Engage Student Learning with Demanding Fraction Tasks Promoting Discourse and Feedback

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NCTM 2018
CBA Levels of Sophistication in Students’ Understanding About Fractions

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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<tbody>
<tr>
<td>1</td>
<td>Student has no concept of the meaning of fractions, but may understand portioning.</td>
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<tr>
<td>2</td>
<td>Student recognizes only familiar pictures of fractions.</td>
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<tr>
<td>3</td>
<td>Students understands fractions as counting all parts and shaded parts.</td>
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<tr>
<td>4</td>
<td>Student understands fractions as portioning a quantity into equal parts and selecting some parts.</td>
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<tr>
<td>5</td>
<td>Student can manipulate or imagine visual representations of fractions to solve simple fraction arithmetic problems.</td>
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<tr>
<td>6</td>
<td>Student uses and has some intuitive understanding of symbolic fraction computations.</td>
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<tr>
<td>7</td>
<td>Student uses pictures or materials to solve difficult fraction arithmetic problems and to understand more precisely why fraction computations work.</td>
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(Michael Battista, 2012)
Introduction
One misconception students often have when learning about fractions is that to half something you must cut it into only two pieces of the same shape. This task develops the idea of different ways of creating half of a whole. It allows students to be artistic and create interesting patterns of their own. It also allows students to describe and justify their thinking. In this task students are asked to consider ways to prove that a square is split in half and also to think of other ways to split a square in half and convince others that it is split in half.

Activity
This problem is a great way to get students thinking about fractions and what it means to have half of something. Students often see half cut only in a certain ways. This allows students to see that a whole can be split in half in many different ways. Allow students to discuss their thinking and explain how they are seeing the problems.

Begin class by asking students how they know when something is split in half. What do halves look like? What needs to be true for something to be split in half? Show them one of the shapes from the Halving Handout. Tell them it shows halves. Their task is to describe how they know that the square has been split in half. They should convince themselves and someone else that they know there are two halves. Once they have discussed with their partner, they can use color-coding, pictures, words and diagrams to show off what they find. They should think about what needs to be true in order for a square to be split in half.

Having students work with a partner, give each pair of students a copy of all the squares that are split in half. Have them work through the different squares one at a time: cutting out the square and then convincing each other that each is \( \frac{1}{2} \) in whatever way they chose. They can record their thinking on their blank piece of paper, using color-coding, words, visuals and other ways to show why the shape shows halves. They should be able to convince themselves and a neighbor. As students work, have students consider the following questions - What does it mean to split a shape in half? What needs to be true in order for them to be halves?

Invite students to share with the class their strategy for convincing others that the square has been made into halves. Pick students with different strategies from each other to show their approach to the class. Have students ask questions if they are not convinced. Have students share their ideas about what it means to split a shape in half and what needs to be true in order for someone to know they are halves. Record their ideas on the board.

Following your whole class conversation, give each individual student a square and colored pens and have them create a design that splits the square in half. Have students exchange their design with their neighbor. Their neighbor must decide if they agree that the design splits the square in half. Encourage students to use color coding to prove that they have split the square in half. They can then talk through if they agree that the square has been split in half.
Consider having a few students share their designs with the class or have students do a gallery walk around the room where they look at the different ways the students split their squares in half.

**Extensions**
Have students make a design that is ¼ of the square. Have them convince their neighbor that they have covered one quarter of the square.

**Materials**
Halving Handouts  
Scissors  
Glue  
White paper  
Colored pens/pencils  
Origami paper or patty paper

Each of these images shows squares split in half. How can you check that each is correct? How can you convince someone that each is split in half? (Use the additional Halving Handout.) What does it mean to split a square in half? What needs to be true in order for it to be half? Think of another way to split a square into two halves. Make your own design that splits a square in half.

