Equitable Access to the SMPs through Purposeful Number Talk Progressions

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METROCHICAGO MATHINITIATIVE

www.mcmi.uic.edu

What is Quantitative Reasoning?

How do you define or describe quantitative reasoning?

CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Common Core State Standards, 2010, http://www.corestandards.org/Math/Practice/

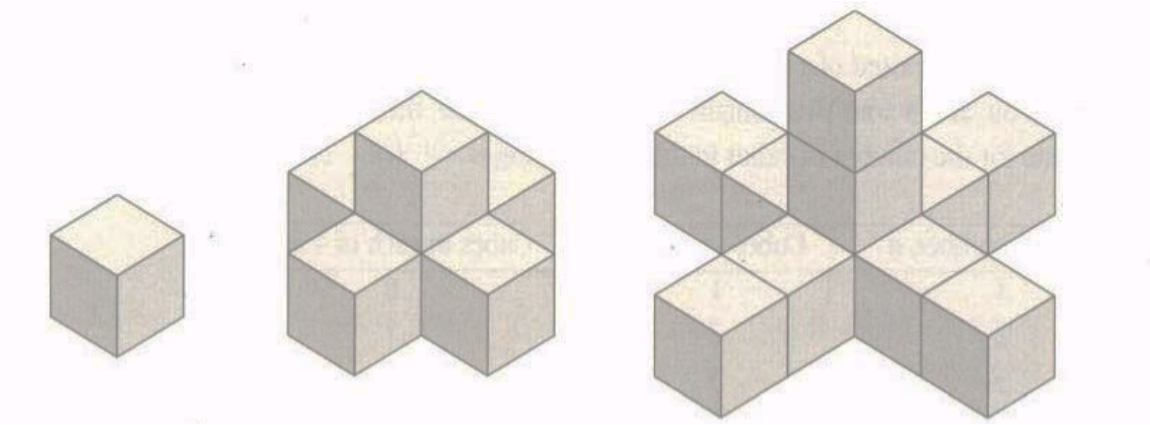
2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

- For example, they can apply ratio reasoning to convert measurement units and proportional relationships to solve percent problems.
- For example, they can solve problems involving unit rates by representing the situations in equation form.
- For example, in middle school, students use properties of operations to generate equivalent expressions and use the number line to understand multiplication and division of rational numbers.

http://commoncoretools.me/wp-content/uploads/2014/05/2014-05-06-Elaborations-6-8.pdf

Counting Cubes Task



Building 1 Building 2 Building 3

1. What do you notice and wonder about the buildings?

2. Describe a pattern you see in the cube buildings.

Adapted from “Counting Cubes”, Lappan, Fey, Fitzgerald, Friel, & Phillips (2004*). Connected MathematicsTM, Say it with symbols: Algebraic reasoning* [Teacher’s Edition]. Glenview, IL: Pearson Prentice Hall.

Counting Cubes Task

Teacher: Peter Dubno

District: New York City School District

Grade: 8

Teacher: --come up and present what they came up with. Arden, come on. Now remember, one person speaks because we can't hear many people speaking at once.

Student: Okay, everybody, like, on the things there were arms, right, which are the extended parts, right? So our pattern was that every time you add one…a cube to each arm, the total volume increases by 5 cubic centimeters, which I think was 5 cubes or whatever you called it. And then on our whatever…

Student: Equation.

Student: --equation, was that $5n + 1$ equals the volume and n equals the length of one individual arm. So that, like, there's the middle cube, excluding the middle cube, you would multiply that by 5 because there are 5 arms and then add 1 for the middle cube and that will give you the volume and number of cubes.

Student: For the 3rd question, it was, what's the volume of the 5th stage? Well, for the 5th stage, there will be 4 cubes on each arm so the equation will be 5 x 4, + 1 equals the volume. So 5 x 4 is 20, + 1 is 21, so the volume will be 21.

Student: Yeah.

Teacher: Any questions for the presenters? Did anyone come up with a different solution? All right, Cassie come on. Do you want to show us? Cassie, Deirdre?

Student: Originally, we got what Arden got, but we tried it out and it didn't always work.

Student: With the 1st one.

Student: With the first one, so what happened is we came up with $5n - 4$, so it's 5 arms and then n would be the building number.

Student: Yeah, and then you subtract it.

Student: And then you subtract 4 and then you get the number of—

Student: Cubes in the building.

Student: --cubes in the building. So we wrote up an example and $n = 2$, so then 5 x 2 - 4 is 10 – 4, and we got 6 cubes in the building.

