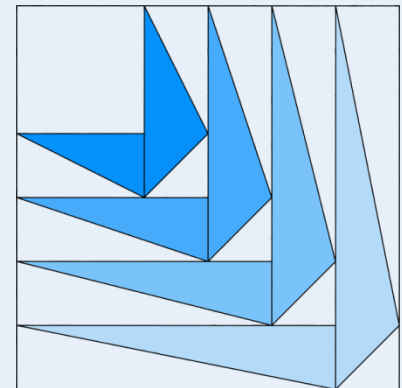


# Origami Boxes

Arsalan Wares

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**If people never did silly  
things nothing intelligent  
would ever get done.**

**Ludwig  
Wittgenstein**

**1889-1951  
Austrian-British  
Philosopher**

**I was always very scientific  
about how I did my art, and  
I was always very artistic  
about how I did my science.**

**Robert Lang**

**American Physicist and Artist**

Origami  
(from *ori* meaning *folding*, and *kami* meaning *paper*)

Origami is the Japanese art of paper folding.

It started in the 17th century AD and was popularized in the mid-1900s.

In 1930 Akira Yoshizawa, a Japanese origami artist/writer, comes up with a way of illustrating the steps. This revitalized origami throughout the world.

My definition of origami...

Origami is mathematics in action.

In origami the goal is to turn a flat piece of paper into a three dimensional sculpture.

Cutting and gluing are not acceptable.

Traditionally a square sheet of paper is used.

But it is okay to break this rule!

Boxes made from rectangular sheets can be very interesting because of two variables involved, length and width.

Rectangular sheets are more readily available.

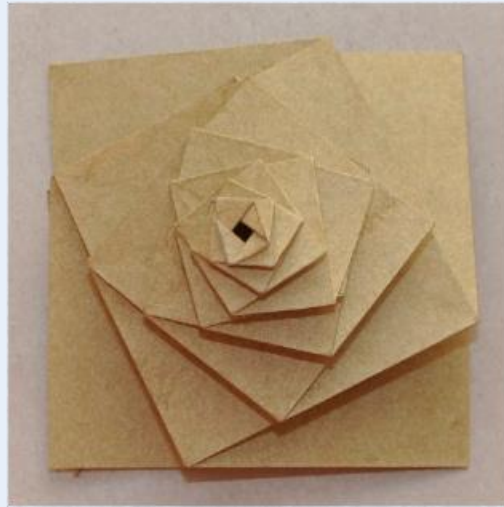
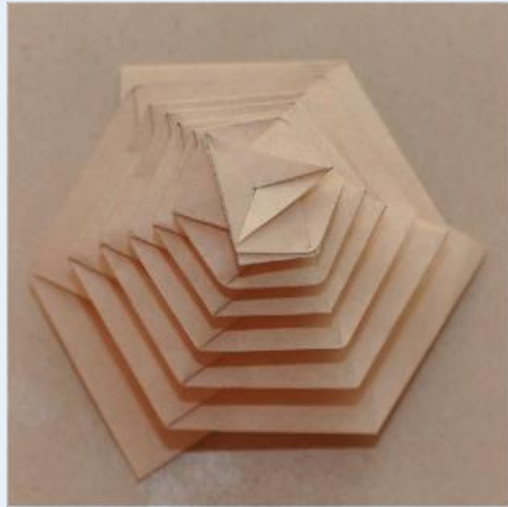




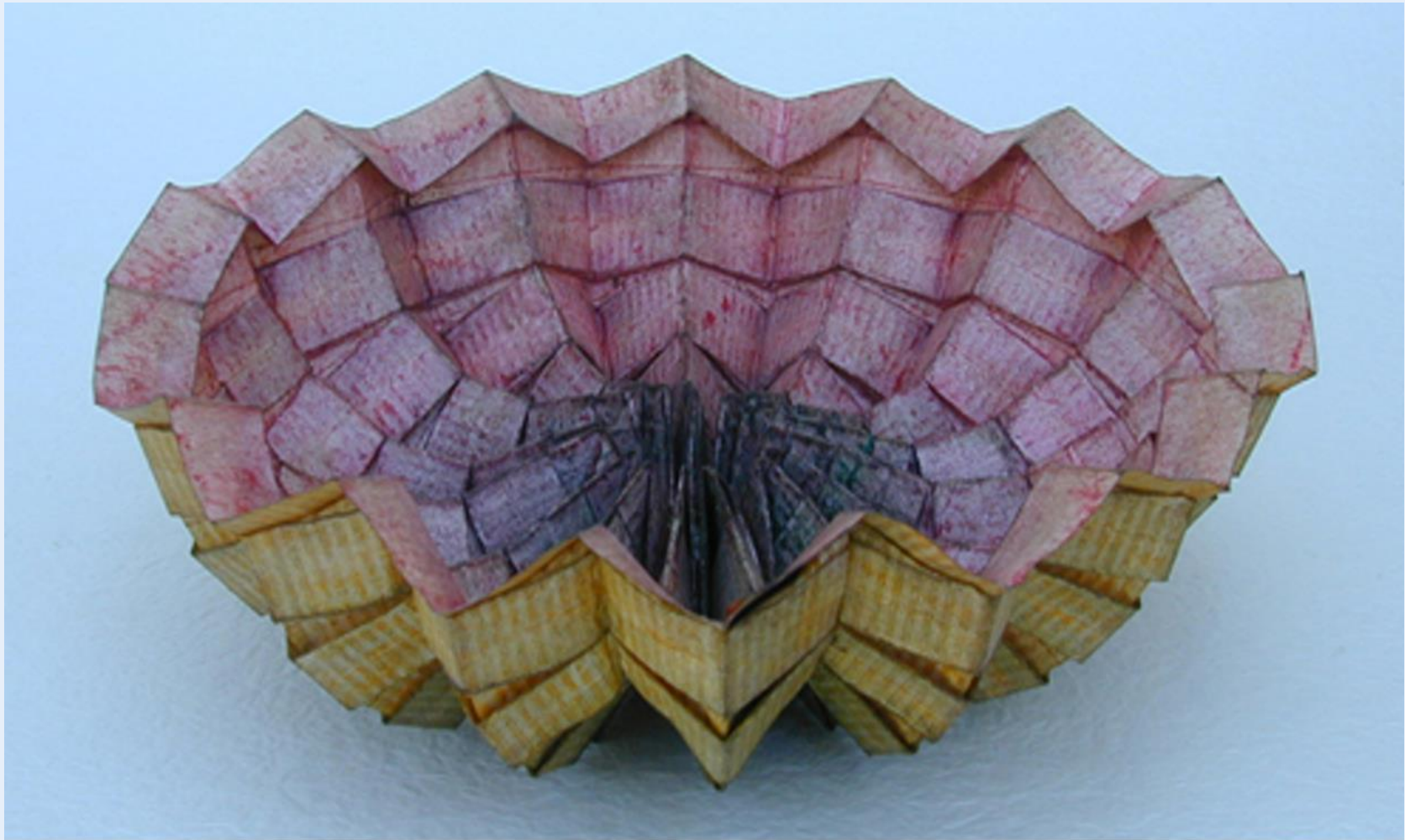
*Hugging Circles* by Erik and Martin Demaine



*Green Cycles* by Erik and Martin Demaine



*Whirlpools* by Tomoko Fuse



*Bowl* by Paul Jackson



Can origami save someone's life?



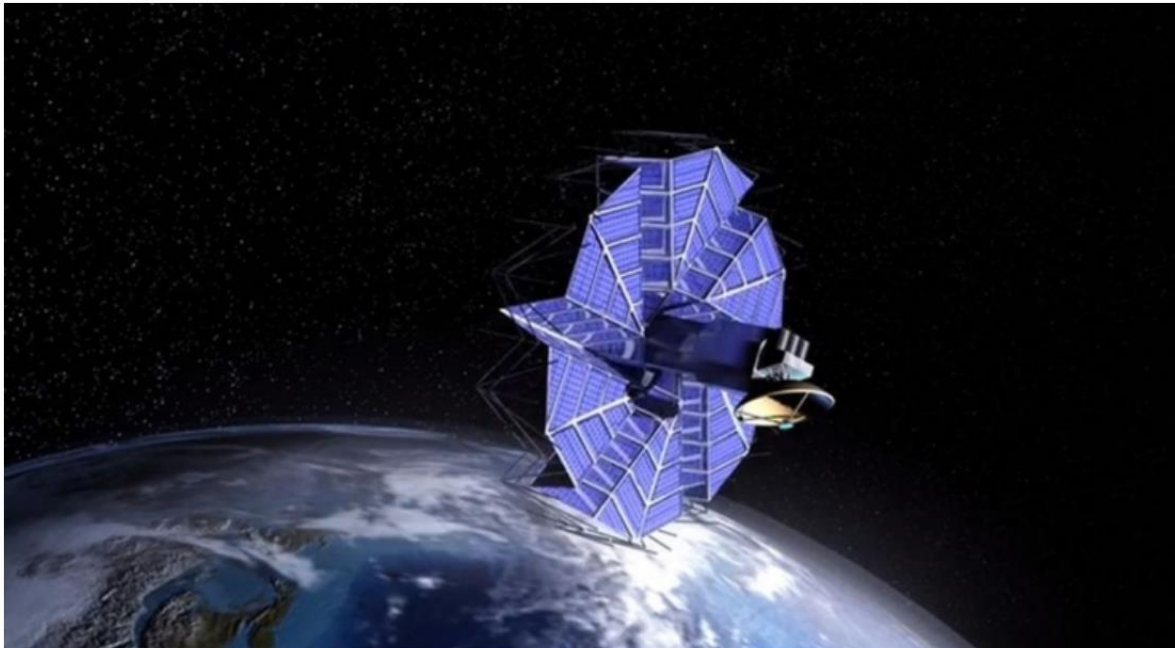
Zhong You and Kaori Kuribayashi

Department of Engineering Science  
University of Oxford  
Parks Road  
Oxford, OX1 3PJ  
UK

SPACE | SOLAR PANELS | ORIGAMI

## Origami-inspired solar panel unfurls in space like a paper flower

Michael Trei Monday, December 16, 2013 - 5:32pm



Launching anything into space is really, really, really expensive. So, the number one concern when building something that needs to be launched skyward is keeping it small, and keeping it light. Most of the solar panels used on spacecraft are designed to fold open once they reach their destination, but just *how* they're folded can have a big effect on how small they are at launch time. To get the maximum possible efficiency in packing, a group of spacecraft engineers at Brigham Young University have sought out the advice of **origami expert** and physicist Robert Lang.

The BYU team has been working with NASA's Jet Propulsion Laboratory to develop a solar array that folds open to nearly ten times its packed size, and which is capable of generating 250 kilowatts of power. When folded, the array is designed to wrap around the core of the spacecraft. While a specific application for the array has yet to be determined, NASA expects to continue developing the origami-inspired designs for the next several years.



NASA mechanical engineer Brian Trease holds the prototype of the origami-inspired solar panel arrays.

Source:

<http://www.space.com/27485-origami-space-solar-panels-video.html>

When you are doing origami, your brain acts as if you are meditating! And meditation is good for your mind and body.

Source:

<https://www.psychologytoday.com/us/blog/urban-survival/201609/5-ways-origami-boosts-mindfulness>

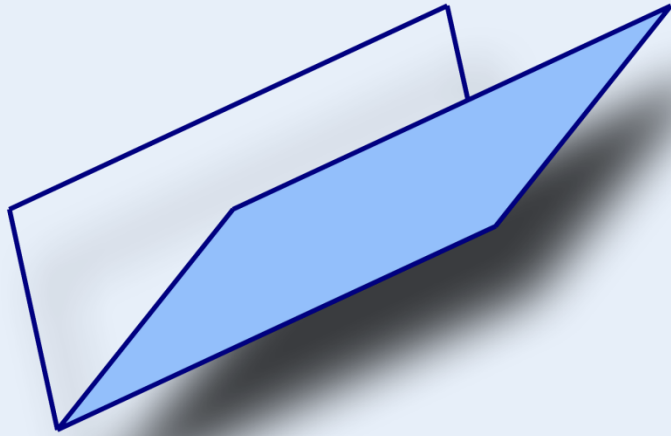
According to Robert Lang,

98% of the innovations in origami came in the last 2% of the art's existence (Lang, 2012).

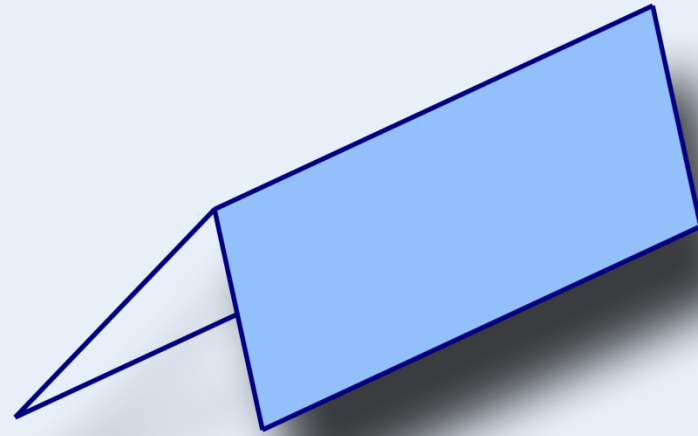
Lang, R. (2012). *Origami design secrets*. Boca Raton, FL: CRC Press.

When you are folding make sure the creases are **ACCURATE** and **SHARP**.

The role of **precision** cannot be overemphasized in origami.



