Promote Statistical Understanding by Connecting Data Collection, Data Analysis, and Inference

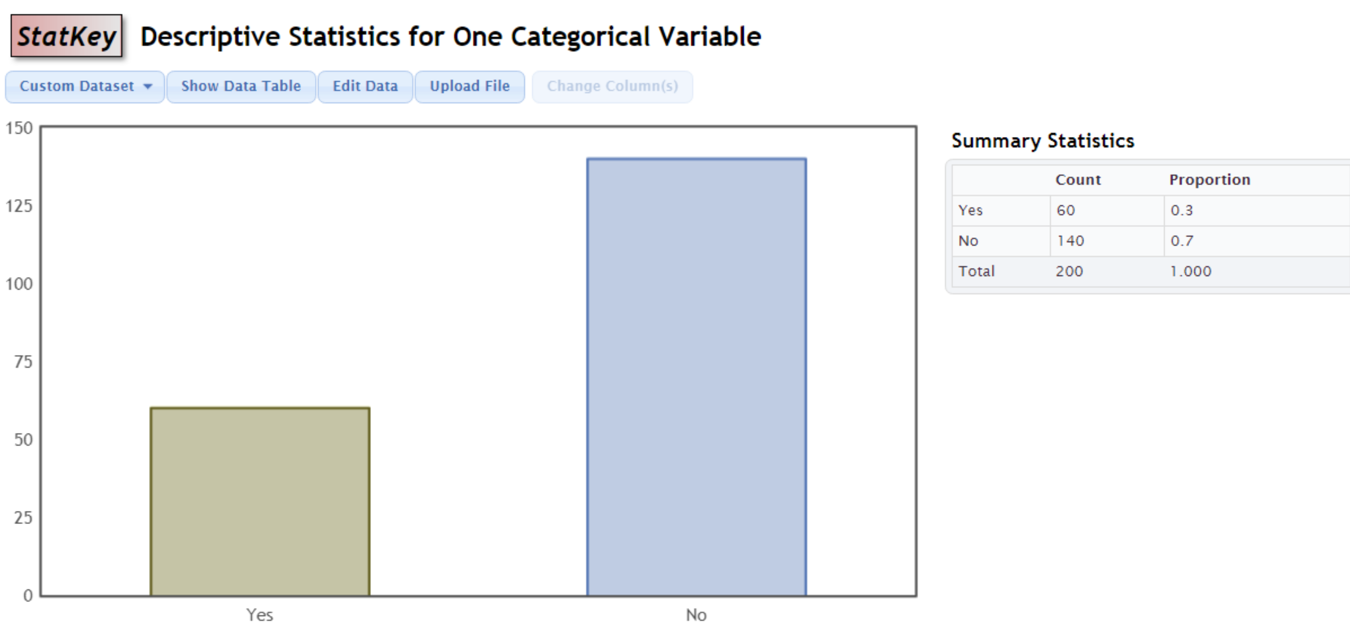
I. Inference for sampling *Who Watches Survivor?*

Television executives and companies who advertise on TV are interested in how many viewers watch particular shows. According to Nielsen ratings, Survivor was one of the most-watched television shows in the United States during every week that it aired. An avid Survivor fan and high school statistics teacher (we'll call him Daren) wants to estimate the proportion of all students at his large high school who have watched Survivor. In a random sample of 200 students from Daren's school, 60 have watched Survivor.

Data Collection: Sample survey (observational study) using random sampling

Data Analysis: Variable of interest that's recorded for each individual in the sample: whether or not they have watched Survivor. This is a *categorical variable* with two possible values: Yes or No.

- Point your browser to www.lock5stat.com/statkey.
- Choose One Categorical Variable in the Descriptive Statistics and Graphs menu.
- Click on Edit Data and enter the number of Yes (60) and No (140) responses in the sample. When finished, click OK to display a *bar graph* and summary statistics (*counts* and *proportions*).



Inference: Daren wants to estimate the true proportion *p* of all students at his large high school who have watched Survivor.

