Conditional Probability
What do we know, and when do we know it?

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We live in a world of Conditional Probability

- Weather Prediction—***Given*** *conditions similar to today*, in the past it has rained x% of the days—so the probability of rain today is _____
- Medicine—***Given*** *the rate of occurrence of a disease in a population*, the probability you have that disease if you test positive is _____
- Statistical Hypothesis Testing—the probability of obtaining a result a least this rare ***under the assumption (given)*** *that there is no initial difference* between a treatment group and a control group is _____
- Insurance—The probability that a person will live at least until age N ***given*** *their current age and health level* is _____

Conditional probability is all about the "*Given*"

- What are our initial assumptions?
- What do we know, and, when do we know it?
- What Is the 'conditioning' event?

Goals for Today

- Explore several Conditional Probability Tasks
- Estimate empirical probabilities using data gathered from simulations for the tasks, or by using a representation of a theorized simulation for the task
- Engage in some Mathematical Teaching Practices, some Mathematical Habits of Mind and Mathematical Habits of Interaction as we engage in the tasks

NCTM Mathematics Teaching Practices

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

--from *Principles to Actions* (NCTM, 2014)

Mathematics Teaching Practices Prominent in Conditional Probability Investigations

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
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Today's Mathematical Habits of Mind

- Use Mathematical Representations
- Justify Why
- Persevere, Seek More
- Engage in Metacognition, Reflection
- Experience Disequilibrium

Today's Mathematical Habits of Interaction

- Private Think Time
- Genuine Listening
  - --Elbow pair shares, Table shares, Group Broadcasts
- Critique and Debate

Task 1. Knowing Before, Knowing After

- You have 2 orange and 2 blue chips in a container. Your partner, unable to see the chips, will pull two of the chips out of the container, one at a time.

- Suppose the first chip pulled is blue. What is the probability that the second chip pulled will be orange?

- Suppose second chip pulled is blue. What is the probability that the first chip pulled was orange?

Habits of Interaction

- Private think time
- Elbow Pair Share
- Some Group Broadcasts

Gather Data

One partner holds the four chips, the other draws two out (blindly).

Record the colors in the order pulled.

Repeat at least 15 times (20 is better).

Probability Based on our Simulation Data

- # times first chip was blue____
- Probability that second chip was orange, *given that* the first one was blue is_____

- # times second chip was blue____
- Probability that first chip was orange, *given that* the second one was blue is _____

Any Surprises?

What is the ‘condition’ in this task?

- What did we know, and, when did we know it?
- What was the “*Given*?”

Task 2. Monty's Dilemma

- You are playing a TV game show where you are shown 3 doors. Behind one of the doors is a valuable prize, behind the other two there is nothing valuable. You pick one of the doors for Monty the game show host to open to see if you win the prize.

- Instead of opening your door, Monty opens one of the other doors that has nothing behind it. Then, Monty says to you, "Do you want to Stick with your original door, or Switch to the other unopened door?"
- What do you do? Stick, or Switch?

Habits of Interaction

- Private think time
- Elbow Pair Share, sharing at table
- Some Large Group Broadcasts and Discussion

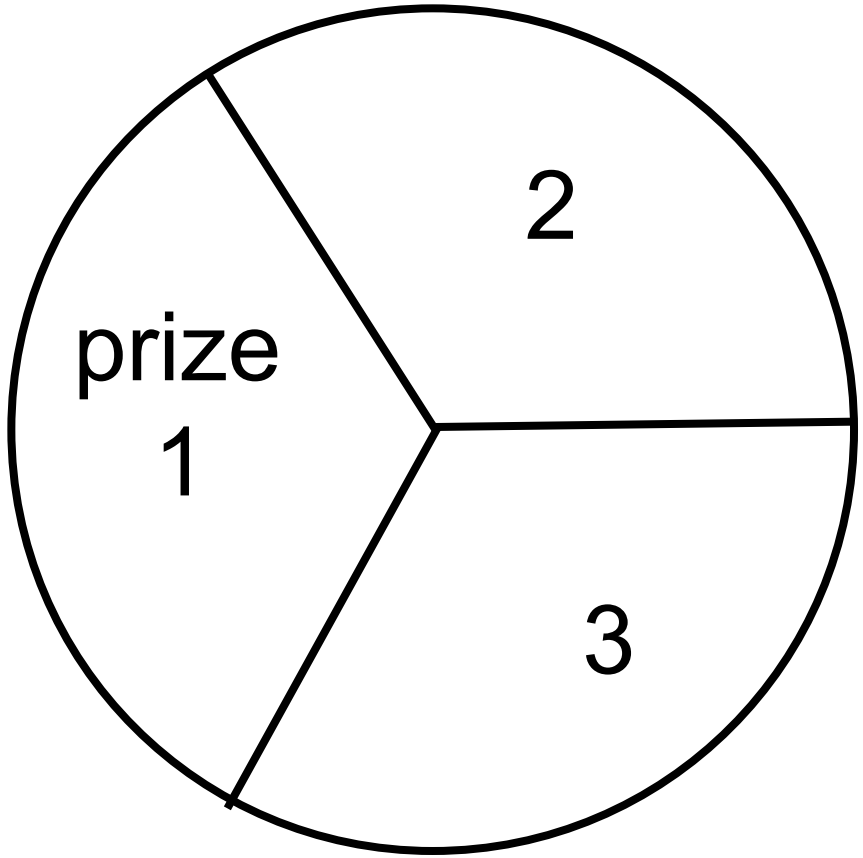
What do people say?

