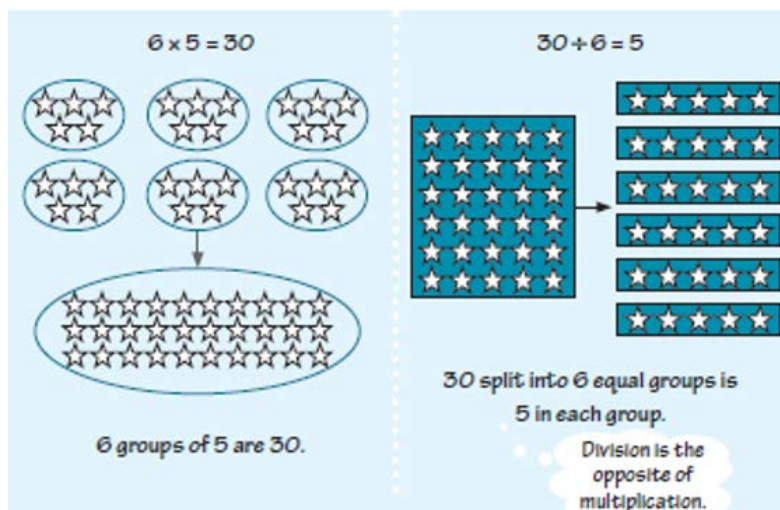
| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |



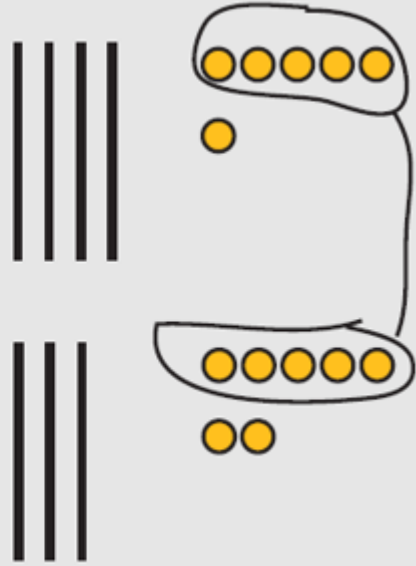
$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$



Multiplication is repeated addition.

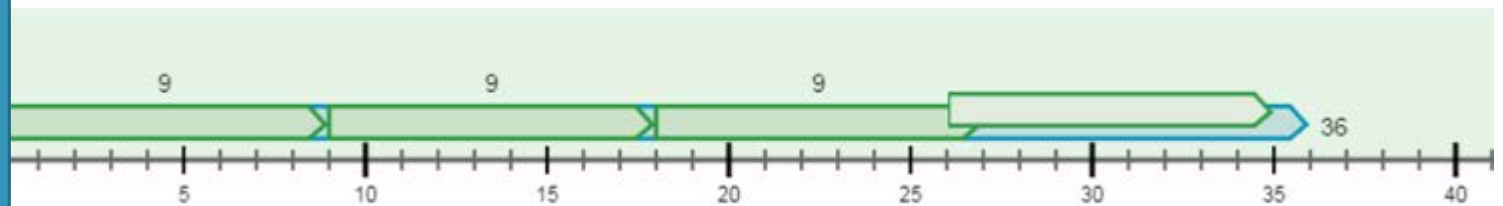
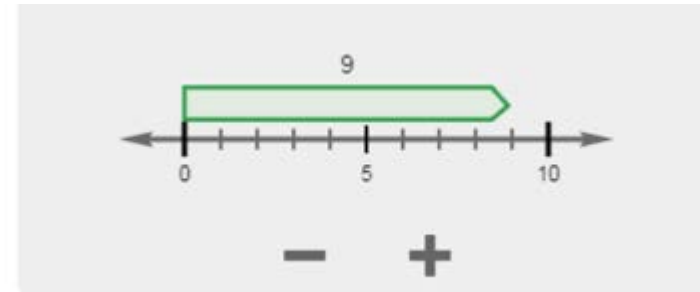


