

Ori-Money in Three Dimensions

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The term, “oru (折る)” in Japanese means “to fold,” so “ori-money” literally means, “money folding.” Polyhedra can be built by using squares folded into modular units and assembled. However, there is a property of the American dollar bill which allows two bills to be folded to form units which can be used to form the regular tetrahedron. By extension, four bills can be used to form the regular octahedron.

Professor Jean Pedersen of Santa Clara University is a master at modular origami polyhedra building and was one of the first to describe the mathematics behind her folding techniques. She was also the first to discover the following amazing dollar bill property and describe its construction. Welcome to the world of unit or modular origami using non-square paper currency – unit ori-money!

Materials needed:

Two crisp American bills (per person or per pair). These do not need to be \$1 bills; they could be of any denomination.

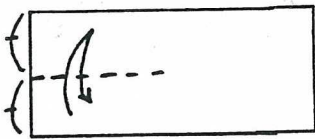
Grouping:

It is optimal if you work with a partner. Each of you can make your own individual model, or you can make one model together.

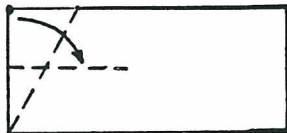


Folding/Assembly Instructions with Questions

1. You will be folding each of two dollar bills the same way. Orient each of your bills horizontally and locate a midline of the dollar bill by folding it in half widthwise and unfold. It is not necessary to crease the entire midline.

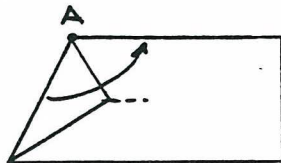


2. Fold the upper left hand corner to the midline while pivoting on the lower left corner. Make sure the angle in the lower left corner is well-defined and sharp.



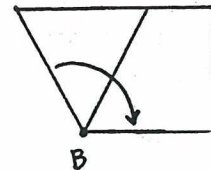
How would you find the slope of the left edge?

3. Fold the slanted left edge to the top edge while pivoting on vertex A.

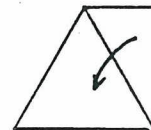


What is the slope of the left edge now?

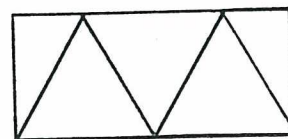
4. Fold the slanted left edge to the bottom edge while pivoting on vertex B. Notice that the upper left corner will nearly or exactly touch the lower right corner of the dollar bill!



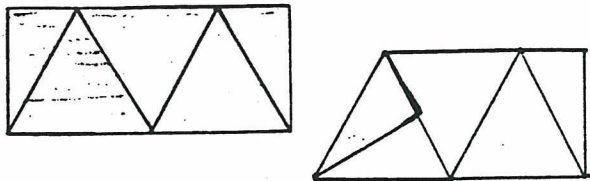
5. Fold down the remaining right triangle at the right of the bill. An equilateral triangle is formed.



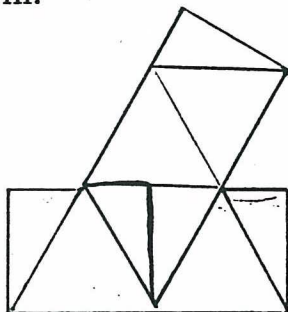
6. Unfold each bill with the curl of the folds facing up. Notice that each bill has three congruent equilateral triangles flanked by two 30-60-90 triangle tabs.



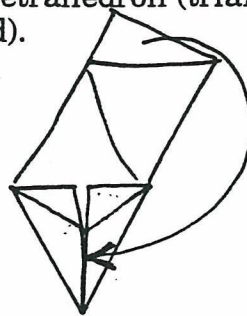
7. Orient one bill so that the creases form an "M". Fold down the 30-60-90 triangle tab on the left side of the other bill.



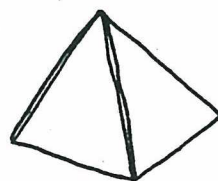
8. Lay the 60° angled vertex of the second bill on the central equilateral triangle of the other bill.



9. Fold up the underlying bill to form a tetrahedron (triangular pyramid).



10. Wrap the other bill around the first and tuck the 30-60-90 triangular tab in the available slit.



You have now formed a unit orimoney tetrahedron!

Exploration

Record your answers in your math journal.

1. For all practical purposes is the tetrahedron you constructed a "regular" tetrahedron? Why?
2. This dollar bill unit contains three equilateral triangles flanked by two 30-60-90 triangles. Will this be true of any rectangle folded in the same manner? Will this be true of any similarly proportioned rectangle folded in the same manner? What are the constraints on the length and width of the rectangular American dollar bill which cause it to be folded into three equilateral triangles flanked by two 30-60-90 triangles?