Retrieved January 2018: https://www.youcubed.org/?s=Halving
Paper Folding

Work with a partner. Take turns being the skeptic or the convincer. When you are the convincer your job is to be convincing! Give reasons for all of your statements. Skeptics must be skeptical! Don’t be easily convinced. Require reasons and justifications that make sense to you.

For each of the problems below one person should make the shape and then be convincing. Your partner is the skeptic. When you move to the next question switch roles. Start with a square sheet of paper and make folds to construct a new shape. Explain how you know the shape you constructed has the specified area.

Construct a square with exactly \( \frac{1}{4} \) the area of the original square.

Construct a triangle with exactly \( \frac{1}{4} \) the area of the original square.

Construct another triangle with exactly \( \frac{1}{4} \) the area of the original square that is not congruent to your first triangle.

Construct a square with exactly \( \frac{1}{2} \) the area of the original square.

Construct another square that is exactly \( \frac{1}{2} \) the area of the original square that is oriented differently than your previous square.

Making Fraction Strips

Carefully cut each paper strip from this page. These strips will be used to fold fractions.
Work with a partner to solve the following problems. You will need to use two sets of fraction strips. Write a number sentence or sentences for each problem.

12. Irma and her sister collect shells. Irma has $1 \frac{1}{2}$ jars full of shells, and her sister has $\frac{3}{4}$ jar full. When they put their collections together, how many jars of shells do they have? How much more do they need to fill 2 whole jars?

13. Michael served 4 pizzas at his birthday party. The pizzas were cut into thirds. Six girls each ate $\frac{1}{3}$ of a pizza and three boys each ate $\frac{2}{3}$ of a pizza. Which group ate more pizza? Explain your thinking.

14. Maya practices shooting free throws for $\frac{1}{2}$ hour a day. If she practices Monday, Tuesday, and Thursday, how long has she practiced?
   A. Solve using Luis’s strategy.
   B. Solve using Shannon’s strategy.

15. Maya said that $\frac{7}{6} + \frac{3}{6} = \frac{10}{15}$. Does her answer make sense? Why or why not? If the sentence is false, make it true.

16. The students at Bessie Coleman School are having a Read-A-Thon. 4 students in Mrs. Dewey’s class each read $\frac{1}{4}$ hour and 3 students in another fourth-grade class each read $\frac{1}{3}$ hour. When their hours are combined, how long have the students read?
   A. Solve using Luis’s strategy.
   B. Solve using Shannon’s strategy.
Fraction Hex

The object of this game is to move two game markers from matching hexagons to opposite matching hexagons that have the same number. This is a game for two or three players.

Materials
- Fraction Hex Game Board 1 and the Halves, Thirds, and Sixths Models or Fraction Hex Game Board 2 and the Halves, Fourths, and Eighths Models
- Fraction Hex Spinner
- two game markers for each player (e.g., same color centimeter cubes)
- one clear plastic spinner or pencil with paper clip

Directions

1. Choose the Fraction Hex Game Board 1 with Halves, Thirds, and Sixths Models or Fraction Hex Game Board 2 with Halves, Fourths, and Eighths Models.

2. Each player places both of his or her game markers on two adjacent hexagons with the same number and shading pattern (solid black, stripes, or stipbles). The target hexagons are the pair with the same pattern on the other side of the game board.

3. The first player spins the Fraction Hex Spinner.

4. If “Greater Than or Equal To” shows, the player can move one marker to a neighboring hexagon with a number that is greater than or equal to the number in the hexagon where the marker is now.

5. If “Less Than or Equal To” shows, the player can move one marker to a neighboring hexagon with a number that is less than or equal to the number in the hexagon where the marker is now.

