Developing Mathematical Fluency:
What is it? and How do we create systems to support it?

Christine Roberts
christine.roberts@tcoe.org
@mathschristine

What is math fact fluency?

Initial Definition	Revised Definition

Phase 1 –

Phase 2 –

Phase 3 –

Fluency Standards in Grades K - 6

Grade	Fluency Standard
K	K.OA.5 Fluently add and subtract within 5.
1	1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
2	2.OA.2 Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers. (Footnote: 2. See standard 1.OA.6 for a list of mental strategies.) 2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
3	3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. 3.NBT.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
4	4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5	5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.
6	6.NS.2 Fluently divide multi-digit numbers using the standard algorithm. 6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

National Governors Association Center for Best Practices, Council of Chief State School Officers Common Core State Standards for Mathematics National Governors Association Center for Best Practices, Council of Chief State School Officers, Washington D.C.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Procedural Fluency in Mathematics

A Position of the National Council of Teachers of Mathematics

Question

What is procedural fluency, and how do we help students develop it?

NCTM Position

Procedural fluency is a critical component of mathematical proficiency. Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. To develop procedural fluency, students need experience in integrating concepts and procedures and building on familiar procedures as they create their own informal strategies and procedures. Students need opportunities to justify both informal strategies and commonly used procedures mathematically, to support and justify their choices of appropriate procedures, and to strengthen their understanding and skill through distributed practice.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures. Although conceptual knowledge is an essential foundation, procedural knowledge is important in its own right. All students need to have a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations (NRC, 2001, 2005, 2012; Star, 2005).

In computation, procedural fluency supports students' analysis of their own and others' calculation methods, such as written procedures and mental methods for the four arithmetic operations, as well as their own and others' use of tools like calculators, computers, and manipulative materials (NRC, 2001). Procedural fluency extends students' computational fluency and applies in all strands of mathematics. For example, in algebra, students develop general equation-solving procedures that apply to classes of problems and select efficient procedures to use in solving specific problems. In geometry, procedural fluency might be evident in students' ability to apply and analyze a series of geometric transformations or in their ability to perform the steps in the measurement process accurately and efficiently.

Procedural fluency builds from an initial exploration and discussion of number concepts to using informal reasoning strategies and the properties of operations to develop general methods for solving problems (NCTM, 2014). Effective teaching practices provide experiences that help students to connect procedures with the underlying concepts and provide students with opportunities to rehearse or practice strategies and to justify their procedures. Practice should be brief, engaging, purposeful, and distributed (Rohrer, 2009). Too much practice too soon can be ineffective or lead to math anxiety (Isaacs & Carroll, 1999). Analyzing students' procedures often reveals insights and misunderstandings that help teachers in planning next steps in instruction. In the same way, worked examples can serve as a valuable instructional tool, permitting teachers to understand how students analyze why procedures work or don't work and consider what procedure might be most appropriate in a given situation (Booth, Lange, Koedinger, & Newton, 2013).

References and Additional Resources

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction, 25*, 24–34.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30*(1), 3–19.

Isaacs, A. C., & Carroll, W. M. (1999). Strategies for basic-facts instruction. *Teaching Children Mathematics, 5*(9), 508–515.

King, G., & Bay-Williams, J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics, 20*(8), 488–497.

Lange, K. E., Booth, J. L., & Newton, K. J. (2014). Learning algebra from worked examples. *Mathematics Teacher, 107*(7), 535–540.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics. Common core state standards (college- and career-readiness standards and K–12 standards in English language arts and math).* Washington, DC: Author. http://www.corestandards.org.

National Research Council (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.

National Research Council. (2005). *How students learn: History, mathematics, and science in the classroom*. Washington, DC: National Academies Press.

National Research Council. (2012). *Education for life and work: Developing transferable knowledge and skills for the 21st century*. Washington, DC: National Academies Press.

Rohrer, D. (2009). The effects of spacing and mixed practice problems. *Journal for Research in Mathematics Education, 40*(1), 4–17.

Russell, S. J. (2000). Developing computational fluency with whole numbers. *Teaching Childdren Mathematics, 7*(3), 154–158.

Star, J. R. (2005). Reconceptualizing conceptual knowledge. *Journal for Research in Mathematics Education, 36*(5), 404–411.

Fluency comes about when students develop number sense, when they are mathematically confident because they understand numbers.

Boaler, Jo. 2015. “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts.” https://www.youcubed.org/fluency-without-fear/.

Unfortunately, the word fluency is often misinterpreted.

Boaler, Jo. 2015. “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts.” https://www.youcubed.org/fluency-without-fear/.

Some students are not as good at memorizing math facts
as others. That is something to be celebrated;
it is part of the wonderful diversity of life and people.

Boaler, Jo. 2015. “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts.”
https://www.youcubed.org/fluency-without-fear/.

Students rarely cry about other subjects, nor do they believe that other subjects are all about memorization or speed.

Boaler, Jo. 2015. “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts.” https://www.youcubed.org/fluency-without-fear/.

When students work on rich mathematics problems, they develop number sense and they also learn and can remember math facts.

Boaler, Jo. 2015. "Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts." https://www.youcubed.org/fluency-without-fear/.

For about 1/3 of students, the onset of timed testing
is the beginning of math anxiety (as early as 5 years old).