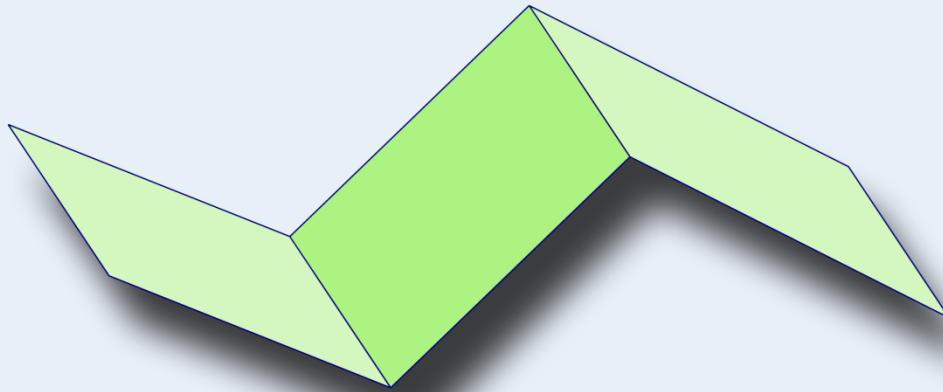
Valley Crease



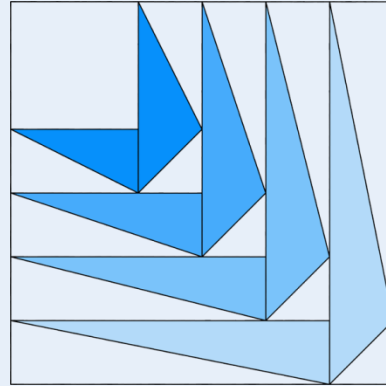
Mountain Crease

# LOOKING FROM ABOVE

CONVEX  
CREASE



CONCAVE  
CREASE

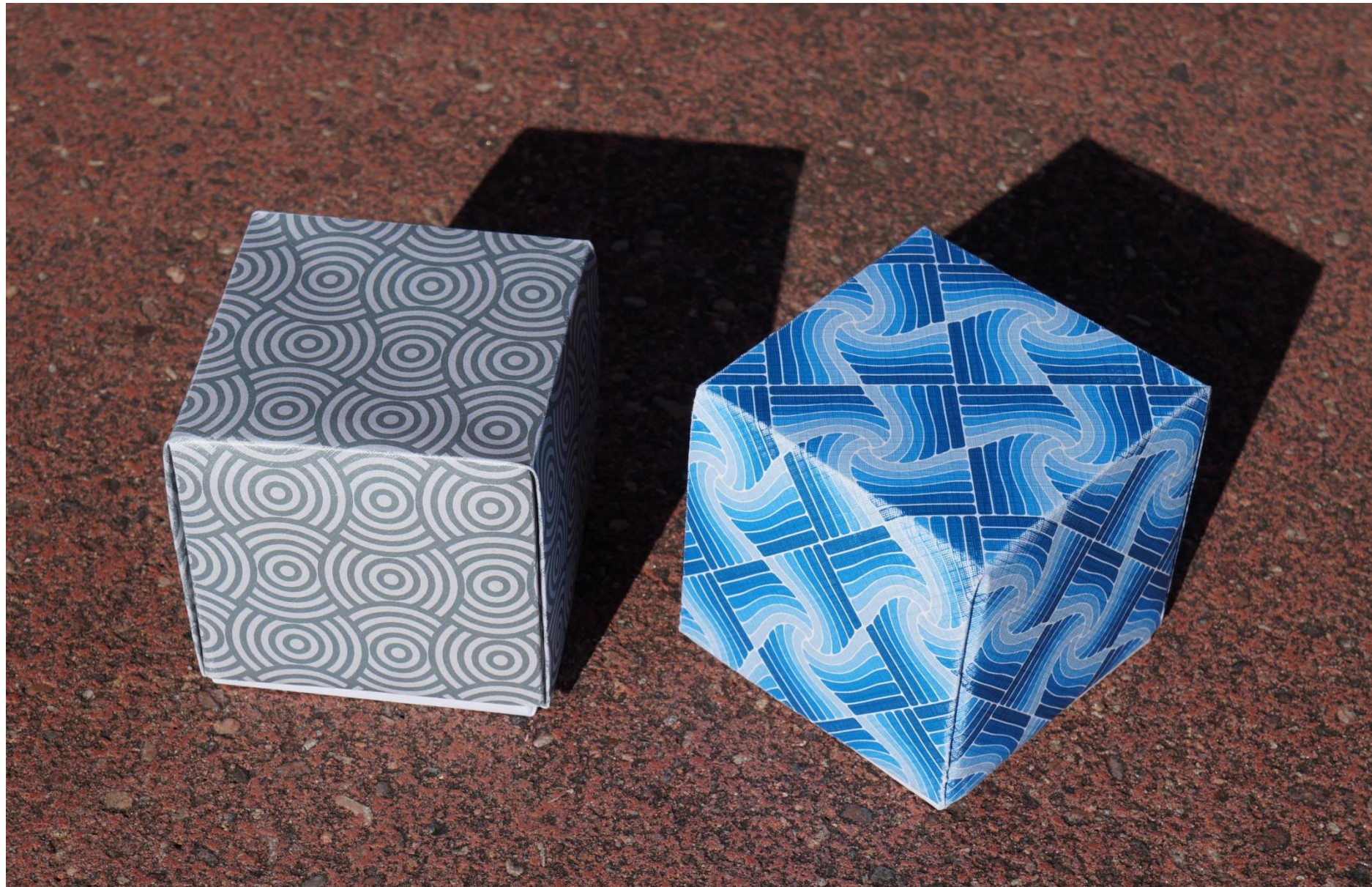


Arsalan Wares

Valdosta State University

Valdosta, Georgia

We will be making a box like the ones shown below.



An 8th grader made the box shown in the picture on March 04, 2017.

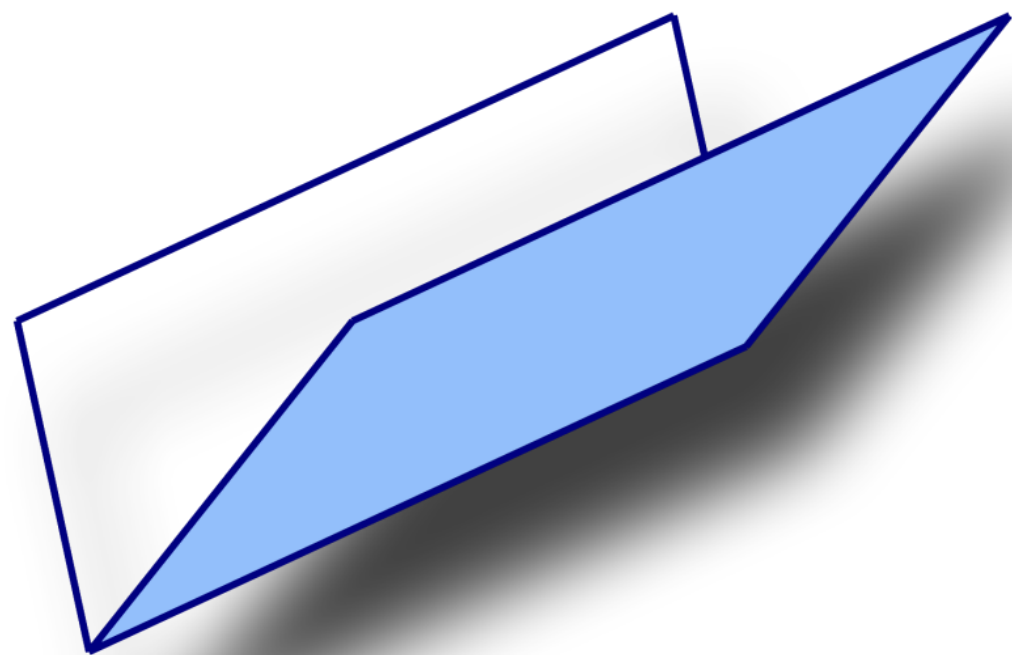


A 3rd grader made the box shown in the picture on March 04, 2017.

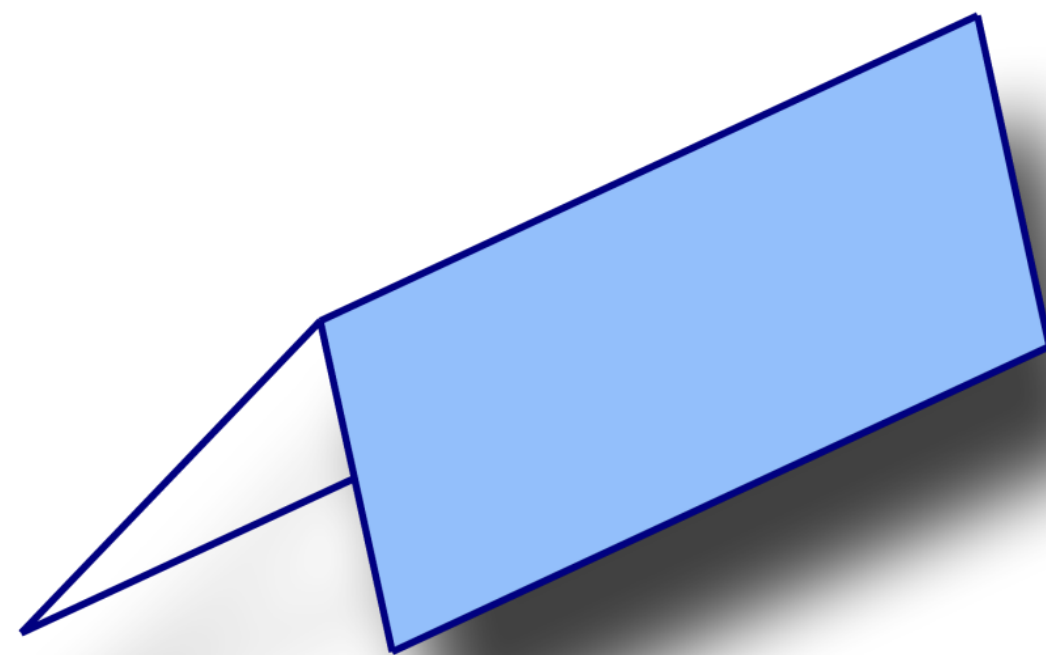






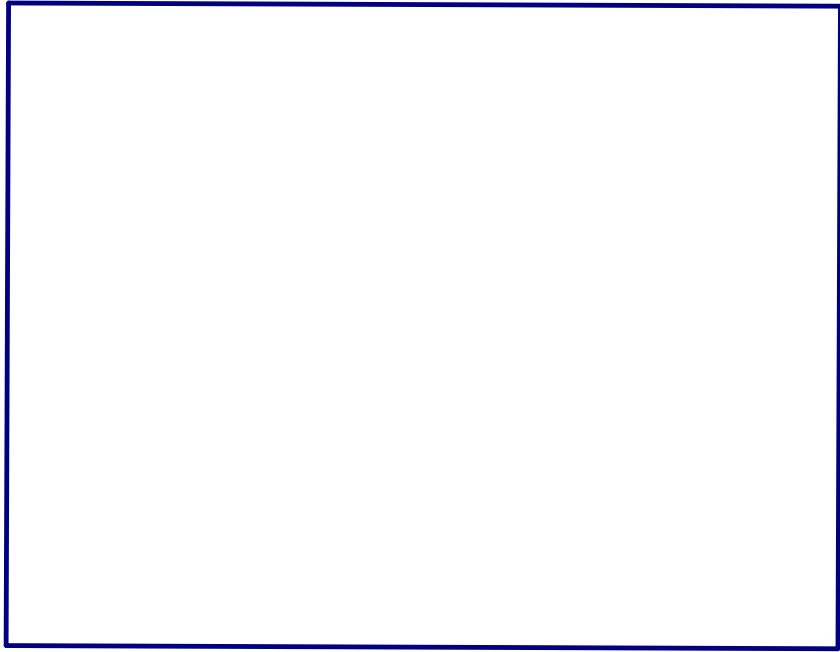


Valley Crease

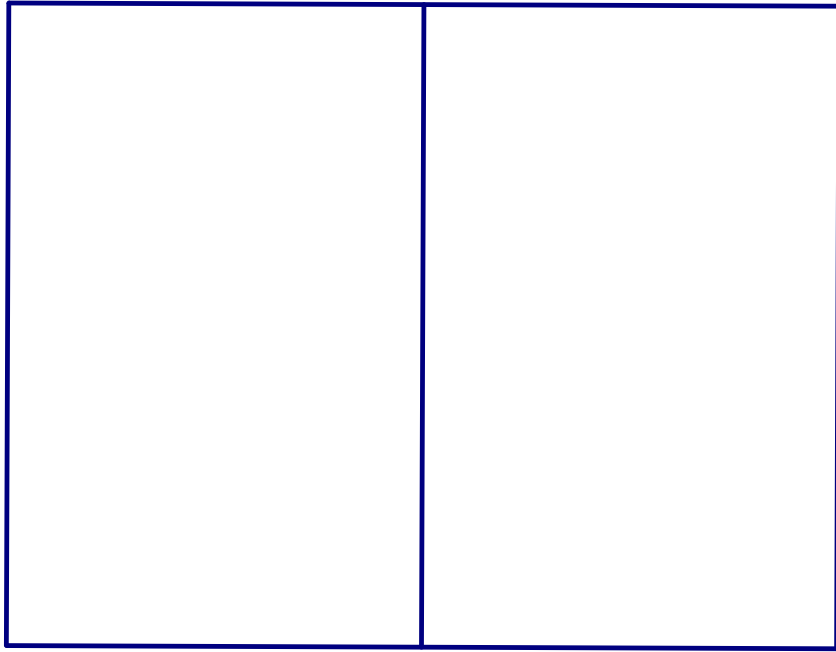


Mountain Crease

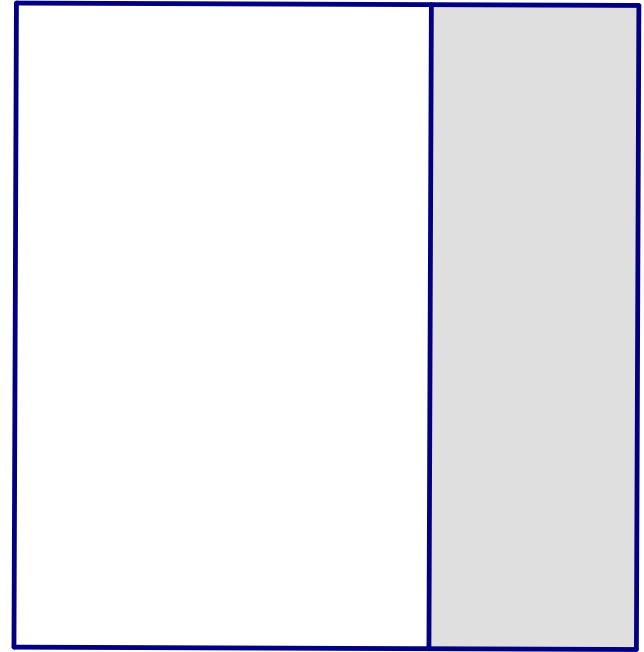
We will be using two sheets of the same size (8.5 inches by 11 inches). One of them is the **origami sheet** (the fancy one), and the other one is the **measurement sheet** (the ordinary one could be from the recycling bin).



