Ratios with Cartons and Beans, Counters and Cups, Tables, Tape Diagrams, and Double Number Lines

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American Institutes for Research

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The Center for English Language Learners (ELL Center) at AIR is committed to improving instruction and outcomes for ELLs by conducting relevant research and applying what we know about what works for ELLs in schools and districts across the country.

Our services include conducting rigorous studies of instructional interventions and evaluating district programs; evaluating federal, state, and district policies that affect ELLs and crafting evidence-based recommendations for policymakers; and providing technical assistance and professional development to help schools and districts improve instruction and learning for ELLs.

Project MELD

- Math and English Language Development
- IES-funded, three-year study to enhance sixth grade, Spanish-speaking ELs' math knowledge and academic language associated with math

Presentation Objectives

By the end of the presentation, participants will be able to:

- model ratios and rate situations with manipulatives in two different ways;
- relate the manipulative models to ratio tables, tape diagrams , and double number lines; and
- compare and contrast the different representations.

Project MELD – Language Scaffolds

- Oral and written English language is integrated with academic content
- Adapted LearnZillion curriculum materials
- Vocabulary – lesson glossaries and language lessons
- Slide-by-slide notes and activities with scaffolded problems
- Partner talk, response frames
- Graphic organizers

Project MELD – Develop Conceptual Knowledge

- Many ELs many need additional time to develop their conceptual knowledge in mathematics.
- Hands-on activities facilitate conceptual development while not requiring a high level of academic language.

Physical Models for Ratios and Rates

Useful Representations

- tables
- tape diagrams
- double number lines

For ratios between "like" quantities there is a set of associated ratios.
"Like" here means the quantities can be combined in an additive way.

A to B, B to A	Part to Part
A to Total, B to Total	Part to Total (whole)
Total to A, Total to B	Total to Part

Associated Ratios – Lots of Different Relationships

There are 3 apples and 4 oranges. How many pieces of fruit?____

The ratio of apples to oranges is _____ to _____.

The ratio of oranges to apples is _____ to _____.

The ratio of apples to fruit is _____ to _____.

The ratio of oranges to fruit is _____ to _____.

The ratio of fruit to apples is _____ to _____.

The ratio of fruit to oranges is _____ to _____.

3 4 7

Associated Ratios – Lots of Different Relationships

There are 3 apples and 4 oranges. How many pieces of fruit? **7**

The ratio of apples to oranges is **3** to **4**.

The ratio of oranges to apples is **4** to **3**.

The ratio of apples to fruit is **3** to **7**.

The ratio of oranges to fruit is **4** to **7**.

The ratio of fruit to apples is **7** to **3**.

The ratio of fruit to oranges is **7** to **4**.

3 4 7

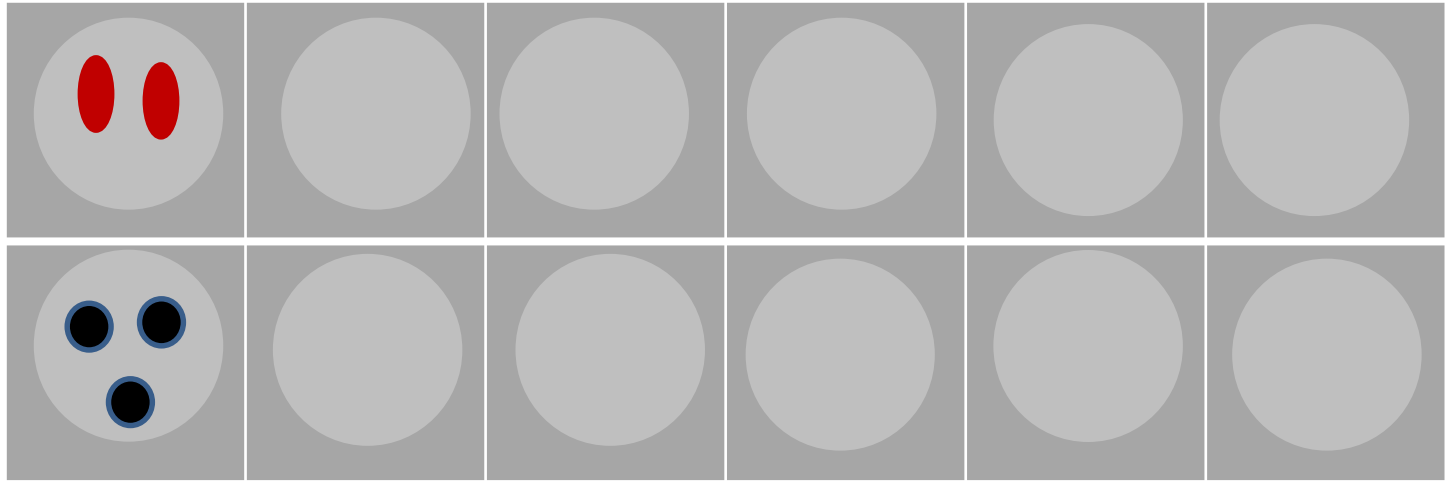
What are the associated rates for "unlike" quantities?

X : Y

Example: 21 miles in 3 hours

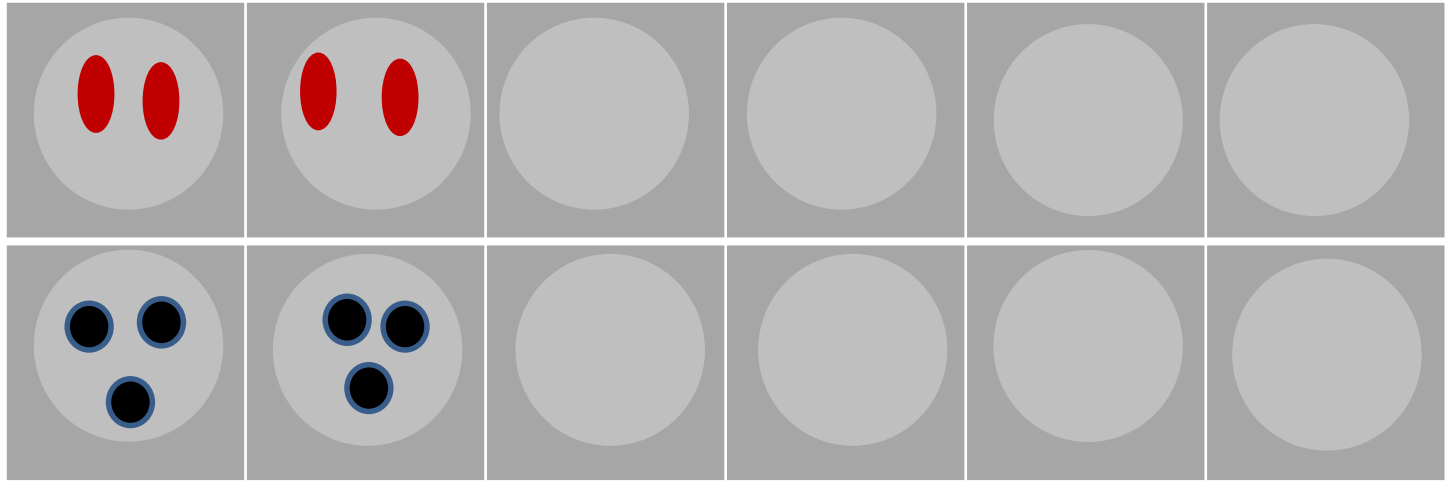
Building Ratios in Egg Cartons

Equivalent ratios can be built using a pattern.



This ratio is 2 red to 3 black.

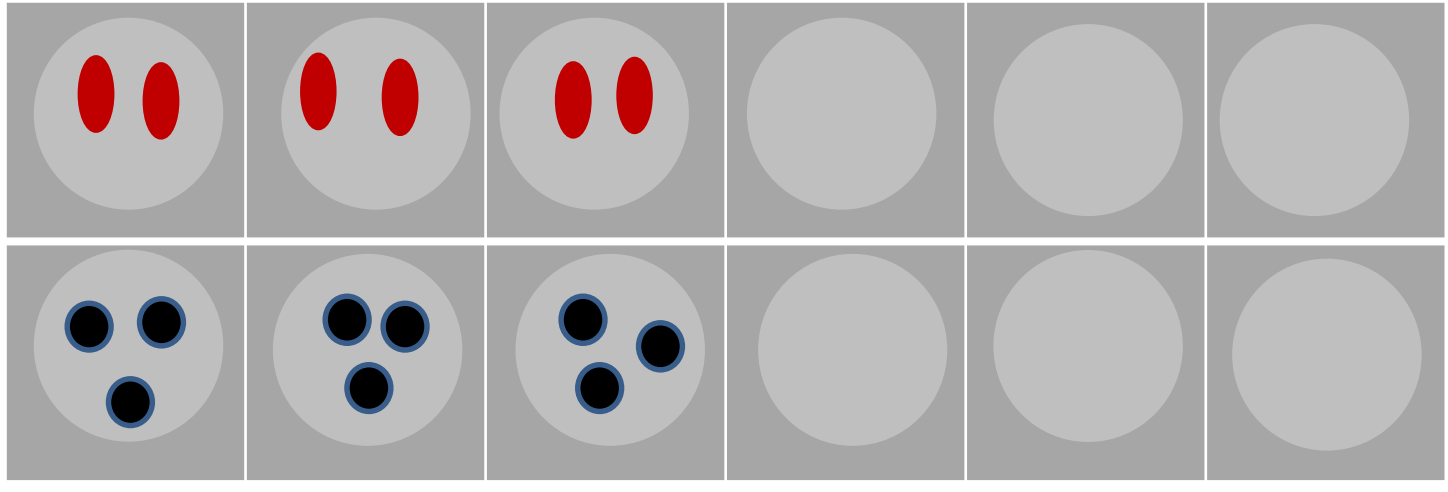
Equivalent ratios can be built using a pattern.



This ratio is equivalent to 2 red to 3 black.

If all the beans are counted it is 4 red to 6 black.

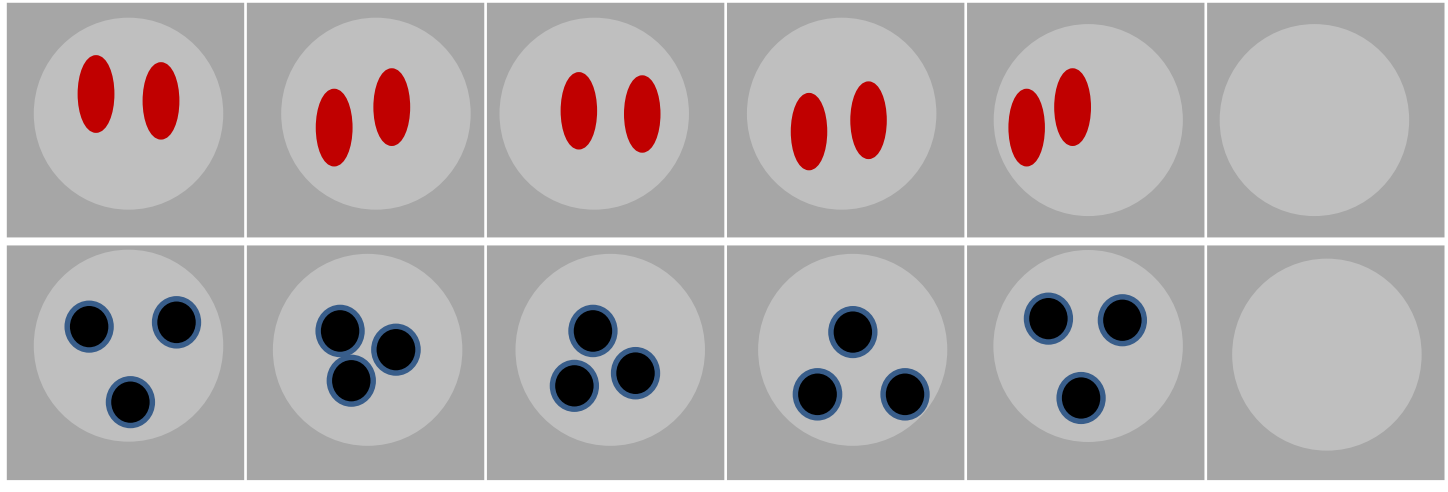
Equivalent ratios can be built using a pattern.



This ratio is equivalent to 2 red to 3 black.

If all the beans are counted the ratio is 6 red to 9 black.

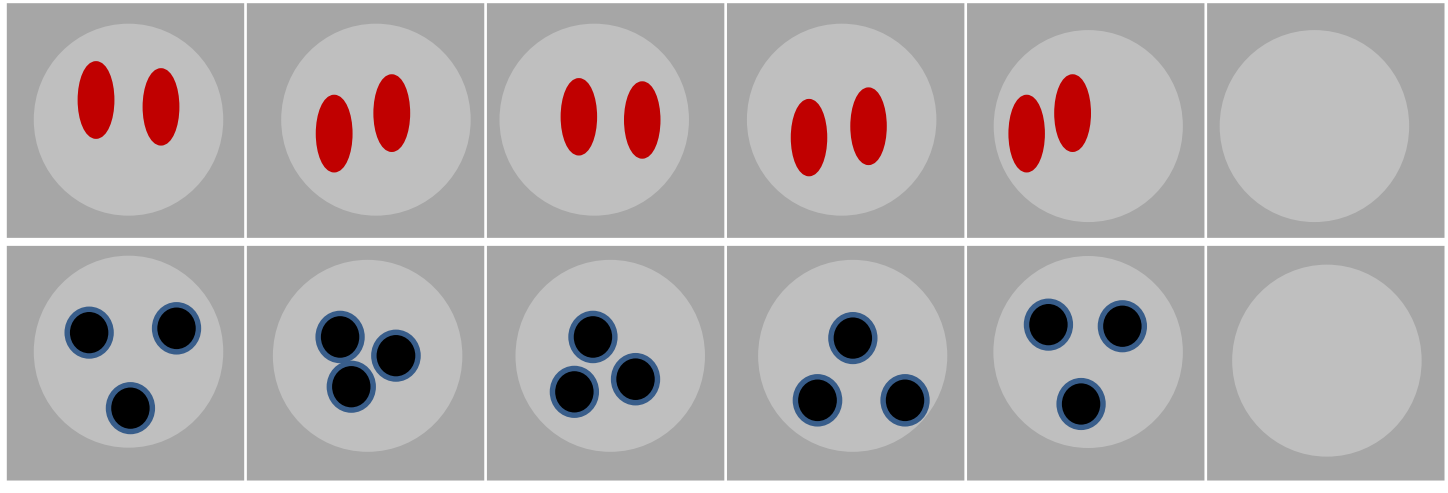
Equivalent ratios can be built using a pattern.



This ratio is equivalent to 2 red to 3 black.

What is the ratio counting all the beans?

We can keep track with a ratio table.

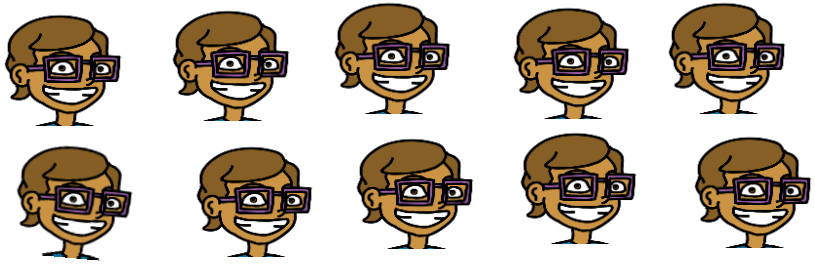


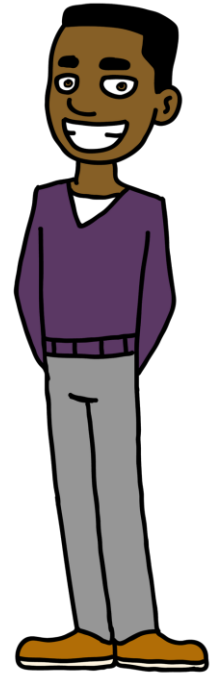
RED	2	4	6	8	10	
BLACK	3	6	9	12	15	

Mr. Nelson says the ratio of boys to girls in his class is 2 to 3.