Student: And that's the number of the cubes in the building.

Student: And it's on the sheet also.

Teacher: So let me ask you. Of that formula, if I asked you how many cubes would be in the 7^{th} building—

Student: It would be 31.

Teacher: And how would you get that?

Student: You'd replace *n* with the 7 and then you do the work, you'd multiply 7 times 5 and then you get 35, and the you minus 4 and you get 31.

Teacher: Okay. How is yours different or the same as what Arden did and Yoshio did?

Student: Both of us used 5*n*, 5 times the building number for each arm but—

Student: The only thing that was different was that we subtracted and he added.

Teacher: Does that make it different or is it the same or what?

Student: No…No, we did the middle square for each arm and then you subtracted 4 middle squares. But what we did is we just added the actual length of the arm excluding the middle square.

Student: But the reason why it didn't work for us was because we were trying to figure out an equation that would fit all buildings instead of just each every 1 except for the 1^{st} one, because it doesn't really work for the 1^{st} one.

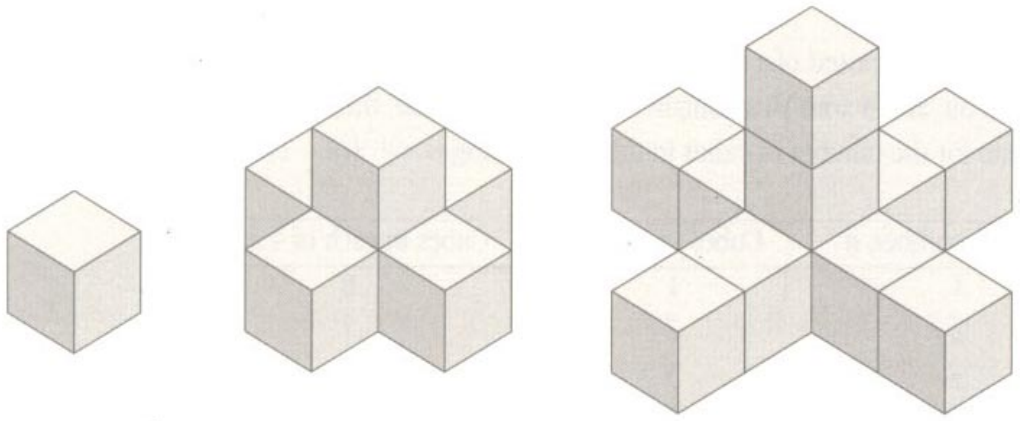
Student: Yea. Because there's only 1 cube.

Student: Yeah, there's no—

Student: 5 x 0 = 0, + 1 = 1.

Student: It's the 1^{st} building, though.

Counting Cubes Task



Building 1 Building 2 Building 3

1. Describe a pattern you see in the cube buildings.

2. Use your pattern to write an expression for the number of cubes in the nth building.

3. Use your expression to find the number of cubes in the 5th building. Check your results by constructing the 5th building and counting the cubes.

4. Look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the nth building.

Adapted from "Counting Cubes", Lappan, Fey, Fitzgerald, Friel, & Phillips (2004). *Connected MathematicsTM, Say it with symbols: Algebraic reasoning* [Teacher's Edition]. Glenview, IL: Pearson Prentice Hall. © Michigan State University

Sample Progression - Prompt 1

Would you rather have….

Pennies to match your weight

Quarters to match your height

Source: http://www.wouldyourathermath.com/height_weight/

Sample Progression - Prompt 2

Which is the better deal for potatoes?

Source: de la Cruz, Garney, *MTMS* 2016

Sample Progression - Prompt 3

Mrs. Leimberer wants to buy oranges to pack in her daughters' lunches. She sees the ads below in the newspaper.

Stanley's Produce Market	Franklin's Fresh Fruit
Fresh oranges on sale! 10 for $2.00	Delicious oranges for you! 8 for $1.75

Where should Mrs. Leimberer buy her oranges?

Sample Progression - Prompt 4

Convince me that...

cereal box B is the better buy.

A

$3.79 ea.

Rice Krispies Cereal, Toasted Rice (12 oz)

B

$4.29 ea.