Boaler, Jo. 2015. "Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts."
https://www.youcubed.org/fluency-without-fear/.

One of the methods for teaching number sense
and math facts at the same time
is a teaching strategy called “number talks”
developed by Ruth Parker and Kathy Richardson.

Boaler, Jo. 2015. “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts.”
https://www.youcubed.org/fluency-without-fear/.

Number sense is the foundation of all higher-level mathematics.

Boaler, Jo. 2015. “Fluency Without Fear: Research Evidence on the Best Ways to Learn Math Facts.” https://www.youcubed.org/fluency-without-fear/.

In fact, growing evidence suggests that timed testing has a negative impact on students (Boaler 2012, Henry and Brown 2008, Ramirez et al. 2013). Surprisingly, the anxiety that many children experience over timed testing is unrelated to how well they do on the tests.

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

Formative assessments—including observations, interviews, performance tasks, and journaling—have become common practice in many classrooms, with a recognition that by using different ways to assess children, we gain a more comprehensive, accurate picture of what they know, what they do not know, and their misconceptions. These data are then used to design instruction accordingly (Wiliam 2011).

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

Yet, in spite of this trend in other areas of education, timed, skill-based assessments continue to be the prevalent measure of basic mathematics facts achievement.

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

With an eye on the aspects of fluency (accuracy, efficiency, flexibility, and appropriate strategy selection), we can use various assessment strategies to see what students know (and do not know) and determine what our next instructional steps might be.

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

Using the range of assessments described above accomplishes these goals, as they provide an opportunity for meaningful, targeted feedback to students that far exceeds the “right or wrong, fast or slow” feedback provided by timed testing.

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

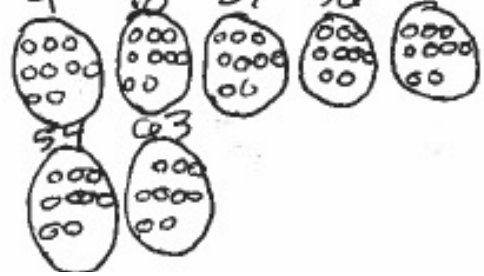
Interviews, journals, and quizzes on basic facts can and should encourage students to reflect on which facts and strategies they know well and which ones are tough for them. This self-assessment can be effectively followed up by having children identify and record strategies that could be used to efficiently determine the “tough” facts in the future. Over time, this self-assessment practice encourages children to instinctively apply effective strategies for challenging facts they encounter.

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

If timed mathematics assessments have questionable value and potentially negative psychological, emotional, and educational impact, why are they still so frequently used? We commonly hear three reasons. First, fluency is interpreted as synonymous with speed. We have already addressed that fluency is more comprehensive than speed. Second, some feel that timed tests prepare children for high-stakes tests. The research shared here convincingly shows it may do the opposite. Third, timed tests are the only assessments widely available for assessing fluency of basic facts.

“Developing and Assessing Fact Fluency,” Amanda Ruch and Gina Kling, and Gina Kling and Jennifer Bay-Williams, Presentation at NCTM 2015.

Student Work for 9 x 7

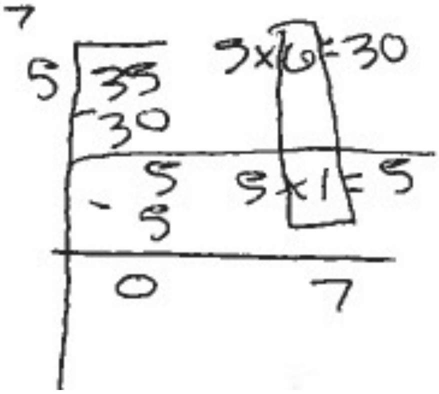
<table>
<tr>
<td>Student A
9 × 7 = ? 9+9+9+9+9+9+9=63</td>
<td>Student B
9 × 7 = ? 63

</td>
</tr>
<tr>
<td>Student C
9 × 7 = ? 63
in my head

Observational Notes: Student appeared to be counting.
T: What were you thinking when you solved this in your head?
S: I was skip counting.</td>
<td>Student D
9 × 7 = ? 9x7=63
in my head

Observational Notes: No action observed.
T: What were you thinking when you solved this in your head?
S: I read the problem and knew the answer right away in my head.</td>
</tr>
</table>

Note: Students also completed 7 x 4, 5 x 6, and 8 x 8 and the same strategies represented above were also evident. The accuracy rate for all problems was about the same.

Student Work for 35 ÷ 5

Student E 35 ÷ 5 = ? 7 	Student F 35 ÷ 5 = ? 5 x 7 = 35 Observational Notes: No action observed. T: What were you thinking when you solved this problem? S: I counted by fives until I got to 35. Then, I counted how many times it took me.
Student G 35 ÷ 5 = ? 7  T: What were you thinking when you solved this problem? S: I drew five circles and then put dots in them until I got to 35.	Student H 35 ÷ 5 = ? 35 ÷ 5 = 7 knew it Observational Notes: No action observed. T: What were you thinking when you solved this problem? S: I just knew the answer. T: Did you skip count or use another strategy? S: No, because I knew it automatically.

Note: Students also completed 32 ÷ 4 and the same strategies represented above were also evident; however, there was a lower accuracy rate.