- "What difference does it make whether you stick or switch? After Monty opens a door, you have a 50% chance of getting the door with the prize either way, because there are only two doors left. I'd stay with the one I picked at first, because I felt it was the right one from the start."
- "Things are better for you after they open a door, because there are only two doors now. At this point, it doesn't matter whether you flip the coin, or just switch, it's all the same."
- "I don't see how opening one of the doors improves your chances, it just adds suspense to the game. Your chances were 1 in 3 that you'd pick the right door in the first place, and your chances are still 1 in 3 after you are shown a door."

The Three Simulation Strategies

- STICK—stay put with your original door after Monty shows you a door without a prize behind it
- FLIP—flip a coin to decide whether to Stick or Switch after Monty shows you a door without a prize behind it
- Switch—always switch to the other door after Monty shows you a door without a prize behind it

Spinner for gathering data for Monty's Dilemma



Gather data for each of the 3 strategies

	Won	Lost
Stick Strategy		
Flip Strategy		
Switch Strategy		

(*min 15 trials for each strategy)*

Any Surprises?

What's the Best Strategy?

WHY?

Results of 10,000 Trials for each Strategy

Strategy	Won	Lost
Stick	32,595	67,405
Flip	51,163	48,837
Switch	67,403	32,597

What is the ‘condition’ in this task?

- What did we know, and, when did we know it?
- What was the “*Given*?”

Task 3. The Blood Test

- There is a blood marker that indicates a potential genetic disorder. The marker shows up in 1% of the population.

- There is a blood test that is 90% accurate for the presence of the marker.

- What is the probability that a person has the genetic marker *given that their test was positive*?

Habits of Interaction

- Private think time—what do you think the probability is?
- Elbow Pair Share
- Sharing out your at table with other pairs

Building a representation of a simulation

Imagine that 1000 people from the population are tested for the marker in their blood.

How many of them would we expect to fall into each of these categories…..

2x2 Contingency Table for 1000 test cases

Presence of Marker	Test is Positive: Yes	Test is Positive: No
Yes		
No		
Total	1000	

2x2 Contingency Table for 1000 test cases

• Presence of Marker	Test is Positive: Yes	Test is Positive: No
Yes	10	
No	990	
Total	1000	

1% of population has the marker in their blood

2x2 Contingency Table for 1000 test cases

- Presence of Marker

	Test is Positive: Yes	Test is Positive: No
Yes	10	9	1
No	990		
Total	1000		

The test is 90% accurate when marker present

2x2 Contingency Table for 1000 test cases

• Presence of Marker	Test is Positive		
	Yes	No	
Yes	10	9	1
No	990	99	891
Total	1000		

The test is 90% accurate when marker is absent

2x2 Contingency Table for 1000 test cases

• Presence of Marker	Test is Positive		
	Yes	No	
Yes	10	9	1
No	990	99	891
Total	1000	108	892

2x2 Contingency Table for 1000 test cases

• Presence of Marker		Test is Positive **Yes**	No
Yes	10	**9**	1
No	990	**99**	891
Total	1000	**108**	892

Given a positive test:

9 of the 108 tested positive

The Probability you have the marker, *given you tested positive, is* 9/108 ~ .083. Note: This test is not very sensitive, too many false positives!

What is the ‘condition’ in this task?

- What did we know, and, when did we know it?
- What was the “*Given*?”

Life is full of conditions! So is Probability!

Keep the '*given*' in mind!

Thanks for coming!

Mike

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