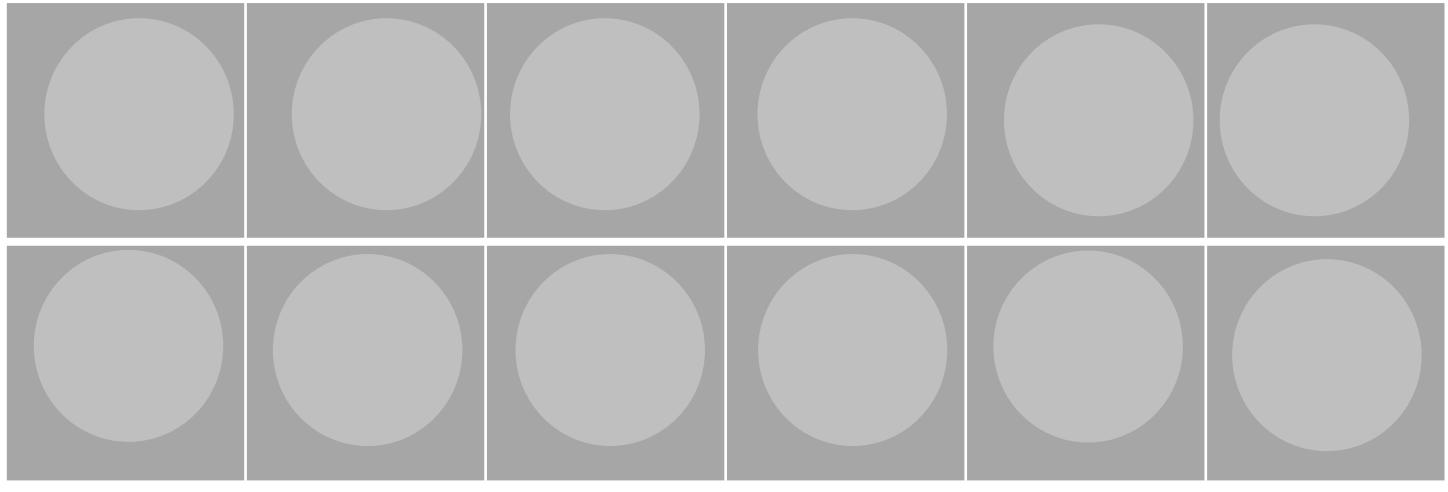
There are 10 boys in his class.

How many girls are in his class?



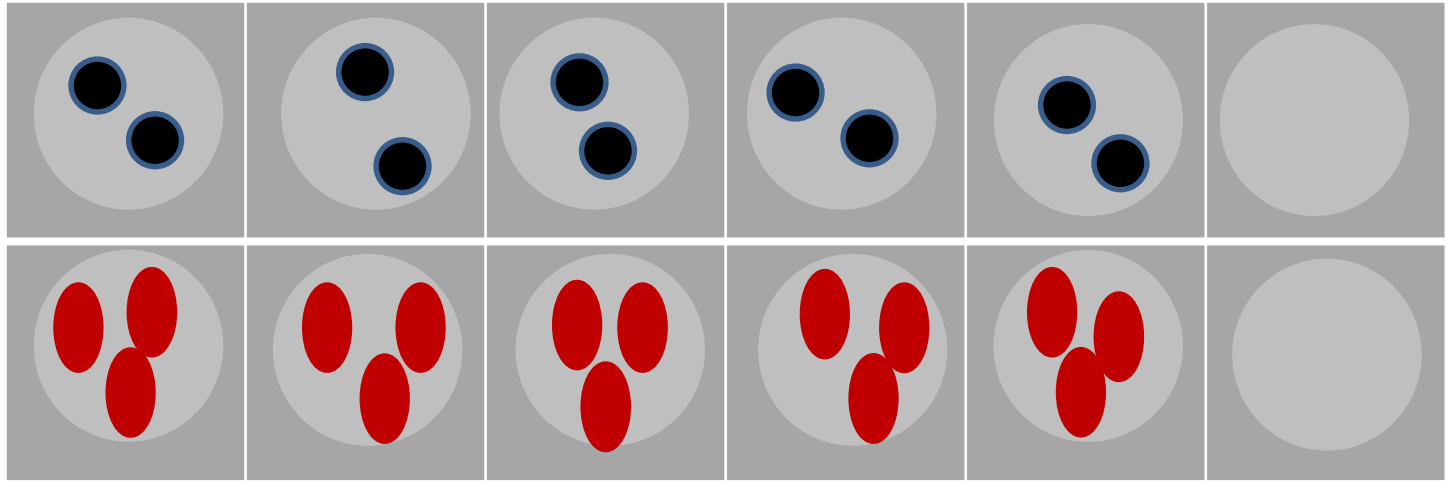


Use beans to model the problem.
Build 2 to 3 ratios until you get to 10 boys.



Use ● to represent boys. Use ⬮ to represent girls.

The equivalent ratio with 10 boys tells how many girls are in the class.

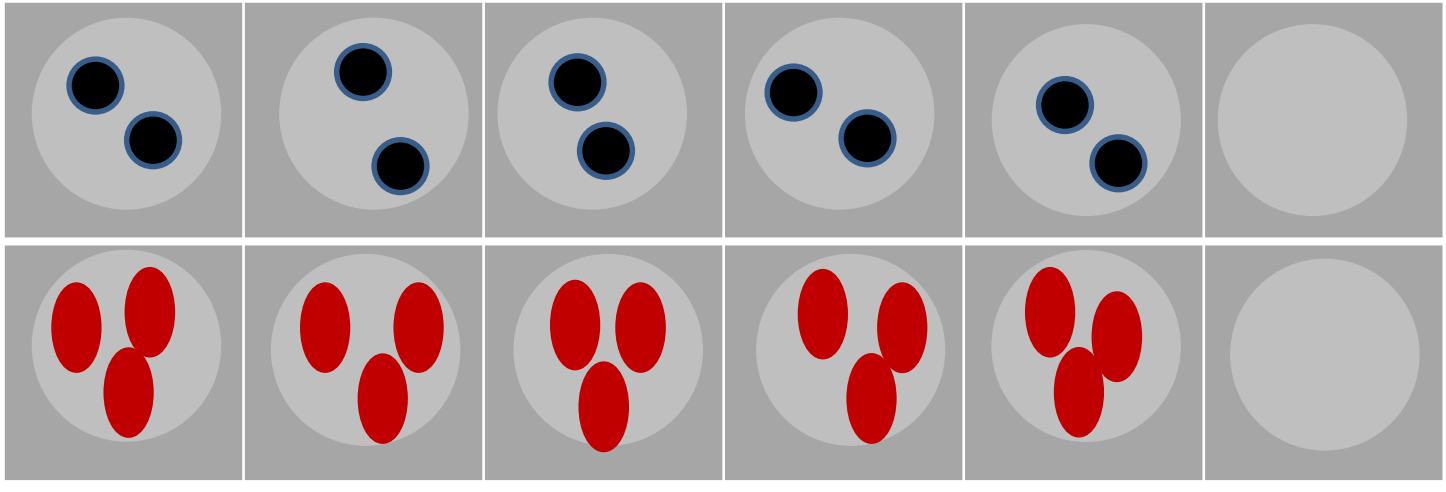


Use ● to represent boys. Use ⬮ to represent girls.

A ratio table can represent the class.

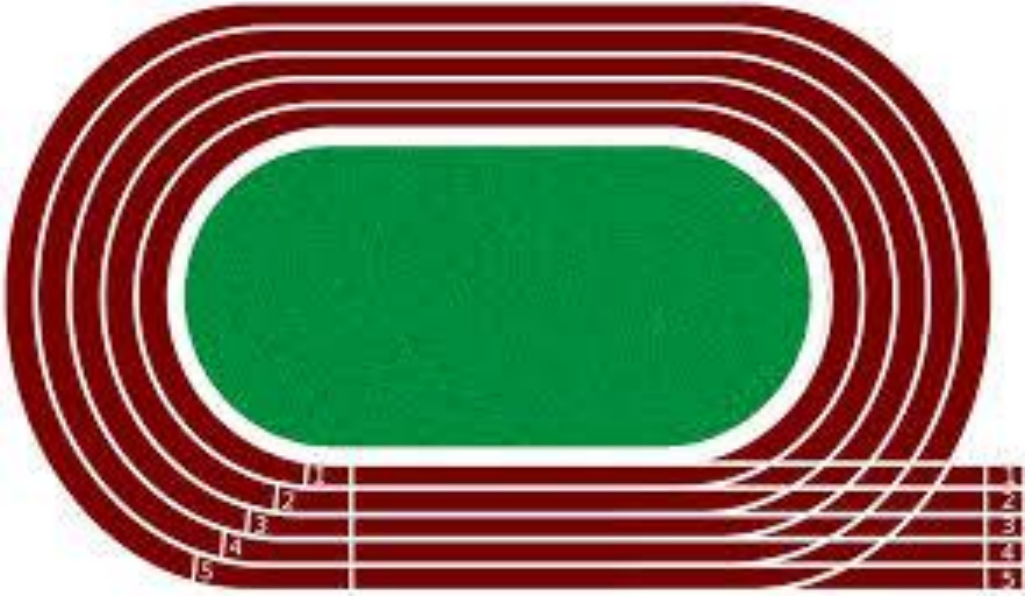
boys

girls

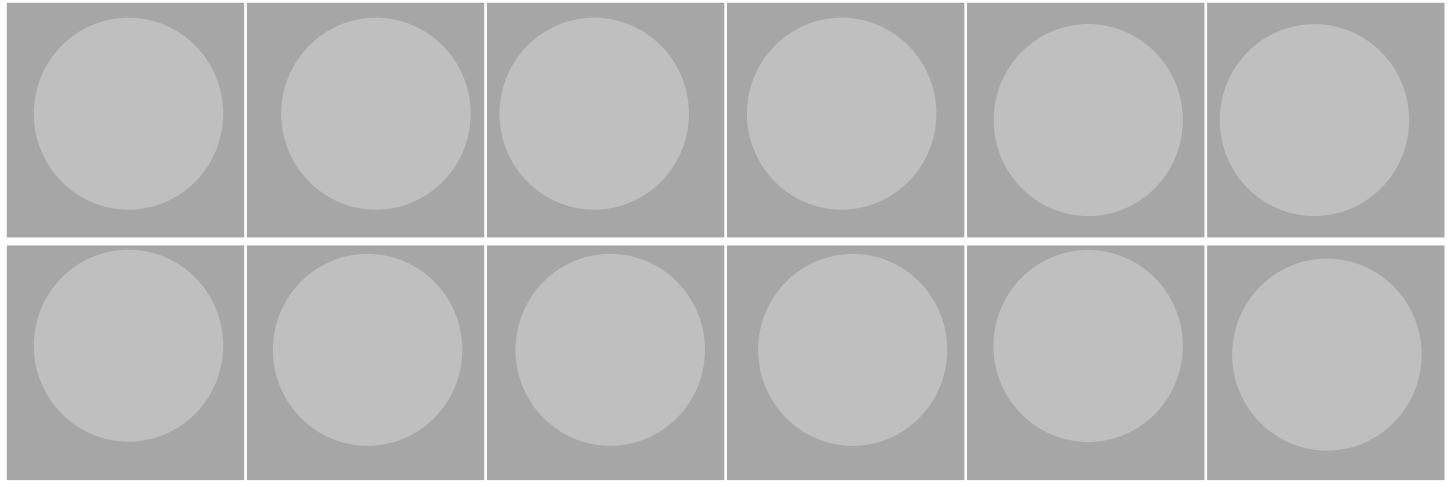


boys	2	4	6	8	10	
girls	3	6	9	12	15	

Meg can run 2 laps in 5 minutes.

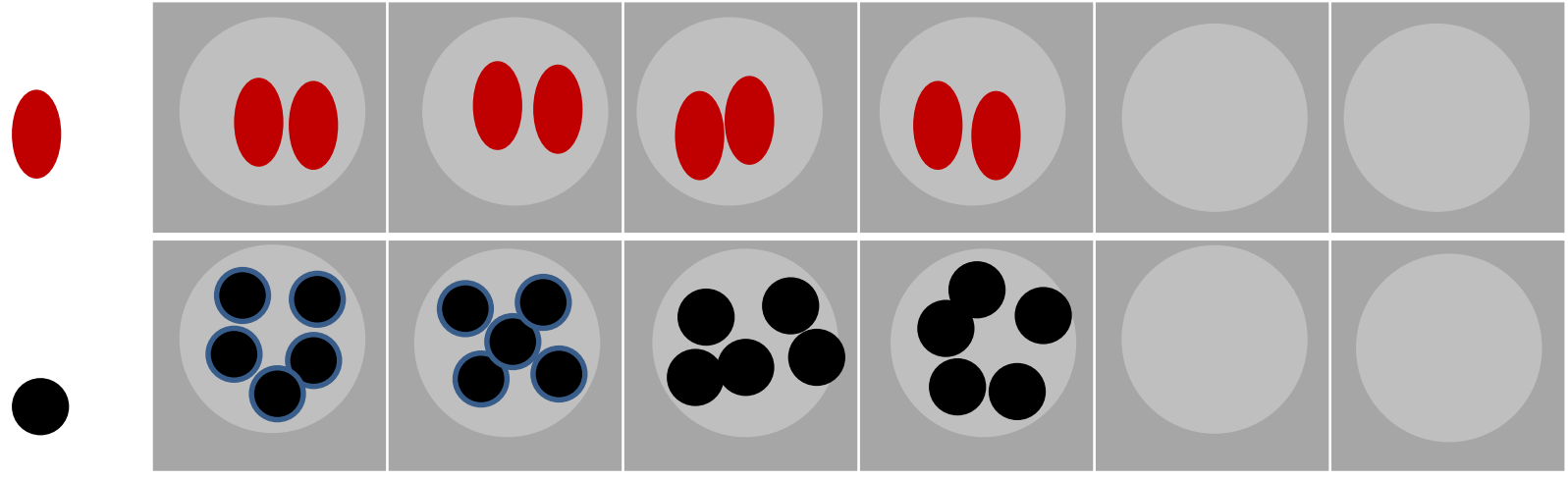


Use beans to model the problem.



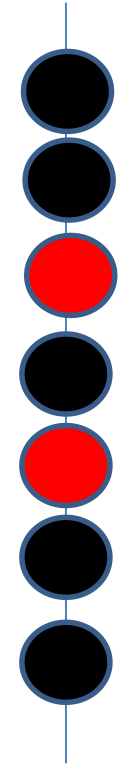
Use ● to represent minutes. Use ⬮ to represent laps.