Rice Krispies Cereal, Toasted Rice (18 oz)

Source: https://tinyurl.com/ConvinceMeThat

Sample Progression - Prompt 5

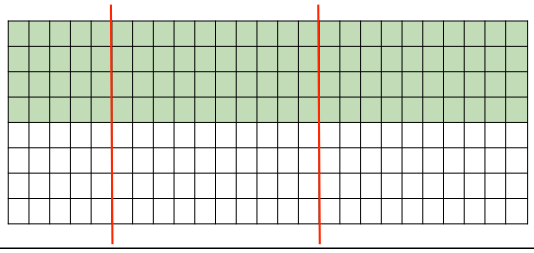
Source: http://www.wouldyourathermath.com/would-you-rather-44/

Sample Progression - Debrief

- Turn and talk to a neighbor:
  - What do you think the goal was of this progression?
  - What big idea(s) came up for you as you were thinking about the prompts?
  - Did you see evidence of "ramping" (building from one task to the next in a purposeful way)?

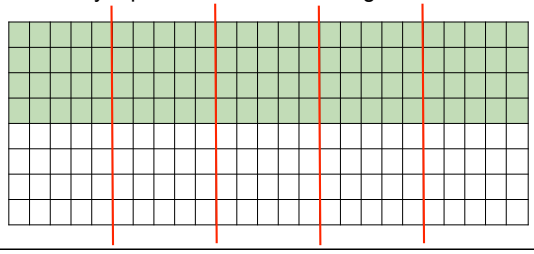
Jigsaw Progression #2 - Prompt 1b

How many squares are in the rectangle?



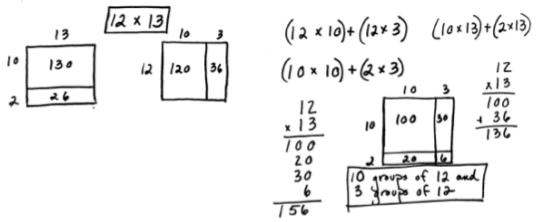
Jigsaw Progression #2 - Prompt 1c

How many squares are in the rectangle?



Jigsaw Progression #2 - Prompt 2

Which represent 12 x 13?



Jigsaw Progression #2 - Prompt 3

15 x 2 =

15 x 4 =

15 x 6 =

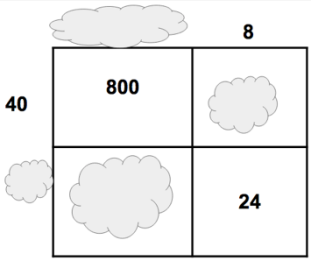
10 x 6 + 5 x 6 =

15 x 10 - 15 x 4 =

Jigsaw Progression #2 - Prompt 3

What relationships do you see?

What values belong in the clouds?



Jigsaw Progression #2 - Prompt 5

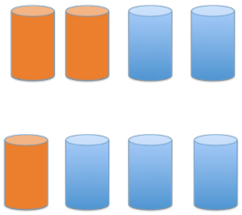
14 x 18

Jigsaw Progression #3 - Prompt 0

The orange represents one part juice. The blue represents one part water.

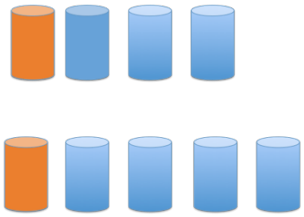
Jigsaw Progression #3 - Prompt 1

Which recipe is "orangier?"



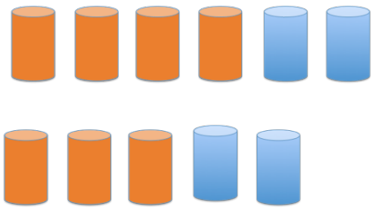
Jigsaw Progression #3 - Prompt 2

Which recipe is "orangier?"



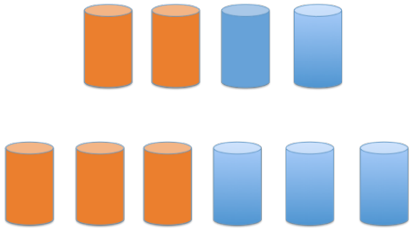
Jigsaw Progression #3 - Prompt 3

Which recipe is "orangier?"



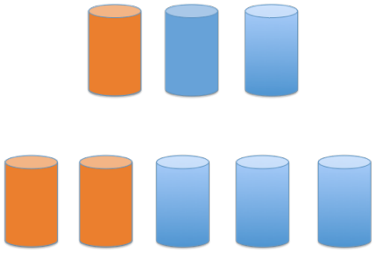
Jigsaw Progression #3 - Prompt 4

Which recipe is "orangier?"



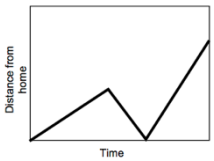
Jigsaw Progression #3 - Prompt 5

Which recipe is "orangier?"



