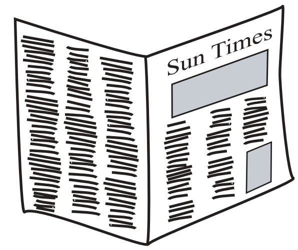
Origami: It's Not Just For Squares

Nelson Norton—a newsy, noisy, and very hip reporter—was an industrious and thrifty fellow. He had a lot of old newspapers lying around and wanted to make something useful—and of course noisy. Of course, he planned to recycle the newspapers after he finished his project.



He decided to try the following activities to explore geometric principles by using a specialized form of origami, called *rectangular origami. Origami* is the Japanese term for paper folding (*ori,* a folding, plus *kami,* or *gami,* meaning paper. Most origami constructions are paper models of objects that vary from animals to spacecraft and that are constructed from square sheets of paper. However, a genre of origami uses paper from such common rectangular sources as newspapers or dollar bills. Although nonsquare papers are used for these constructions, many geometric properties of square-paper origami are still evident.

Take a sheet of newspaper—approximately 27 inches by 23 inches, or 68.5 centimeters by 58 centimeters—and tear it in half along its center fold. See **figure 1.**

1. What are the dimensions of one of these sheets of paper?______________________
2. Is the ratio of the length of this sheet to its width the same as the ratio of the length of the original rectangle of newspaper to its width? How can you tell?
3. What is the area, in square inches and in square centimeters? __________________

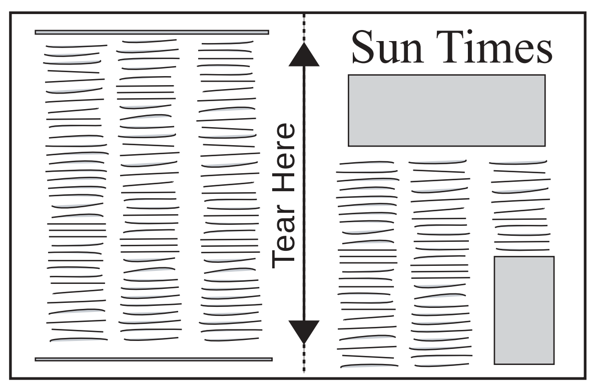


Fig. 1

Take one of the torn sheets of newspaper, and orient it so that its length lies vertically, that is, in portrait format, in front of you, as shown in **figure 2a.** Then fold the top corners (*A* and *B*) together and fold the bottom corners (*C* and *D*) together, as shown in **figure 2b,** forming triangles I, II, III, and IV. See **figure 2c.**

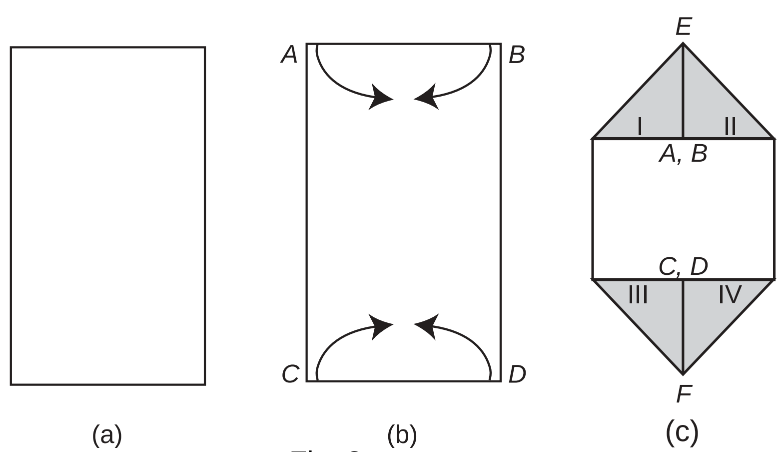


Fig. 2

The editors wish to thank David Masunaga, Iolani School, Honolulu, HI 96826, for the ideas used in this issue of *Student Math Notes.*

4. Assuming that the corners of the newspaper form 90 degree angles, what kind of triangles did you fold in **figure 2c?** ____________________________________

5. What do you notice about the shape and size of the triangles in **figure 2c?**

6. What are the measurements of the nonright angles in the triangles, for example, in the close-up of triangle I, shown in **figure 3?**

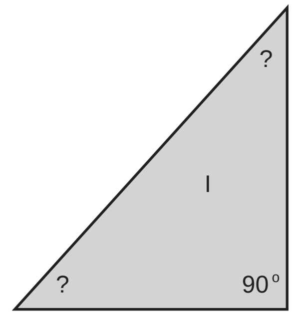


Fig. 3

Triangles similar to triangles I, II, III, and IV are called *isosceles* triangles, since they have two sides that are the same length. This term is from the Greek word *isoskeles*, with *isos* meaning *equal* and *skelos* meaning *leg*.

