Exploring Mathematical Modeling:Estimating the Number of Fish in a Pond with Sampling

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NCTM Annual Conference
Washington D.C.
April 28, 2018

Global Math Department

www.bigmarker.com/communities/GlobalMathDept/conferences

9 PM Eastern Time

@GlobalMathDept

If you weren't able to attend #NCTMannual, no worries. Come hear a few of the favorite take-aways from several NCTM sessions.

What Is Mathematical Modeling?

From Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) Report

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

From Society for Industrial and Applied Mathematics (SIAM) Math Modeling Getting Started and Getting Solutions

Mathematical modeling is a representation of a system or scenario that is used to gain qualitative and/or quantitative understanding of some real-world problems and to predict future behavior.

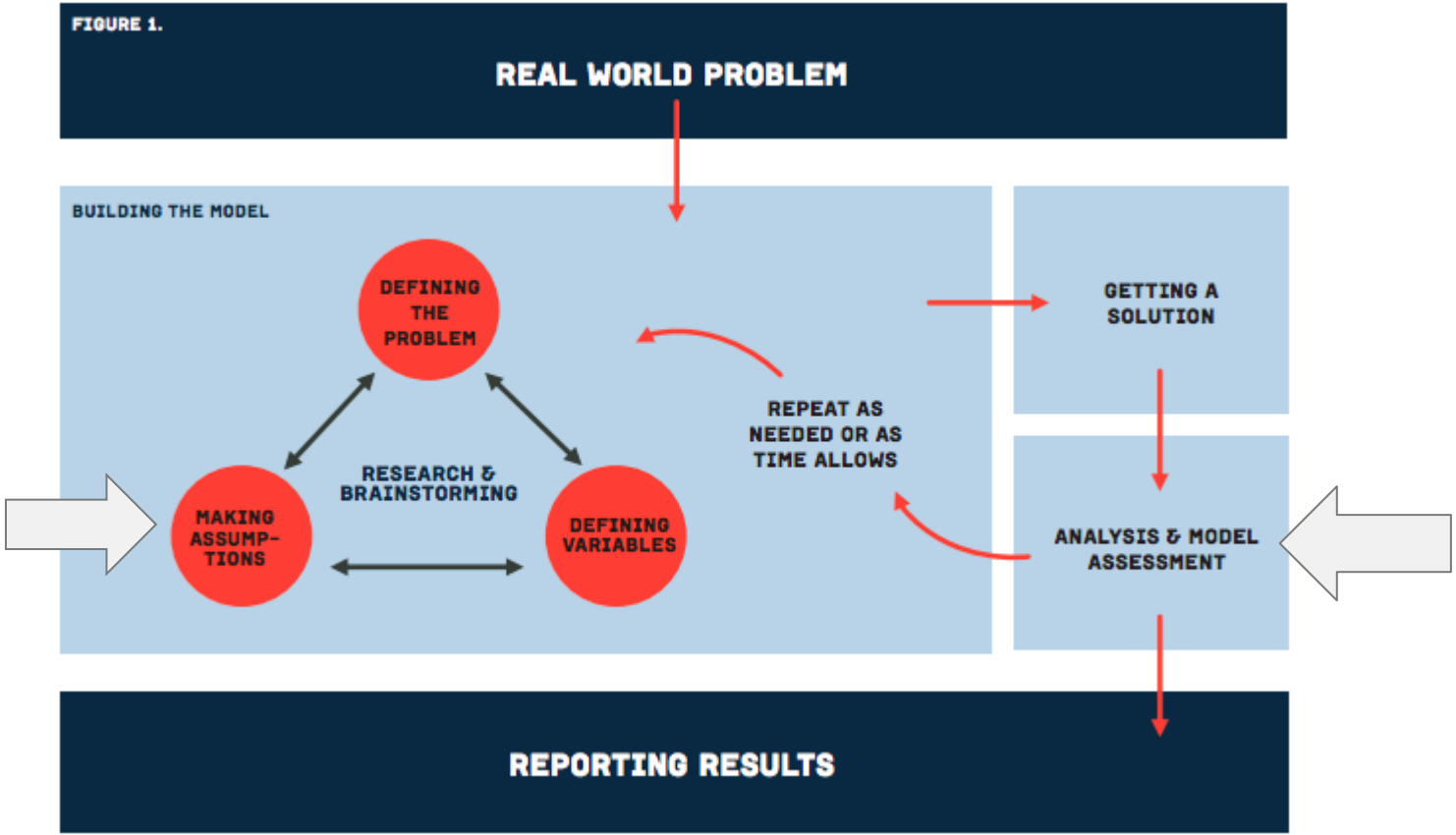
Examples of Math Modeling Questions

- Is it worth it to drive across town to buy gas?
- Is it possible to make a school lunch for $2 per student that is nutritious?
- Does replacing unprofitable hospitals with clinics and ambulance service save money while still providing a reasonable level of medical care access?
- Where are college classrooms trending (traditional, online hybrid) and what type of classroom is best for students?

Components of Modeling

1) Identify the problem
2) Make assumptions and identify variables
3) Do the math
4) Analyze and assess the solution
5) Iterate
6) Implement the model

Note: The full process is open-ended and messy.



From: Math Modeling Getting Started & Getting Solutions

How many fish are there in your pond?

Capture-Recapture Method

1) Estimate wildlife populations
2) Estimate number of children with a genetic disease
3) Estimate number of drug addicts in a city

Estimating the total number of “Bead” fish

