Engage Student Learning with Demanding Fraction Tasks that Promote Discourse and Feedback

Teaching Integrated Math and Science Project
University of Illinois at Chicago

www.mathtrailblazers.uic.edu
Engage Student Learning with Cognitively Demanding Fraction Tasks: Promoting Discourse and Feedback from Peers and Teachers
ENGAGE STUDENT LEARNING WITH COGNITIVELY DEMANDING FRACTION TASKS: PROMOTING DISCOURSE AND FEEDBACK FROM PEERS AND TEACHERS

Sandra Niemiera
Elizabeth Cape
Jennifer Leimberger

Teaching Integrated Mathematics and Science (TIMS) Project
Learning Sciences Research Institute, University of Illinois at Chicago
Participants will
• Explore multiple strategies, flexibility and perseverance in problem solving;
• Analyze student work to identify thinking and uncover mathematical thinking as well as misconceptions;
• Engage in solving cognitively demanding tasks focused on Numbers and Operations with Fractions; and
• Develop strategies that supports discourse and feedback among peers and teachers.
MATHEMATICS TEACHING PRACTICES

- Establish Mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.
UNDERSTANDING FRACTIONS

Work with the members at your table:

• Make a list of what your students need to know in order to understand fractions.

• List the major misconceptions that your students might have as they begin to develop deeper understanding of fractions.
THINKING ABOUT FRACTIONS

• Work with your table to place the following fractions on your number line.

  1/4, 1 2/3, 3/8, 3/4, 2/2, 7/8, 7/6

• Justify the placement of each fraction.

• Name the fraction indicated by the X on the number line.
UNDERSTANDING LEVELS OF STUDENT REASONING

<table>
<thead>
<tr>
<th>Level</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Student has no concept of the meaning of fractions but may understand partitioning.</td>
</tr>
<tr>
<td>1</td>
<td>Student recognizes familiar pictures of fractions.</td>
</tr>
<tr>
<td>2</td>
<td>Student understands fractions of counting all parts and shaded parts.</td>
</tr>
<tr>
<td>3</td>
<td>Student understands fractions as partitioning a whole shape into equal parts and selecting the parts.</td>
</tr>
<tr>
<td>4</td>
<td>Student understands partitioning a quantity into equal parts and selecting some parts.</td>
</tr>
<tr>
<td>5</td>
<td>Student can manipulate or imagine visual representations of fractions to solve simple fraction arithmetic problems.</td>
</tr>
<tr>
<td>6</td>
<td>Student uses and has some intuitive understanding of symbolic fraction computation.</td>
</tr>
<tr>
<td>7</td>
<td>Student uses pictures or materials to solve difficult fraction arithmetic problems and to understand more precisely why symbolic fraction computations work.</td>
</tr>
</tbody>
</table>

Battista, 2012
Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

NCTM Principles to Actions
Review the Halving Activity Handout.

At your table discuss the following:

• What strategies might your students use to complete this task.
• What do students need to understand to complete this task?
• What possible misconceptions might students have?
"In order to be half, they need to have equal area."

—Collin
Even what we an Collin did was we cut out the pieces, as you can see. Then we tapped them like this 📕, once we did that we tapped it in a square.
FOLDING FRACTIONS

Review the Paper Folding Handout

At your table discuss the following:

• What strategies might your students use to complete this task.
• What do students need to understand to complete this task?
• What possible misconceptions might students have?
\[
\frac{1}{2} \text{ of } 64 = 32 \\
64 \times \frac{1}{2} = 32 \\
\frac{1}{4} \text{ of } 64 = 16 \\
64 \times \frac{1}{4} = 16
\]
Cognitively Demanding Tasks

Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

NCTM Principles to Actions
Discussions should achieve a mathematical goal.

Students need to know what and how to share.

Teachers need to orient students to one another and to the mathematical ideas.

Teachers must communicate that all students are sense makers and that their ideas are valued.

Kazemi & Hitz, 2014
POSITIVE NORMS TO ENCOURAGE IN MATH CLASS

- Everyone can learn math to the highest level.
- Mistakes are valuable—They make your brain grow.
- Failure and struggle do not mean that you can’t do math—these are the most important parts of math and learning.
- Questions are really important.
- Math is about creativity and making sense.
- Math is about connections and communication.
- Depth is much more important than speed.
- Math class is about learning not performing.