6. The player does not have to move a marker during his or her turn.

7. More than one marker can be on the same hexagon at the same time.

8. Players take turns spinning the spinner and moving game markers.

9. The first player to get both game markers to his or her target hexagons is the winner.

Variation
- Only one marker can be on the same hexagon at the same time.
Fraction Hex Spinner

Less Than or Equal To

Greater Than or Equal To
Fraction Hex Game Board 2

366 SAB · Grade 3 · Unit 9 · Lesson 6 Comparing Fractions
Halves, Thirds, and Sixths Models

Fraction Chart

1 unit whole

\[
\begin{array}{ccc}
\frac{1}{2} & \frac{1}{2} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6}
\end{array}
\]

Fraction Circle Pieces

Comparing Fractions
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The Charles A. Dana Center at The University of Texas at Austin

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Fraction Games Spinners

Spinner 1

Spinner 2

Fraction Sentences
Spinner 3

Fraction Sentences
Fraction Fill Games

Fraction Fill 1

The object of this game is to be the first team to earn 6 points by filling 6 circles (unit wholes) with fraction pieces. Points are recorded using number sentences to represent each filled circle. This game is for two teams of two players each.

Materials
- pink, yellow, and blue circle pieces from two circle pieces sets
- one Fraction Fill 1 Game Board
- Spinner 1 on the Fraction Games Spinners pages
- clear plastic spinner or paper clip and pencil

Directions
1. For this game, the unit whole is the full circle.
2. To begin, place one piece of each color (pink, yellow, and blue) on the Fraction Fill 1 Game Board as shown.
3. The first team spins one time. The team must place a piece or pieces on the game board equal to the fraction shown on the spinner. For example, if a team spins \( \frac{1}{4} \), they can choose to place either 1 yellow or 2 blue pieces.

The outside edges of the pieces must line up with the lines on the circle. This means that:

- Only pink pieces can go in the lines on the halves circle.
- Only pink and yellow pieces can go on the fourths circle.
- Pink, yellow, and blue pieces can go on the eighths circle.
4. Teams earn one point each time they complete a circle. They record their points by writing a number sentence for the completed circle. For example, \( \frac{1}{2} + \frac{1}{2} = 1 \) or \( \frac{1}{2} \times 2 = 1 \) represents a filled circle. The other team can ask players to explain why a move is correct before a point is recorded.

5. When a team fills a circle, they cannot use that circle again until their next turn. They remove the pieces to reset the circle with one piece, so the circle can be used by the other team.

6. Teams take turns spinning and adding pieces. The first team to earn 6 points wins the game.

Example:
As shown on the sample game boards, Team 1 spins \( \frac{5}{8} \). They can place 5 blue pieces on the eighths circle. Or, because \( \frac{5}{8} \) is the same as \( \frac{1}{2} + \frac{1}{8} \), the team can choose to place 1 pink and 1 blue piece on the circles. The pink piece can go on the halves circle and complete a whole to earn a point. The blue piece can go on the eighths circle. To record their point, the team writes \( \frac{1}{2} + \frac{1}{2} = 1 \).

Team 1 takes off the pink piece, so Team 2 can use the halves circle on their turn.
Fraction Fill 2

The object of this game is to be the first team to earn 6 points by filling 6 circles (unit wholes) with fraction pieces. Points are recorded using number sentences to represent each filled circle. This game is for two teams of two players each.

Materials
- orange, aqua, and black circle pieces from two circle pieces sets
- one Fraction Fill 2 Game Board
- Spinner 2 on the Fraction Games Spinners page
- clear plastic spinner or paper clip and pencil

Directions
Follow the Fraction Fill 1 directions, but use orange, aqua, and black pieces.

1. For this game, the unit whole is the full circle.
2. To begin, place one piece of each color (orange, aqua, and black) on the Fraction Fill 2 Game Board.
3. The first team spins one time. The team must place pieces on the game board that are equal to the fraction shown on the spinner. For example, if a team spins \( \frac{5}{6} \), they can choose to place either 5 aqua pieces or 2 orange pieces and 1 aqua, or 10 blacks.

The outside edges of the pieces must line up with the lines on the circle. This means that:
- Only orange pieces can go on the thirds circle.
- Only orange and aqua pieces can go on the sixths circle.
- Orange, aqua, and black pieces can go on the twelfths circle.