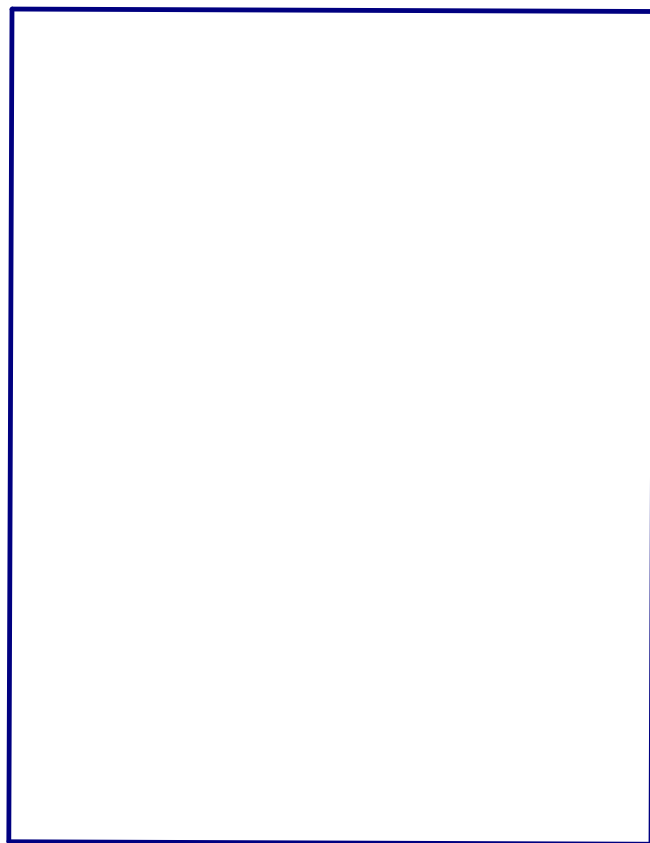
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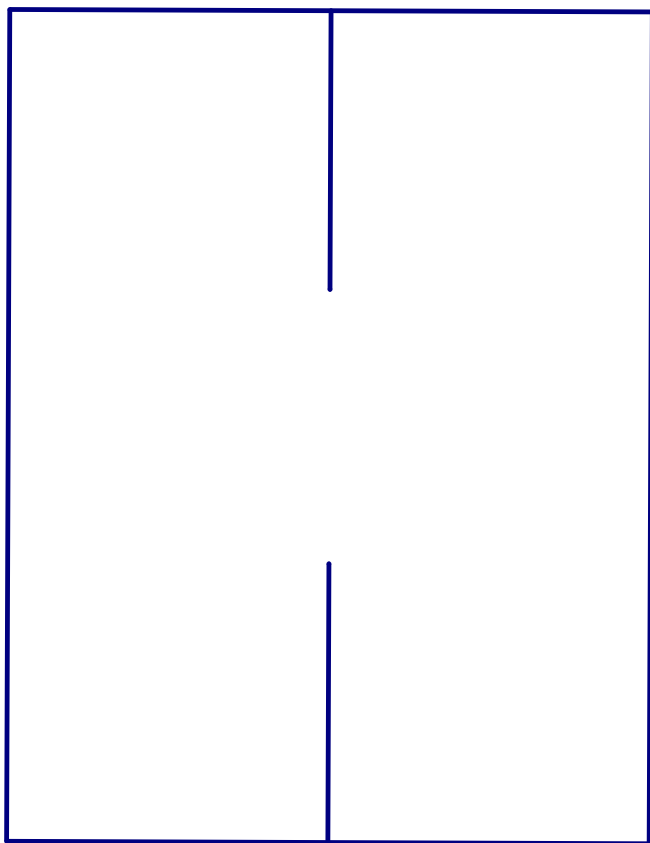
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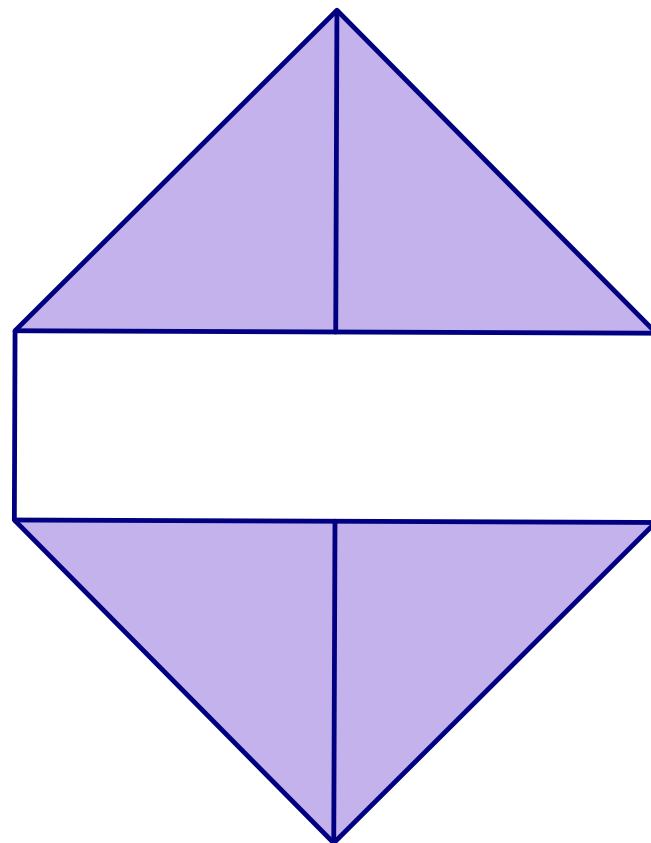
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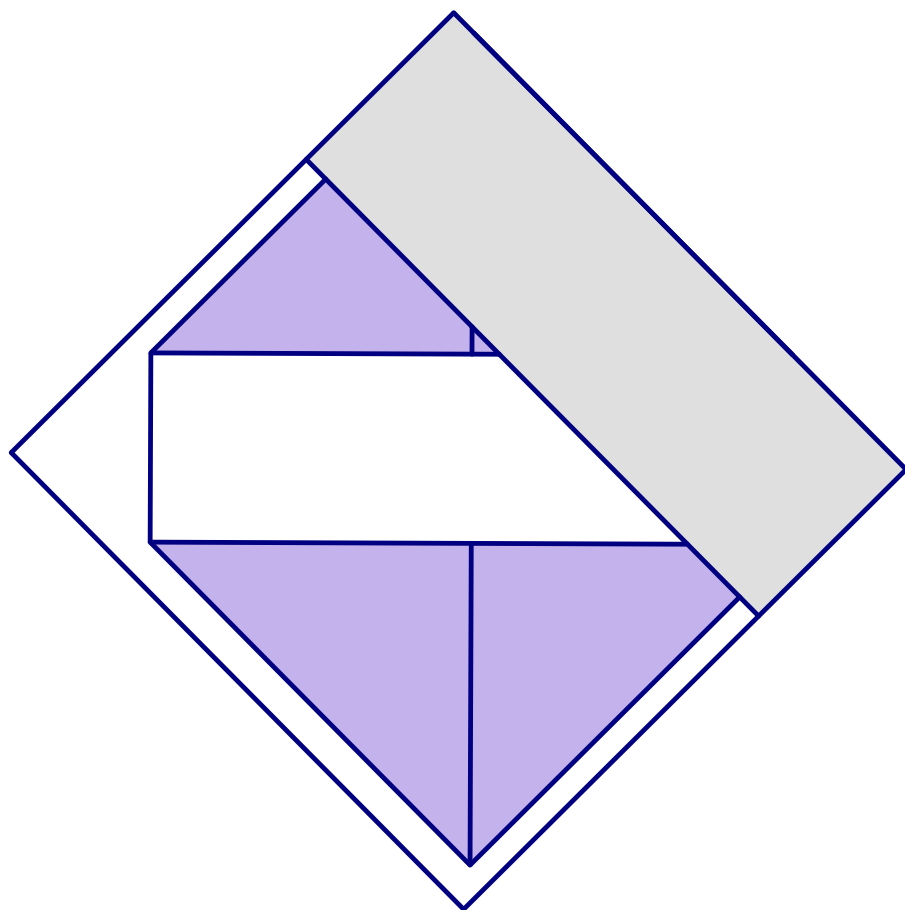
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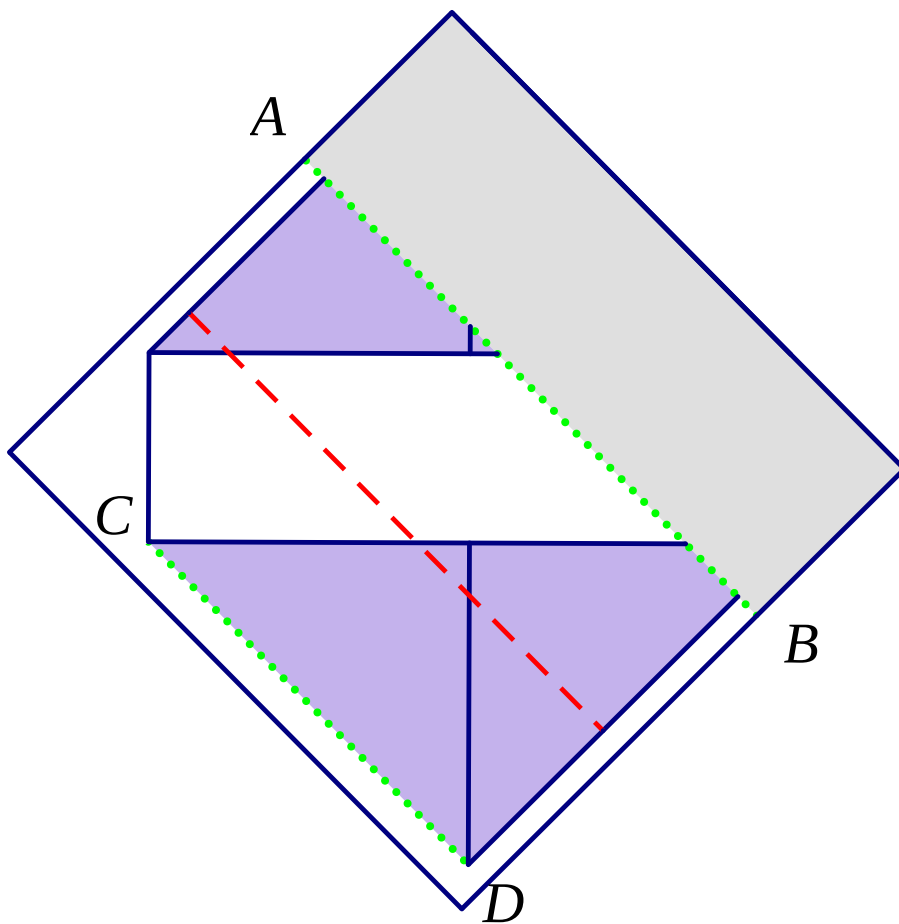
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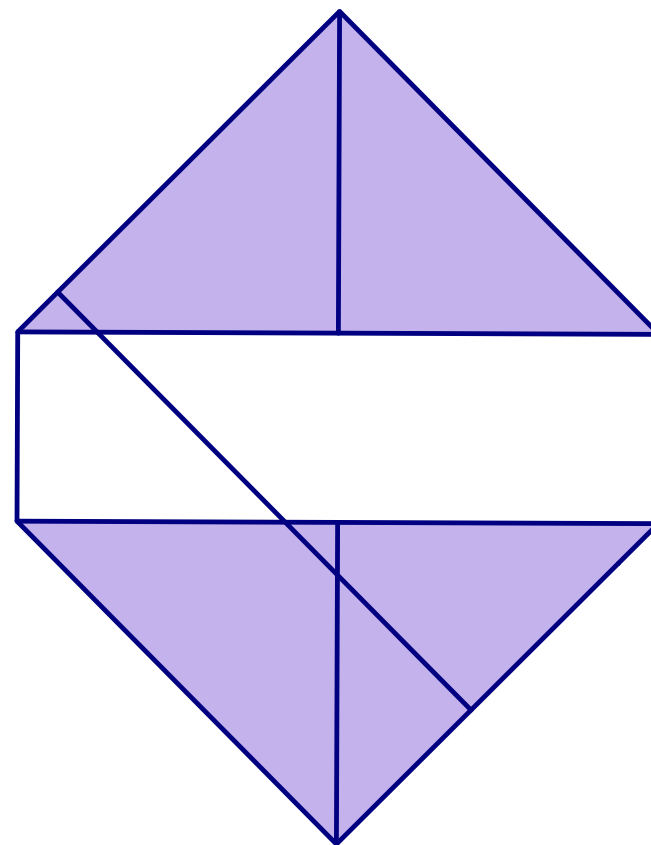
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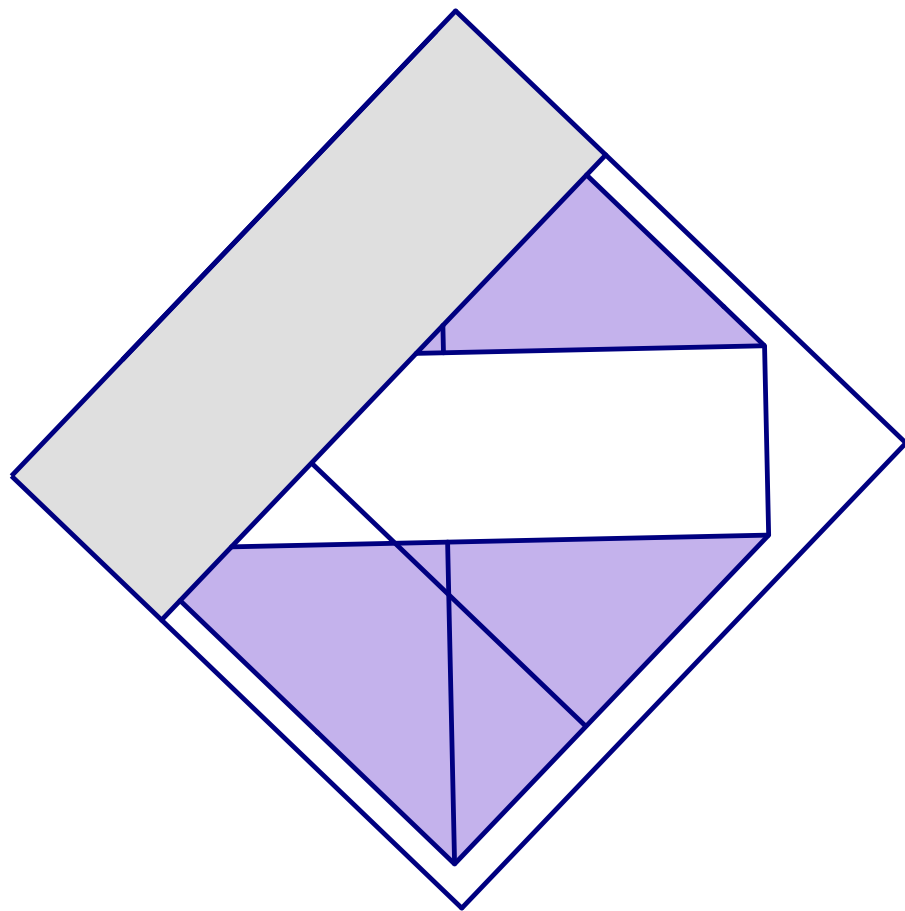
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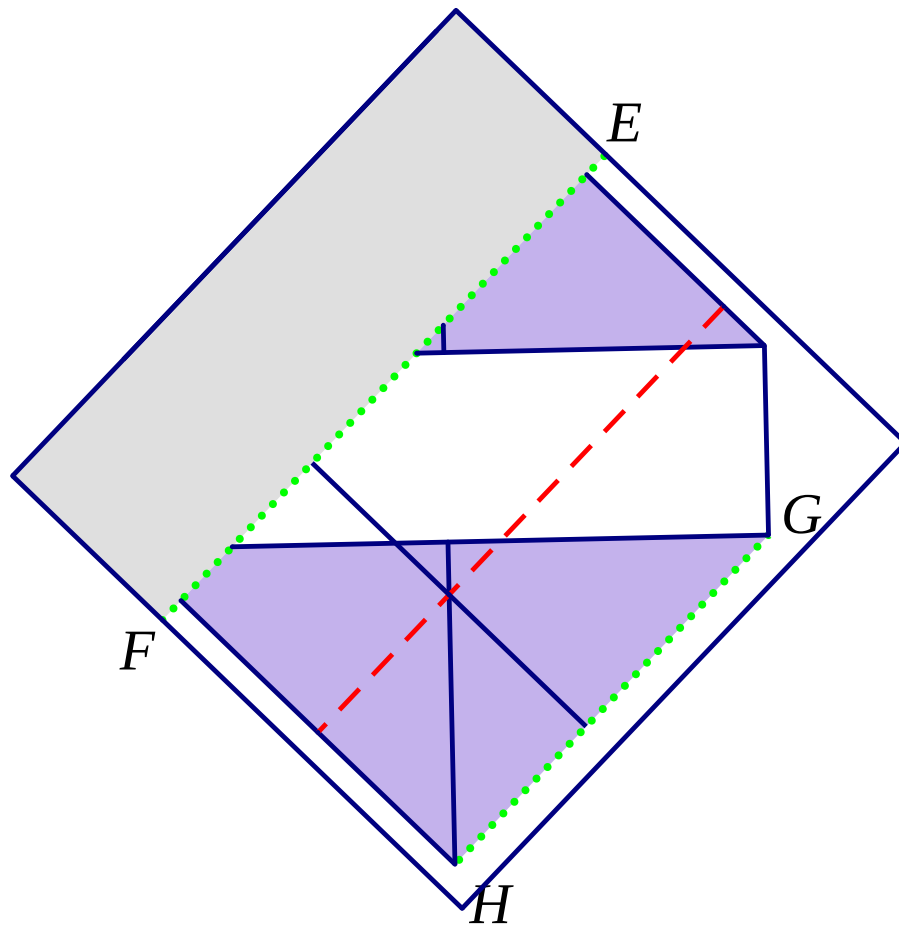
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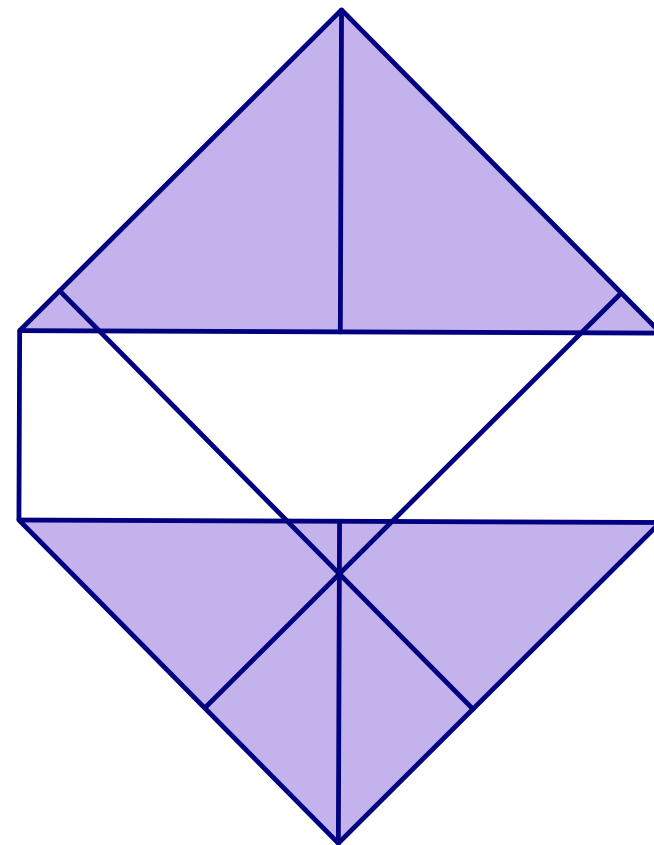