7. What name describes the largest polygon in **figure 2c** and **4a?** ________________________

A "regular" polygon is a polygon that is equilateral, that is, its sides are all the same length, and equiangular, that is, its angles all have the same measure.

8. Is the polygon in **figure 4a** a "regular" polygon? Why?

In **figure 4a,** draw the horizontal line of symmetry on the figure. Remember that both regions across the line of symmetry are congruent. Fold along the horizontal line of symmetry to match points *E* and *F,* as shown in **figures 4a** and **b.**

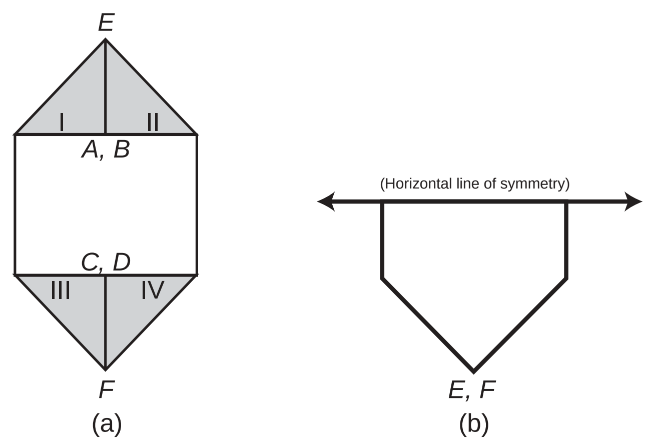


Fig. 4

9. What kind of polygon is displayed in **figure 4b?** ____________________________________

10. Is this polygon "regular"? Why?

11. What things in the real world does this polygon remind you of?

12. What do all the things that you mentioned in your answer to question 11 have in common?

Fold the polygon in **figure 5a** in half (right edge to left edge) along its vertical line of symmetry, to form **figure 5b.**