8 laps in 20 minutes



laps	2	4	6	8	10	12
minutes	5	10	15	20	25	30

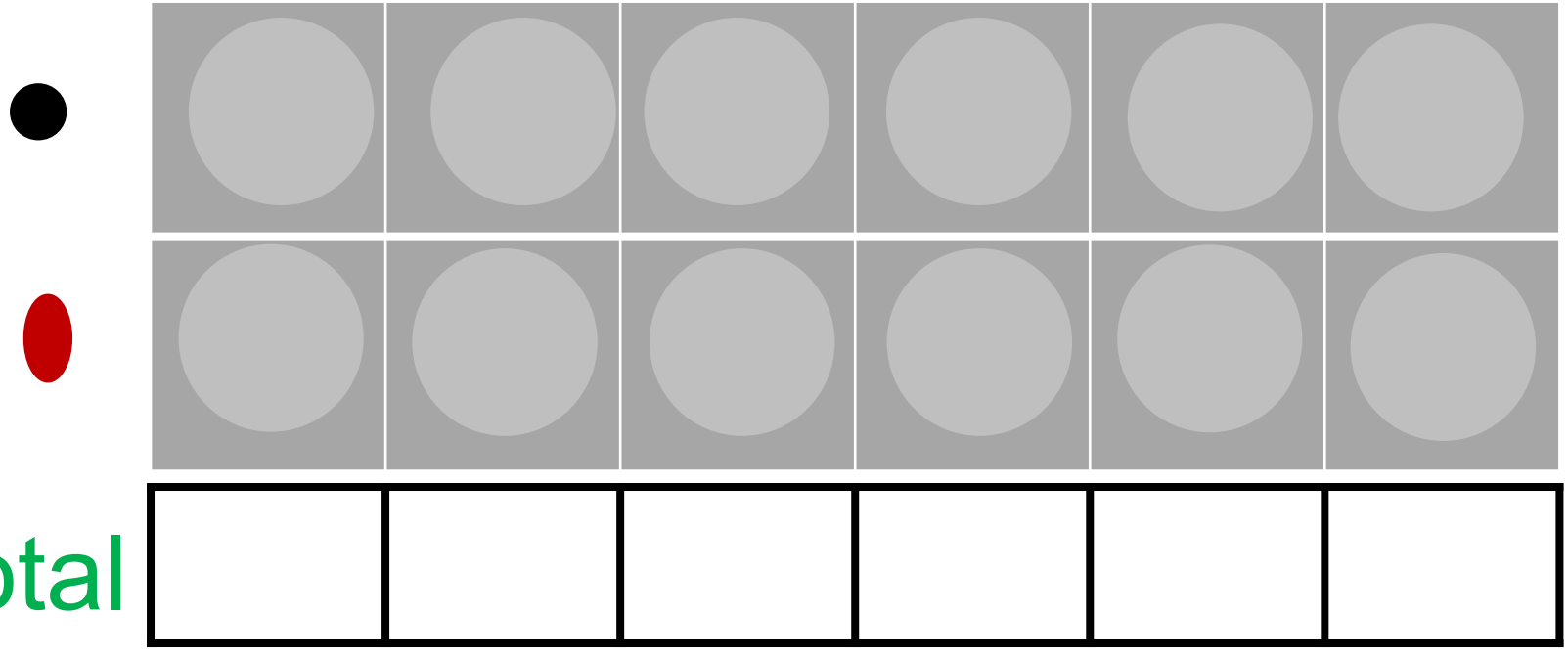
Susan makes necklaces and bracelets with black and red beads. The ratio of black beads to red beads is 5:2.



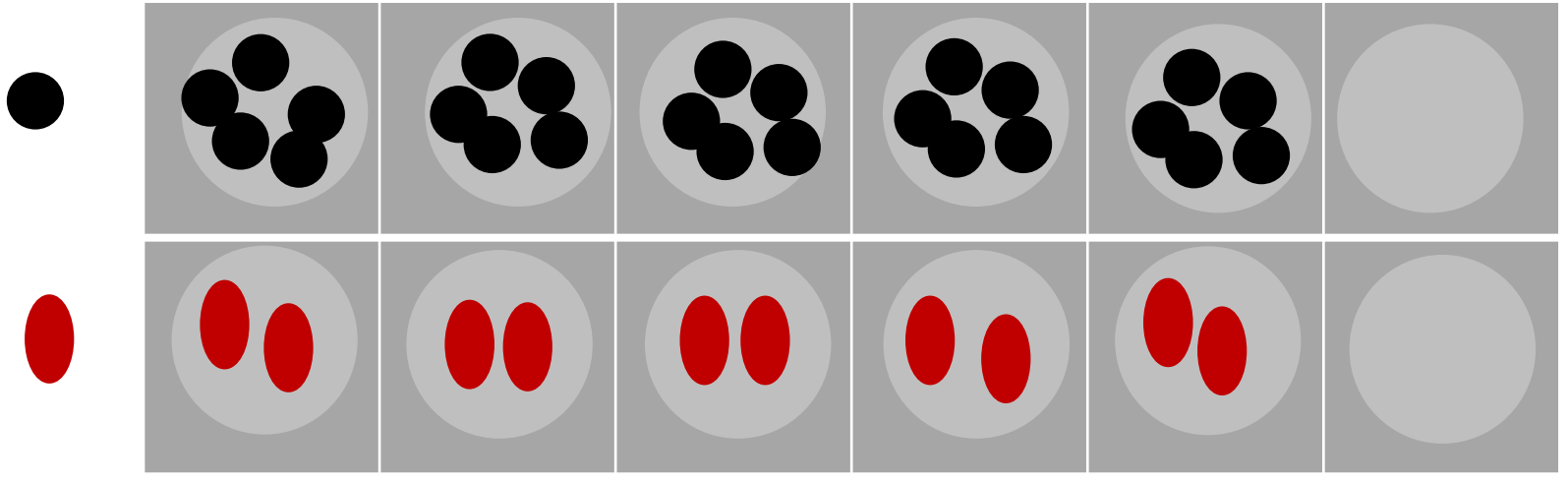

Necklaces have 35 beads.
How many of each color will I need?

The ratio of black beads to red beads is 5 to 2.
There are 35 beads in each necklace.

Use beans to model the problem.
Record the total number of beads in each group.
How many groups of beads until you get to 35?

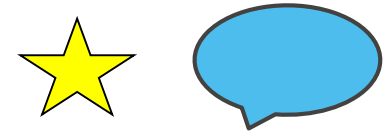


Use beans to model the problem.
Record the total number of beads in each group.
How many groups of beads until you get to 35?



Total	7	7	7	7	7	

An extended ratio table with 3 rows can be used to model the problem. The extended ratio table is started. Complete the table to answer Susan's question.

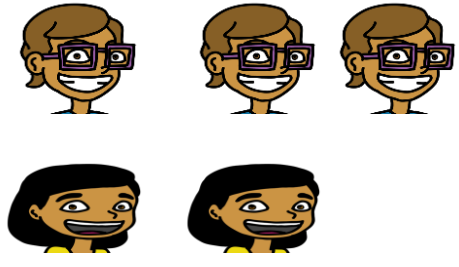


Black	5	10	15	20		
Red	2	4	6			
Total	7	14				

An extended ratio table with 3 rows can be used to model the problem. The extended ratio table is started. Complete the table to answer Susan's question.

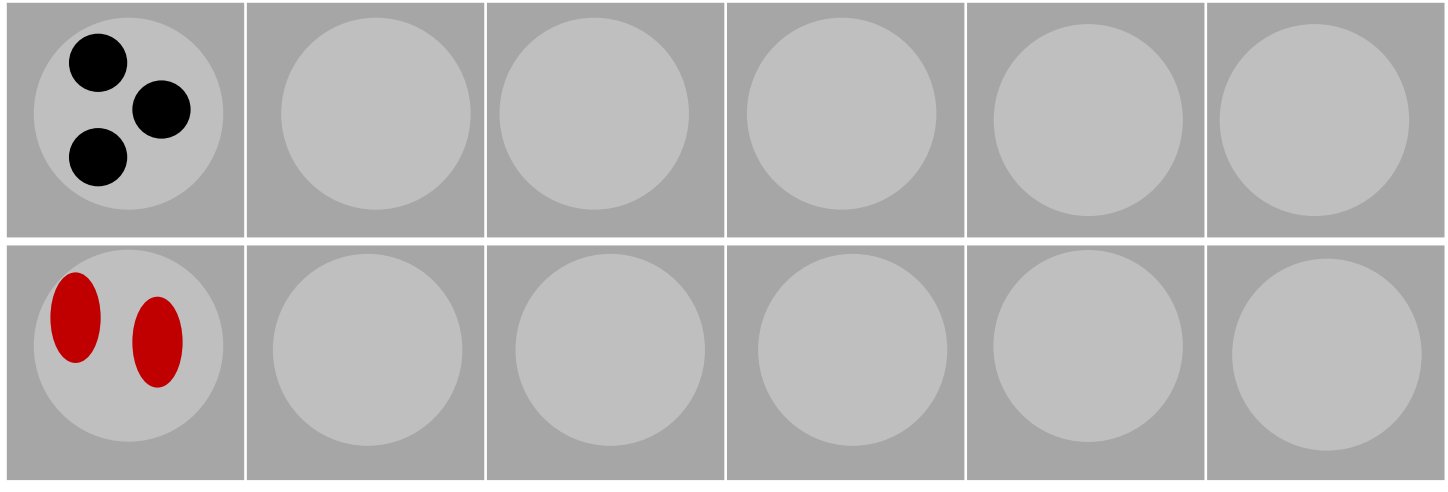
Black	5	10	15	20	25	30
Red	2	4	6	8	10	12
Total	7	14	21	28	35	42

The ratio of boys to girls in Ms. Aziz's 3rd period class is 3 to 2.
18 students are boys.
What is the total number of students in Ms. Aziz's class?



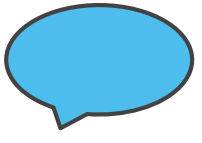
Ratio of Girls to Boys is 3:2. There are 18 boys.
How many students in the class?

Use beans to model the problem.



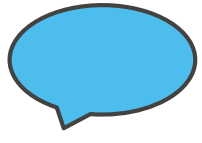
Use a ratio table.
Complete the table to find the number of students.

Boys	3					
Girls	2					
Total	5					



Use a ratio table.
Complete the table to find the number of students.

Boys	3	6	9	12	15	18
Girls	2	4	6	8	10	12
Total	5	10	15	20	25	30



Try a few of the Additional Practice problems with your partners.

What do you notice as you work?

Share your thoughts, please.

Moving to larger values.

Ashley is asked to cook omelets for 40 people.

A table counting by 2s will be way too long!

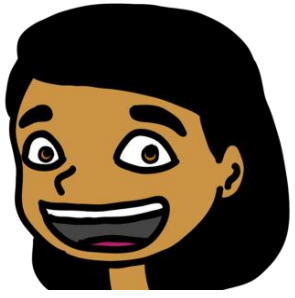
Five 2s make 10 and then I can count by 10s to 40!

eggs	5	25	50			
people	2	10	20	30		

I need 100 eggs for 40 people.

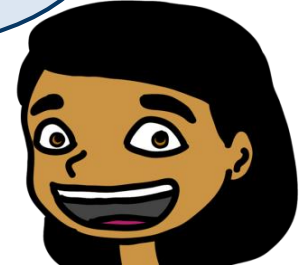
eggs	5	25	50	75	100	
people	2	10	20	30	40	

I wonder if there is a faster way?



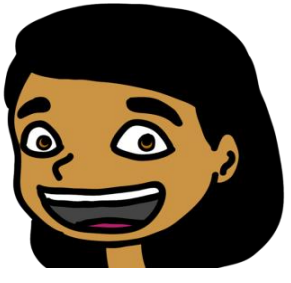
eggs	5	25	50	75	100	
people	2	10	20	30	40	

Ratios are multiplicative relationships. You can use multiplication!



I know I need 20 groups of 2 people to get to 40 people.

So you need 20 groups of 5 eggs!
20 x 5 = 100



eggs	5	100
people	2	40

Suppose I had to cook omelets for 100 people?

eggs	5	?
people	2	100

You need 50 groups of 2 people to get 100 people; so you need 50 groups of 5 eggs. 50 x 5 = 250

eggs	5	250
people	2	100

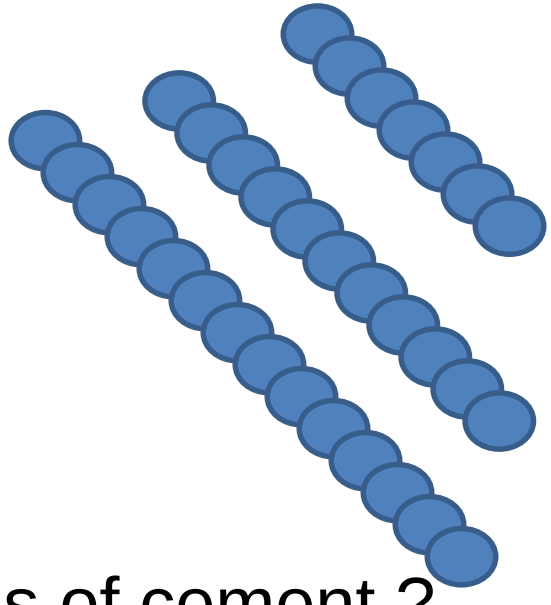
Growing Parts of Ratios

Example

Model the number of parts for each quantity.

Sand

Cement



The ratio of sand to cement is 3 : 2.

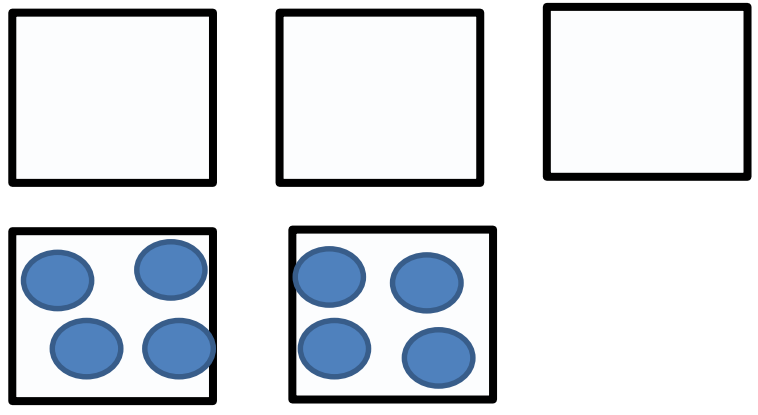
How many tons of sand are needed for 8 tons of cement ?

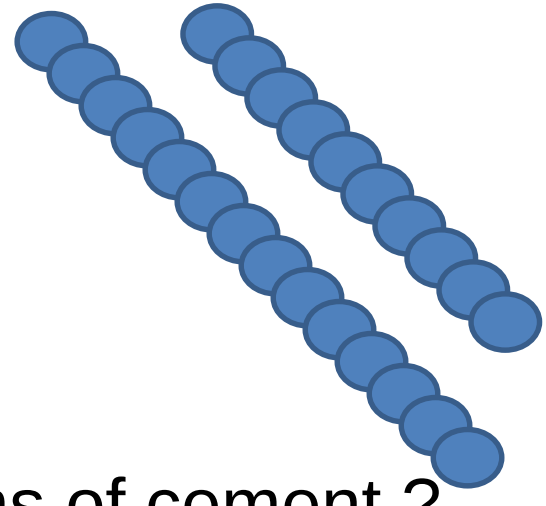
Example

Deal out known quantities to the parts.

Sand

Cement





The ratio of sand to cement is 3 : 2.

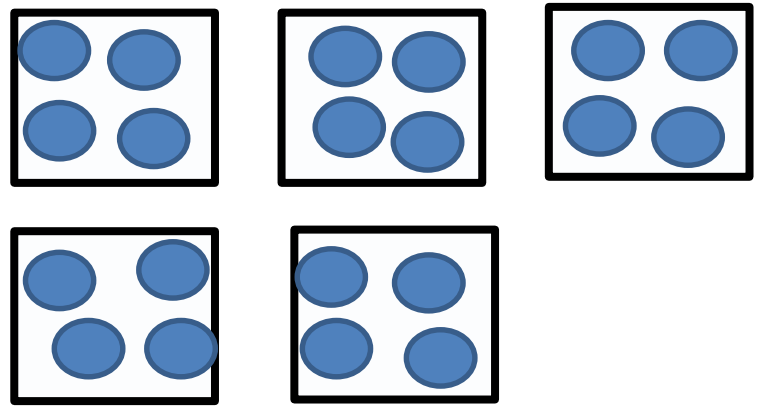
How many tons of sand are needed for 8 tons of cement ?

