Advancing Algebraic Thinking in Your Classroom
K-5

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WHERE DISCOVERIES BEGIN

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A Little Background…

Focus Questions

- What do we mean by “early algebra”, or algebra in the elementary grades?
- How can we develop students’ algebraic understandings?
- What role does Functional Thinking play in the advancement of my students’ algebraic understanding?

What is Early Algebra?

It’s NOT:

- Pre-algebra
- Algebra Early

What is early algebra?-Big Ideas

- **Generalized Arithmetic** (understanding structure and properties)
- **Expressions, Equations, Equality and Inequality (EEEI)**(developing a relational understanding of the equal sign and operations)
- **Functional Thinking**(generalizing relationships between co-varying quantities)
- **Variable** (using symbols to express generality)

What is early algebra?- Four Key Math Practices

- **Generalizing mathematical relationships**
  - Finding a "function rule"
- **Representing generalizations**
  - Representing a rule in words or letters (variable notation)
- **Justifying generalizations**
  - Using the problem context to justify the nature of a rule
- **Reasoning with generalizations**
  - Using a rule as an object to reason about a novel situation

- Take a few minutes to solve the task
- What do you see in this task that you would describe as “algebra” or “algebraic thinking”?

GROWING TRAIN PROBLEM

There was a train that ran the same route everyday. As it went along, it picked up two train cars at each stop.

1. How many train cars did it have at stop 1? How many train cars did it have at stop 2? How many train cars did it have at stop 3?

2. Organize your information. Write an equation that shows the relationship between the values in your table for each set of values.

3. Find a relationship between the number of stops and the total number of cars on the train. Represent your rule in words and letters (variable notation).

4. If we count the engine, how would this affect your function table? How would this affect your rule?

- How would students in your class solve this task?
- What knowledge do you wish they would bring to this task…
  - about functional relationships?
  - about representing relationships with variables?
  - about reasoning with relationships?

One Student's Thinking-Growing Train

REAGAN CLIP

Representing unknown quantities

- In function tables
- in natural language
- with variables

Variables

- as a varying unknown
- understanding what the variable notation represents – quantity, not object
- using different letters to represent different quantities
- choice of letter and why they might choose it

Noticing relationships between co-varying quantities

- Recursive pattern – "add 2 more every time"
- Co-variation: "If I make one more stop, the number of cars increases by two"
- Functional relationships

Reasoning with relationships

- Predict far function values
- Changing the problem and using the original relationship to find a new relationship

How do we Nurture Students' Functional Thinking?

- Introduce the task with a real-life scenario or story - (manipulatives may also help) - make the task meaningful. Problems are always understood better when students can relate to them.
- Start Simple. Diving in to algebraic/functional thinking requires student knowledge and understanding to be built over time.

Our Instructional Sequence

- Grades K-2
- 8 weeks
- Two 30-45 minute lessons each week (16 lessons total)

Our instructional sequence

Week 1: Repeating and growing geometric patterns, with an introduction to making the independent variable explicit

Example: (Portion of Growing Pattern Task)

***Follow-Up Partner Activity*:**

- Draw the next few figures. Label the position numbers.
- What is different about this pattern?
- How would you describe how to make this pattern to a friend? What is the "rule" to build this pattern?
- If a figure has 18 tiles in it, what is the position number? How do you know?

Some of the key features of this task…

- Identifying "near" and "far" figures
- Connecting the figure (position) number to the figure
- Understanding (and quantifying) how successive figures differed
- Understanding a general rule that could be used to build any figure
- Reversibility - given a dependent value (number of tiles in a given figure), find the independent value (position number)

Weeks 2-4:

Functions of the form $y = mx$ (represented as additive; for example $y = x + x$)

Weeks 5-8:

Functions of the form $y = x + b$

Task Focus	Functional Relationship
Pennies in a Jar – If Sara gets one penny each day from her Grandmother, find a relationship between the number of pennies and the number of days.	$y = x$
Attached Tables – If Brady can seat two people at each square desk joined end-to-end, find a relationship between the number of desks and the number of people who can be seated.	$y = 2x$ $y = x + x$
People and Ears – Find a relationship between the number of people and the total number of ears.	$y = 2x$ $y = x + x$
Relationships of Two – Identify a quantity that occurs in twos (e.g. number of eyes) and find a relationships between that quantity and the number of people for which the attribute is measured.	$y = 2x$ $y = x + x$
Dogs and Legs – Find a relationship between the number of dogs and the total number of dog legs.	$y = 4x$ $y = x + x + x + x$
Find the Rule– Using information from the previous tasks, students played a game where, in groups, they matched different representations (rule in words, rule in variables, function table) of the functional relationship.	Functions of the form $y = mx$

