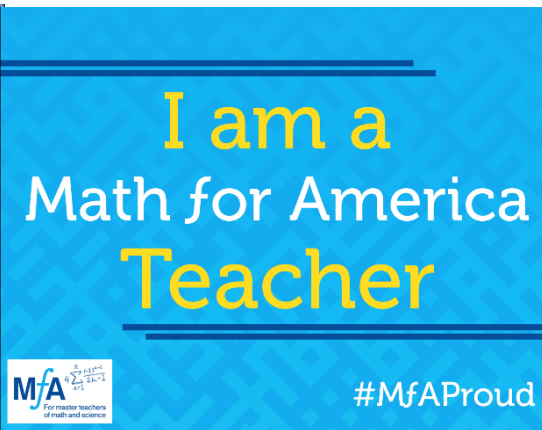


MASTER PROOFS through GAMES

NCTM Annual Conference 2018 - Washington D.C.



Peter and Quinn



Presented by
Peter Sell and
Paul Winston



Paul and KenKen inventor
Tetsuya Miyamoto

DIGITS

(Mastermind with Numbers)

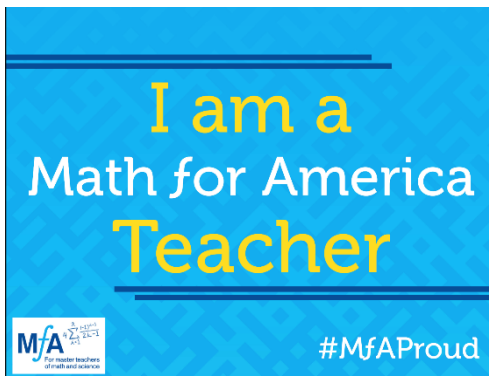
Presented by Peter Sell

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DIGITS

- D = Digits: This means that a digit in the guess is in the number
- P = Place: This means one of the digits is in the correct place in the number.

If the secret number was 290, this is how we would score it.

GUESS	D (Digits)	P (Place)
349	1	0
564	0	0
892	2	1
710	1	1

Copy the table and score 815

Guesses	D (Digits)	P (Place)
253		
897		
730		
451		

Copy the table and score 815

Guesses	D (Digits)	P (Place)
253	1	0
897	1	1
730	0	0
451	2	0

From each individual guess, what can we conclude?

Guess	D	P
249	0	0

Guess	D	P
123	3	0

Guess	D	P
123	3	1

What can we conclude?

Guess	D	P
123	0	0
709	0	0
465	0	0

What can we conclude?

Guess	D	P
123	1	0
709	1	0
465	1	0

What can we conclude?

Guess	D	P
213	2	0
790	1	0

What can we conclude?

Guess	D	P
249	0	0
194	1	1

And now?

Guess	D	P
249	0	0
194	1	0

Class Challenge

The class will try to determine the number that I have selected. See how many guesses it would take.

Line	GUESS	DIGITS	PLACE
1			
2			
3			
4			
5			
6			
7			
8			

Sample Puzzle:

Can you determine the number?

GUESS #	GUESS	D	P
1	178	0	0
2	295	1	0
3	414	0	0
4	663	0	0
5	304	1	1
6	489	1	0
7	816	0	0
8	452	0	0

Sample Proof

	Statement	Reason
Line 1	No 1's, 7's or 8's	Guess #1
Line 2	No 4's or 1's	Guess #3
Line 3	No 6's or 3's	Guess #4
Line 4	No 5's or 2's	Guess #8
Line 5	Only digits are 9 and/or 0	Lines 1, 2, 3, 4
Line 6	There is a 9	Line 5 and guess 2
Line 7	9 can only be in the hundreds place	Line 6, guess #2, and guess #6
Line 8	The other two digits are 0's	Line 5 and line 7
Line 9	The # is 900	Line 7 and line 8

Determine the numbers on your puzzle. Justify your answer to your partner.

Extensions

Given a set of clues with a mistake. Can students locate the contradiction?

Can students devise a set of guesses that can guarantee them the right answer on the 9th guess? 8th guess? 7 guesses? (for 3, 4 digit numbers)

Have the students work out the number of possibilities remaining after a certain number of guesses.

Is there an optimal strategy? Try it out.

Combinatorics & Probability.

Design a program (for CS students) for the game.

Digits as an introduction to Proofs

Guess	D	D	P	Statement	Reason
guess 1	1 2 3 4	1	0	No 5, 6, 7, 8	guess 2
guess 2	5 6 7 8	0	0	No 3, 4, 5, 6	guess 4
guess 3	9 0 1 2	3	1	There is no 1	guess 1, 3, 6, 4
guess 4	3 4 5 6	0	0	There are numbers 9, 0, 2	guess 3, 5, 6, 7, 8
guess 5	7 8 9 0	2	0	2 is the 1 st digit, and 0 is the 2 nd and 3 rd digit	guess 7, 8, 3
guess 6	6 1 9 8	1	0		
guess 7	4 2 0 7	2	1		
guess 8	2 0 4 5	2	2		
guess 9	4 0 8 5	1	1		

Solution:

2 0 0 9

	C	D	P
guess 1	123	1	1
guess 2	456	1	0
guess 3	789	0	0
guess 4	861	1	0
guess 5	452	1	1
guess 6	297	1	0
guess 7	304	0	0
guess 8	678	1	1
SOLUTION	622	3	3

By:
Gray for
Fashion
John

Statement

No 7, 8, 9

No 3, 0, 4

6 is in the number
(hundreds place)

2 is in the #

NO 5

2 is in the number
(ones place)

NO 1

2 is in the tens place.