Example

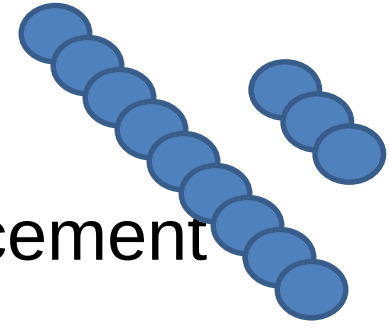
Fill the other parts with equal amounts.

Sand

Cement



The ratio of sand to cement is 3 : 2.



How many tons of sand are needed for 8 tons of cement

There are 12 counters in the sand parts so 12 tons are needed.

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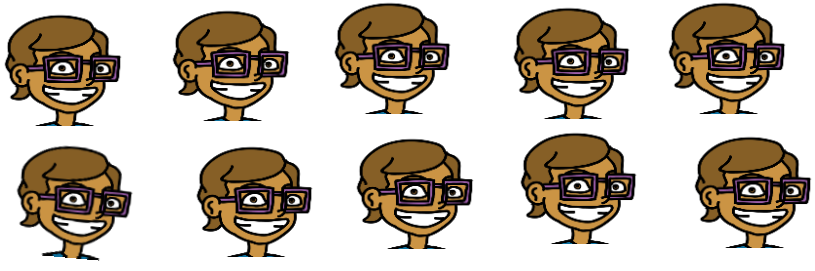
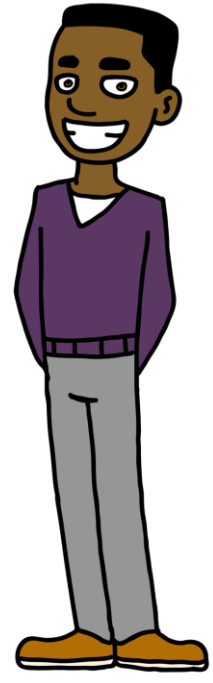
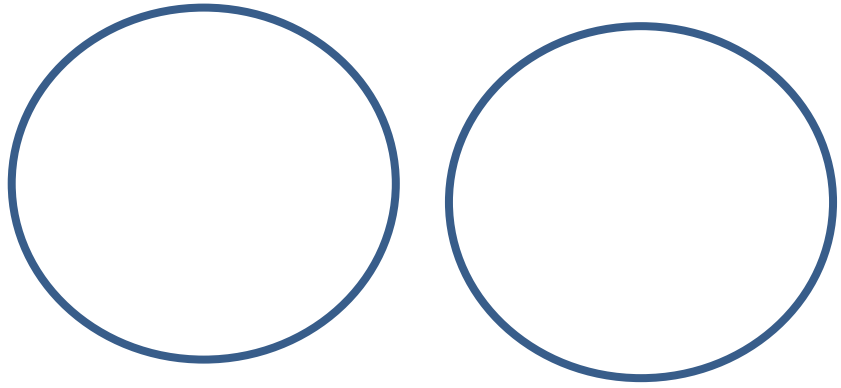
There are 10 boys in his class.

How many girls are in his class?

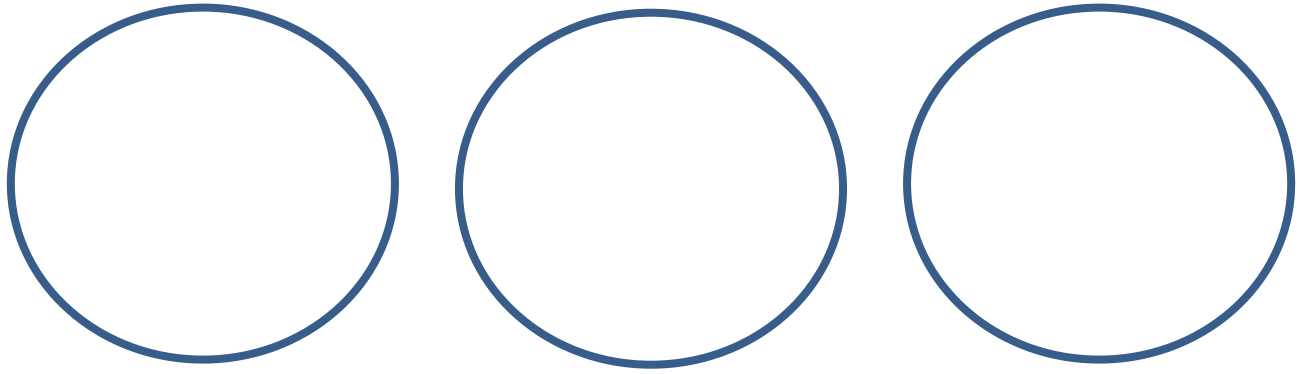




Boys



Girls



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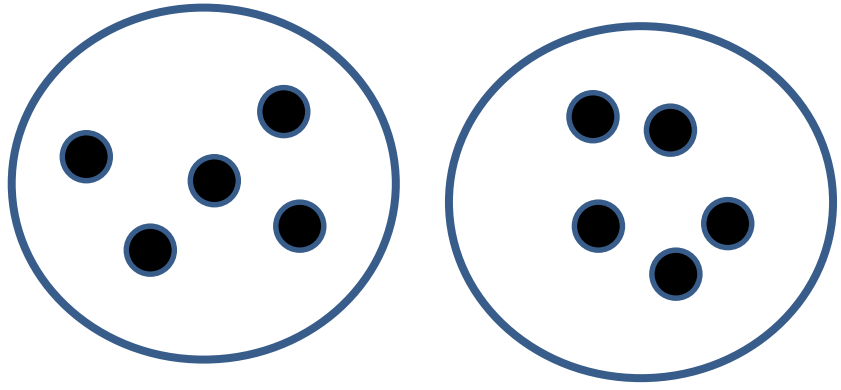
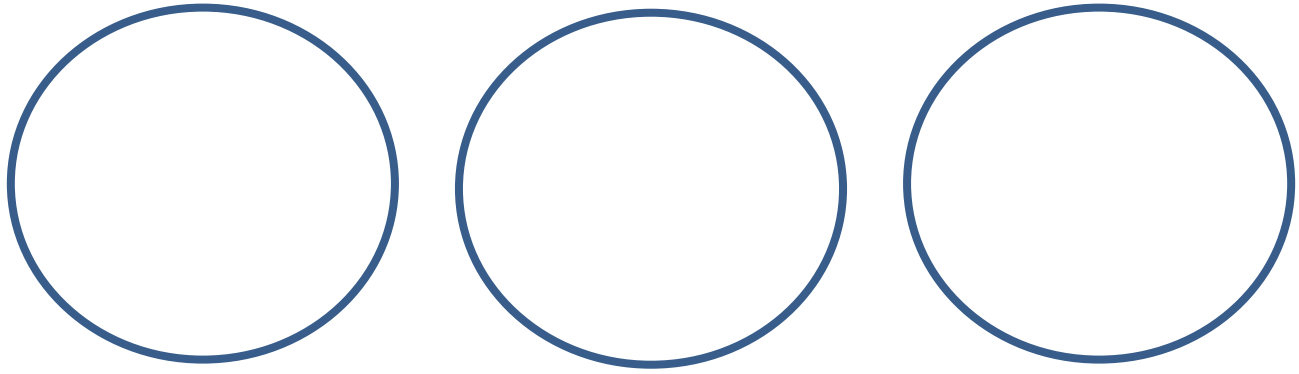
Boys



Girls



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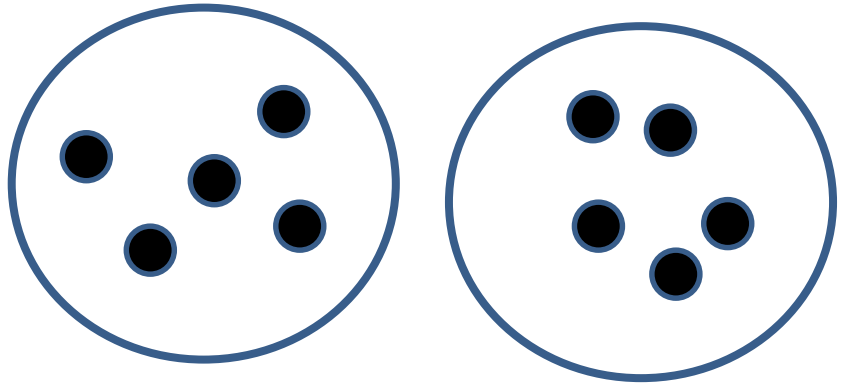
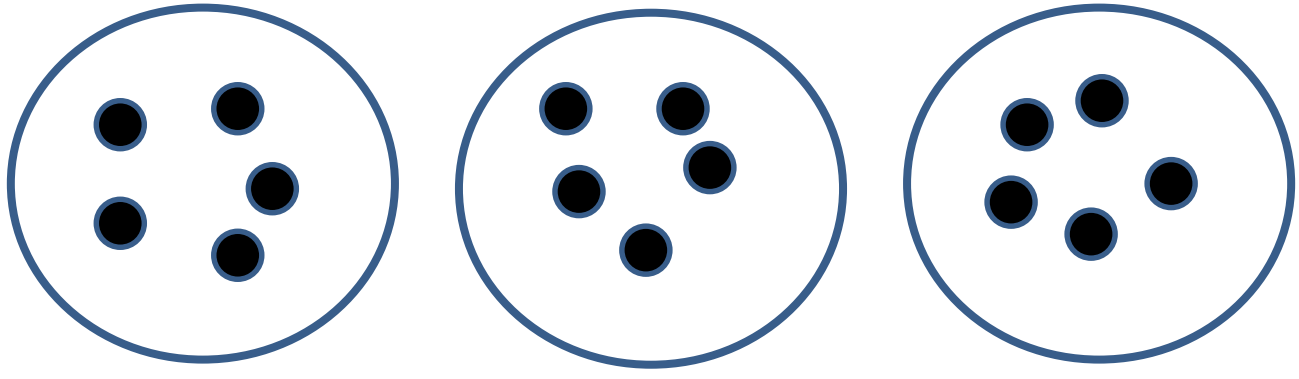
Boys



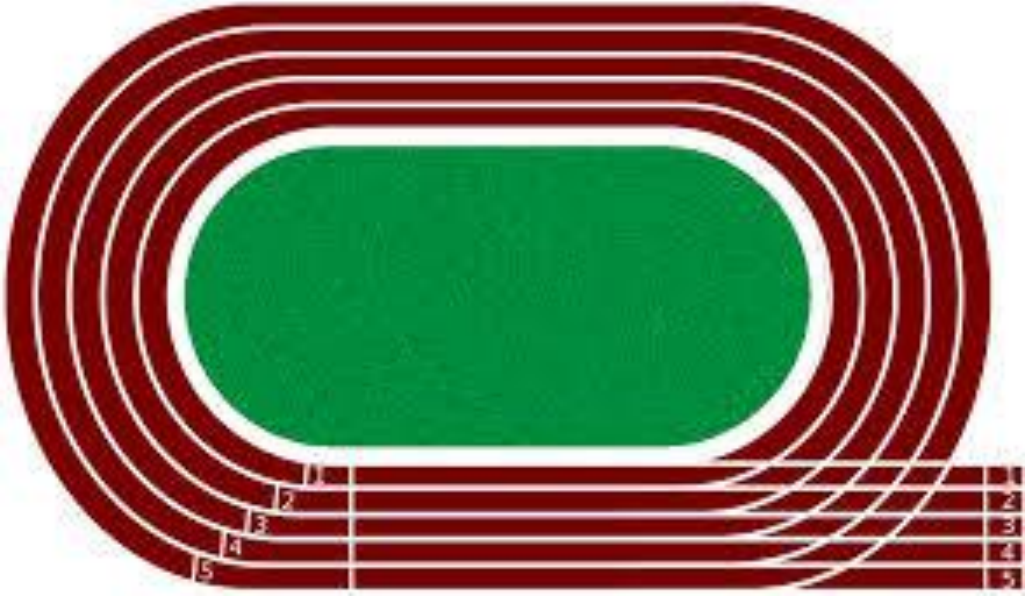
Girls



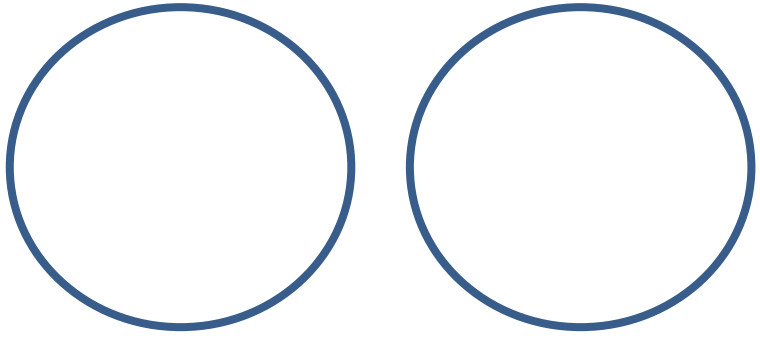
How many girls are in his class?

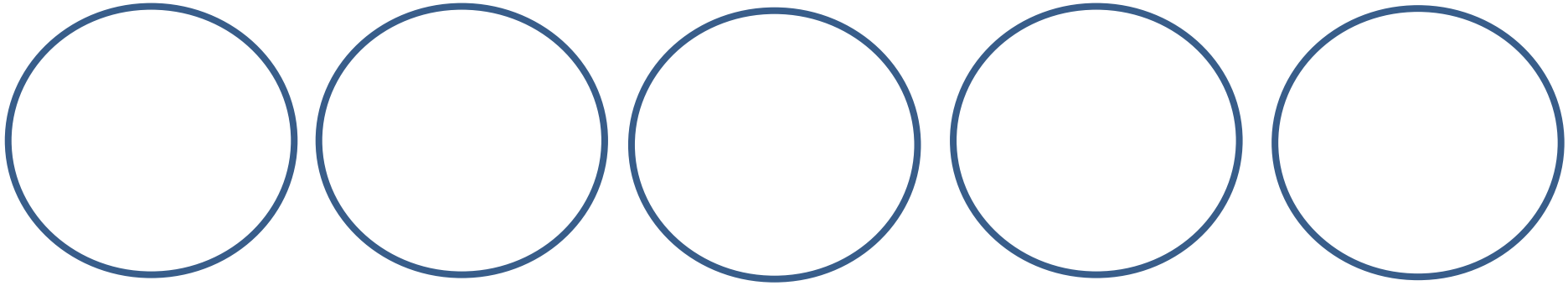
There are 15 girls in his class.

Meg can run 2 laps in 5 minutes.