$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$

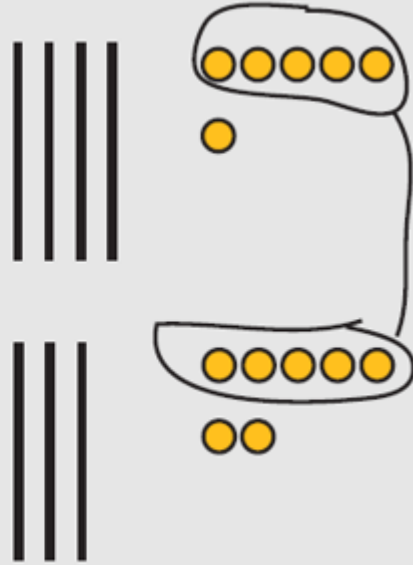


| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

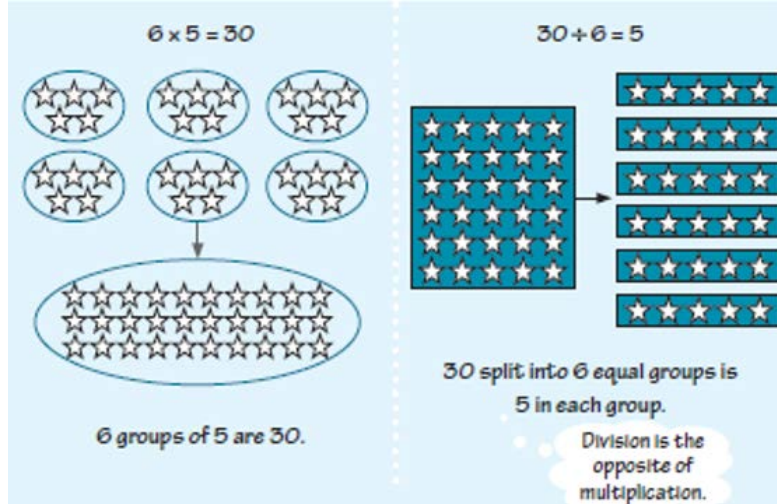
Multiplication and
division are the
same.



$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$

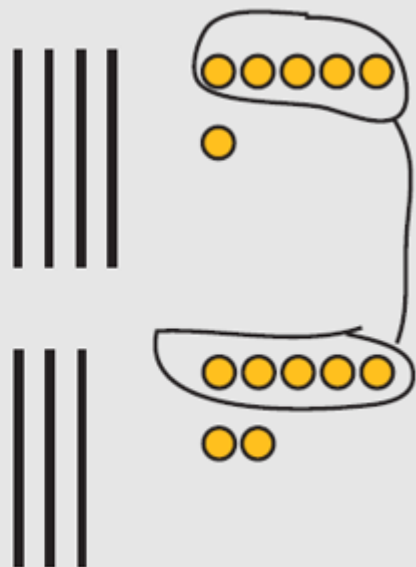


| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |



Division is repeated subtraction.