Number is 622

Reason

Guess 3

Guess 7

Guess 8 + Line 1

Guess 6 + Line 1

Guess 2 + Line 3

Guess 5 + Line 5

Guess 1 + 4

Guess 1 + Line 8

All of the above

Ken Ken Proofs--background

In New York State, we used to teach logic proofs in the first two weeks of Geometry in order to get students thinking logically, and understanding the form of a Euclidean two-column proof. There were only 6 laws of logic used, in addition to “Givens.” Logic proofs are no longer part of the curriculum.

Ken Ken, is a square number puzzle. In an $n \times n$ Ken Ken, the numbers 1 through n appear once each in every column and row. “Cages” separate some cells from others, and require the mathematical operation indicated. When no operation is indicated, the cage value is given.

Ken Ken proofs--utility

- With a small number of valid “reasons,” Ken Ken proofs offer the student a chance to think logically, explain their process, and to facilitate much more challenging Euclidean proofs (i.e. low entry, high ceiling, easy to differentiate.)

An easy 3 x 3 with addition only; ideal for 1st proof.

7+	4+	
		1
1	5+	

Solutions to our first puzzle.

In this puzzle, what is the sum of the numbers in each row? In each column?

How about in a 4 x 4?

A 5 x 5?

7+ 2	4+ 1	3
3	2	1
1 1	5+ 3	2

This is puzzle 3 from KenKenPuzzle.com
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A first proof

Statement	Reason
1) $B3 = 1$	1) Given
2) $C1 = 1$	2) Given
3) $A2 + A3 = 4$	3) Given
4) $A2$ and $A3 = 1,3$ or $3,1$	4) No other way (to make 4) with our numbers (NOW WON.)
5) $A3 = 3$	5) 1 is already used in Column C
6) $A2 = 1$	6) Steps #4 and 5 (If $A2$ is not 3, it must be = 1.)
7) $A1 = 2$	7) Last Cell in Row (LCIR)
8) $B1 = 3$	8) Last Cell in Column (LCIC)
9) $B2 = 2$	9) LCIR
10) $C2 = 3$	10) LCIC
11) $C3 = 2$	11) LCIC or LCIR

Alternate proof

Statement	Reason
1) $B3 = 1$	1) Given
2) $C1 = 1$	2) Given
3) $A2 + A3 = 4$	3) Given
4) $A1 + A2 + A3 = 6$	4) The sum of the numbers of any row of a 3 by 3 is 6.
5) $A1 = 2$	5) Subtracting (St #4 - St #3)
6) $B1 = 3$	6) LCIC
7) $B2 = 2$	7) LCIR
8) $C2 = 3$	8) 1, 2 already used in column, row.
9) $A2 = 1$	9) LCIC
10) $A3 = 3$	10) LCIR
11) $C3 = 2$	11) LCIR or LCIC

2	4	
6	3	
		3

William Solomon 9/13/16

	Statement	Reason
1)	$A1=2$	1) Given
2)	$C3=3$	2) Given
3)	$B2+B3=3$	3) Given
4)	$B1+B2+B3=6$	4) The sum of the #s of any row is 6 (in a 3 by 3, pw)
5)	$B1=3$	5) Subtraction (Steps 4 and 3)
6)	$C1=1$	6) Last cell in column
7)	$C2=2$	7) Last cell in row
8)	$B2=1$	8) 3 and 2 are used in row and column
9)	$B3=2$	9) Last cell in row
10)	$A2=3$	10) Last cell in column
11)	$A3=1$	11) Last cell in row

(25/25)

Excellent
Proof
William!
(nice use of
subtraction!)

Extensions

- Grid size: 3 x 3, 4 x 4, 5 x 5, etc
- Operations: addition only, addition/subtraction, 4 basic operations, no operations specified.
- Number Systems: Counting Numbers, Negative Numbers.
- Fill in the blank: Statements provided & reasons required, vice-versa
- Challenge your students to create a KenKen puzzle with a unique solution.
- Competitions: Class, School, or National

3+		7+	
4+	6+		3+
	7+		
6+		4+	

2÷		3
9×		1—
2		

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2÷		7+	3
2−			8×
4	6+		
4+			

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SELECT PUZZLE TYPE & DIFFICULTY ▼

ENTER PUZZLE NO.:

Go!

Step 1: Select Puzzle Size

3x3

4x4

5x5

6x6

7x7

8x8

9x9

Step 2: Select Operation(s)

+

+ -

× ÷

+ -
× ÷

NEW!
?
NO-OP

Step 3: Select Difficulty

Easiest

Easy

Medium

Hard

Expert

PLAY NOW

Thank you!

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