Developing Mathematical Fluency:
What is it?
How do we create systems to support it?
Grades 3 - 5

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Table Introductions

▷ Introduce yourself to the people at your table (name, district, role).

▷ Discuss: What are you hoping to learn about fluency today?
Session Learning Goals

Develop an understanding of what fluency is and ways to develop & assess it.

Learn about ways to value and use student thinking to guide fluency instruction.

Explore ways that systems can be created to support work with fluency.

1. What is fluency?
Procedural fluency is defined as “skill in carrying out procedures flexibly, accurately, efficiently and appropriately” (CCSSM, 2010, p. 6).

What is fluency?

▷ Select a paragraph (1-4) to read from NCTM’s position on procedural fluency.

▷ Be prepared to share a quote that stands out to you.
Two Ways to Learn Basic Facts

- Strategies
- Memorization

From "Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts" by Jo Boaler, 2015, p. 3
Knowing from Memory ≠ Memorization

Fluency Across the Grade Levels

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
</tr>
</tbody>
</table>
Why focus on fluency?

To support each and every student in accessing rich mathematics content.

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2.

How can we develop and assess fluency?
“In meaningful learning of basic number combinations, students progress through well-documented phases toward fluency (Baroody 2006; Baroody, Bajwa, and Eiland 2009; Carpenter et al. 1999). Students begin by using objects, visual representations, and verbal counting, and then they progress to reasoning strategies using number relationships and properties.”

Principles to Actions: Ensuring Mathematical Success for All, NCTM, 2014.

Phases of Fluency

<table>
<thead>
<tr>
<th>Phase 1 Counting</th>
<th>Phase 2 Deriving</th>
<th>Phase 3 Mastery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counts or counts on with objects or mentally</td>
<td>Reasoning strategies based on known facts</td>
<td>Efficient production of answers</td>
</tr>
</tbody>
</table>

Adapted from Baroody, 2006
## Developing Fluency

<table>
<thead>
<tr>
<th>Number Talks &amp; Routines</th>
<th>Games</th>
<th>Meaningful Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>● Dot Images/Quick Images</td>
<td>● Targeted Practice</td>
<td>● Story Problems</td>
</tr>
<tr>
<td>● Computational problems</td>
<td>(focus on a particular group of facts)</td>
<td>● Contextual problems</td>
</tr>
<tr>
<td>● Number strings</td>
<td>● General Practice</td>
<td>● Visual representations</td>
</tr>
<tr>
<td>● True/False Equations</td>
<td>(all facts for a particular operation)</td>
<td>(Ten Frames, Arrays)</td>
</tr>
<tr>
<td>● Relational Thinking</td>
<td>● Computer games/Apps</td>
<td>● Multiple Representations</td>
</tr>
<tr>
<td>● Incredible Equations</td>
<td></td>
<td>● Fact Relationships</td>
</tr>
<tr>
<td>● Counting Routines</td>
<td></td>
<td>(Triangle Flash Cards, Multiple Representations)</td>
</tr>
</tbody>
</table>

### Math Cards

- Lay cards face down.
- Take turns picking cards.
- You can continue picking up cards that have the same solution (shown through any representation).
- Explain how you know that the different cards are equivalent.
Task Debrief

▷ In what ways does this task develop fluency?
▷ What mathematical ideas and connections can be strengthened through this task?
Task Debrief

▷ In what ways does this task develop fluency?
▷ What mathematical ideas and connections can be strengthened through this task?

Teacher & Student Actions

Describe teacher actions that develop student understanding and fluency.

Describe student actions that support their development of fluency.
### Build procedural fluency from conceptual understanding

**Teacher and student actions**

<table>
<thead>
<tr>
<th>What are teachers doing? What are students doing?</th>
<th>What are teachers doing? What are students doing?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Providing students with opportunities to use their own reasoning strategies and methods for solving problems.</td>
<td>Making sure that they understand and can explain the mathematical basis for the procedures that they are using.</td>
</tr>
<tr>
<td>Asking students to discuss and explain why the procedures that they are using work to solve particular problems.</td>
<td>Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.</td>
</tr>
<tr>
<td>Connecting student-generated strategies and methods to more efficient procedures as appropriate.</td>
<td>Determining whether specific approaches generalize to a broad class of problems.</td>
</tr>
<tr>
<td>Using visual models to support students’ understanding of general methods.</td>
<td>Striving to use procedures appropriately and efficiently.</td>
</tr>
<tr>
<td>Providing students with opportunities for distributed practice of procedures.</td>
<td></td>
</tr>
</tbody>
</table>

*Principles to Actions: Ensuring Mathematical Success for All, NCTM, 2014.*

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**Too often we hear that the kids don’t know their basic facts. . .**

Instead ask: Which kids? & Which facts?
Assessing Fluency

▷ Observation
▷ Interviews
▷ Journal/Writing prompts
▷ Strategy quizzes
▷ Self-assessment

Phases of Fluency

Phase 1
Counting
Student uses counting strategies to solve. The student counts or counts on with objects or mentally.

Phase 2
Deriving
Student uses reasoning strategies to solve. The student uses strategies based on known facts to solve for a given problem.