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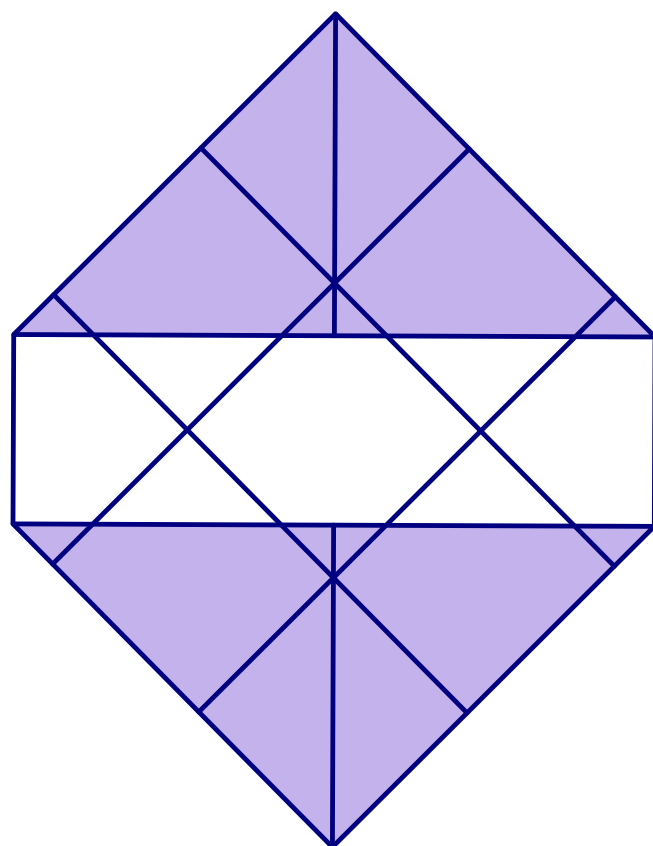
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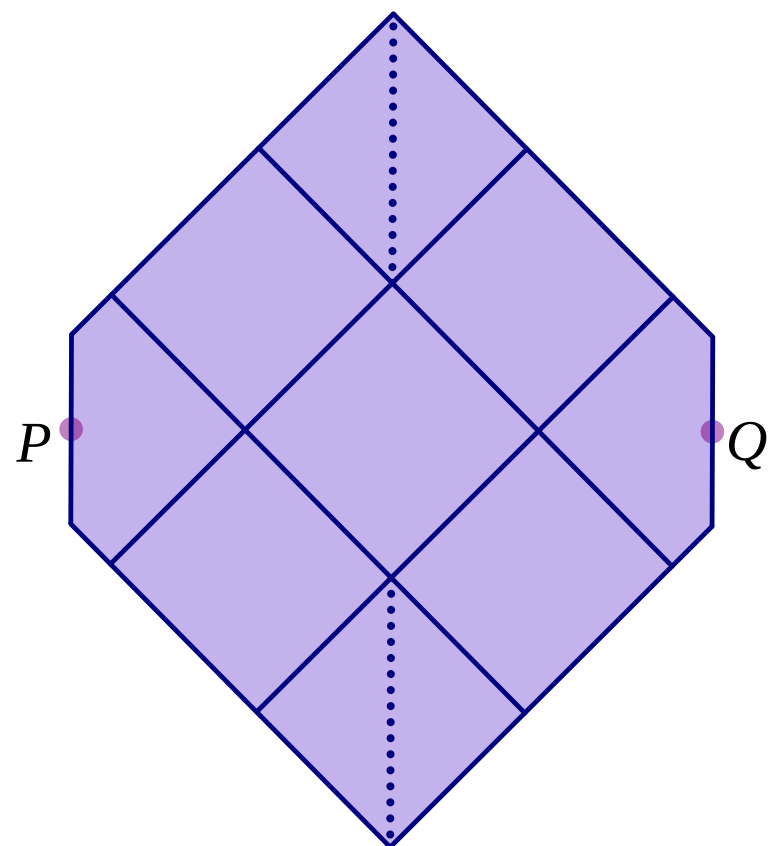
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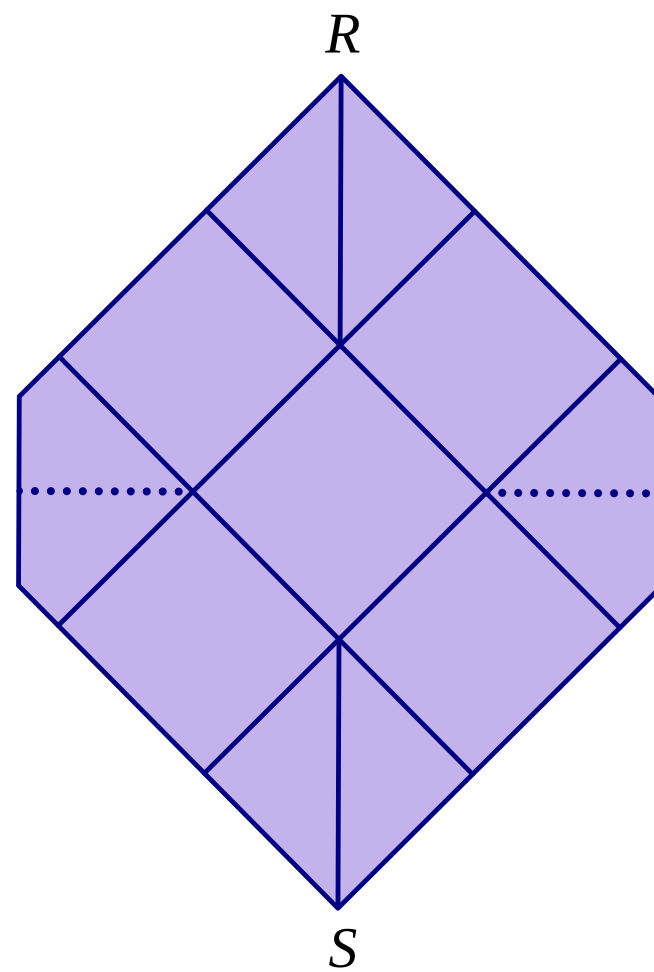
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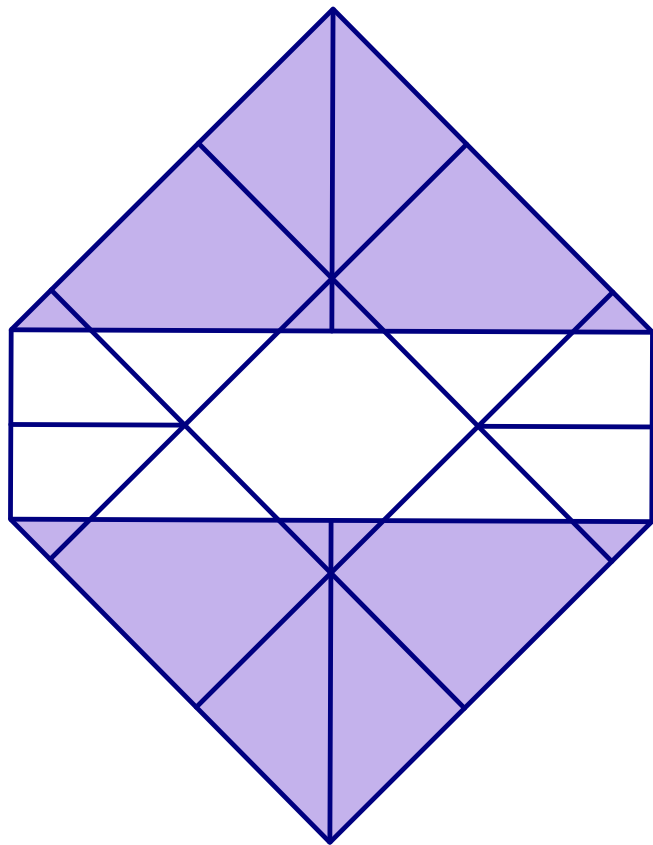
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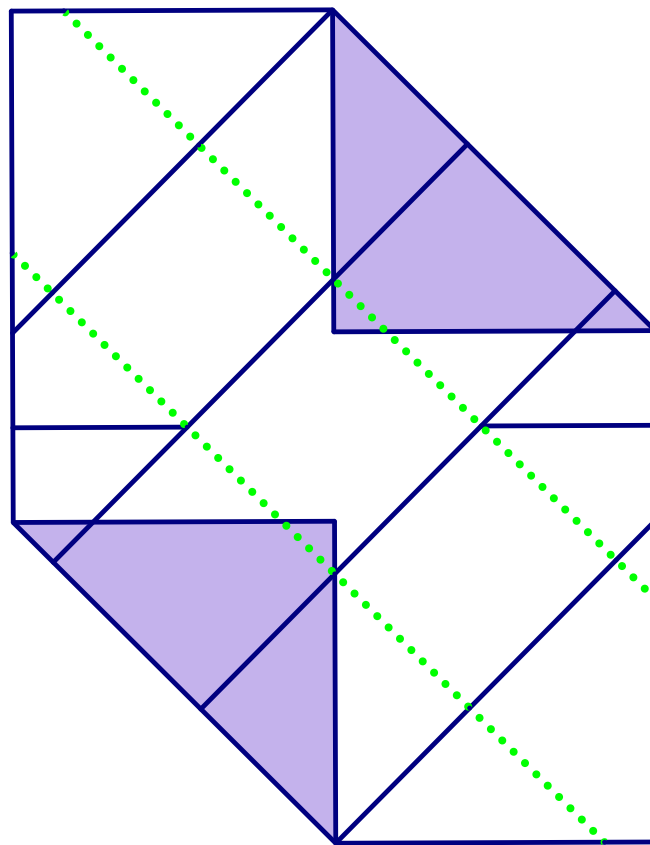
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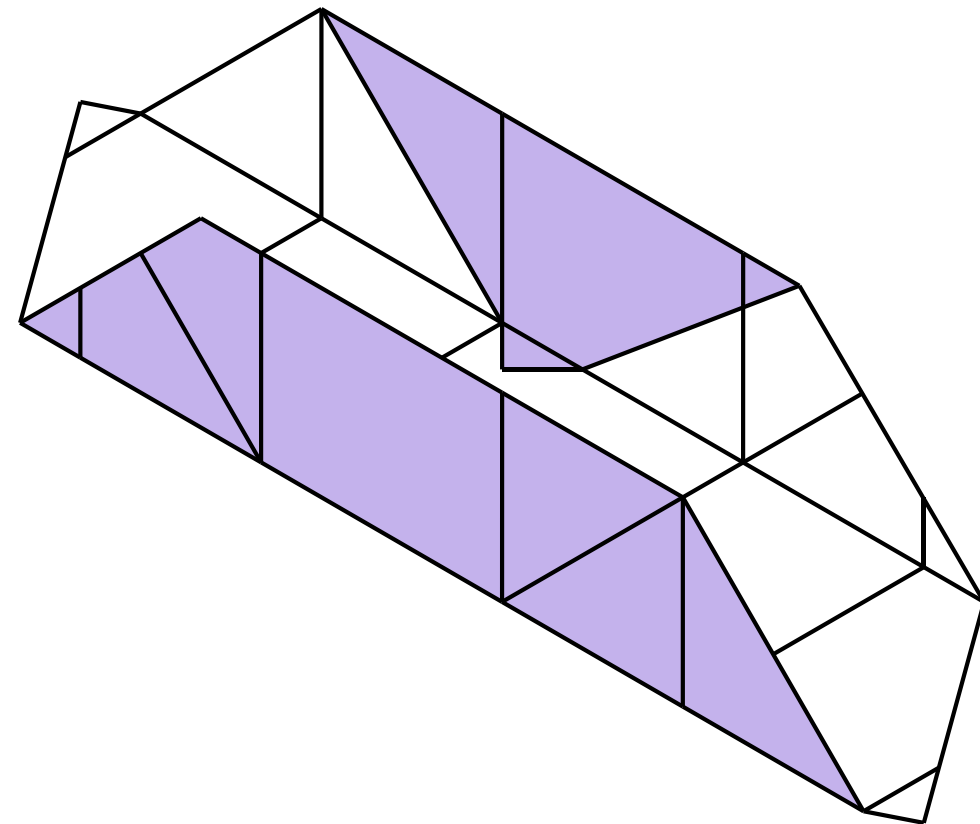
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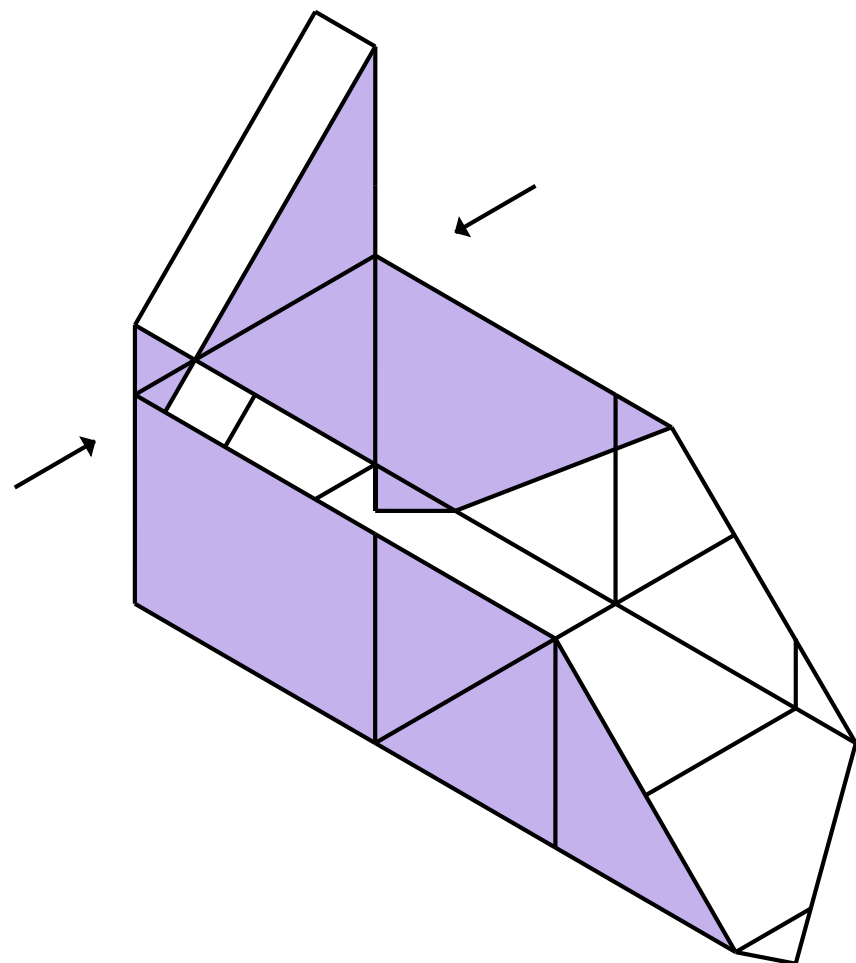
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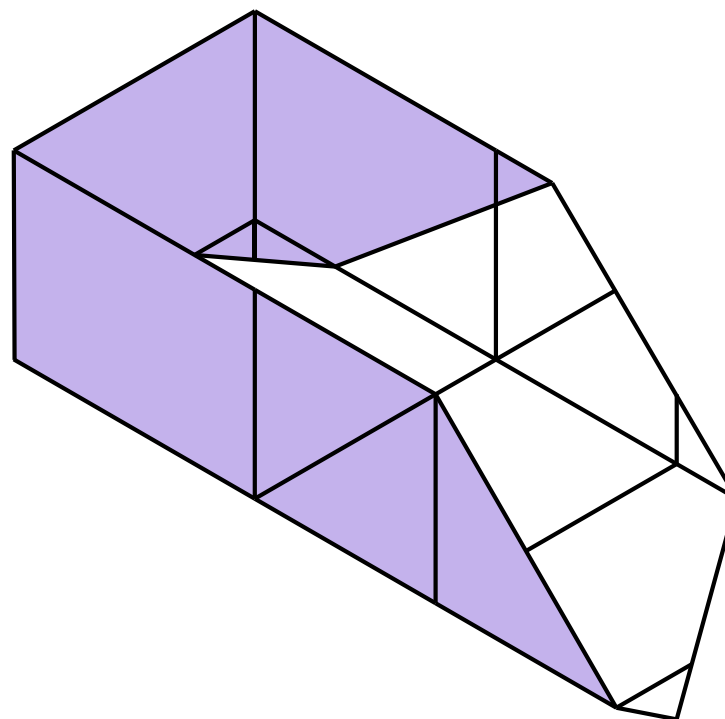
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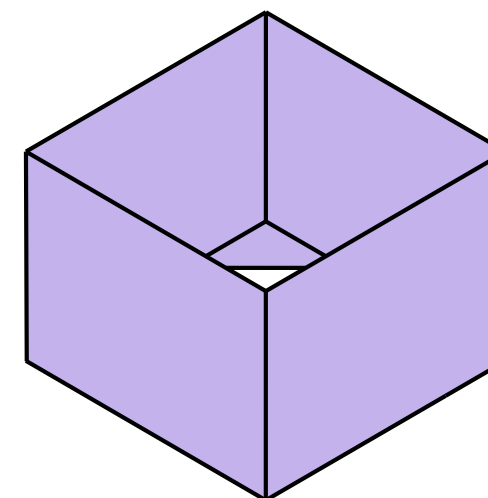
18