1) Remove 10 beads. Replace with red beads.
2) Mix the beads.
3) Remove 15-18 beads. Record number that were red.
4) Repeat previous step 10 times. Find the average # of red beads. (Be sure to re-capture the same amount each time.)
5) Create and solve an equation to get an estimate. Use average # of red beads, original number tagged and number removed.
6) Count your population of “bead” fish to assess the validity of your model.

Testing Model with a Simulation: FATHOM

- Start with population with known number of fish and certain number tagged
- Take a sample from the population.
- Count number of tagged. Count = statistic
- Repeat sampling process to get a partial sampling distribution. Calculate mean number tagged.
- Use mean number tagged to estimate the total population.
- Verify your result.

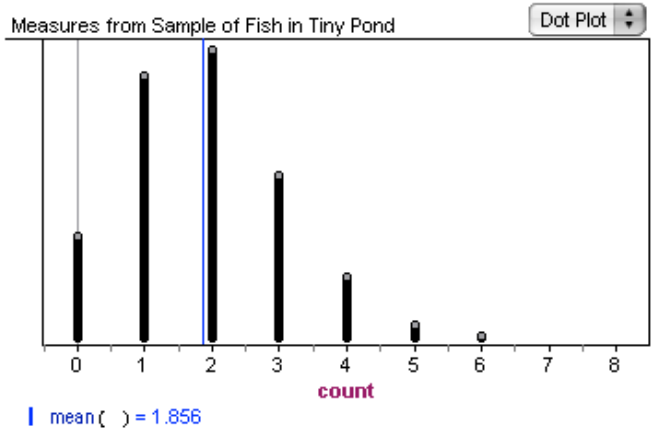
Let’s go to Fathom!

Fathom: Sampling Distribution of Counts

N = 82

(15*10)/1.856

About 81 fish!



15 sampled
10 tagged

What could go wrong? (Assumptions)

1) The proportion of tagged and untagged hasn’t changed between initial tagging and recapture.
2) All have an equal chance of recapture.
3) Sufficient time for tagged to mix with the whole population.
4) Animals do not lose their marks.
5) If we only do this once, will we get it right?

Resources for Teaching Mathematical Modeling

From the M3Challenge Website:
https://m3challenge.siam.org/resources/modeling-videos

https://m3challenge.siam.org/resources/modeling-handbook

https://m3challenge.siam.org/sites/default/files/uploads/siam-cards-final-press.pdf

The Guidelines for Assessment and Instruction in Mathematical Modeling and Education http://www.siam.org/reports/gaimme-for_print.pdf
Note: Appendix A is rich with resources!!

Free trial of Fathom for simulations: http://fathom.concord.org/

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