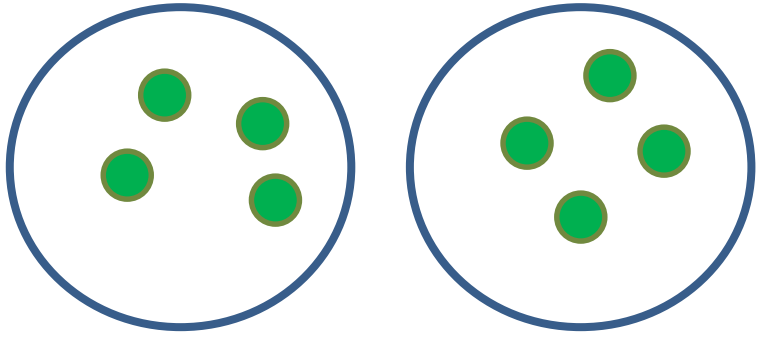
laps



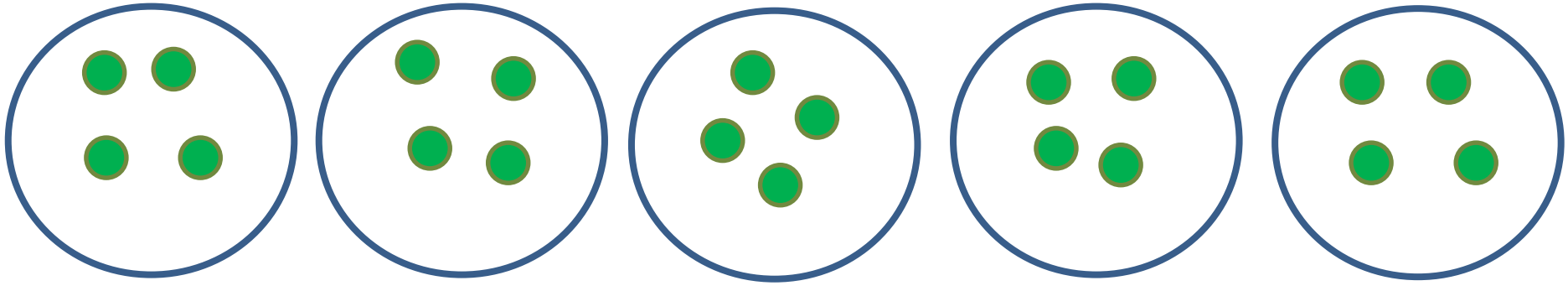


minutes

laps

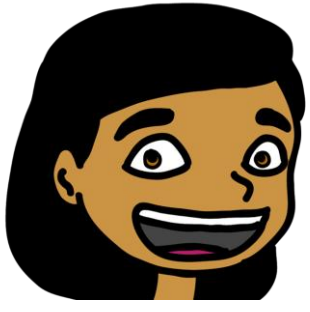


Meg can run 8 laps
in
20 minutes.



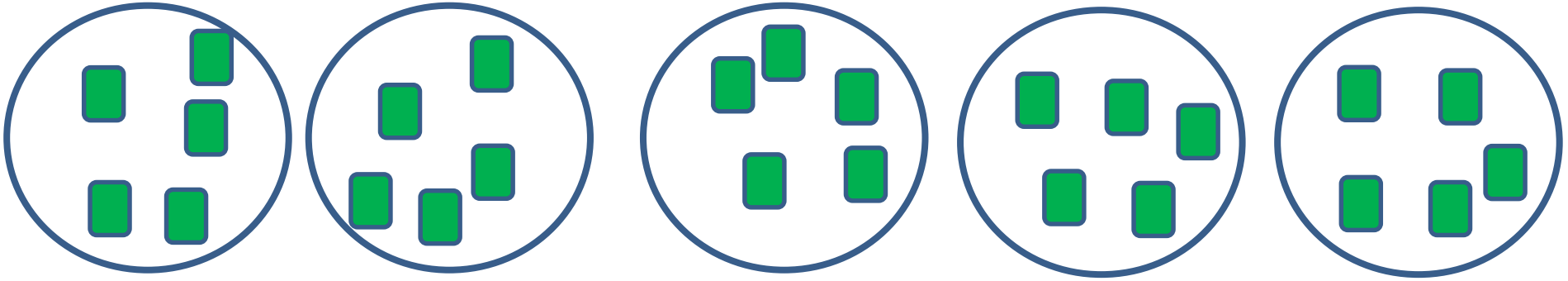
minutes

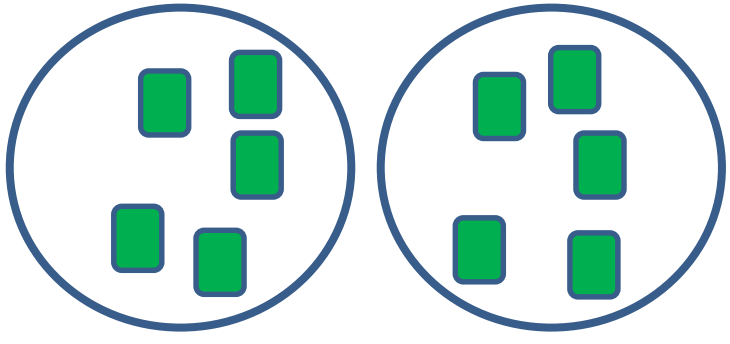
Necklaces have 35 beads.
How many of each color will I need?



The ratio of black beads to red beads is 5:2.

black beads





35 beads

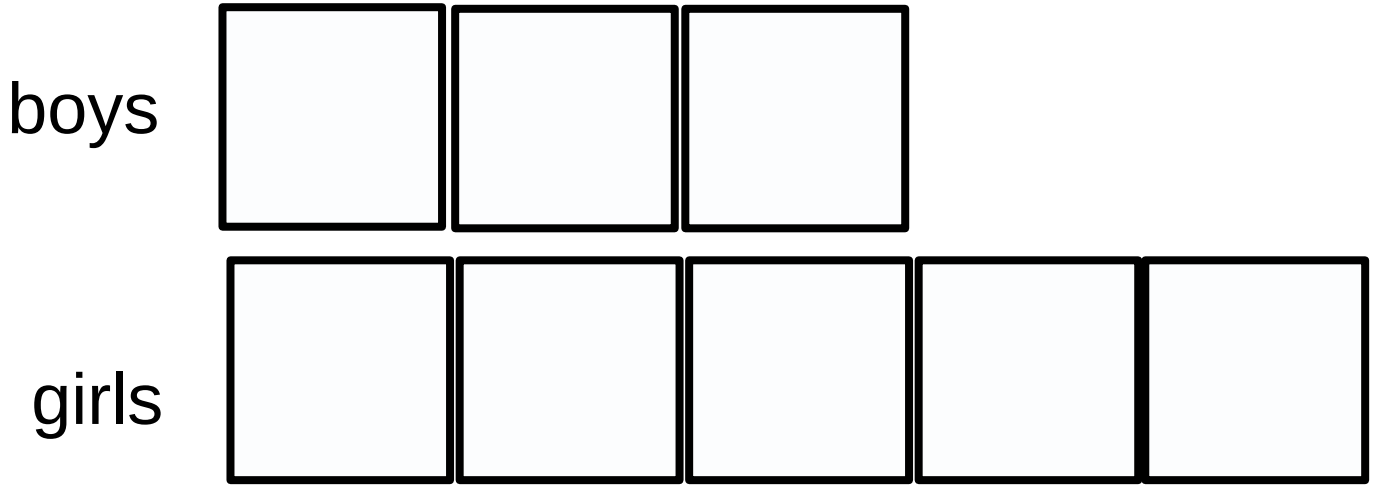
red beads

Try a few others with your partners.

What do you notice as you work?

Problem Number 1

In a class the ratio of boys to girls is 3 to 5.
There are 6 more girls than boys.
How many students are in the class?



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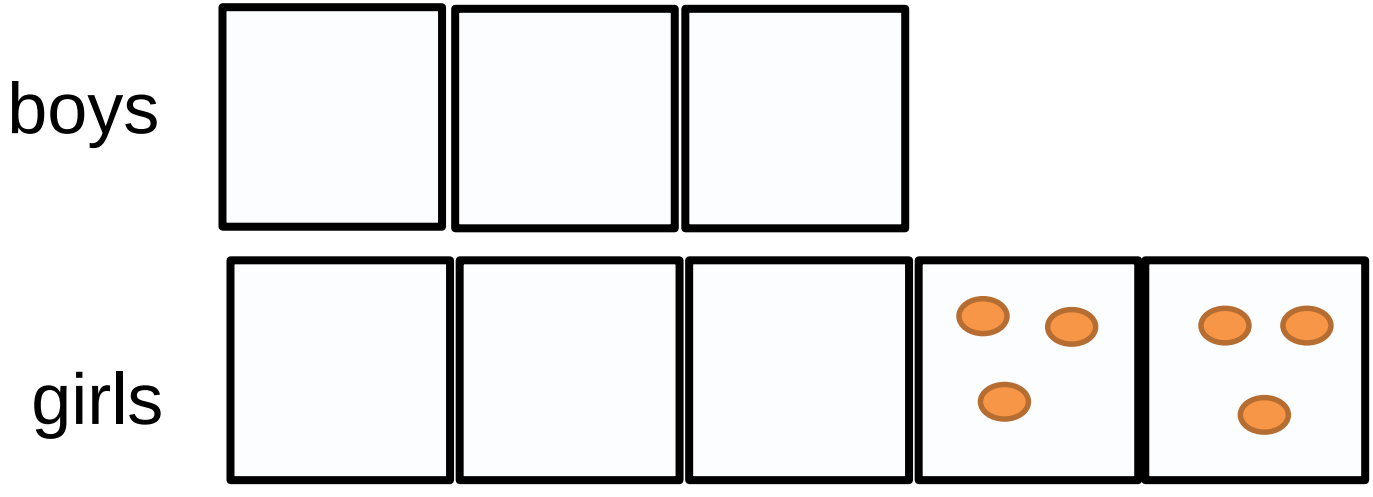

Problem Number 2

At the end of 8 hours, the manager said the snack shop had sold 2 hamburgers for every 3 cheeseburgers.
They sold 84 hamburgers.
How many more cheeseburgers did they sell than hamburgers?

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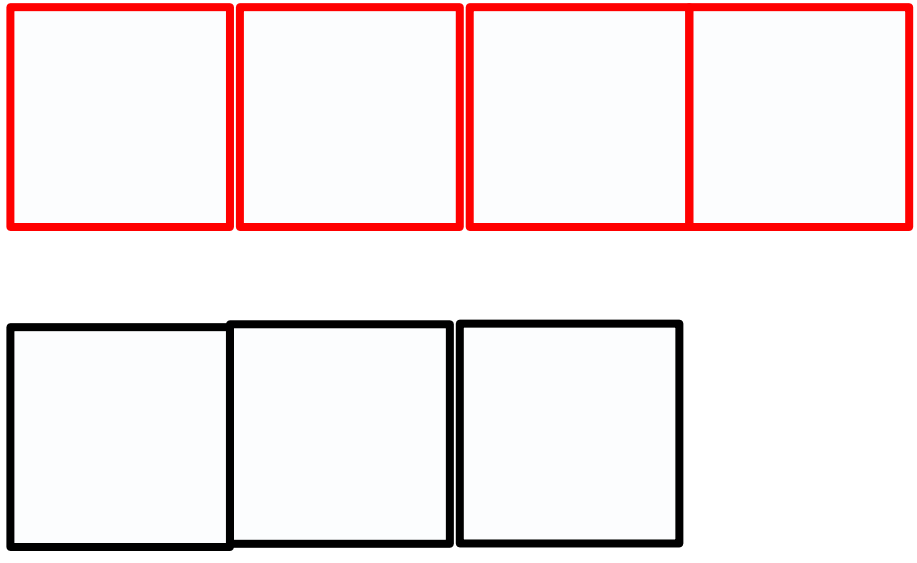
42 42

42 42 42

Problem Number 3

The ratio of boys to girls in Grade 6 is 4 to 3.
There are 175 students in Grade 6.
How many of the students in Grade 6 are girls?



$175 \div ?$

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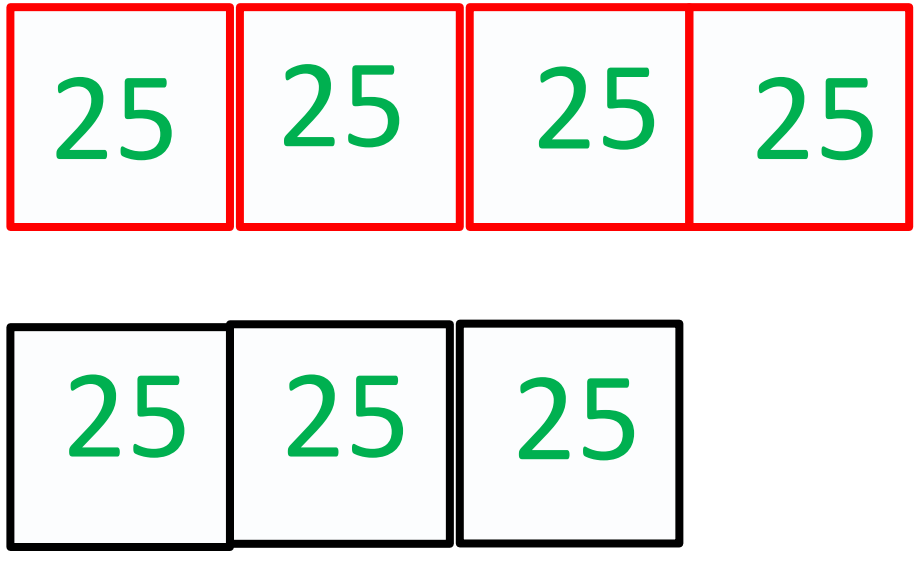
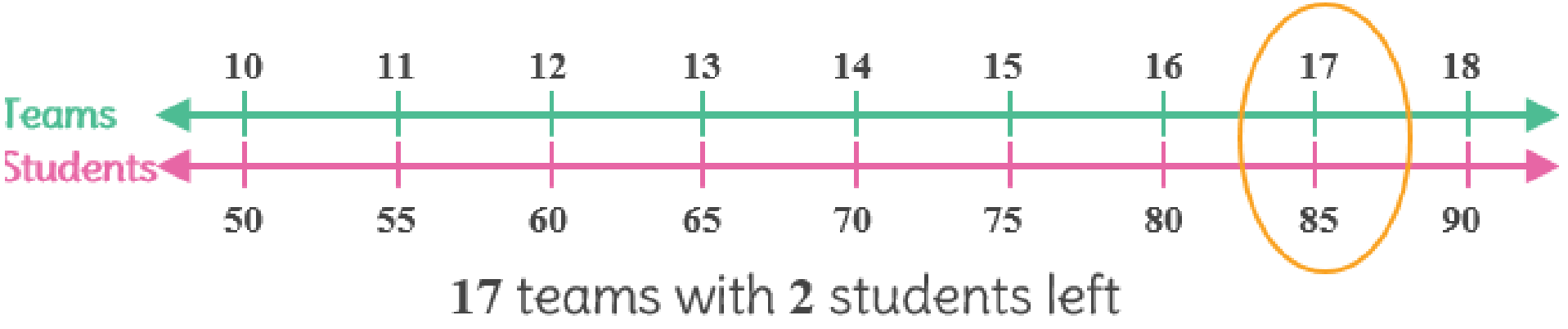