Boaler, 2016
How many avocados do you see?
FRACTION STRIPS
Daniel has a bowl of six orange halves. If he eats 5 of them, what fraction of an orange does he eat?
Mrs. Dewey asked the class the following question.

“Daniel has a bowl of six orange halves. If he eats 5 of them, what fraction of an orange did he eat?”

Luis explained his solution to the class.

“I know the unit whole is one orange. Since he ate 5 of the halves, I added one-half 5 times. \(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{2}\).” He showed the class with fraction strips, and counted each half.
“I can say it another way, too. I know $\frac{1}{2} + \frac{1}{2}$ equals 1 whole, and then $\frac{1}{2} + \frac{1}{2}$ equals another whole, and then there was $\frac{1}{2}$ more. So Daniel ate $2\frac{1}{2}$ oranges.”

Shannon said, “I thought of it in a different way. I knew that I was going to add $\frac{1}{2}$ again and again, so I just multiplied 5 times $\frac{1}{2}$. $5 \times \frac{1}{2} = \frac{5}{2}$. That is the same as adding one-half 5 times. I can call it five-halves or two and one-half.”

Mrs. Dewey wrote this on the board: $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 5 \times \frac{1}{2}$. Is this a true number sentence? Why or why not?
The object of this game is to move two game markers from matching hexagons to opposite matching hexagons that have the same number. This is a game for two or three players.
COMPARING FRACTION CARDS

The object of this game is to select a card and compare the two fractions on the card. Students will justify their answer.
The object of this game is to be the first team to earn 6 points by filling 6 circles (unit wholes) with fraction pieces. Points are recorded using number sentences to represent each filled circle. This game is for two teams of two players each.
Finding Equivalent Fractions and Ratios Menu

Using Models

Using Tables and Graphs

Dimes to Nickels

<table>
<thead>
<tr>
<th>Dimes</th>
<th>Nickels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Using Multiplication and Division

\[
\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}
\]

\[
\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2} \times 2 = \frac{4}{8} = \frac{6}{12} \times 2
\]

\[
\frac{4 \text{ cups}}{10 \text{ servings}} = \frac{4 \div 2}{10 \div 2} = \frac{2 \text{ cups}}{5 \text{ servings}}
\]

\[
\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100}
\]

1 dime = 3 dimes = 6 dimes
2 nickels = 6 nickels = 12 nickels
Adding and Subtracting Fractions Menu

Using Area Models

Using Number Lines

Using Paper and Pencil

Find a common denominator.
Multiplying Fractions Menu

Using Area Models

\[3 \times \frac{2}{5} = \frac{2}{5} \times 3 = \frac{6}{5} \text{ or } 1 \frac{1}{5}\]

\[\frac{1}{4} \times \frac{2}{3} = \frac{1}{4} \times \frac{2}{3} = \frac{2}{12} \text{ or } \frac{1}{6}\]

\[\frac{1}{4} \text{ of } \frac{2}{3} \text{ is } \frac{2}{12}\]

Using Number Lines

\[3 \times \frac{3}{4} = \frac{9}{4} \text{ or } 2 \frac{1}{4}\]

\[\frac{1}{2} \times \frac{4}{5} = \frac{4}{10} \text{ or } \frac{2}{5}\]
Dividing Fractions Menu

Using Area Models

$$4 \div \frac{1}{3} = 12$$

How many \(\frac{1}{3}\)-cups are in 4 cups of flour?

1 2 3 4 5 6
7 8 9 10 11 12

Using Number Lines

$$3 \div \frac{1}{2} = 6$$

6 halves are in 3

$$\frac{1}{2} \div 4 = \frac{1}{8}$$

\(\frac{1}{2}\) divided into 4 parts—Each part is \(\frac{1}{8}\)
QUESTIONS
ENGAGE STUDENT LEARNING WITH COGNITIVELY DEMANDING FRACTION TASKS: PROMOTING DISCOURSE AND FEEDBACK FROM PEERS AND TEACHERS

Email:

Mathtrailblazer@uic.edu

Author Website:

www.mathtrailblazers.edu.uic

Thank you for Attending!
Rate this presentation on the conference app! 
Search “NCTM 2018” in your app store or follow the link at nctm.org/confapp to download.

Join in the conversation! #NCTMannual

Download available presentation handouts from the online planner at: nctm.org/planner
References


http://www.youcubed.org