- What is our "best guess" for the value of the population proportion *p* based on the sample data?

- Do you believe that the population proportion *p* is <u>exactly equal</u> to this value?

BIG QUESTION: What values of *p* are <u>plausible</u> (believable) based on the sample result?

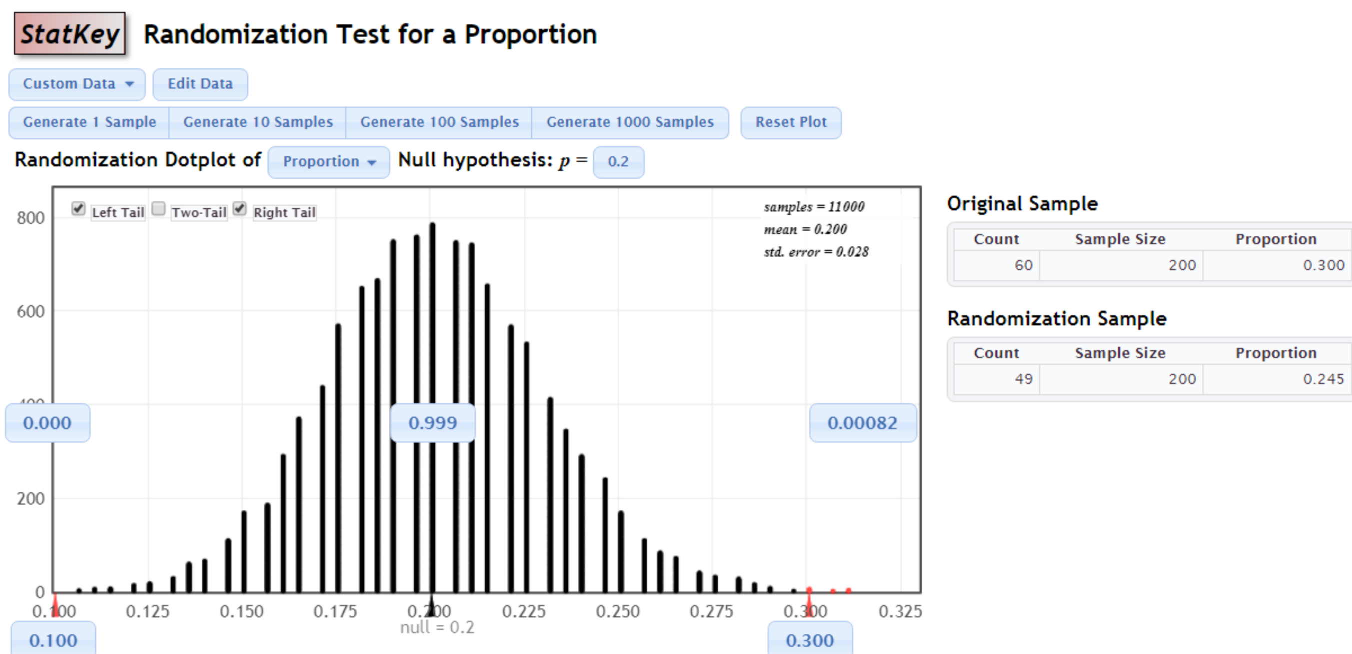
- Is *p* = 0.20 a plausible value of the population proportion?

1. Click on the StatKey logo in the top left corner. Then choose “Test for Single Proportion”.
2. Click on “Edit Data”. Enter 60 for the count of successes and 200 as the sample size.
3. Change the “Null hypothesis” value of *p* to 0.20.
4. Click on Generate 1 Sample. Repeat several times. What is happening?
5. Click on Generate 1000 Samples several times. What do you notice?
6. Click the “Right Tail” button. Then click on the dialog box on the horizontal axis and change the value to 0.30 (to match the observed sample proportion of students who said Yes).

What do the red dots indicate?

If the actual population proportion were 0.20, it is just as surprising to get a sample proportion of 0.30 or greater as it is to get a sample proportion of ______ or less.