Jigsaw Progression #1 - Prompt 1

What do you notice/wonder about this graph?



Tell a story that would fit the graph.

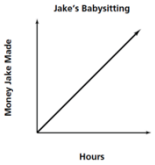
Jigsaw Progression #1 - Prompt 2

These graphs show the distances traveled from home.

1. Which graph shows the fastest travel time?
2. Which graph shows the car is not moving?
3. Which graph is impossible?

Jigsaw Progression #1 - Prompt 3

What's the story here? Put a scale on each axis and explain why they are reasonable.



Jigsaw Progression #1 - Prompt 4

Which line represents y = $5.25x and which line represents y = $8.50x?

How do you know?

What would a line look like for earnings at $12.75 per hour?

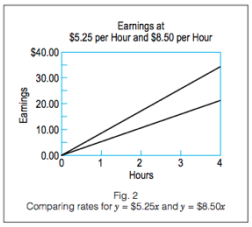
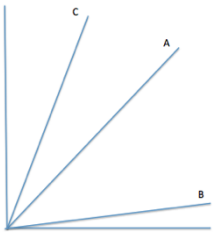


Fig. 2
Comparing rates for y = $5.25x and y = $8.50x

Jigsaw Progression #1 - Prompt 5



If the slope of line A is 1, what is the slope of B?

What is the slope of line C?

Planning a Math Talk Progression

What is the mathematical goal, structure, or big idea that you want to get across in this string?	
Prompt 1	**Rationale, Notes, Anticipated Responses**
Prompt 2	**Rationale, Notes, Anticipated Responses**
Prompt 3	**Rationale, Notes, Anticipated Responses**
Prompt 4	**Rationale, Notes, Anticipated Responses**
Prompt 5	**Rationale, Notes, Anticipated Responses**

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Prompt 5	**Rationale, Notes, Anticipated Responses**

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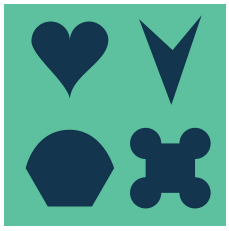
Math/Number Talk Resources

Which One Doesn't Belong?
http://wodb.ca/

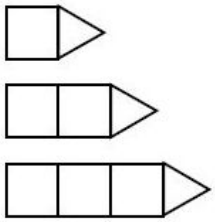


Find a reason why each one doesn't belong.

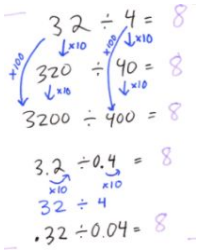
Estimation 180
http://www.estimation180.com/

Would You Rather Math
http://www.wouldyourathermath.com/

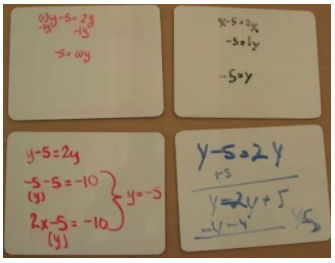
Visual Patterns
http://www.visualpatterns.org/



Number Strings
https://numberstrings.com/

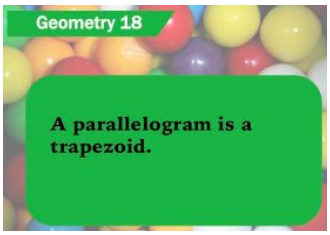


Math Mistakes
http://mathmistakes.org/

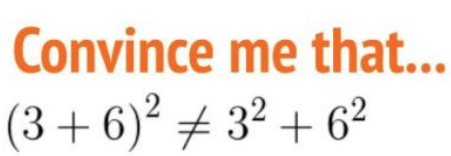


Fraction Talks
http://www.fractiontalks.com/

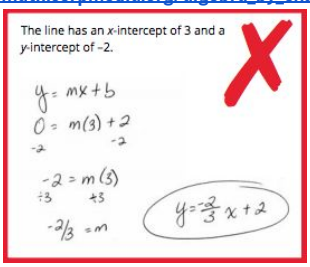
Always, Sometimes, Never
http://asnmath.blogspot.com/



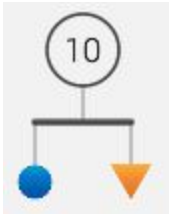
Convince Me That
tinyurl.com/ConvinceMeThat



Algebra by Example
http://math.serpmedia.org/algebra_by_example/



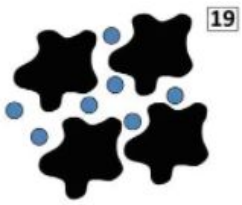
Solve Me Mobiles
https://solveme.edc.org/