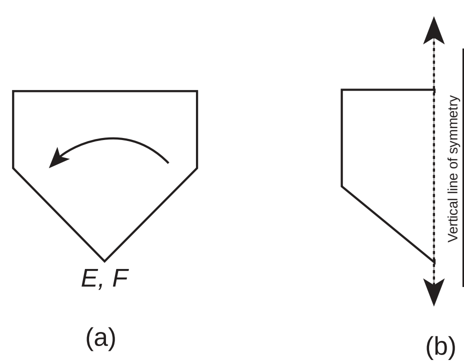


Fig. 5

13. What type of polygon is shown in **figure 5b?** ____________________________________

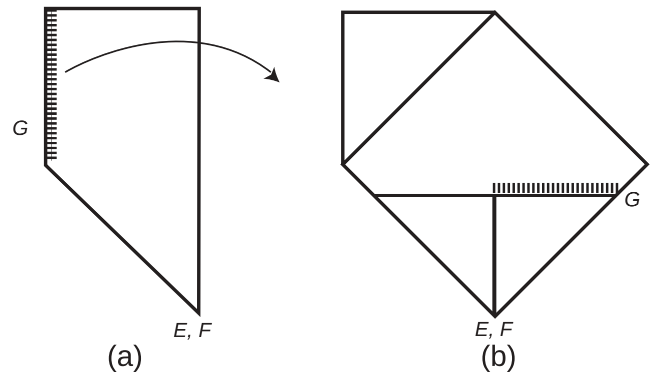


Fig. 6

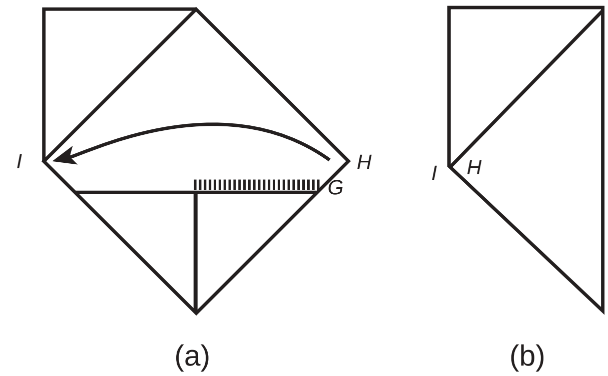


Fig. 7

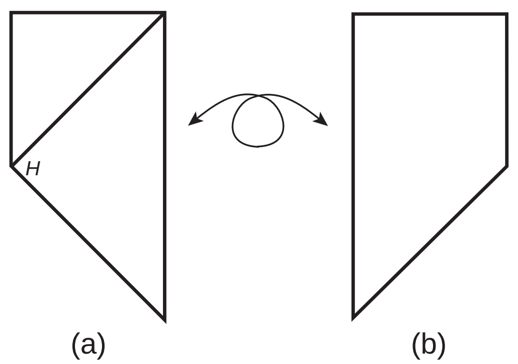


Fig. 8

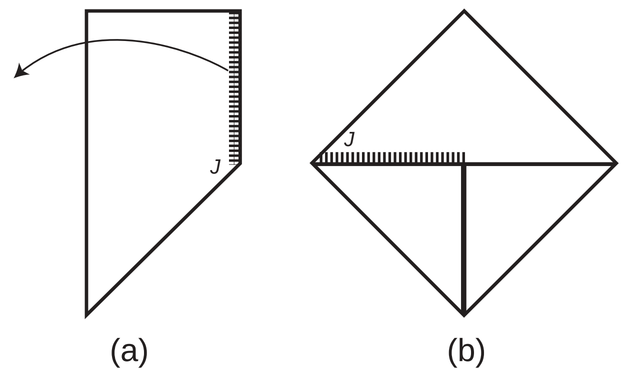


Fig. 9

Place your right thumb between the top and second leaf on the left edge at point *G,* pull the top leaf to the right, and press the paper down to form a square. See **figures 6a** and **b.**

Fold point *H* to point *I,* forming an isosceles right triangle. See **figures 7a** and **b.**

14. What is the relationship between the isosceles right triangles that you have just formed and the square in **figure 6b?**

Next, flip the entire shape over. See **figures 8a** and **b.**

Proceed as you previously did by placing your left thumb between the top and second leaf on the right edge at point *J,* pulling the top leaf to the left, and pressing down to form a square, as in **figures 9a** and **b.**

Fold point *K* to point *L,* as shown in **figure 10a,** forming a thick stack of isosceles right triangles, as **figure 10b** shows.

Next, grab the isosceles triangle firmly between your thumb and forefinger at *M,* and give the entire assembly one sharp downward movement (right angle facing away from you), as if you were using this triangle like a hatchet. You should hear a loud pop. You have been industrious and very noisy, just like Nelson.

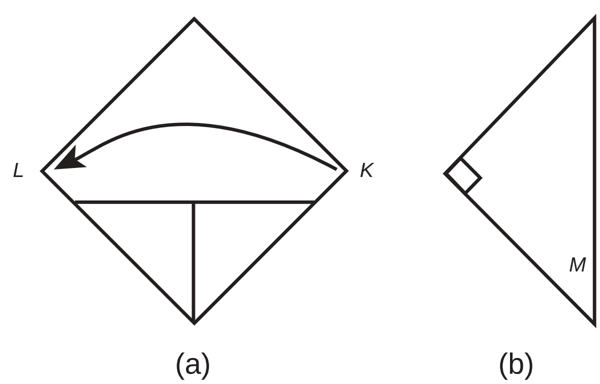


Fig. 10

Nelson was invited to a prestigious dinner. However, in his rush to get ready, he forgot his tie. “No problem,” he thought, “Not only will I have a tie, but I will also show my true value!”

He used his origami skills to fold another object. This time, he used an American dollar bill as his rectangle.

Fold a dollar bill in half, on the horizontal axis, as shown in **figure 11a,** so that the portrait of George Washington is on the outside, as in **figure 11b.** Fold the new region in half, using the vertical axis. See **figure 11c.**

15. Is this new rectangle similar to the original dollar bill; in other words, does it have the same proportions as the original rectangle? Why?