19



20



21

Figures for steps 1-3 show the measurement sheet (this sheet is discarded).

Figures for steps 4-6, 9, and 12-21 show the origami sheet (this sheet is folded into the box).

Figures for steps 4-8, and 10-11 show the origami sheet slid between the layers of the measurement sheet.

Link to the video:

<https://youtu.be/zyAYHDeY0PE>

Do the following search on Youtube. It must be exact.

**“Origami Box from Rectangular Sheet”**



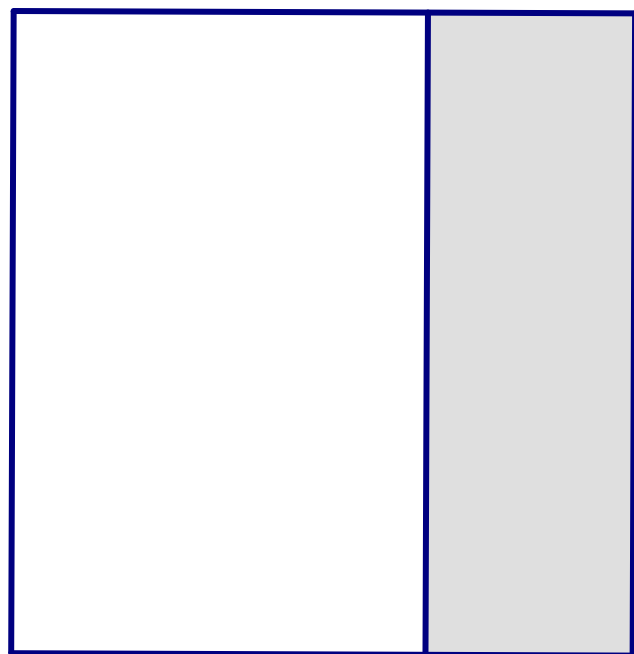
Select the video shown above.

My email:

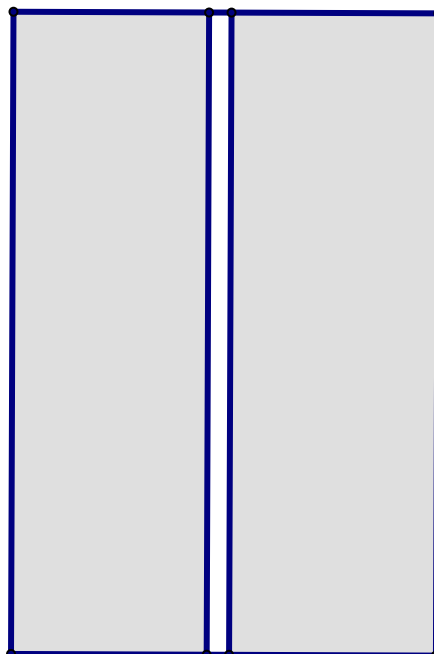
awares@valdosta.edu

Link to the video:

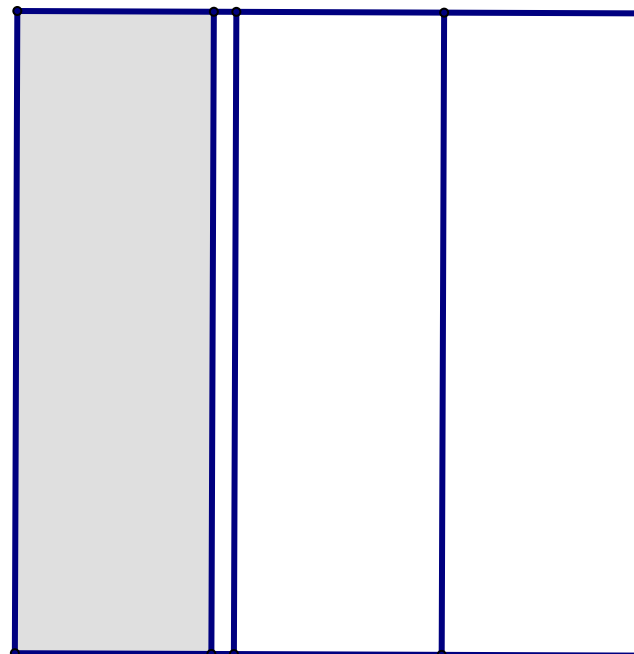
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1