$175 \div 7$

gives the number per part

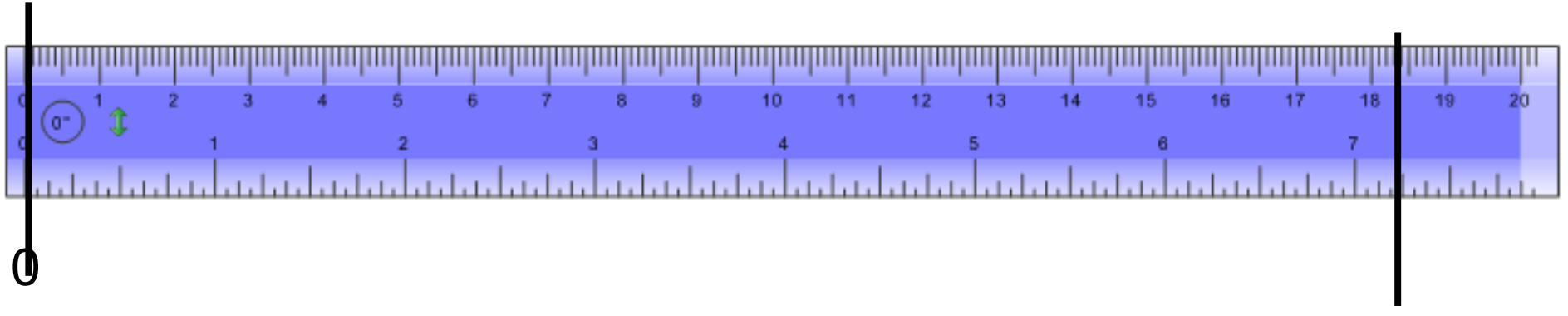
Double Number Lines



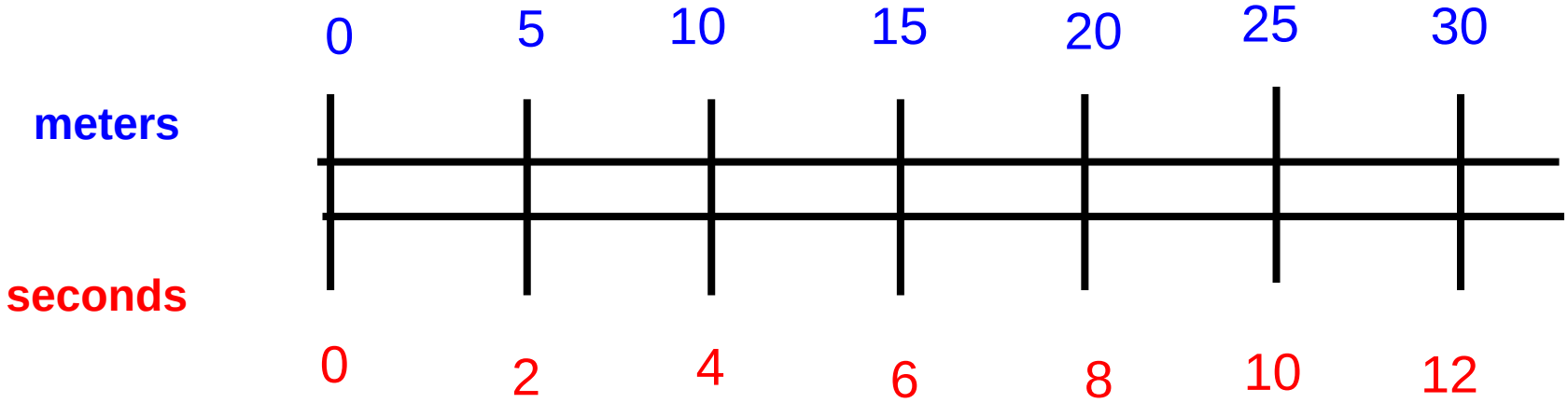
17 teams with 2 students left

Where do we see double number lines?

There are 2.54 cm in one inch. How many cm are in 7¼ inches?

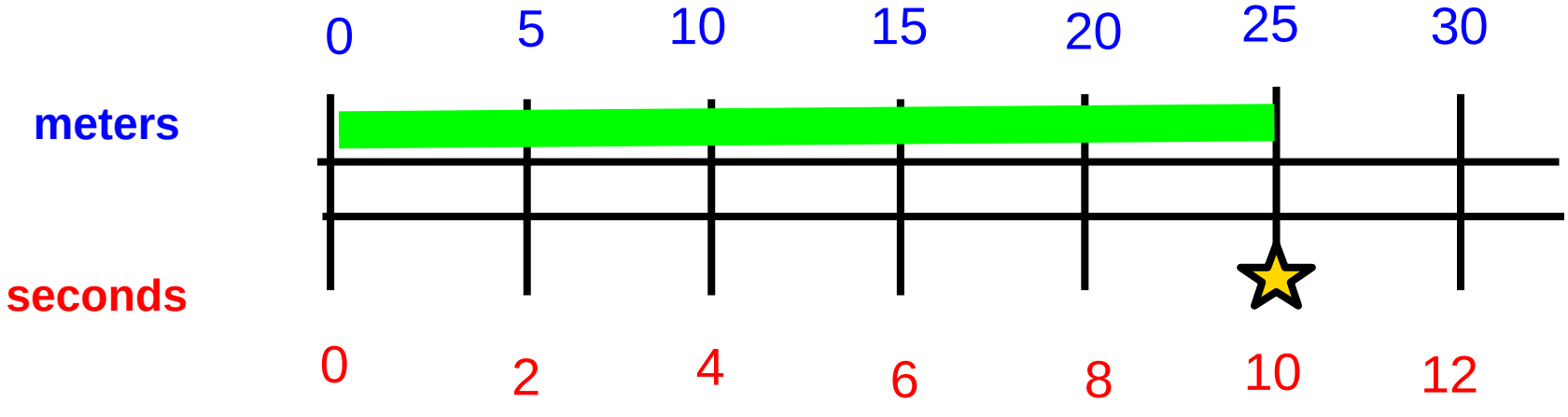


A child runs at a rate of 5 meters every 2 seconds.



How far can she run in 10 seconds?

A child runs at a rate of 5 meters every 2 seconds.



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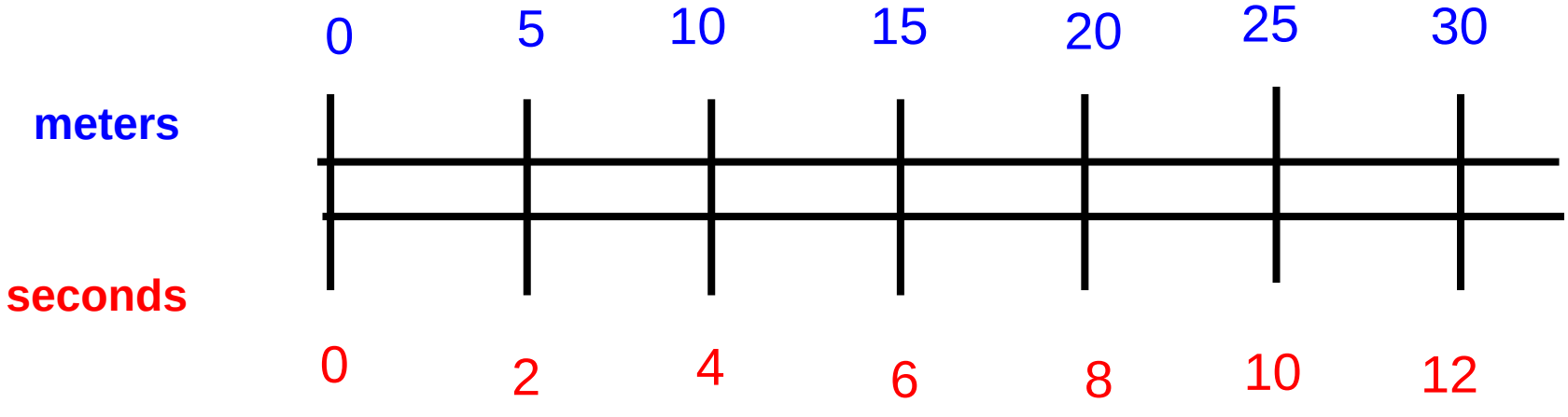

How long will it take her to run 20 meters?

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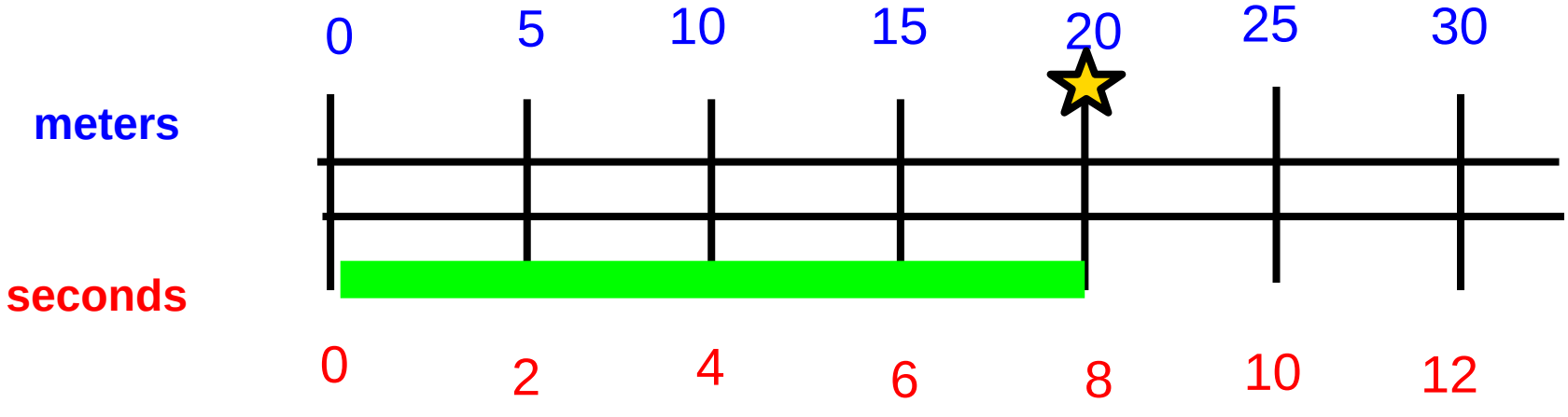

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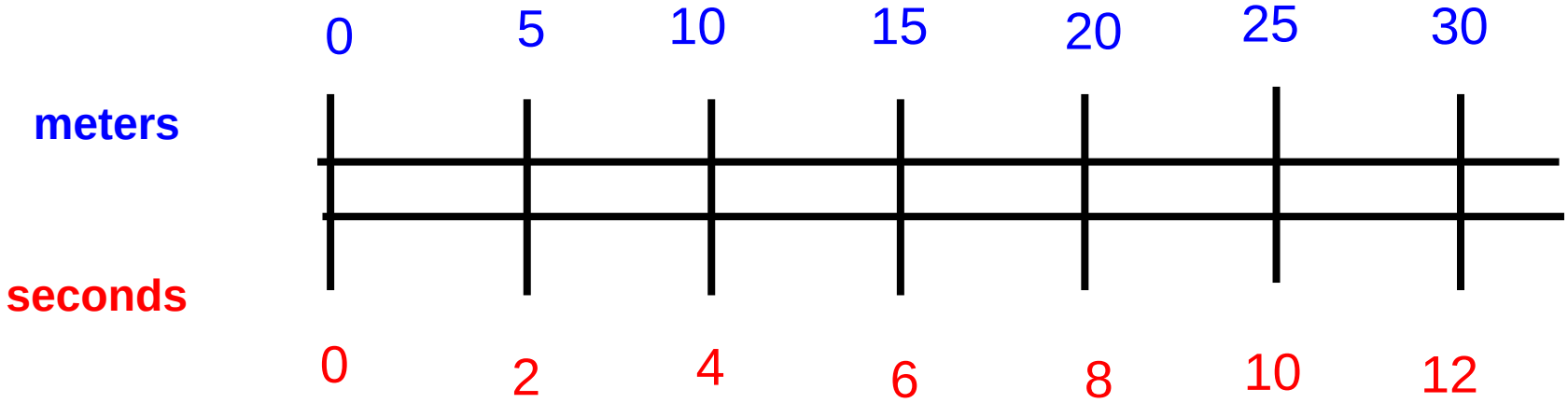
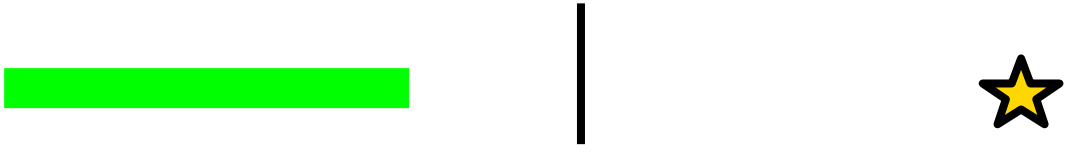


How far can she run in 5 seconds?



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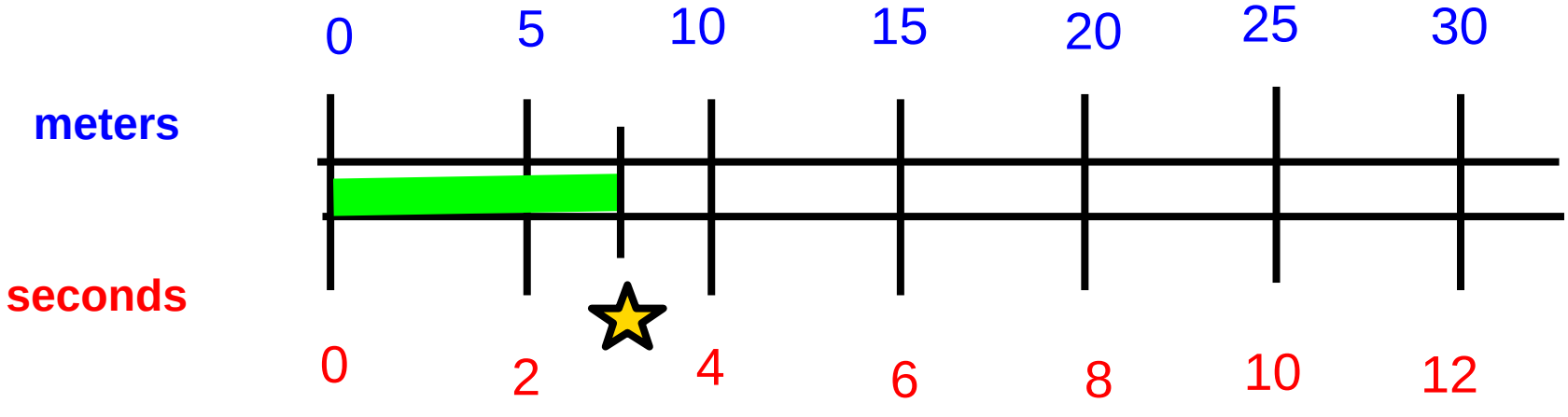
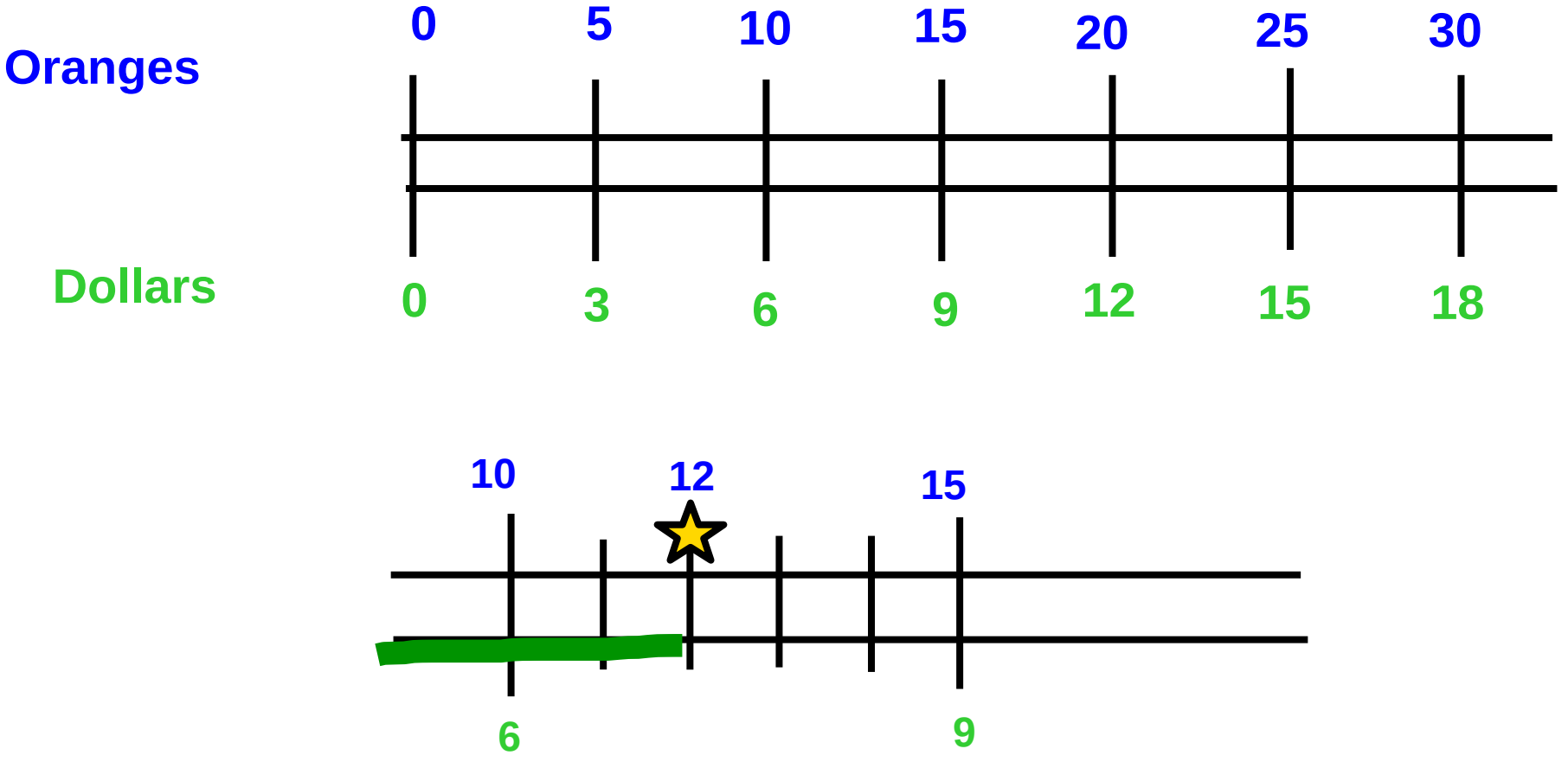