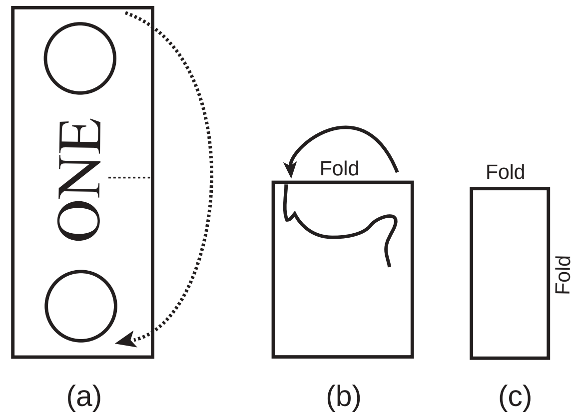


Fig. 11

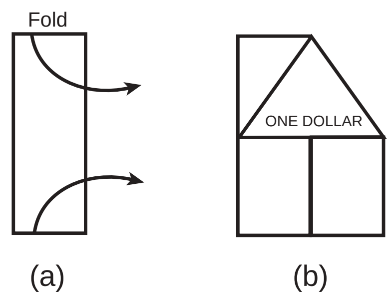


Fig. 12

Place your right thumb between the top and second leaf on the left edge, opening the leaf of paper from the lower-left edge all the way to the right, and press down to form an isosceles triangle. The triangle forms at the top, as pictured in **figures 12a** and **b.**

Then flip the entire assembly over, and make the same fold, except open the one leaf from the right edge instead, as shown in **figures 13a–13c.**

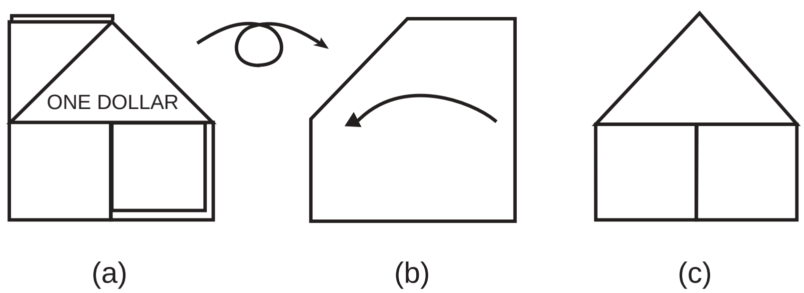


Fig. 13

Fold one leaf from the right edge over to the left. Flip the construction over, and repeat, that is, fold one leaf over from the right edge to the left. See **figures 14a–14d.**

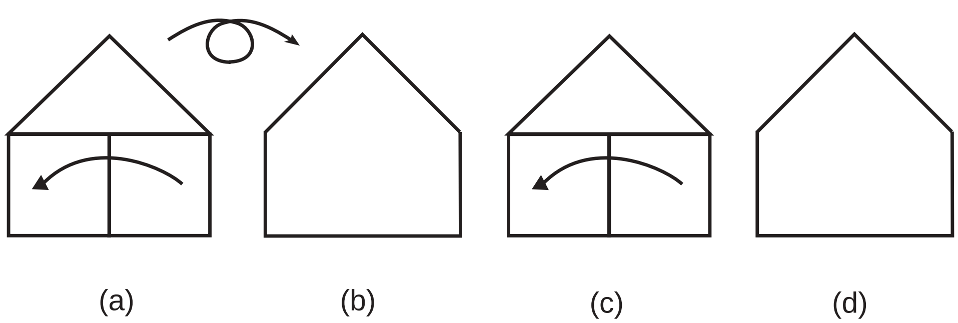
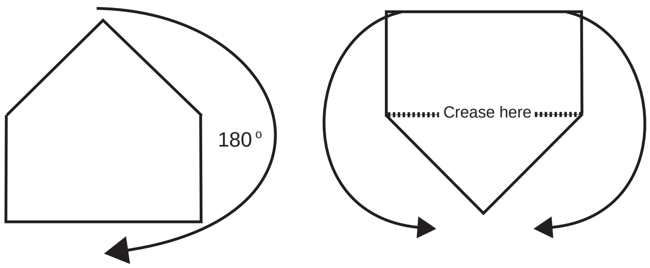


Fig. 14

Rotate the pentagon 180 degrees, so that it appears as if you are looking at home plate. Fold one flap down the front, and crease along the line shown. Lift the dollar bill, and repeat with one flap on the back, making sure the point of "home plate" is toward you. See **figures 15a** and **b.**



(a) (b)

Fig. 15

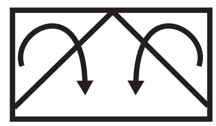
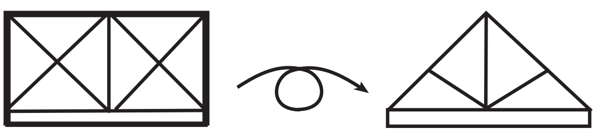


Fig. 16

Fold an upper-right and upper-left corner single thickness down to the center, as shown in **figure 16**. You can now see many isosceles right triangles. Where have you seen these before?