$$\begin{array}{r} 46 \\ +37 \\ \hline 83 \end{array}$$



| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

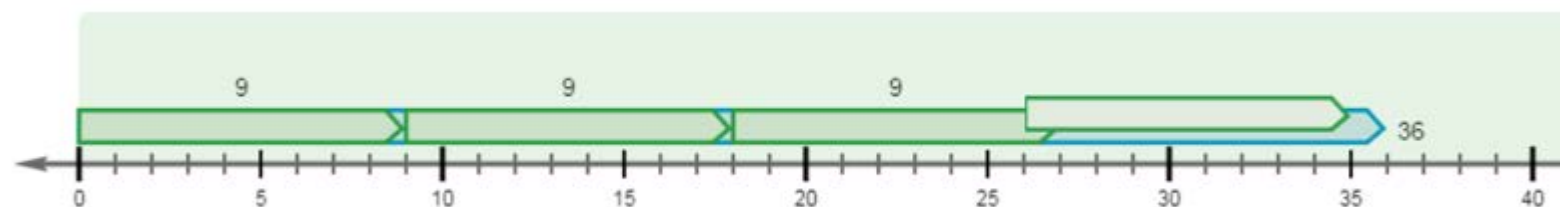
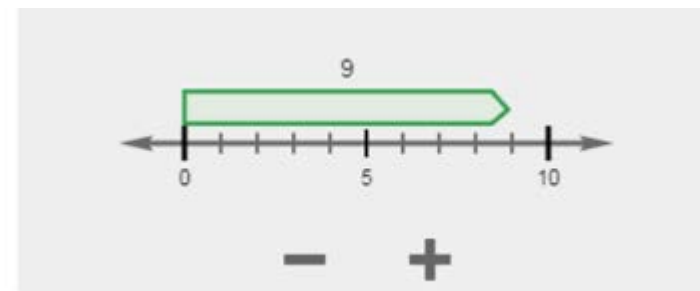
$6 \times 5 = 30$

6 groups of 5 are 30.

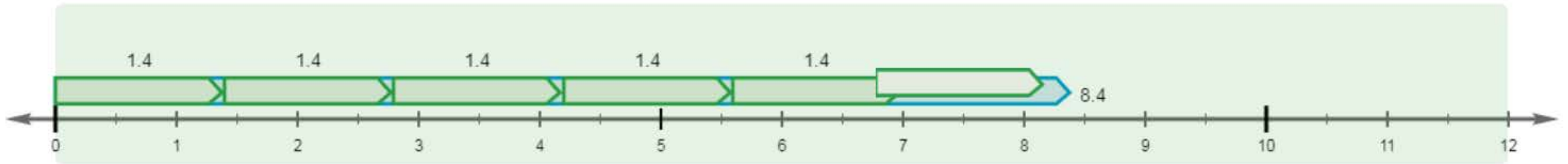
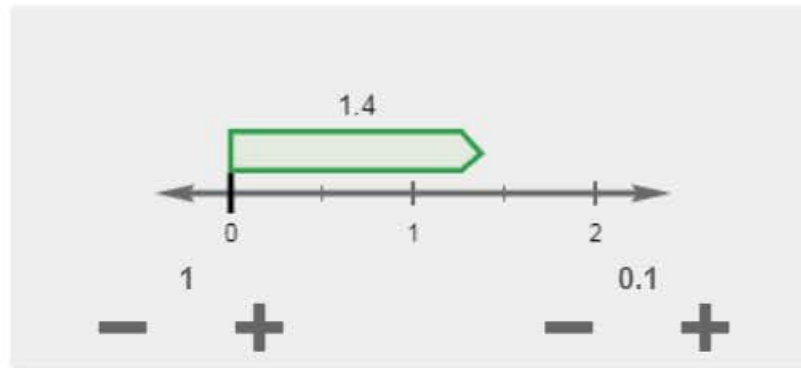
$30 \div 6 = 5$

30 split into 6 equal groups is 5 in each group.

Division is the opposite of multiplication.



$$8.4 \div 1.4 = ?$$



? groups of 1.4 are in 8.4

$$8.4 \div 1.4 = ?$$



Let's call this 1.4.

$$8.4 \div 1.4 = ?$$



Now we have 2.8.

$$8.4 \div 1.4 = ?$$



4.2!