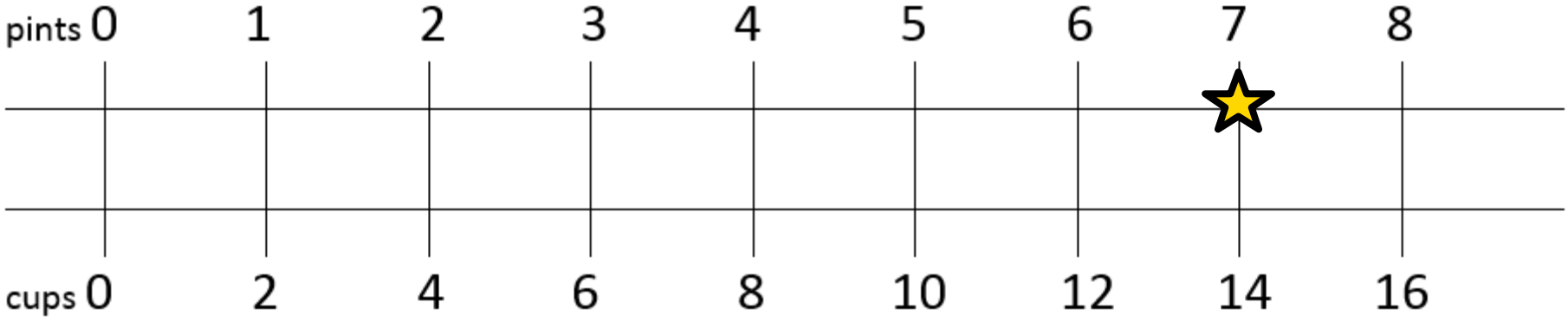
How far can she run in 5 seconds?

Mary can buy 5 oranges for $3.00. How much will she pay for one dozen oranges?

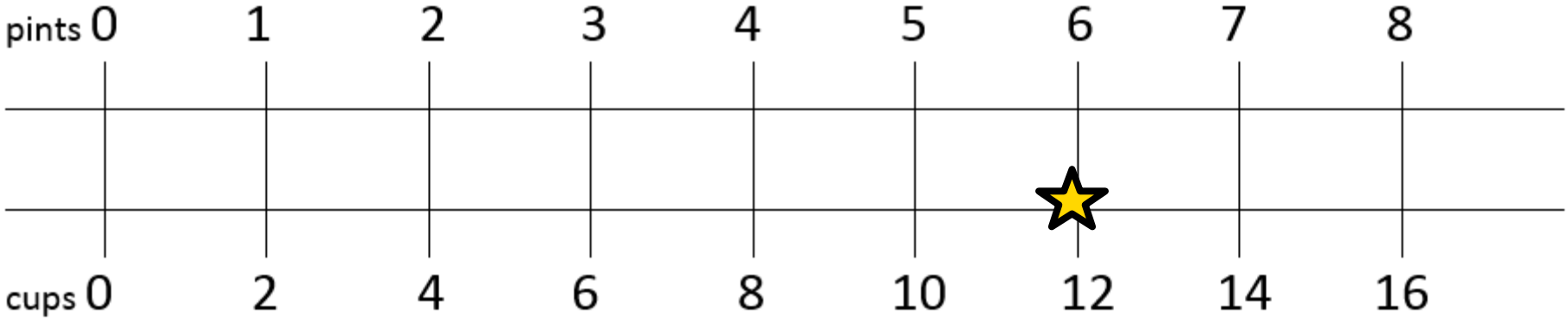


$3.00 per jump divided by 5 means $0.60 per space.
2 spaces = 1.20 so price is $7.20

There are 2 cups in one pint.
Jamal needs 7 pints of juice.
How many cups is this?



There are 2 cups in one pint.
Jamal uses 12 cups.
How many pints is this?



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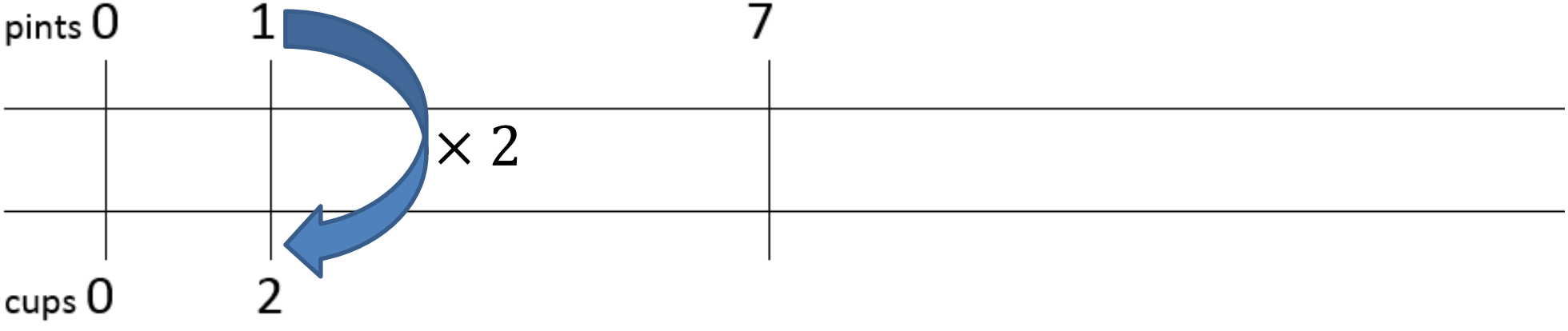

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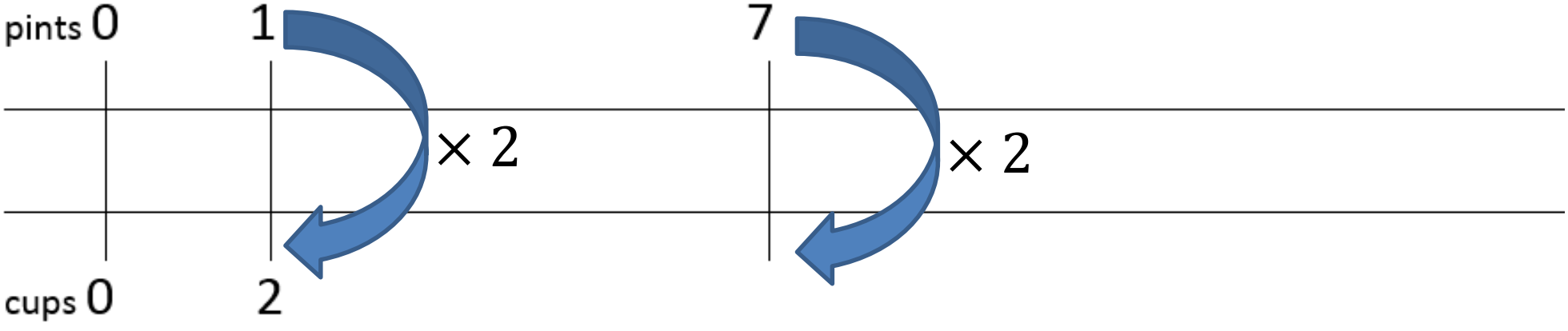

14 cups

There are 2 cups in one pint.
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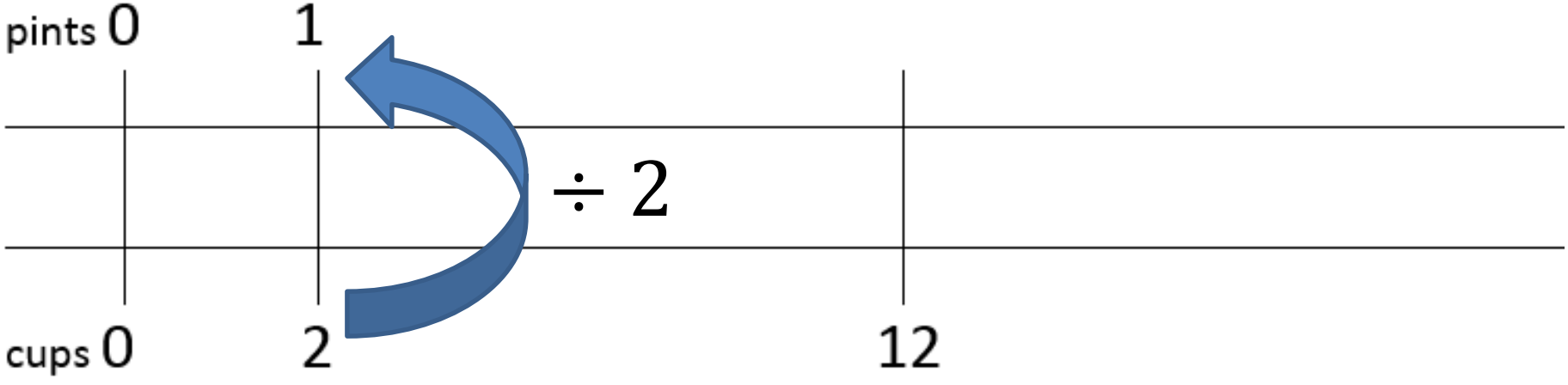

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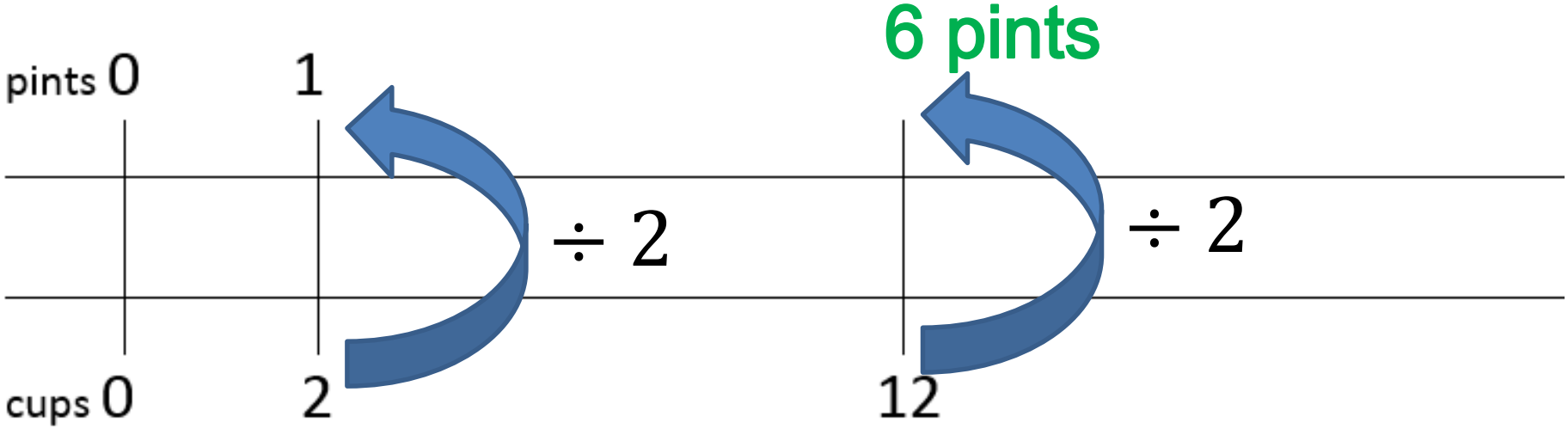
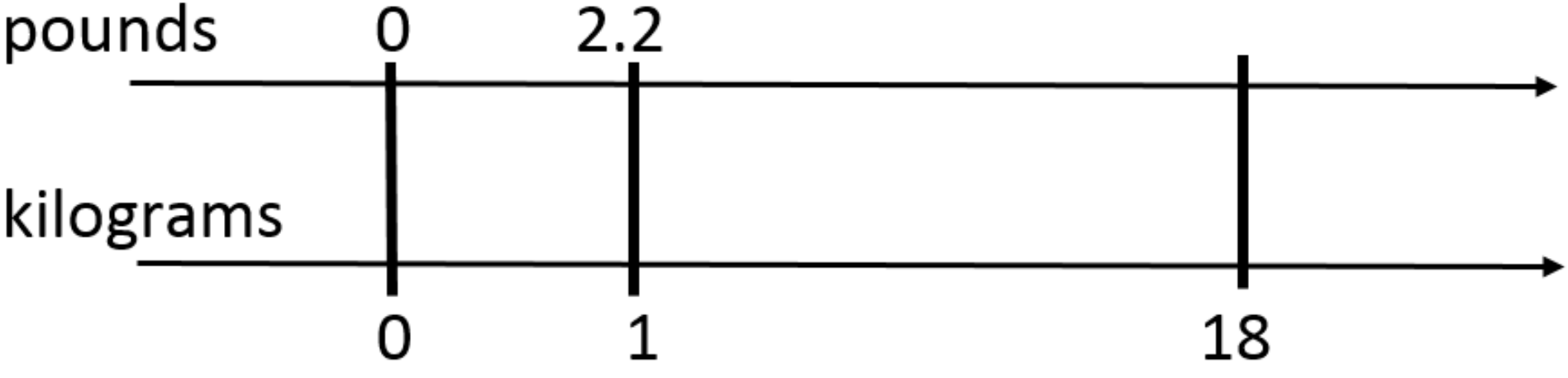


An airline requires that carry on luggage weigh 18 kilograms or less. Kara's scale only measures pounds.

Kara reads that 2.2 pounds = 1 kilogram. How many pounds is 18 kilograms?