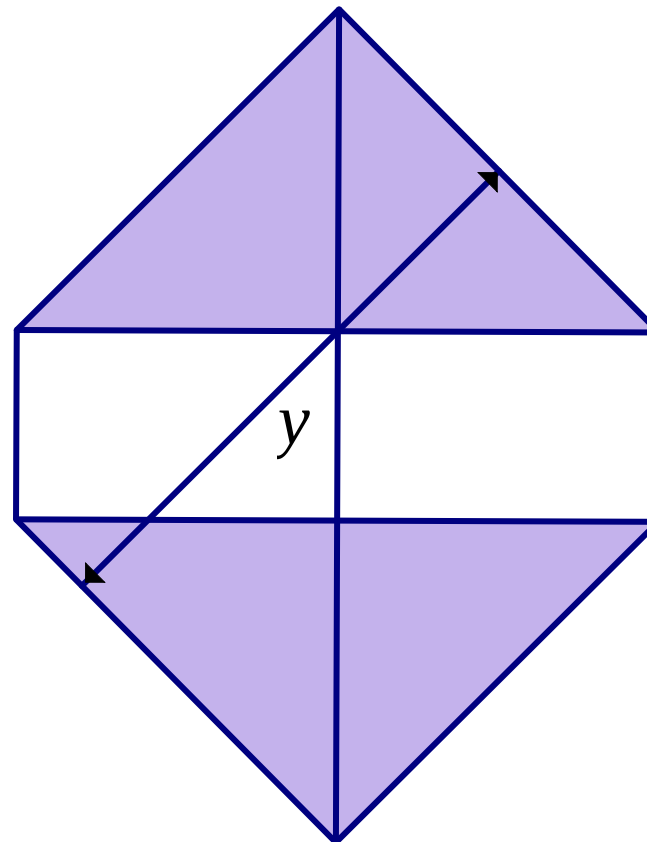
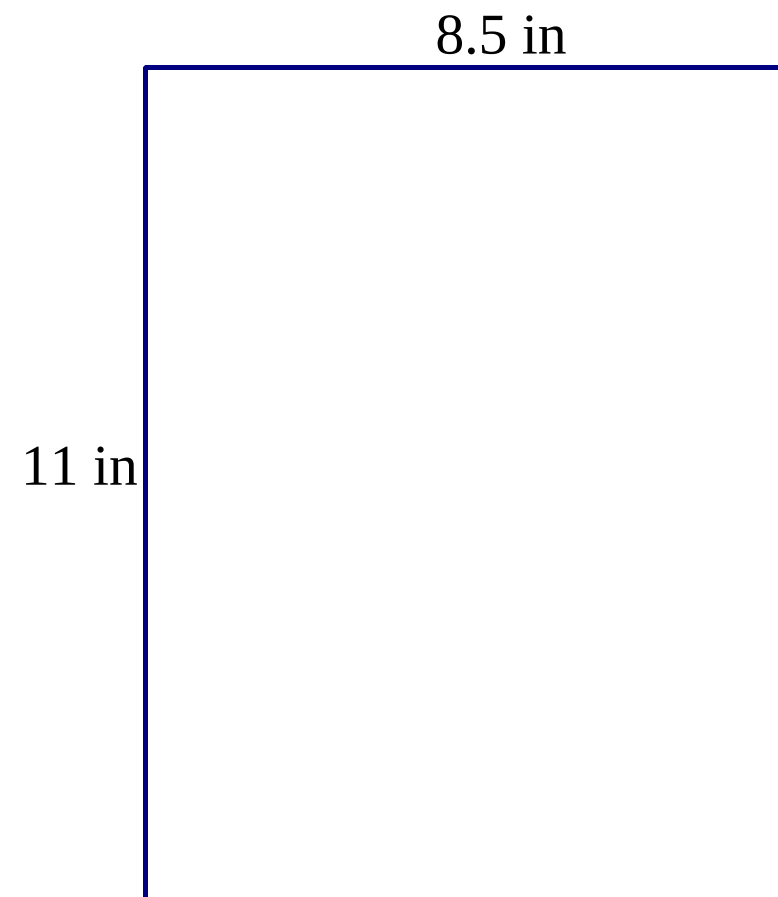


2

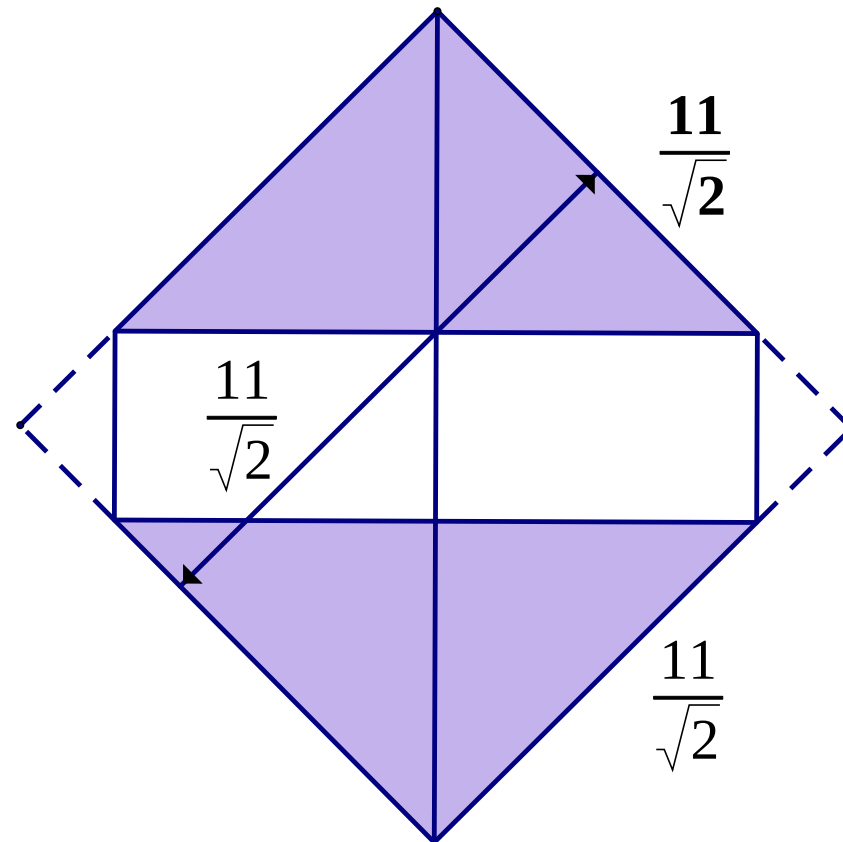
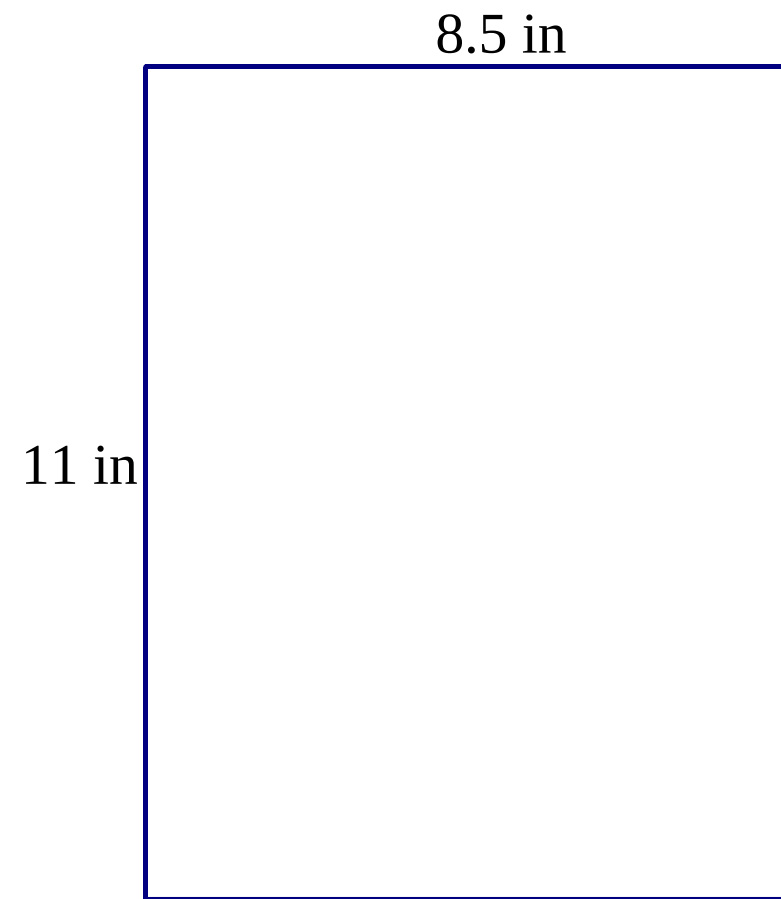


2

We have a problem...

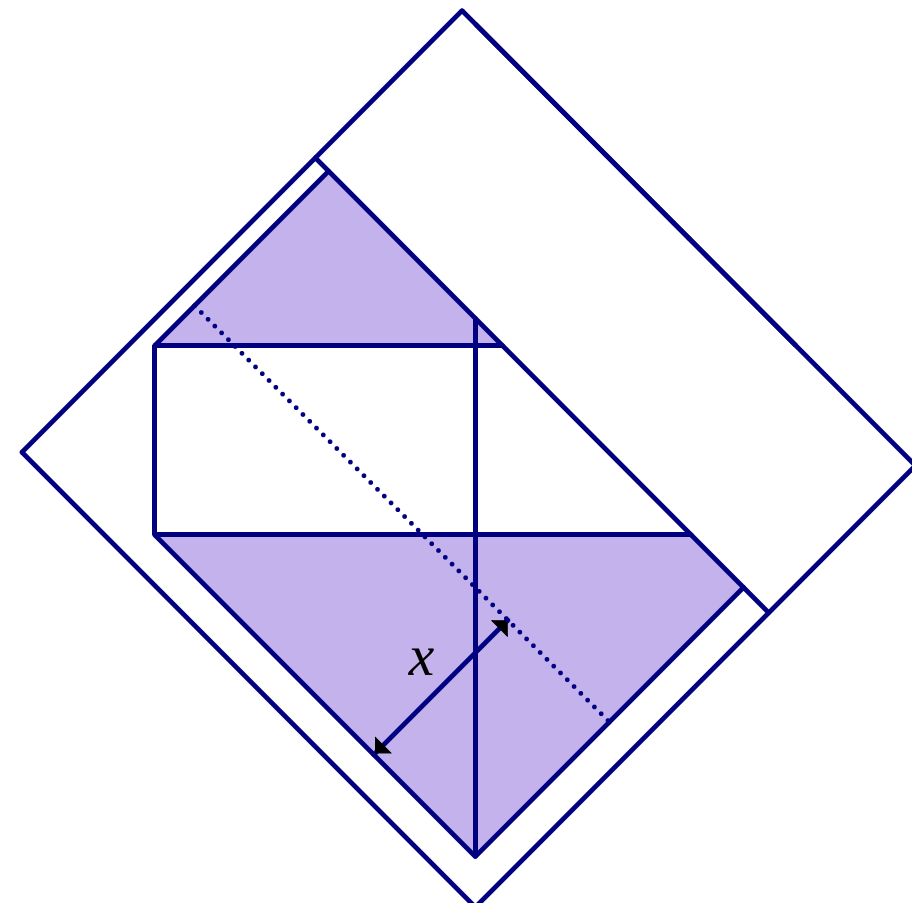
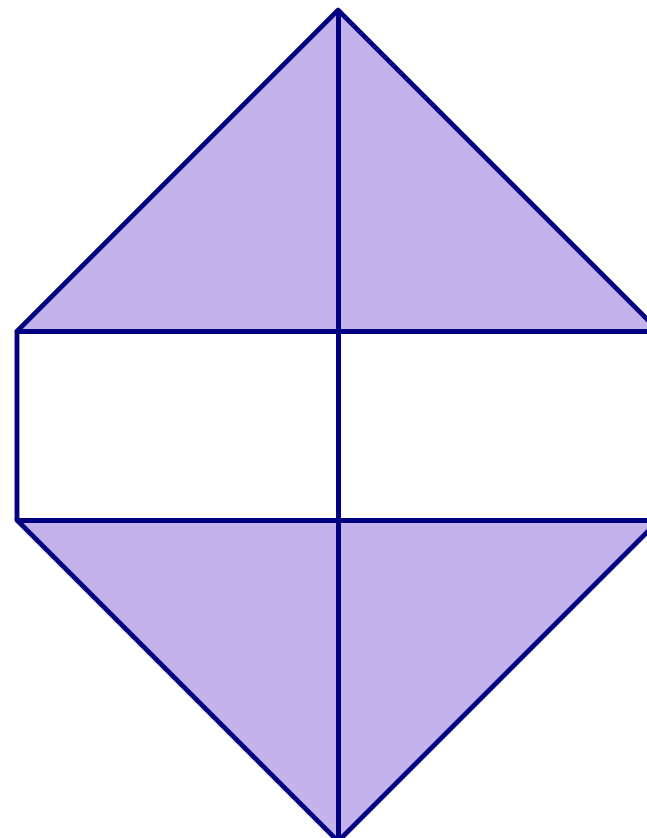
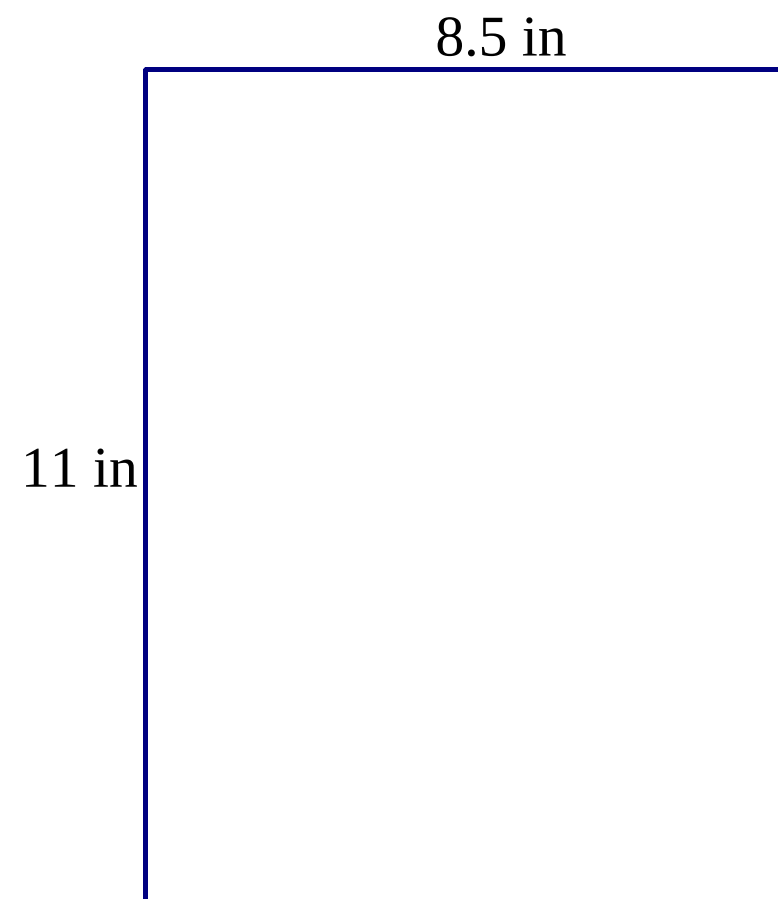


Can you find the value of  $y$ ?