Numberless Word Problems
https://bstockus.wordpress.com/numberless-word-problems/

Number Talk Images
http://ntimages.weebly.com/

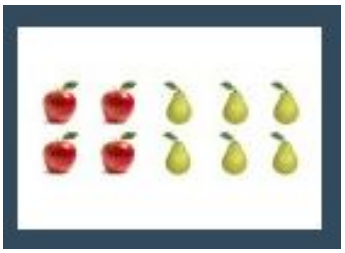
Splat!
http://www.stevewyborney.com/?p=893



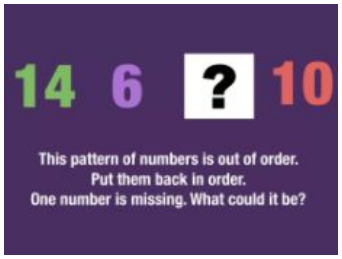
Cube Conversations
http://www.stevewyborney.com/?p=1253

Math/Number Talk Resources

Math is Visual
http://mathisvisual.com/

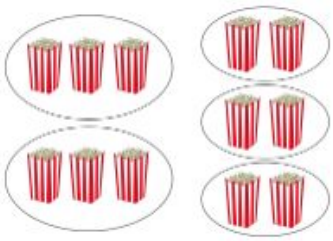


Math Before Bed
https://mathbeforebed.com/

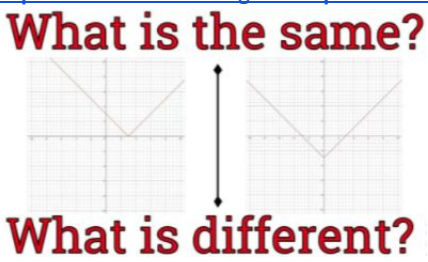


B2N: Between Two Numbers
http://www.between2numbers.com/

Same BUT Different
https://www.samebutdifferentmath.com/



Same OR Different
https://samedifferentimages.wordpress.com/

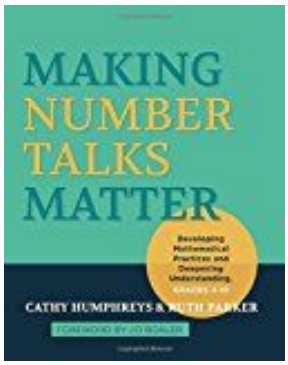


Problem Strings from Pam Harris
http://www.mathisfigureoutable.com/problem-strings/

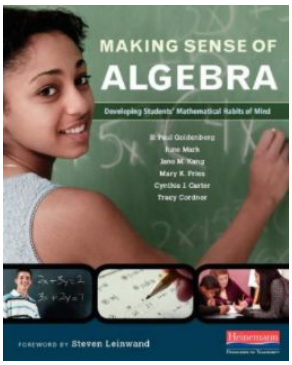
$$x(x+4)=5$$
$$x(x-1)=12$$
$$x(x-2)-8=0$$
$$x^2+2x=15$$

Print Materials

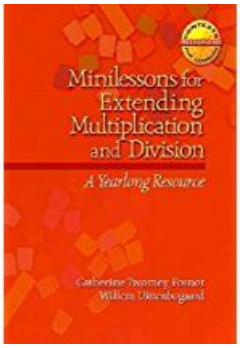
Making Number Talks Matter
by Cathy Humphreys & Ruth Parker



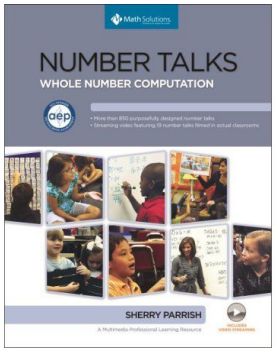
Making Sense of Algebra
Paul Goldenberg, et. al.



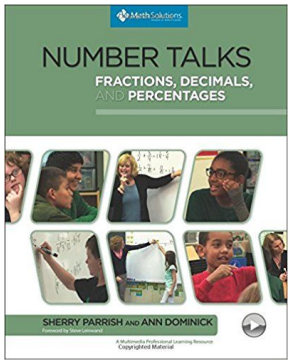
Minilessons for Operations
Cathy Fosnot, et. al.
https://tinyurl.com/FostnotAmazon



Number Talks
by Sherry Parrish



Number Talks: FDP
by Sherry Parrish



Number S.E.N.S.E. Series
by McIntosh, Reys, and Reys