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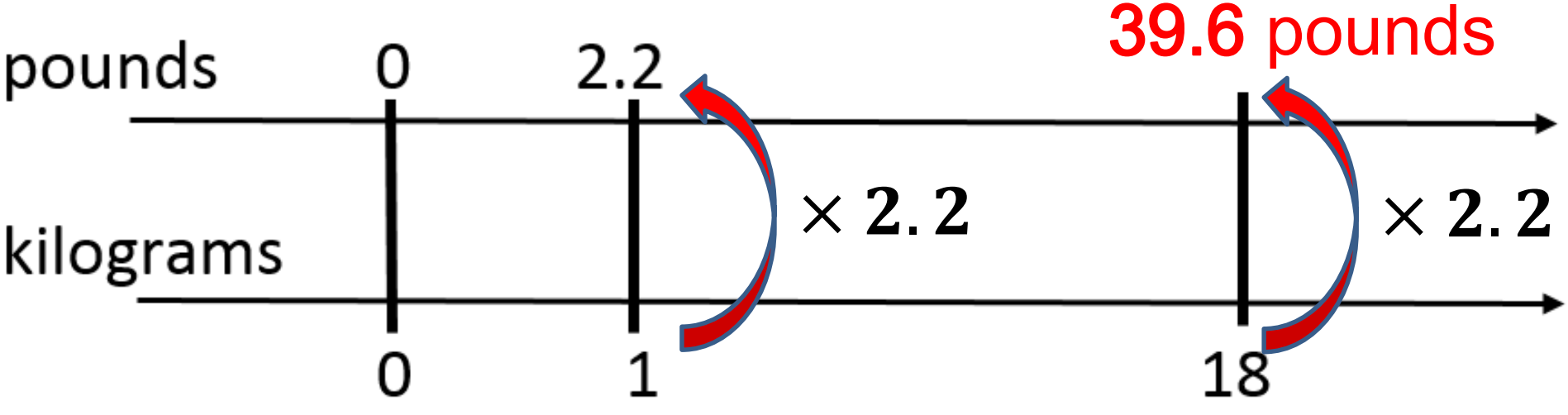
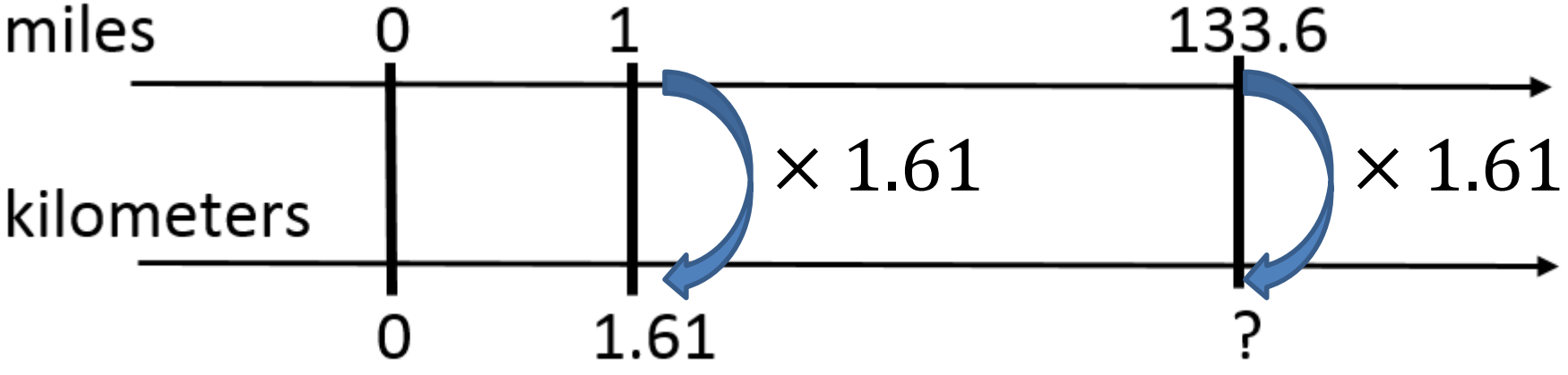
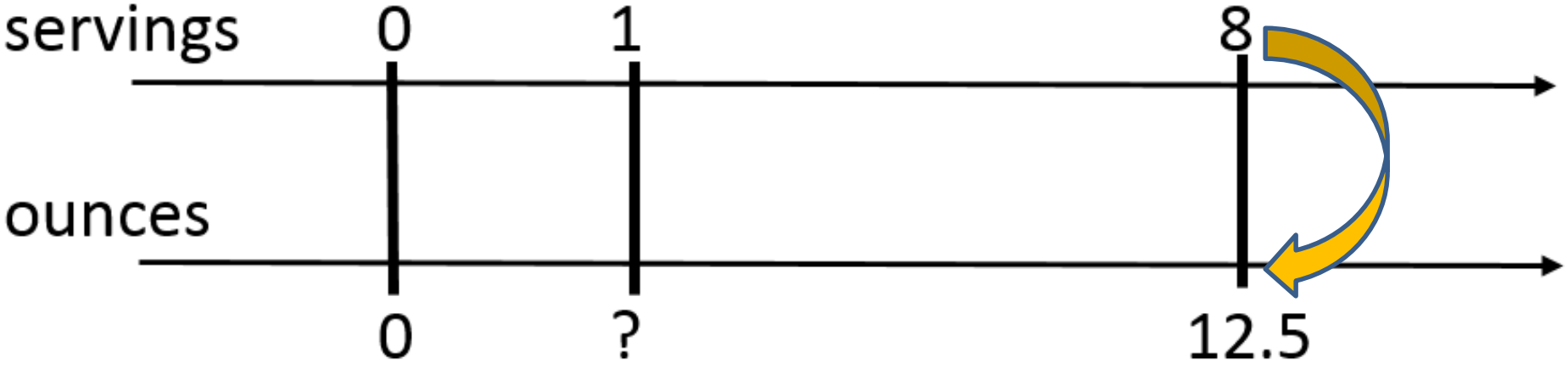


According to Google it is 133.6 miles from my house to Philadelphia.
How many kilometers is this? I mile = 1.61 km



133.6 miles is 214.3 kilometers.

A bag of chips weighs 12.5 ounces.
The label says this is 8 servings.
What is the weight of one serving?



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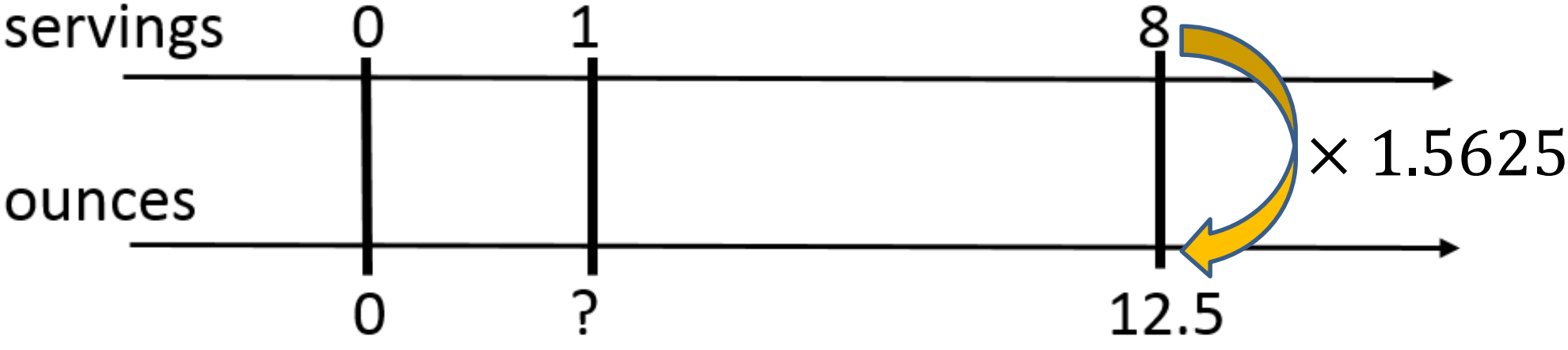

$$8 \times \_ = 12.5 \qquad 12.5 \div 8 = 1.5625$$

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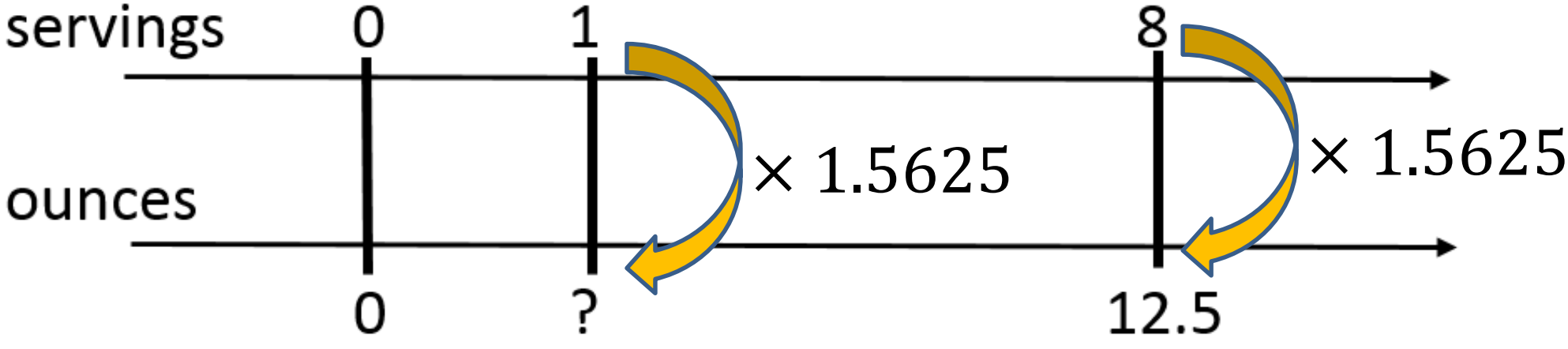
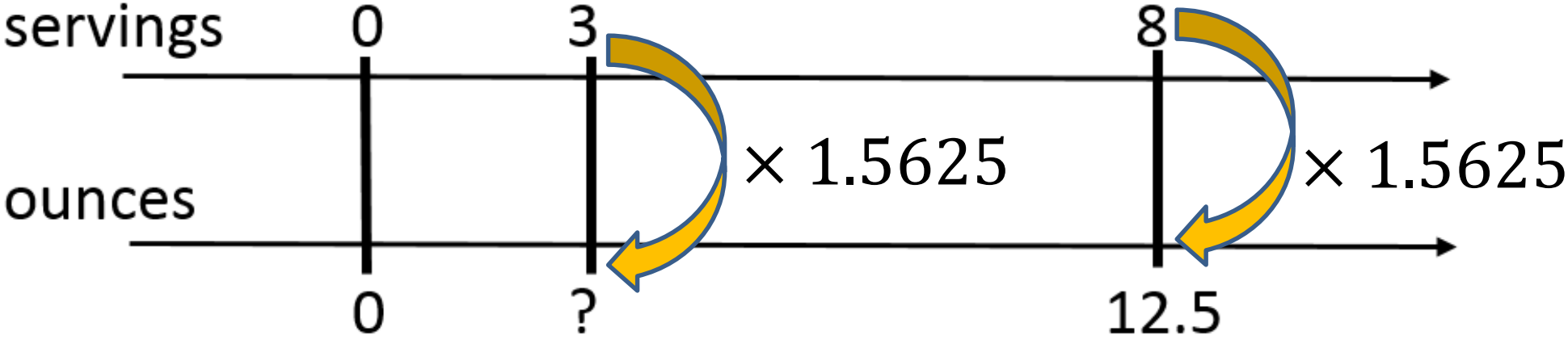


One serving is 1.5625 ounces.

A bag of chips weighs 12.5 ounces.
The label says this is 8 servings.
What is the weight of 3 servings?



Three servings is 4.6875 ounces.

Try a few others with your partners.

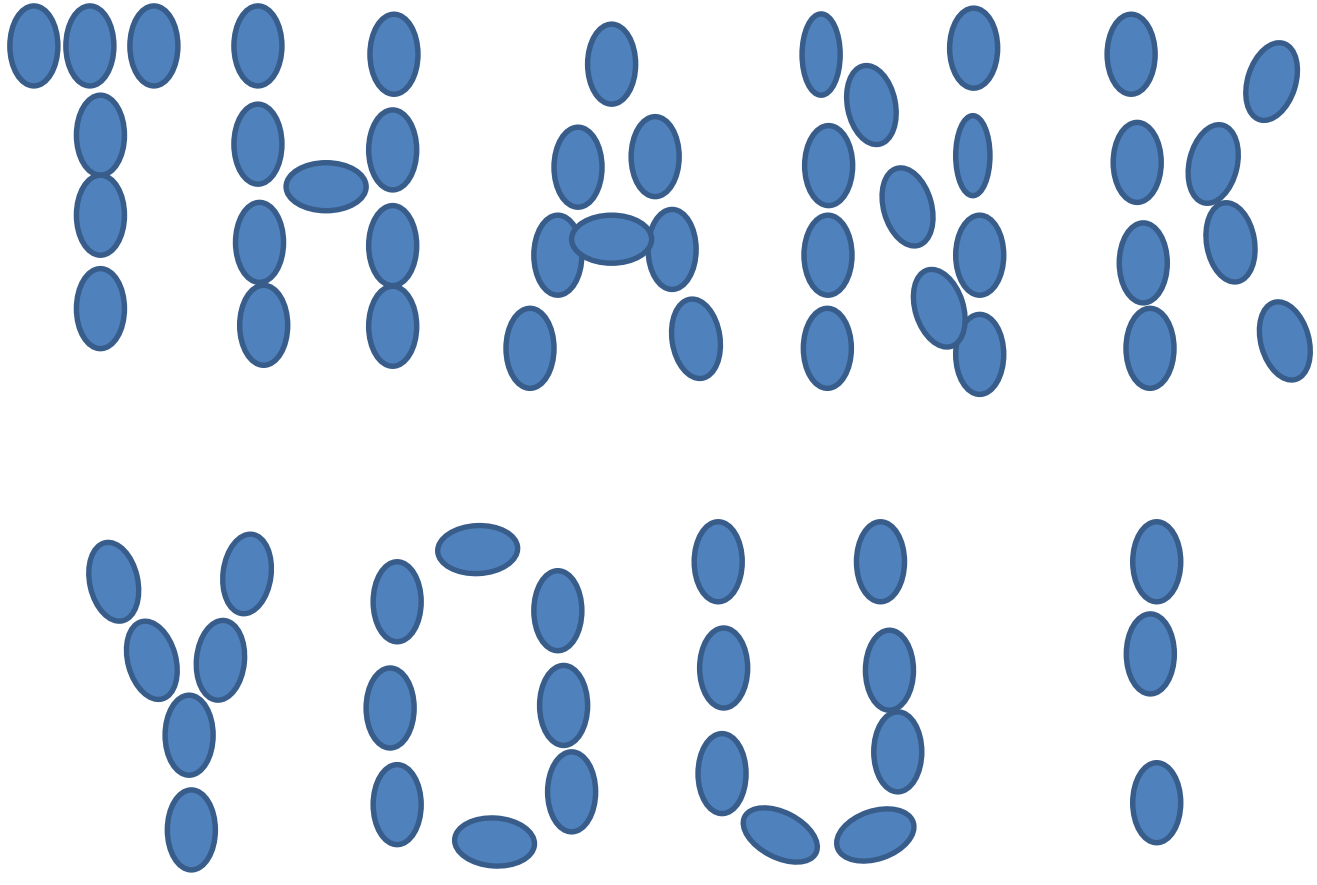
What do you notice as you work?

Reflections and Next Steps

Which method(s) do you prefer?

Which methods(s) do you think your students will prefer?

How will you use what you learned today?


THANK
YOU!

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