Fraction Fill 2 Game Board

![Fraction Fill 2 Game Board](image)
4. Teams earn one point each time they complete a circle. They record their points by writing a number sentence for the completed circle. For example, $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$ represents a filled circle. The other team can ask players to explain why a move is correct before a point is recorded.

5. When a team fills a circle, they cannot use that circle again until their next turn. They remove the pieces to reset the circle with one piece, so the circle can be used by the other team.

6. Teams take turns spinning and adding pieces. The first team to earn 6 points wins the game.

Example:
As shown on the sample game board, Team 1 spins $\frac{1}{6}$ and places 1 orange on the sixths circle. Then Team 2 spins $\frac{5}{6}$. Since $\frac{5}{6} = \frac{2}{6} + \frac{3}{6}$, this team places 3 aquas on the sixths circle and 2 aquas on the twelfths circle. They complete a whole. They write $\frac{4}{6} + \frac{1}{3} = 1$ to record one point.
Fraction Fill 3

The object of this game is to be the first team to earn 6 points by filling 6 circles (unit wholes) with fraction pieces. Points are recorded using number sentences to represent each filled circle. This game is for two teams of two players each.

Materials
- pink, orange, yellow, aqua, blue, and black circle pieces from two circle pieces sets
- one Fraction Fill 1 Game Board and one Fraction Fill 2 Game Board
- Spinner 3 on the Fraction Games Spinners pages
- clear plastic spinner or paper clip and pencil

Directions
1. For this game, the unit whole is the full circle.
2. To begin, place one piece of each color (pink, yellow, and blue) on the Fraction Fill 1 Game Board. Place one piece of each color (orange, aqua, and black) on the Fraction Fill 2 Game Board.
3. The first team spins one time. The team must place a piece or pieces on the game boards equal to the fraction shown on the spinner. For example, if a team spins $\frac{3}{8}$, they can fill 1 yellow and 1 blue on Fraction Fill 1 Game Board or they can fill 1 blue on Fraction Fill 1 Game Board and 3 blacks on Fraction Fill 2 Game Board.

The outside edges of the pieces must line up with the lines on the circle. This means that:
- Only pink pieces can go the lines on the halves circle.
- Only pink and yellow pieces can go in the fourths circle.
- Pink, yellow, and blue pieces can go on the eighths circle.
- Only orange pieces can go in the lines on the thirds circle.
- Pink, orange, and aqua pieces can go in the sixths circle.
- Pink, yellow, orange, blue, aqua, and black can go in the twelfths circle.
4. Teams earn one point each time they complete a circle. They record their points by writing a number sentence for the completed circle. For example, $\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = 1$ or $\frac{1}{4} + \frac{1}{4} + \frac{2}{8} + \frac{4}{12} = 1$ represents a filled circle. The other team can ask players to explain why a move is correct before a point is recorded.
5. When a team fills a circle they cannot use that circle again until their next turn. They remove the pieces to reset the circle with one piece, so the circle may be used by the other team.

6. Teams take turns spinning and adding pieces. The first team to earn 6 points wins the game.

Example:
As shown on the sample game boards, Team 1 spins \(\frac{1}{3}\) and places an orange on the thirds circle on Fraction Fill 2 Game Board. Team 2 spins \(\frac{5}{6}\). They break \(\frac{5}{6}\) into the sum of \(\frac{3}{6} + \frac{2}{6}\). They know \(\frac{3}{6} = \frac{1}{2}\) and \(\frac{2}{6} = \frac{1}{3}\), so they place 1 pink on the halves circle on Fraction Fill 1 Game Board and 1 orange on Fraction Fill 2 Game Board. That fills two circles. They write \(\frac{1}{2} + \frac{1}{2} = 1\) and \(\frac{3}{3} = 1\) to record 2 points.