Task Focus	Functional Relationship
Cutting String Given a piece of string, find a relationship between the number of cuts of the string and the resulting number of pieces of string	$y = x$
Candy Boxes John and Mary each have a box of candies with the same number of candies. Mary has 1 more candies on top of her box. Find a relationship between the number of candies John and Mary have.	$y = x + 1$
Jar of Pennies Sarah has a jar of pennies. She finds 3 more pennies. Find a relationship that represents how the number of pennies Sarah had before she found the pennies relates to the number of pennies she had after.	$y = x + 3$
Age Difference Keisha and Janice are sisters. Keisha is 2 yrs older than Janice. Find a relationship that represents how Keisha's age is related to Janice's age	$y = x + 2$
Height with a Hat Give a hat that is 1-foot in height, find a relationship that represents how a person's height without the hat is related to their height with a hat. How does the relationship change if the hat is 2 feet tall?	$y = x + 1$; $y = x + 2$
(Review)Growing Caterpillar A caterpillar grows by adding 2 circular body parts each day. If we don't count the caterpillar's head, find a relationship between the size of the caterpillar (number of body parts) and the number of days. How does your rule change if we count the head?	$y = x + x$ $y = x + x + 1$
Find Your Rule – Using information from the previous tasks, students played a game where, in groups, they matched different representations (rule in words, rule in variables, function table) of the functional relationship.	Combining $y = mx$, $y = x + b$, and $y = mx + b$

Our Instructional Sequence (3-5)

- 7 -9 Functional Thinking Tasks/Lessons in grades 3, 4 and 5 (once a week)
- 45-60 minute lessons

Type of Function	Problem
Linear Function with one operation y=mx	Growing Circle Problem
Linear Function with two operations y= mx +b	String Problem
Quadratic Functions $y= a^x +c$	Trapezoid Problem
Exponential Functions $y = a^x$	Paper Folding

How do we Nurture Students' Functional Thinking?

- Teach students how to organize data- Organizing data is one of the most important aspects of functional thinking and teaching students to organize their data and their thinking will help in every area of their math understanding.
- This takes teaching, students don't naturally organize.

Function Table

How do we Nurture Students' Functional Thinking?

- Quality Questioning
- Discussion, Discussion, and MORE Discussion - teachers have to be a captive audience for their students. Show them you are interested in what they have to say!
- Opportunities to write about their thinking - students' written productions provide us with additional evidence about their thinking, and also make explicit for themselves the generalizations they are making, allowing them to begin to think about them in a different way.

Questions that Promote Generalizing and Justifying

Questions that keep the conversation going:

- Can you tell me more about that idea?
- Talk me through your thinking…
- How do you know?
- What do you think is happening? Take a moment to share your idea with a buddy and listen to their thoughts.

The Classroom (continued)

Mathematical Questions that Support Functional Thinking:

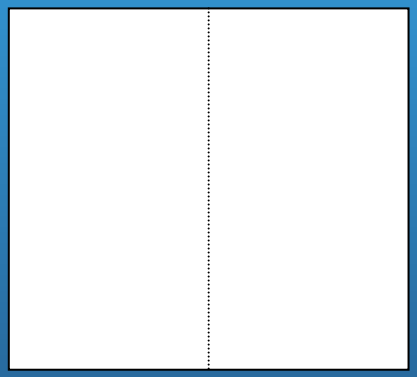
- How can we organize our data?
- What patterns do you see?
- Do you see a relationship between x and y?
- Can you describe your relationship using a number sentence?
- Can you describe your relationship using variable notation?

3 Keys for Successful Classroom Discussion:

1. Students should have the opportunity to discuss ideas with their teachers and their peers.
2. Students should be given the opportunity to write or draw their ideas as a way of explaining the mathematical relationships they notice.
3. At the end of each lesson, it is important that the teacher and students come together to discuss their findings.

Let's Try a Problem

Take a piece of paper and fold it in half to create two regions (see figure).



1 fold yields 2 regions

Take the folded paper and fold it in half again. How many regions does this create in the piece of paper?
Continue this, folding the paper in half a third time. How many regions does this third fold create? What if you fold the paper in half a fourth time?

Thank You!

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Hungry for more?

How Do We Prepare Students for Algebra? Designing Tasks That Build Students' Algebraic Thinking

11:00 AM

Walter E. Washington Convention Center-152A

Speaker: Maria Blanton