$$8.4 \div 1.4 = ?$$

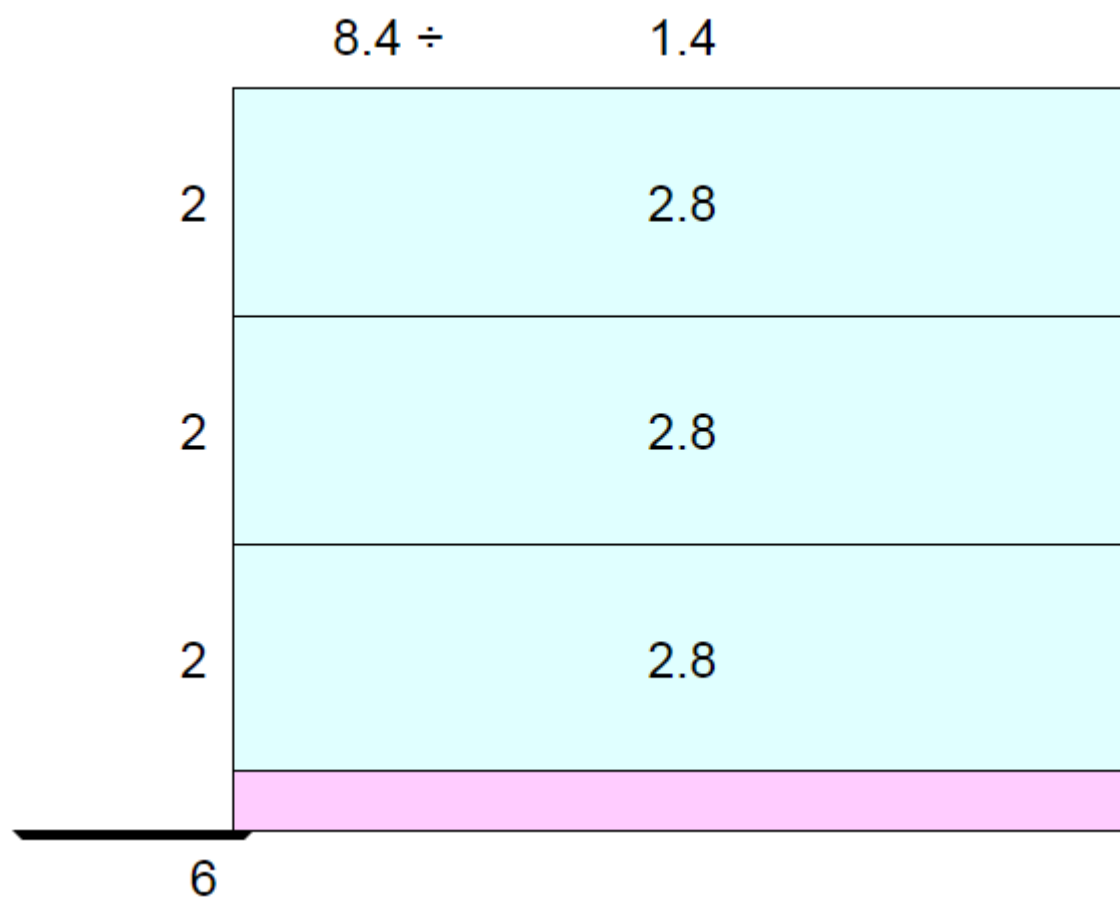


Voila!

$$8.4 \div 1.4 = ?$$



Voila!



$$\begin{array}{r}
 1.4 \\
 \hline
 2 \times 1.4 \\
 \hline
 2 \times 1.4 \\
 \hline
 + 2 \times 1.4 \\
 \hline
 6
 \end{array}
 \quad
 \begin{array}{r}
 6 \\
 \overline{) 8.4} \\
 \underline{- 2.8} \\
 5.6 \\
 \underline{- 2.8} \\
 2.8 \\
 \underline{- 2.8} \\
 0.0
 \end{array}$$

$$8.4 \div 1.4 = 6$$

Videos by Graham Fletcher

<https://gfletchy.com/progression-videos/>



Progression Videos

Early Number & Counting



Addition & Subtraction



1 to 1 Million: Number Sense Progressions for K-5 NCTM 2018

Arjan Khalsa

akhalsa@activatelearning.com

[@arjankhalsa](#)

www.arjankhalsa.com

www.conceptuamath.com

A solid teal horizontal bar at the bottom of the slide.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning