

NCTM 2018 Washington DC

Alternative Algorithms and Place Value Understanding

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Convention Center Room 207 A --Thursday, April 26th 11:30

Place Value Assessment by M. Burns

* **Cena** _ (00-36, 1:08-1:25, 3:21-Jonathan)

[https://www.youtube.com/watch?](https://www.youtube.com/watch?v=_ofQ_WnQiZ4&list=PL1xniZYEEnMdjlD7OkuPzRbhkRMMWzOuiu&index=3)

[v=_ofQ_WnQiZ4&list=PL1xniZYEEnMdjlD7OkuPzRbhkRMMWzOuiu&index=3](https://www.youtube.com/watch?v=_ofQ_WnQiZ4&list=PL1xniZYEEnMdjlD7OkuPzRbhkRMMWzOuiu&index=3)

* **Jonathan** – (after whole class 3:18-4:17 “probably”)

<https://www.youtube.com/watch?v=1puQxclB2aw>

Jacey – first grade

43

+15

Place Value Assessments

- *Counting Skills
- *One More and Ten More, One Less and Ten Less
- *Digit Correspondence
- *Using Tens
- *Using Groups of 10

Counting Skills

- * Count forward starting at 87
- * Count backward starting at 45
- * Count by tens
- * Count by tens starting at 36
- * Count backwards by tens starting at 142

One More and Ten More One Less and Ten Less

- * Student reads a number such as 352
- * Student writes the number that is 1 more
- * Student writes the number that is 10 more

- * One less and ten less is checked the same way

Digit Correspondence

- * Dump out 24 blocks
- * Student counts and writes the number
- * Teacher points to the 4 and asks, “Does this part of your 24 have anything to do with how many blocks there are?”
- * Teacher repeats by pointing to the 2...

Levels of understanding based on digit correspondence task

- *Single number
- *Position names
- *Face value - Cena & Jonathan
- *Transition to place value
- *Full understanding

Place Value Understanding Starts with Four Number Relationships

- * One and Two More, One and Two Less
- * Anchors of 5 and 10
- * Part – Part – Whole
- * Spatial Relationships (and subitizing)

14 “Hard” Addition Facts 28 “Hard” Subtraction Facts

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Number
Relationship
Facts are
represented in
color, remaining
addition and
subtraction
problems are
the white boxes.

Make Ten Strategy based on Place Value

*Ten Frames

*Rekenreks

Ten Frames

***[https://illuminations.nctm.org/
activity.aspx?id=3565](https://illuminations.nctm.org/activity.aspx?id=3565)**



Games

1. How many?

2. Build

3. Fill

4. Add

5. Play all

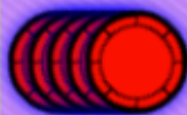
Use frames to find $8 + 5$

next

Bank

5

1



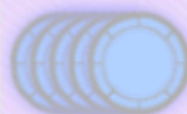
Move 8

Done

Bank

5

1



Move 5

Done



Games

1. How many?

2. Build

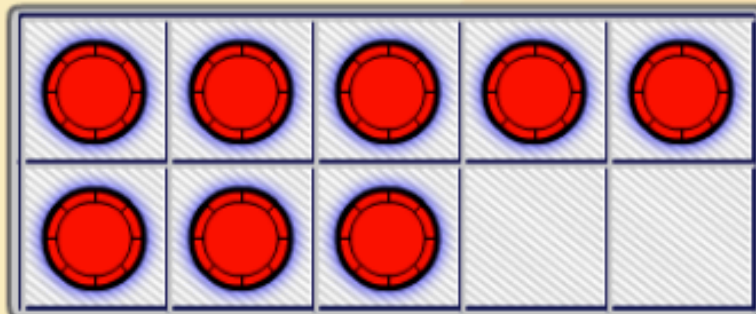
3. Fill

4. Add

5. Play all

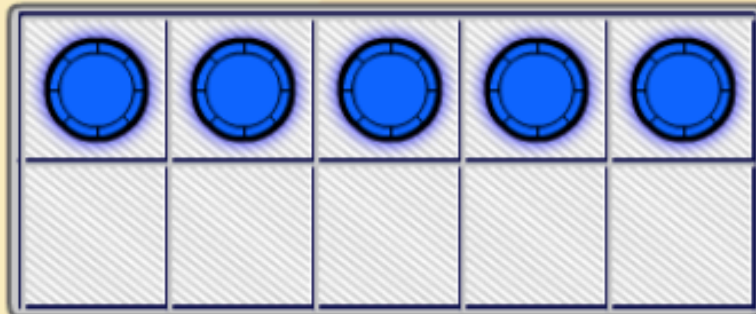
Use frames to find $8 + 5$

next



Done

Now combine  and 





Games

1. How many?

2. Build

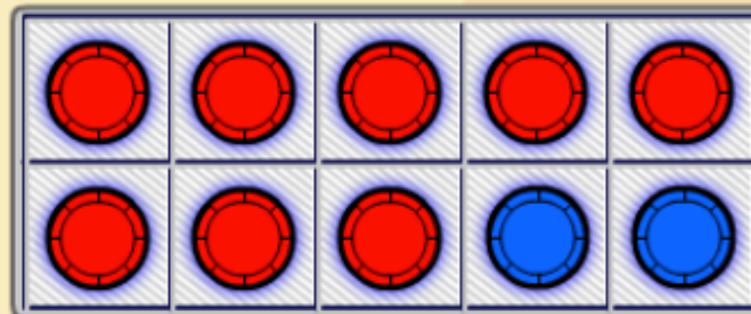
3. Fill

4. Add

5. Play all

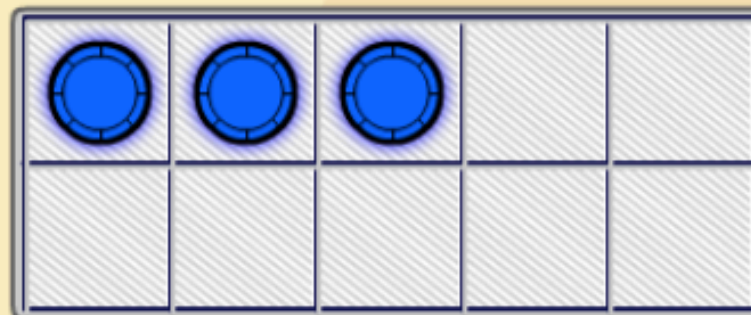
Use frames to find $8 + 5$

next



Done

Now combine  and 

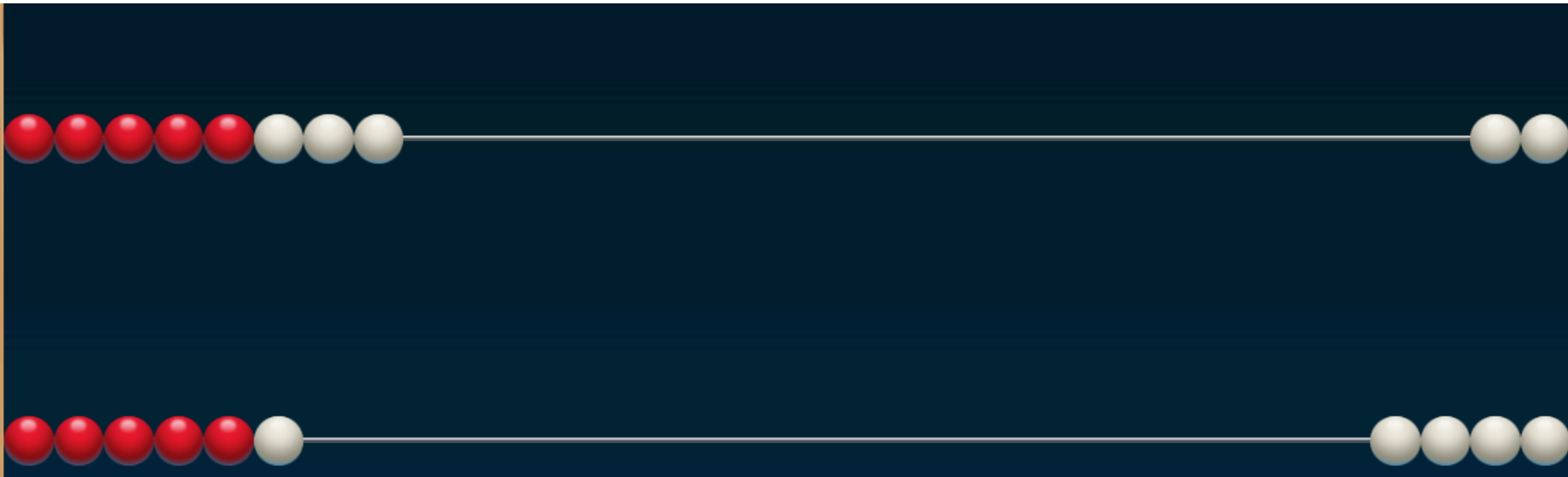


$8 + 5$ becomes $10 + 3$

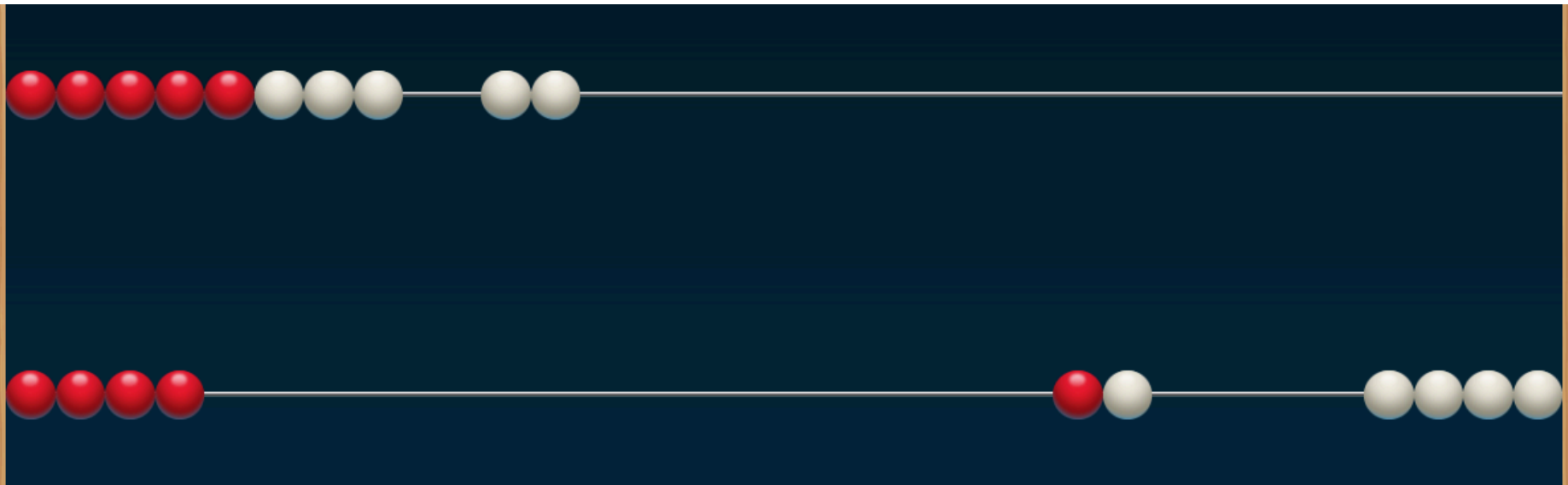
Rekenrek

$$8 + 6$$

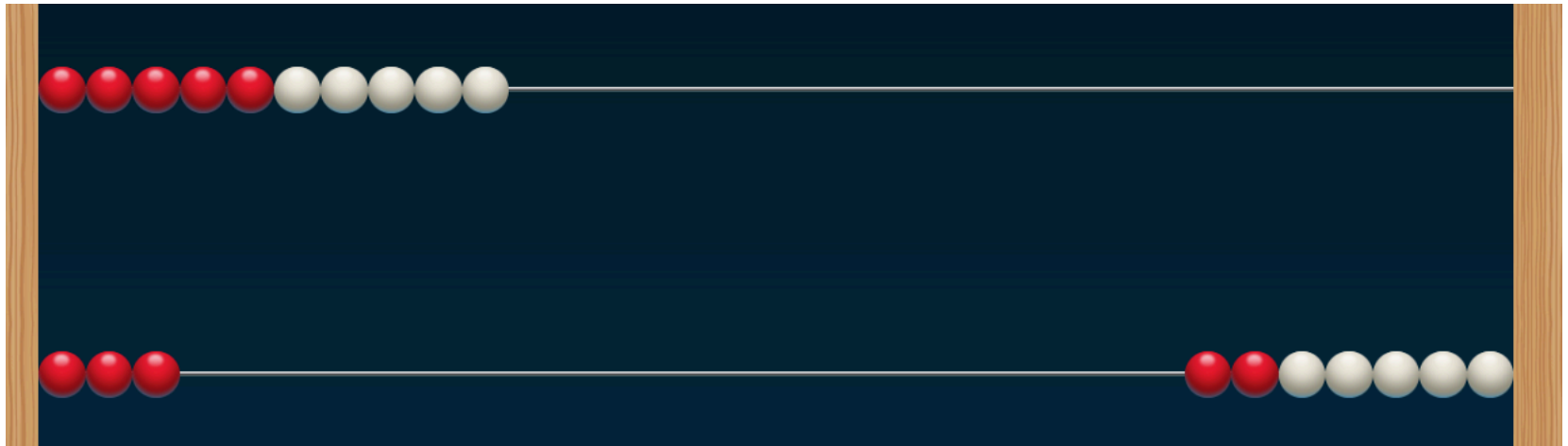
* <https://apps.mathlearningcenter.org/number-rack/>



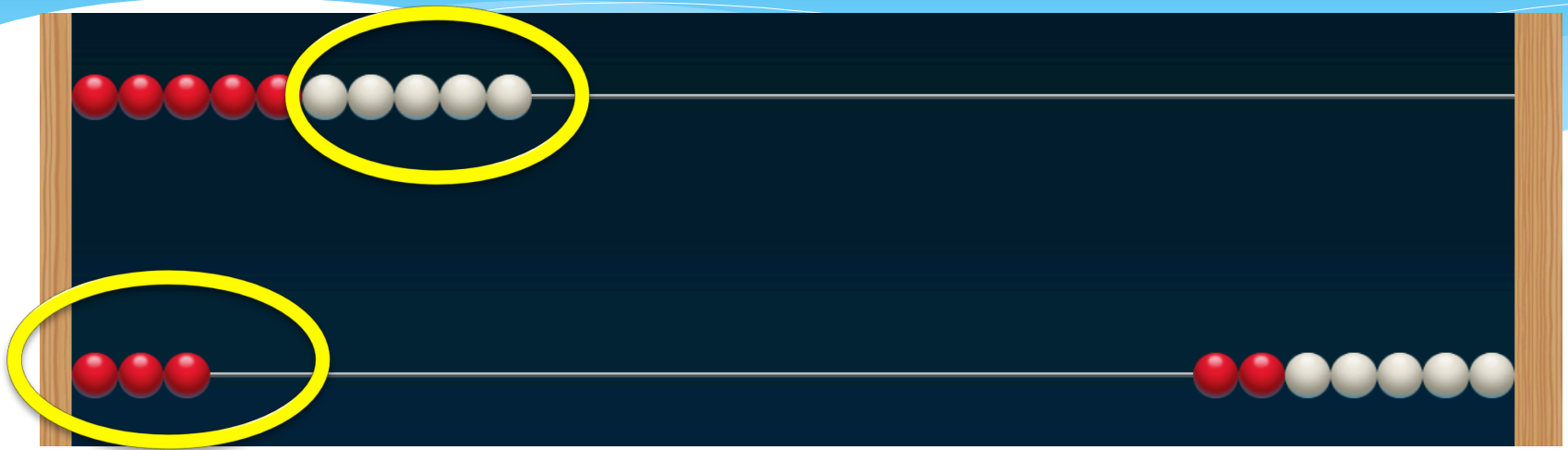
$8 + 6$ becomes $10 + 4$



Take Away Subtraction: $13 - 8$



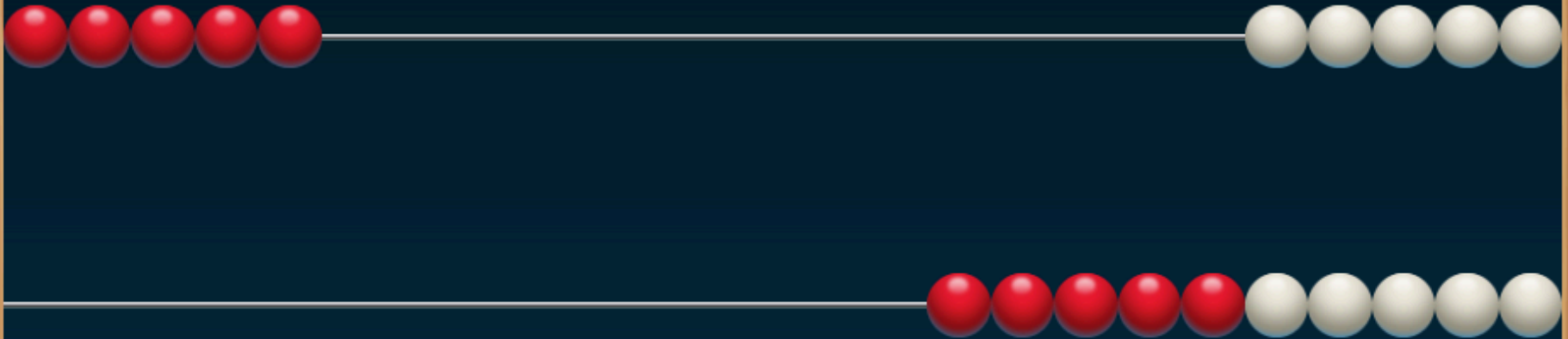
Take Away Subtraction: $13 - 8$



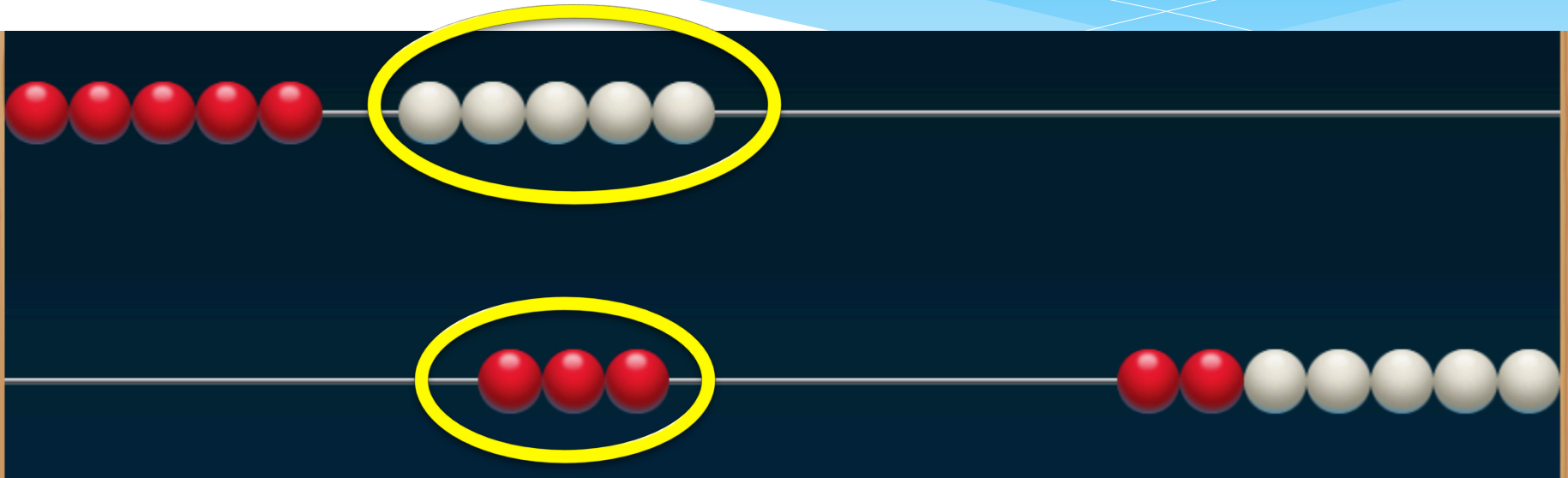
13 take away 3 (of the 8) to get to 10
take away 5 more (of the 8) and 5 are left

$$13 - 3 = 10 \text{ and } 10 - 5 = 5$$

Subtraction by Counting Up: $13 - 5$



Subtraction by Counting Up: $13 - 5$

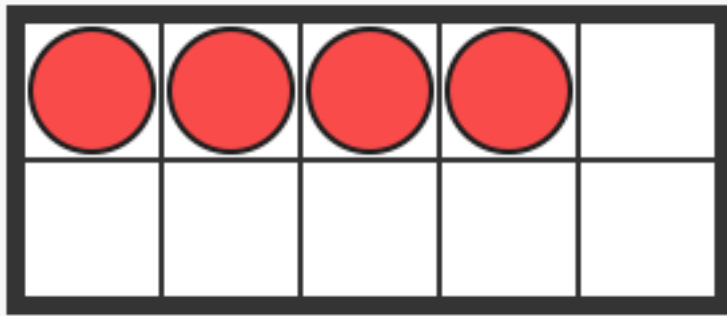
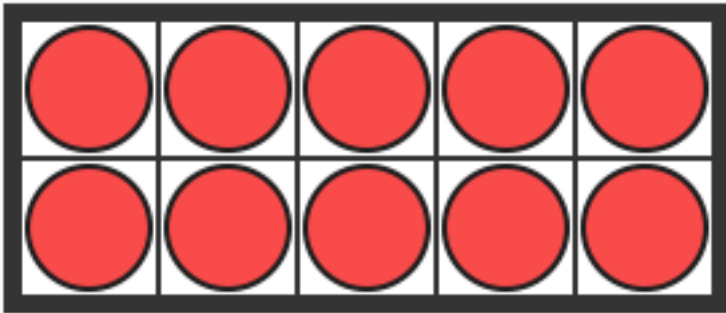


Start with 5 and add 5 more to get to 10

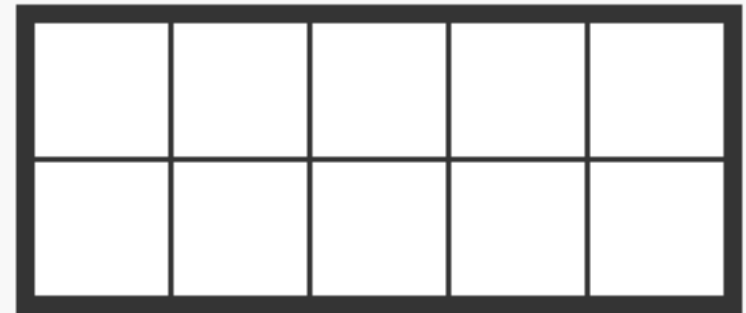
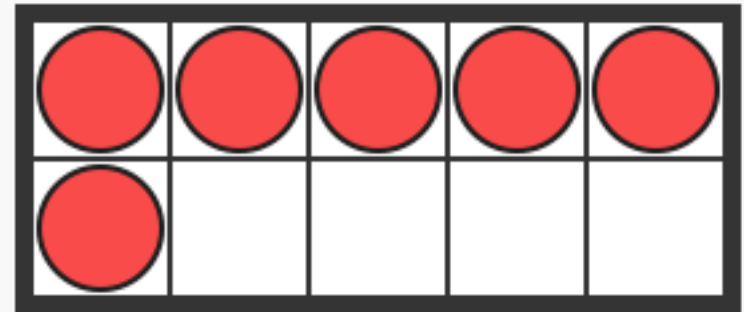
Then add 3 more to get to 13

How many did you add to get from 5 to 13?

Same with Ten Frames: 14 - 6

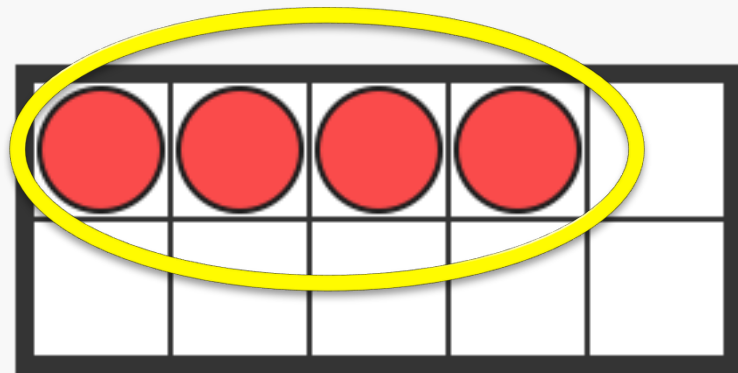
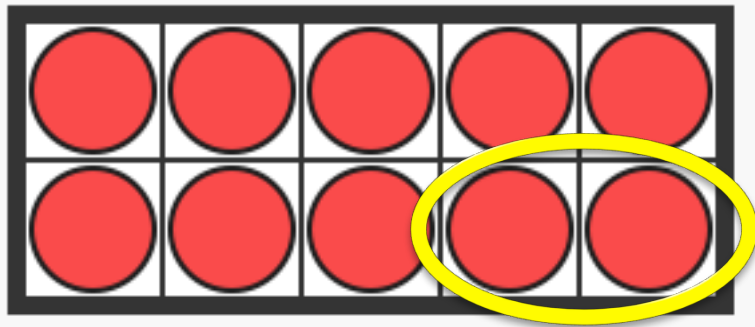


Take Away

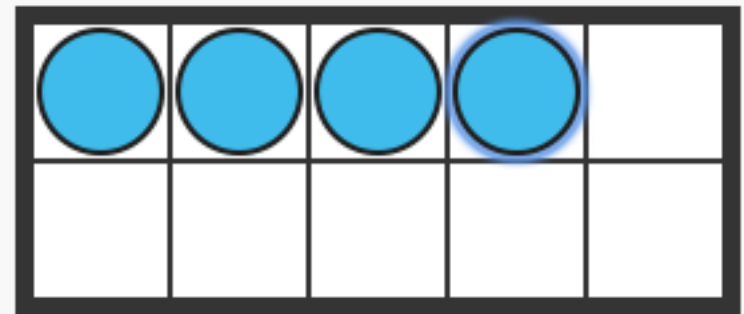
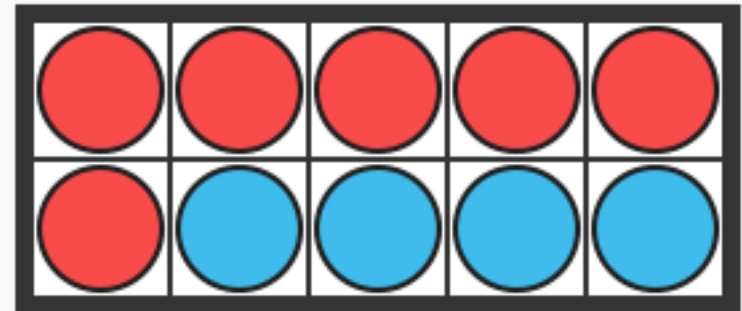


Count up...

Same with Ten Frames: 14 - 6

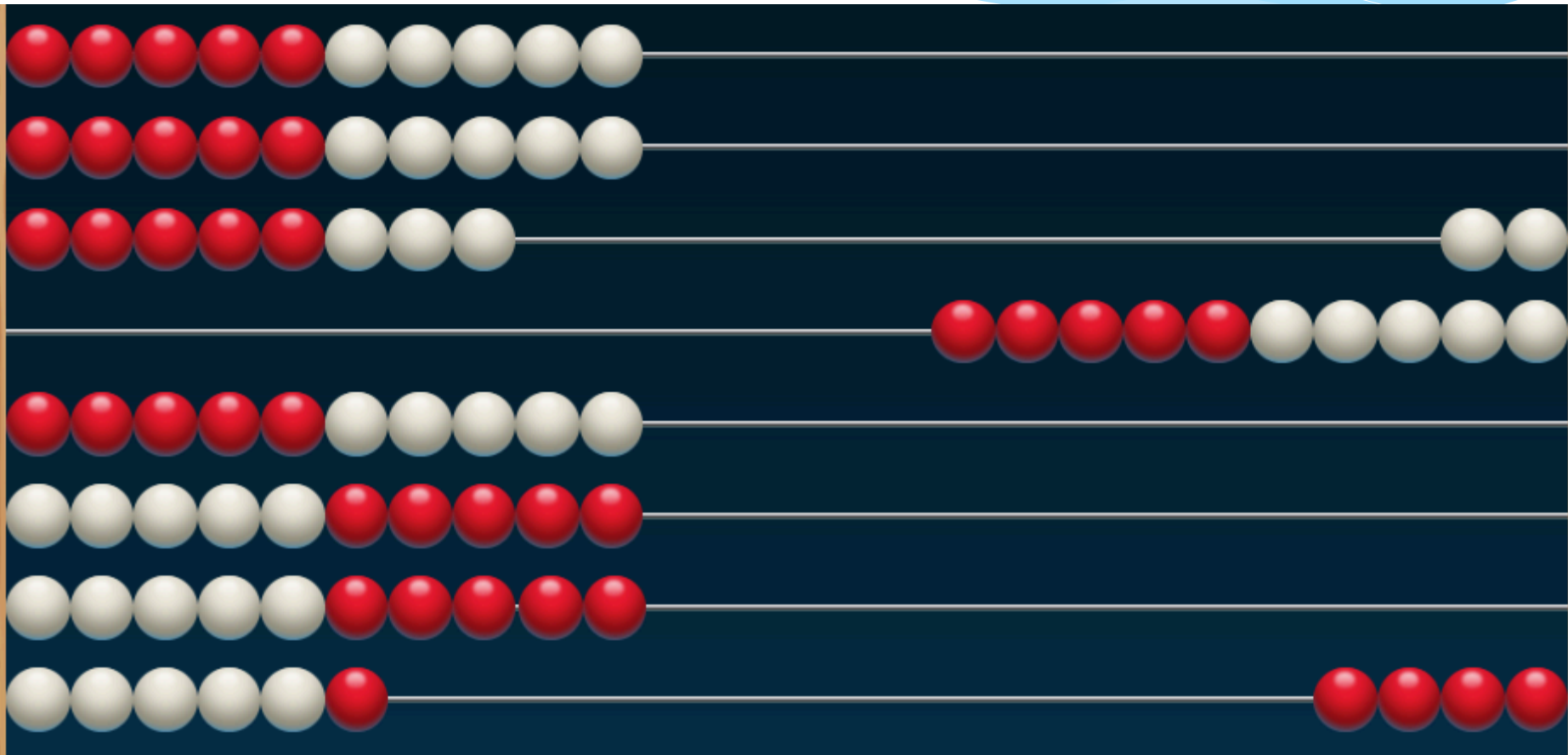


Take Away



Count up...

$$28 + 36$$



2nd Grade Number Talk (Math Solutions)

$$16 + 15$$

<https://www.youtube.com/watch?v=SPEfxPgZJy4>

Search “Number Talks” for your grade level examples

Invented Strategies for Addition

$$46 + 38$$

Invented Strategies for Addition

$$46 + 38$$

*Add Tens, Add Ones, Then
Combine

$$40 + 30 = 70, 6 + 8 = 14$$

$$70 + 14 = 84$$

Invented Strategies for Addition

$$46 + 38$$

*Add on Tens, Then Add Ones

$$46 + 30 = 76$$

$$76 + 8 = 76 + 4 + 4 = 84$$

Invented Strategies for Addition

$$46 + 38$$

***Move some to make tens**

Take 2 from the 46 and put it on the 38 so the problem becomes

$$44 + 40 = 80$$

Invented Strategies for Addition

$$46 + 38$$

*Use a nice number and compensate

Add 2 onto the 38 to make 40

$$46 + 40 = 86$$

Now take the 2 back off $86 - 2 = 84$

Invented Strategies for Subtraction

using Take Away $73 - 46$

*Take away tens, then ones

$$73 - 40 = 33$$

$$33 - 3 = 30$$

$$30 - 3 = 27$$

Jacey – 3rd Grade

43

-5

43

-5

³~~4~~3

-5

Invented Strategies for Subtraction using Take Away $73 - 46$

*Take extra tens, then add back

$$73 - 50 = 23 \quad (\text{took away 4 extra})$$

$$23 + 4 = 27 \quad (\text{add those 4 back})$$

Invented Strategies for Subtraction

using Take Away $73 - 46$

*Shift the number line

$73 - 46$ could be shifted up 4
since the distance stays the same

$$77 - 50 = 27$$

Invented Strategies for Subtraction

by Counting Up.... $73 - 46$

*Add tens to get close, then ones

$$46 + 20 = 66$$

$$66 + 4 = 70$$

$$70 + 3 = 73$$

$$20 + 4 + 3 = 27$$

Invented Strategies for Subtraction

by Counting Up.... $73 - 46$

*Add tens to overshoot, then come back

$$46 + 30 = 76$$

$$76 - 3 = 73$$

$$30 - 3 = 27$$

Invented Strategies for Subtraction

by Counting Up.... $73 - 46$

*Add ones to make a ten, then
tens and ones

$$46 + 4 = 50$$

$$50 + 20 = 70$$

$$70 + 3 = 73$$

2nd Grader: 5×14

- *5 tens is 50
- *Two 4s would be 8
- *8 and 8 is 16
- *And one more 4 is 20
- *50 and 20 makes 70

References

- * Online Number Rack (Rekenrek)
<https://apps.mathlearningcenter.org/number-rack/>
- * Online Number Frames (Ten Frames)
<https://apps.mathlearningcenter.org/number-frames/>
- * Burns, M., Assessment Videos
- * Parrish, S., [Number Talks](#) by Math Solutions
- * Van De Walle, J., [Elementary and Middle School Mathematics: Teaching Developmentally](#)