Phase 3
Mastery
Student knows the fact from memory or uses a procedure efficiently and accurately to solve for an answer.

Adapted from Baroody, 2006
Student Work 9 x 7

▷ Discuss each student’s thinking/strategy.
▷ What questions would you want to ask these students to learn more about their thinking?
▷ What evidence of the phases do you see?

Student Work 35 ÷ 5

▷ Discuss each student’s thinking/strategy.
▷ What questions would you want to ask these students to learn more about their thinking?
▷ What evidence of the phases do you see?
Based on the student work that you have analyzed, what would your next steps with students be and why?

3. How do we create systems to support fluency?

Think about how your system supports fluency.
Creating a System to Support Fluency

1. Shifting Beliefs
2. Changing Practices
3. Using Evidence of Student Thinking
4. Aligning Efforts Across the System
Changing Practices

Using Evidence of Student Thinking

How do we know what they’re thinking?
Aligning Efforts Across the System

Fluency Preview

Fluency Pilot

Fluency Professional Learning

Fluency Forums & Support

Site-Coach PL & Support

Fluency Standards to Develop & Assess

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<th>Grade</th>
<th>Fluency Standards to Develop &amp; Assess</th>
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<tbody>
<tr>
<td>3</td>
<td>3.OA.7 Multiplication</td>
</tr>
<tr>
<td></td>
<td>3.OA.7 Division</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2 Fluently add/subtract within 1000</td>
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<td>4</td>
<td>3.OA.7 Multiplication, 3.OA.7 Division</td>
</tr>
<tr>
<td></td>
<td>4.NBT.4 Add and Subtract</td>
</tr>
<tr>
<td>5</td>
<td>3.OA.7 Multiplication, 3.OA.7 Division</td>
</tr>
<tr>
<td></td>
<td>5.NBT.5 Multiply Multi-Digit Numbers</td>
</tr>
</tbody>
</table>
### Mathematical Fluency

<table>
<thead>
<tr>
<th>Standard</th>
<th>Trimester</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3.OA.7 Fluently multiply within 100. Fluently Divide within 100.</strong></td>
<td>1 2 3</td>
</tr>
<tr>
<td>Phase 1 – Student multiplies by counting objects, counting mentally, or skip counting.</td>
<td></td>
</tr>
<tr>
<td>Phase 2 – Student multiplies using a strategy such as friendly numbers, decomposing/composing or using a known fact.</td>
<td></td>
</tr>
<tr>
<td>Phase 3 – Student can easily recall the division fact from memory.</td>
<td></td>
</tr>
<tr>
<td><strong>4.NBT.4 Fluently add and subtract multi-digit numbers using the standard algorithm.</strong></td>
<td>1 2 3</td>
</tr>
<tr>
<td>Phase 1 – Student adds or subtracts multi-digit numbers by counting objects or counting mentally.</td>
<td></td>
</tr>
<tr>
<td>Phase 2 – Student adds or subtracts multi-digit numbers by using strategies such as counting up, and/or composing/decomposing.</td>
<td></td>
</tr>
<tr>
<td>Phase 3 – Student adds or subtracts multi-digit numbers by using the standard algorithm.</td>
<td></td>
</tr>
</tbody>
</table>

**What ideas do you have for supporting fluency across your system?**
Mathematical Fluency

▷ Revisit your definition of mathematical fluency.
▷ Add to, revise, or adjust your definition based on what you have learned.

Session Reflection

What is something you are going to stop doing?
What is something you want to learn more about?
What is something you want to start doing?
Session Reflection

What is something you are going to **stop** doing?
What is something you want to **learn** more about?
What is something you want to **start** doing?

Identify 1-2 next steps that you will take based on your learning today.

Session Learning Goals

**Develop an understanding of what fluency is and ways to develop & assess it.**

Learn about ways to value and use student thinking to guide fluency instruction.

Explore ways that systems can be created to support work with fluency.
Thank you and enjoy the NCTM Annual Conference!

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@mathschristine

Resources

• NCTM’s Procedural Fluency in Mathematics, A Position of the National Council of Teachers of Mathematics. 2014.
• Mastering the Basic Math Facts in Multiplication and Division by Susan O’Connell and John SanGiovanni.
• Smarter Balanced Fluency Module created by Illustrative Mathematics, https://www.smarterbalancedlibrary.org/content/illustrative-mathematics-fluency-module.
Resources

• “Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.


Resources

• Building Conceptual Understanding and Fluency Through Games for the CCSSM, North Carolina Department of Public Instruction, http://maccss.ncdpi.wikispaces.net/Elementary

• Intentional Talk: How to Structure and Lead Productive Mathematical Discussions by Allison Hintz and Elham Kazemi

• Slide Carnival Templates, www.slidescarnival.com/help-use-presentation-template
Self-Assessment & Self-Monitoring

Self Assessment Index Card/Journal Prompt
▷ Which facts are easy for you?
▷ Which facts are difficult for you?
▷ What is one fact that you would like to work on improving?

Self Monitoring
▷ Fluency folder
▷ Individual fluency graph/log