7. Click the “Left Tail” button. Then click on the dialog box on the horizontal axis and change the value to 0.10 (an equally surprising value for the sample proportion).



8. The red dots show the trials in which the simulated sample proportion was less than or equal to 0.10 or greater than or equal to 0.30 when the actual population proportion is 0.20. Said another way: If the actual population proportion is *p* = 0.20, the probability of getting a sample proportion as far or farther from the population proportion as the one observed (0.30) is about _________.

9. Based on the estimated probability in Step 8, are you willing to believe that the actual population proportion is *p* = 0.20?

10. Now we need to consider other potential values of the population proportion *p*. Let’s agree that if the observed result (0.30) or one even more extreme has <u>less than a 5% chance of occurring</u> when the population proportion has the hypothesized value, we will consider that value implausible.

Is it plausible that p =

0.21?	0.26?	0.31?	0.36?
0.22?	0.27?	0.32?	0.37?
0.23?	0.28?	0.33?	0.38?
0.24?	0.29?	0.34?	0.39?
0.25?	0.30?	0.35?	0.40?

What are the plausible values of the population proportion p based on the sample result of 0.30?

What is the maximum distance that the sample proportion is likely to be from the true population proportion due to random sampling variability? In other words, what's the *margin of error*?

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II. Inference for experiments *Do subliminal messages work?*

A "subliminal" message is below our threshold of awareness but may nonetheless influence us. Can subliminal messages help students learn math? A group of 18 students who had failed the mathematics part of the City University of New York Skills Assessment Test agreed to participate in a study to find out. All received a daily subliminal message, flashed on a screen too rapidly to be consciously read. The treatment group of 10 students (assigned at random) was exposed to "Each day I am getting better in math." The control group of 8 students was exposed to a neutral message, "People are walking on the street." All 18 students participated in a summer program designed to raise their math skills, and all took the assessment test again at the end of the program. The table below gives data on the subjects' scores before and after the program.

Mathematics skills scores before and after a subliminal message

Treatment Group		Control Group	
Pre-test	Post-test	Pre-test	Post-test
18	24	18	29
18	25	24	29
21	33	20	24
18	29	18	26
18	33	24	38
20	36	22	27
23	34	15	22
23	36	19	31
21	34		
17	27		

Source: Data provided by Warren Page, New York City Technical College, from a study done by John Hudesman.

Do the data provide convincing evidence that the subliminal message about getting better at math results in larger increases in scores, on average, than the neutral message for students like these?

Data Collection: Randomized experiment with two treatments: subliminal message and neutral message

Data Analysis: Variable of interest for the subjects in each of the two groups: improvement from pre-test to post-test. This is a *quantitative variable*.

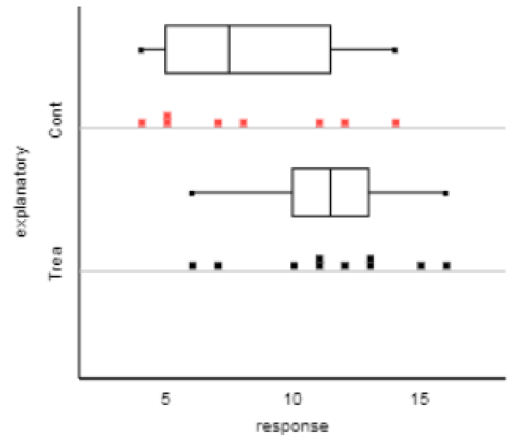
- Point your browser to www.rossmanchance.com/applets and choose the two means applet in the Statistical Inference menu.
- Click on Clear and enter the data values for each of the two groups as shown.
- When finished, click Use Data to display parallel *dotplots* along with summary statistics (*mean* and *SD*) for the two groups. Add *boxplots* by clicking the appropriate button.

Sample data:

(explanatory,response) ☑ Unstacked

```
Treatment Control
6 11
7 5
12 4
11 8
15 14
16 5
11 7
13 12
13 *
10 *
```

Use Data | Clear



☑ Boxplots

Summary Statistics:

	n	Mean	SD
Cont	8	8.25	3.69
Trea	10	11.40	3.17
pooled	18	10.00	3.41

Compare the improvement scores for the two groups. Does it appear that students who received subliminal messages showed more improvement than students who received neutral messages? Explain.