16. How many isosceles right triangles are there? ___________________________

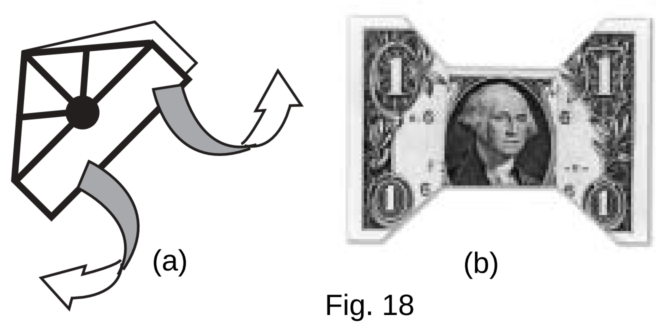
Flip the dollar bill over, and fold down the remaining two corners, as shown in **figures 17a** and **b.**



(a) (b)

Fig. 17

Now comes the tricky part; insert your forefingers inside the unit, on either side of the peaked section. Place your thumbs on the outside, where the dot appears on **figure 18a;** grasp firmly; and pull outward slowly and gently. Your result should be the "George" bowtie, displayed in **figure 18b.**



(a) (b)

Fig. 18

17. What kinds of symmetry are in this bowtie?

18. Compare these symmetries with the symmetries in the folding steps that you did before this final step.

Can you . . .

- explain why the four initial folds in the newspaper popper yielded four congruent isosceles right triangles?
- show why the isosceles right triangle is the only triangle that can be divided into two congruent parts, each of which is similar to the original triangle?
- find other applications of the isosceles right triangle in your origami constructions?
- fold an equilateral triangle?

Did you know . . .

- that the properties of the isosceles right triangle are fundamental to many different manipulations used in origami?

- that origami is an ancient art form and that it probably traces its origins to China at about the first or second century A.D. In Japan, its origins possibly go back to the sixth century A.D., when paper was first introduced to Japan from China?
- that Friedrich Froebel (1782–1852), the German educator and inventor of kindergarten, used origami with children because he believed that it helped their manual dexterity and increased their familiarity with geometric shapes?
- that Peter Engel has found that if many of the more complex origami models are unfolded, the pattern of folds in the paper displays fractal self-similarity?
- that an entire genre of origami uses only the dollar bill?

Mathematical content

Spatial visualization and various aspects of geometry, including symmetry, congruence, similarity, and classification of polygons

Bibliography

Brosterman, Norman. *Inventing Kindergarten.* New York: Harry N. Abrams, 1997.

Engel, Peter. *Origami from Angelfish to Zen.* New York: Dover Publications, 1994.

Johnson, Anne Akers. *The Buck Book*. Palo Alto, Calif.: Klutz Press, 1993.

Kenneway, Eric. *Complete Origami*. New York: St. Martin's Press, 1987.

Ohashi, Koya. "The Roots of Origami and Its Cultural Background." In *Abstracts of the Second International Meeting of Origami Science and Scientific Origami.* Otsu, Japan: Seian University of Art and Design, 1994.

Temko, Florence, and V'Ann Cornelius. *Money Folding*. Torrance, Calif.: Heian Press, 1995.

Answers

1. 13.5 inches by 23 inches; 34.25 centimeters by 58 centimeters
2. No; the ratios of the lengths to the widths are different in the two rectangles.
3. The area is approximately 310.5 square inches, or 1986.5 square centimeters.
4. You folded isosceles right triangles; all of them are similar, and some are congruent.
5. They are all the same shape; the similar, small triangles are congruent, and the large triangles are double the size of the small triangles.
6. They are 45 degrees.
7. It is a hexagon.
8. No; it is neither equilateral nor equiangular.
9. It is a pentagon.
10. No; it is neither equilateral nor equiangular. The angles measure either 90 degrees or 135 degrees.
11. Answers will vary, but the polygon might remind you of a house, a short pencil, home plate, and so on.
12. They are pentagons with a line of reflection.
13. It is a quadrilateral; or more specifically, it is a right trapezoid.
14. A square is made up of two congruent isosceles right triangles.
15. Yes; both the length and the width of the original bill were halved, thus giving this new rectangle the same ratio of length to width that the original rectangle had.
16. The number of isosceles right triangles is eighteen.
17. The bowtie has 180 degree, or half-turn, rotational symmetry and two lines of reflectional symmetry.
18. Most of the folding steps before the final "bowtie" show reflectional symmetry and at most one line of symmetry.

NCTM STUDENT MATH NOTES is published exclusively on the NCTM Web site, www.nctm.org, by the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191. The five issues a year appear in September, November, January, March, and May. Pages may be reproduced for classroom use without permission.