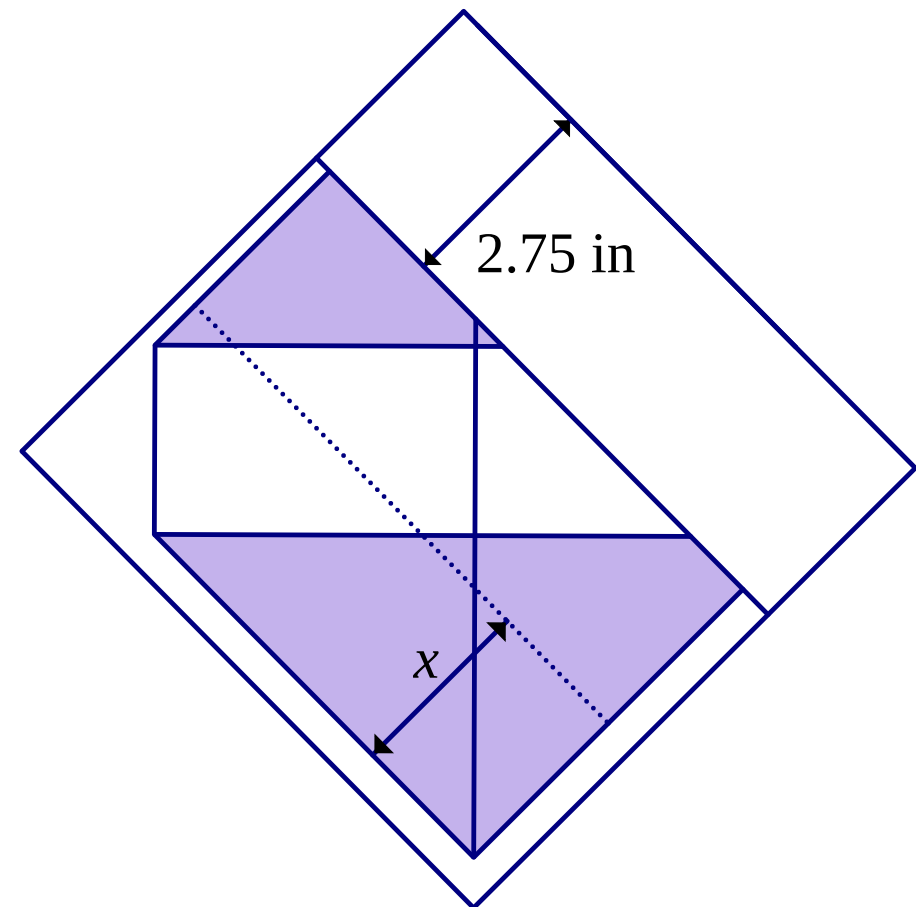
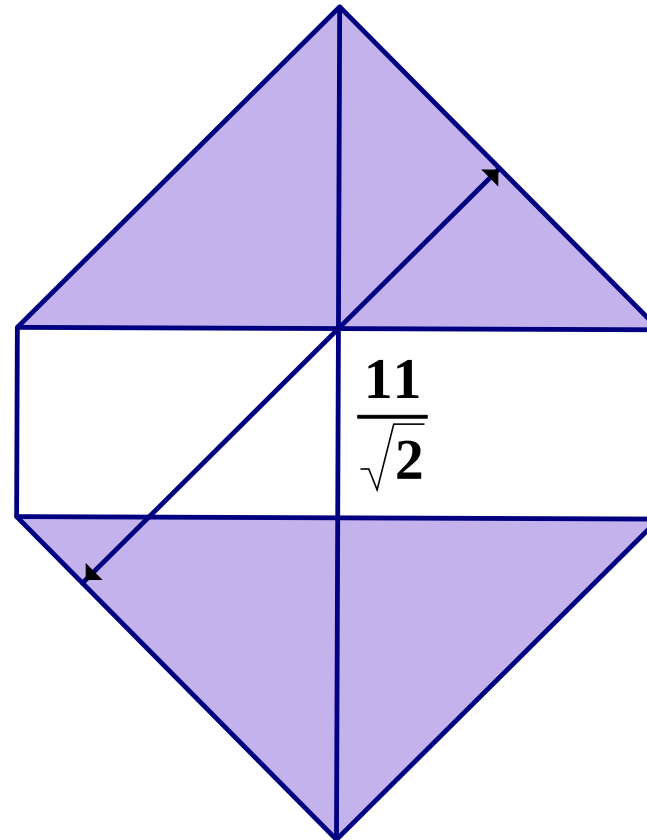
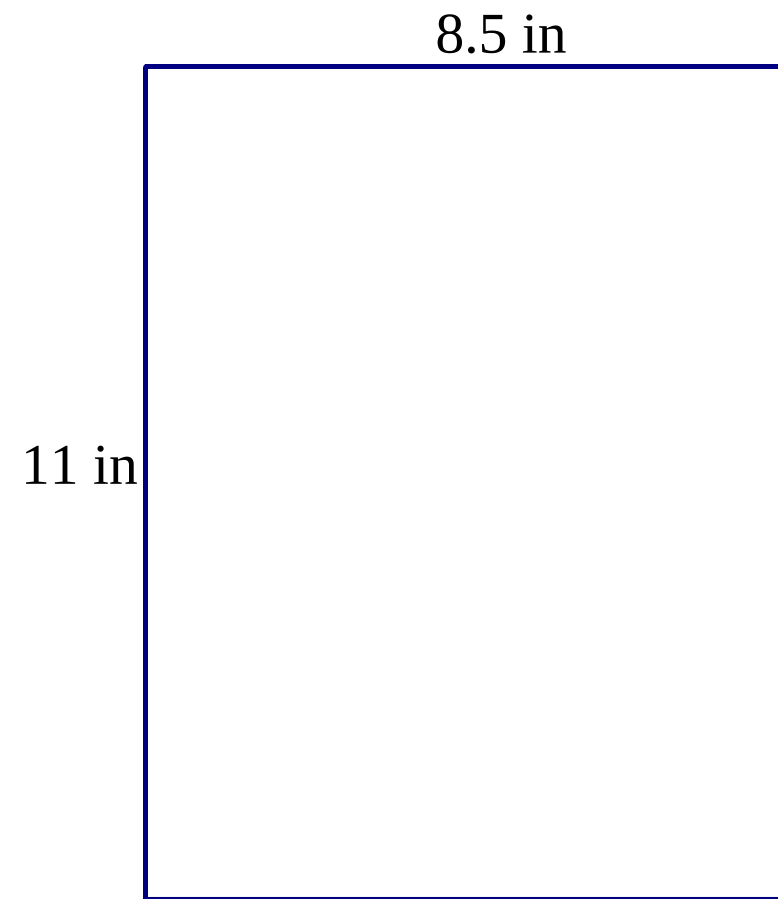


$$y = \frac{11}{\sqrt{2}}$$

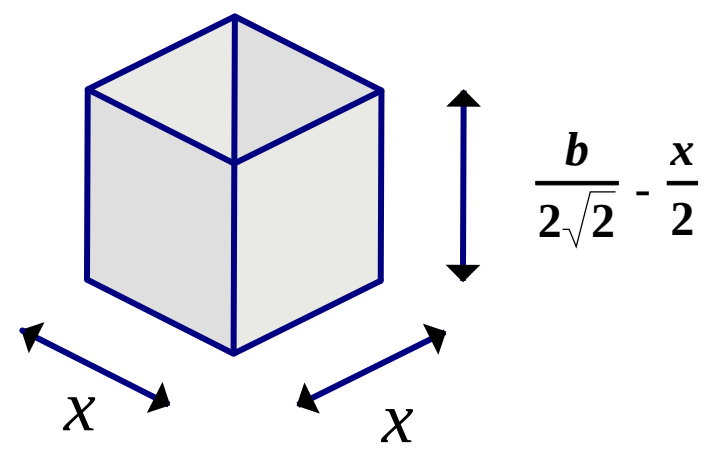
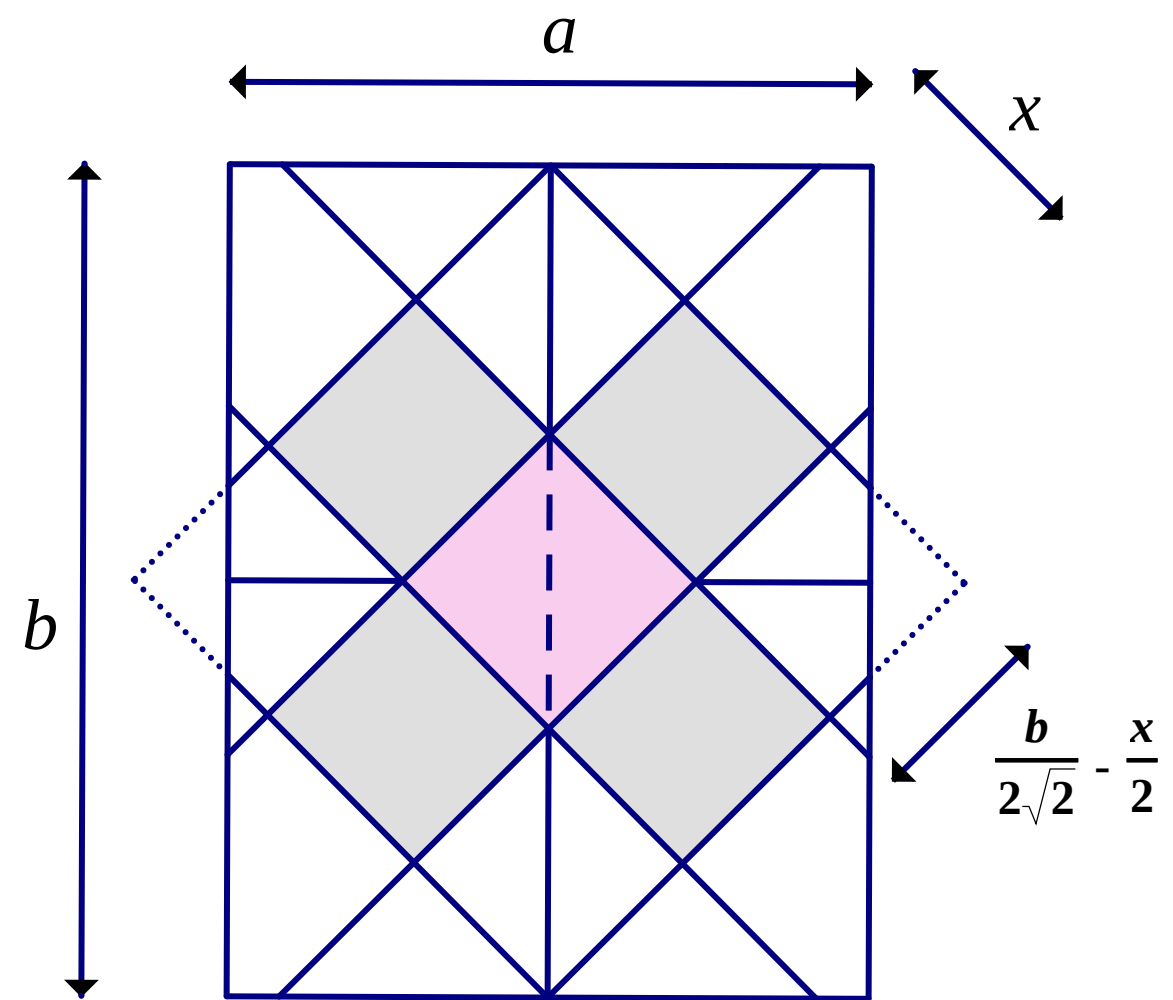
We have another problem...

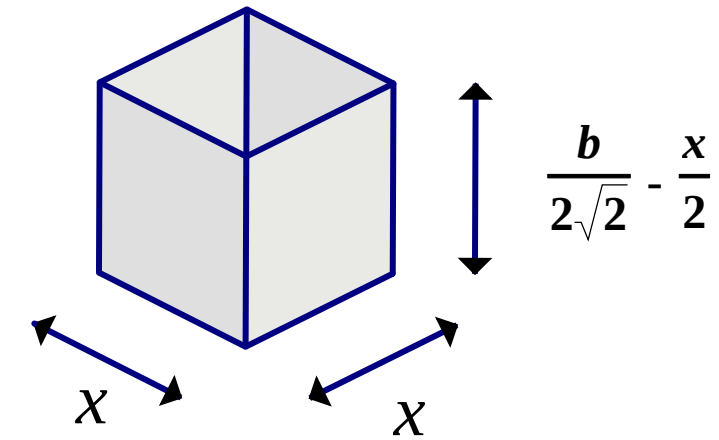


Can you find the value of  $x$ ?