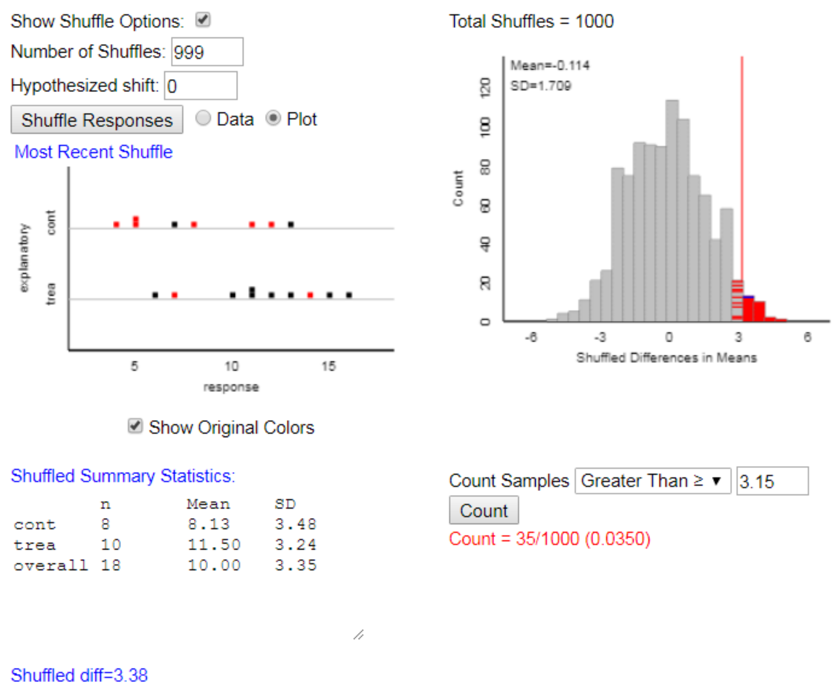
Inference: Do these data give convincing evidence that subliminal messages result in more improvement than neutral messages for subjects like the ones in this study? Or is it plausible (believable) that there's really no beneficial effect of subliminal messages, and that random chance alone produced the observed differences between these two groups?

Simulation of the random assignment assuming no treatment effect:

- If no treatment effect, then increase in test scores value for each subject will be the same as in the original study regardless of whether the subject receives subliminal or neutral message.
- How large a difference in group means with different random assignments?
- Mix the increase in test score values and deal into two groups—one with 10 subjects (subliminal message) and one with 8 subjects (neutral message).
- Calculate the difference (subliminal – neutral) in mean test score increase for the two groups.
- Perform many trials.

Using the applet,

- Click on Show Shuffle Options.
- Choose the Plot button and then click Shuffle Responses. What happens?
- Now add several more shuffles so that you have at least 1000 in all.
- Have the applet count the number of simulated random assignments that result in a difference (Treatment – Control) in means greater than or equal to 3.15.



If there is really no treatment effect, the probability of getting a difference (Treatment – Control) in group means of 3.15 or larger purely due to the chance involved in the random assignment is about _______.

Which is the more believable explanation?

(1) There is no effect of subliminal messages on test score improvement, and the observed difference in group means occurred purely due to the chance involved in the random assignment

(2) Subliminal messages cause a greater increase in average test score improvement than neutral messages for subjects like the ones in this study.

Scope of inference

		Were subjects randomly selected?	
		YES	**NO**
Were subjects randomly assigned?	**YES**	The researcher: —may infer cause and effect; *and* —may generalize findings to the population.	The researcher: —may infer cause and effect; *but* —may *not* generalize findings to the population.
	NO	The researcher: —may *not* infer cause and effect; *but* —may generalize findings to the population.	The researcher: —may *not* infer cause and effect; *and* —may *not* generalize findings to the population.