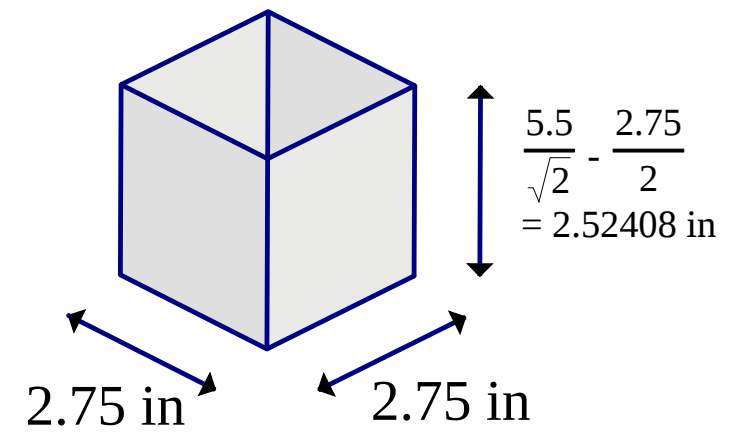
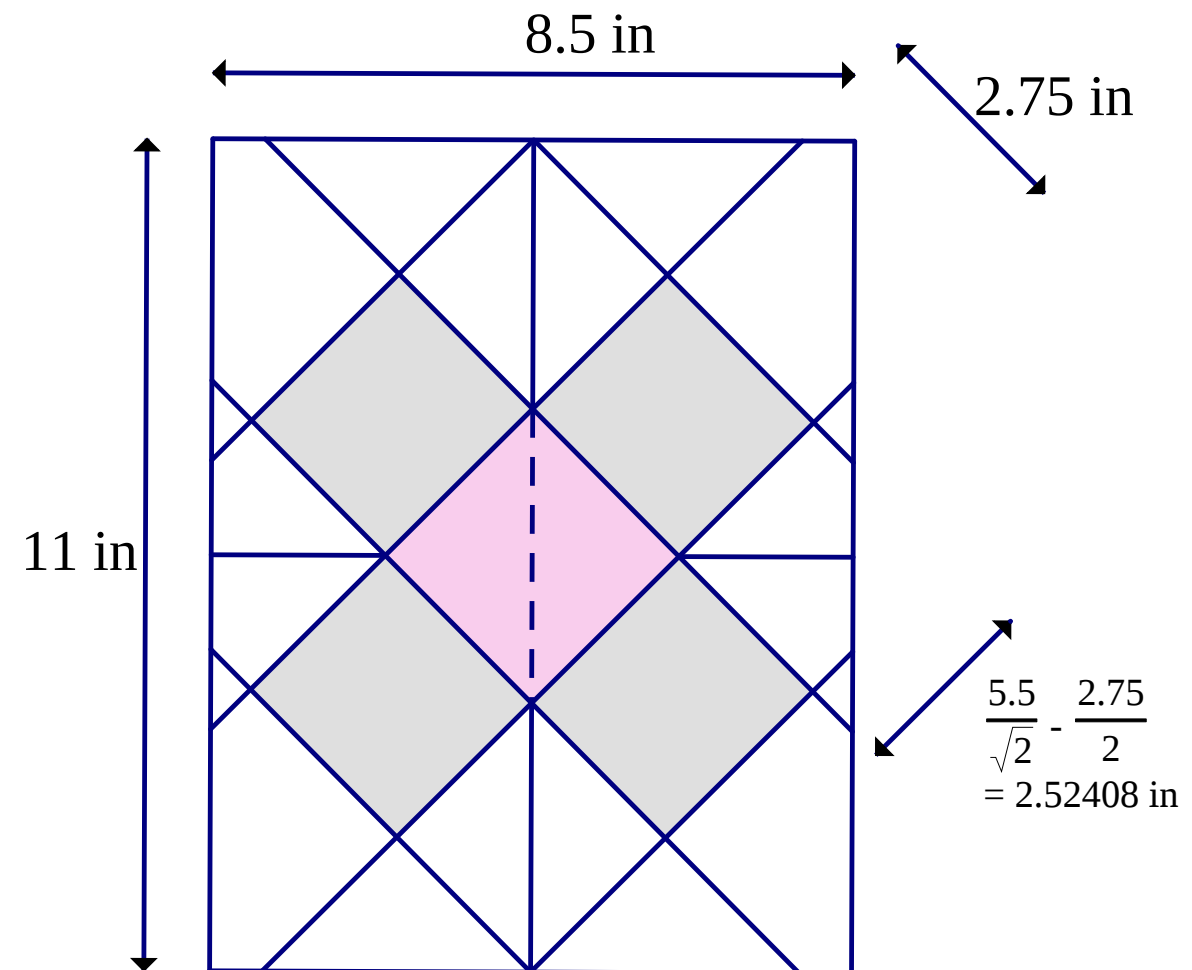


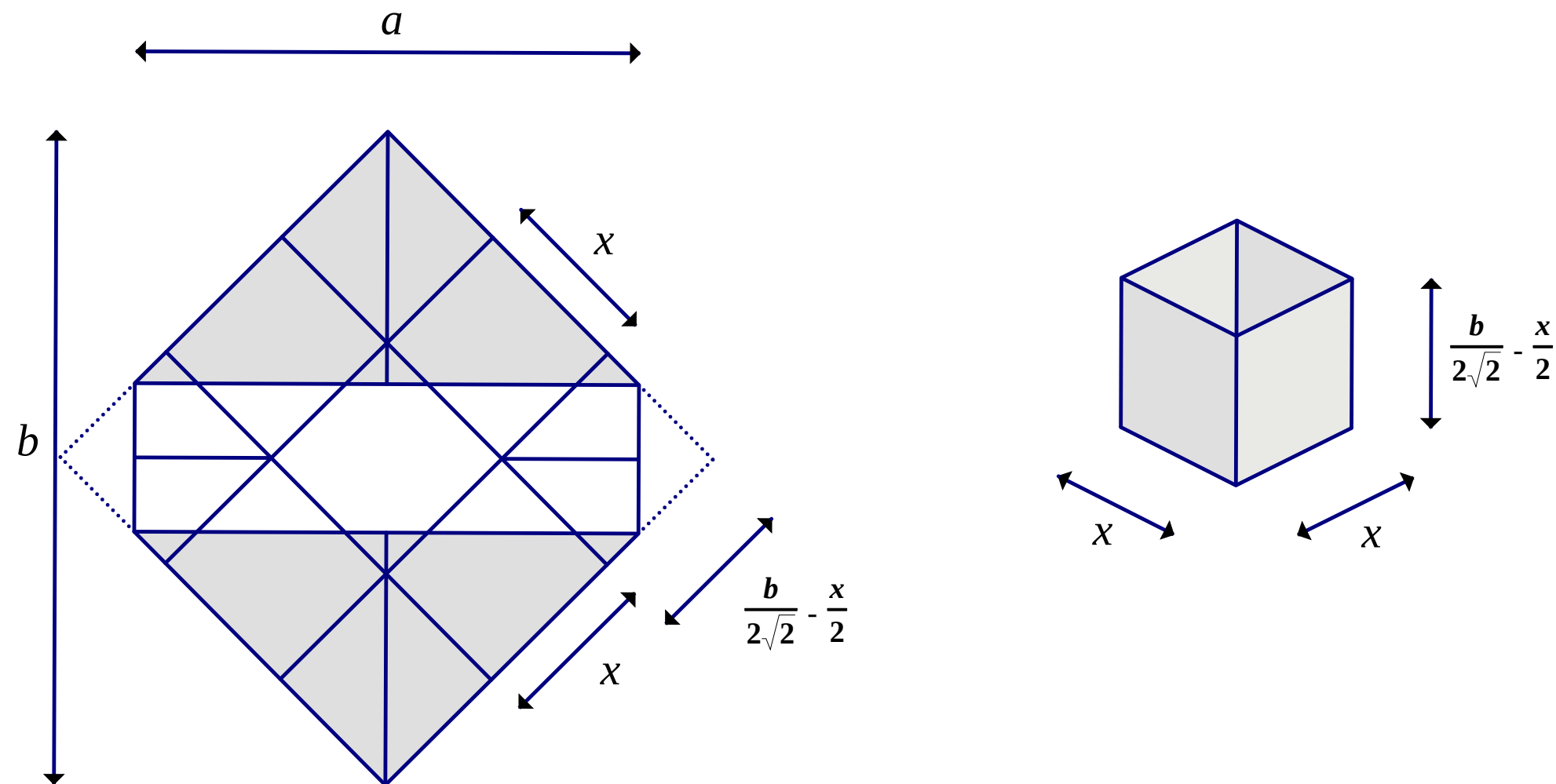
$$x = \frac{1}{2} \left( \frac{11}{\sqrt{2}} - \frac{11}{4} \right) = \frac{11}{2\sqrt{2}} - \frac{11}{8}$$





A 3D diagram of a cube. The front face is shaded light gray. The edges are labeled with blue arrows. The height is labeled  $\frac{b}{2\sqrt{2}} - \frac{x}{2}$ . The width of the front face is labeled  $x$ . The depth of the cube is also labeled  $x$ .





Suppose the volume of the box  $=V(x)=\left(\frac{b}{2\sqrt{2}}-\frac{x}{2}\right)x^2=\frac{bx^2}{2\sqrt{2}}-\frac{x^3}{2}, x\leq\frac{2a-b}{\sqrt{2}}$

$$V(x)=\frac{bx^2}{2\sqrt{2}}-\frac{x^3}{2}$$

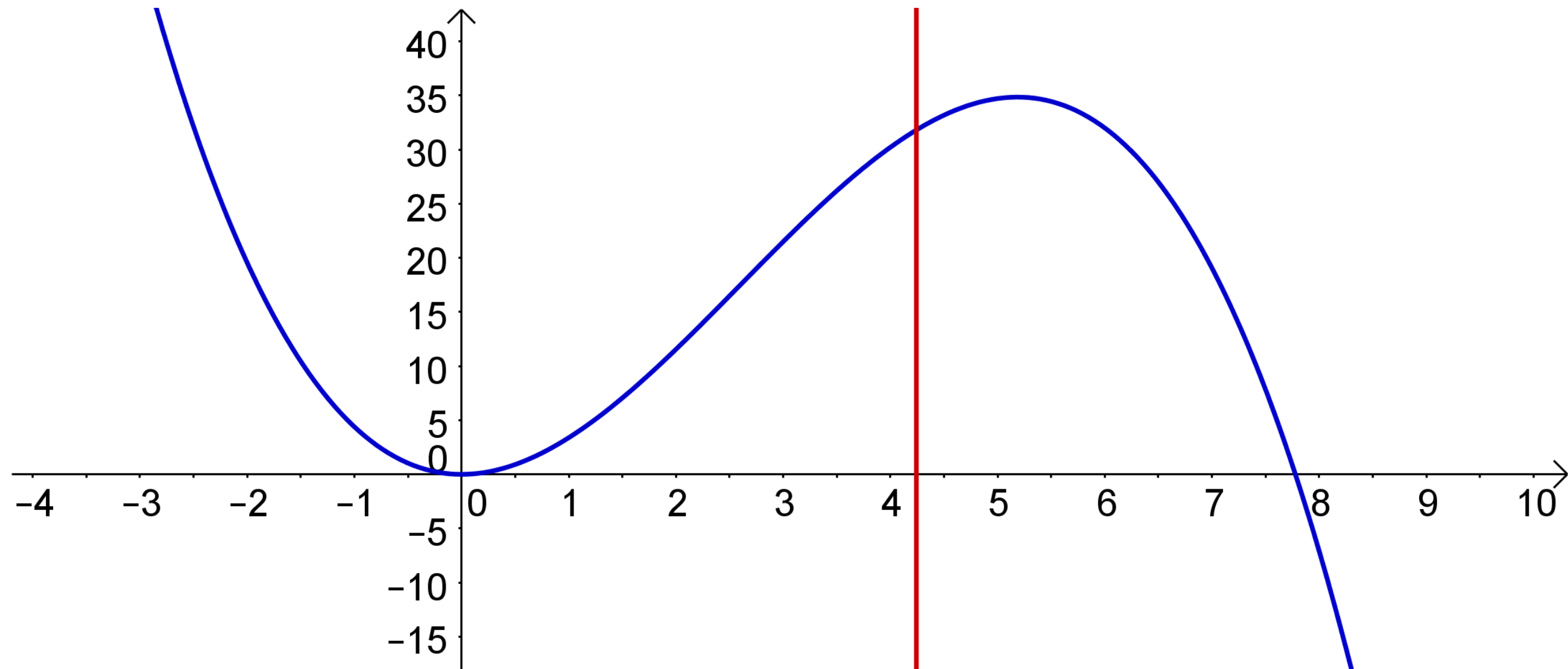
$$\Rightarrow V'(x)=\frac{bx}{\sqrt{2}}-\frac{3x^2}{2}$$

$$\Rightarrow V''(x)=\frac{b}{\sqrt{2}}-3x$$

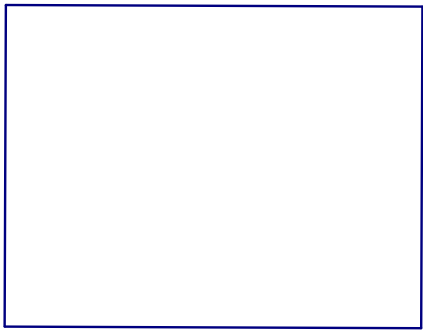
$$V'(x)=0$$

$$\Rightarrow \frac{bx}{\sqrt{2}}-\frac{3x^2}{2}=0$$

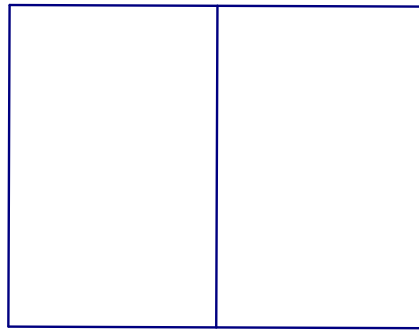
$$\Rightarrow x=0 \text{ or } x=\frac{\sqrt{2}b}{3}$$



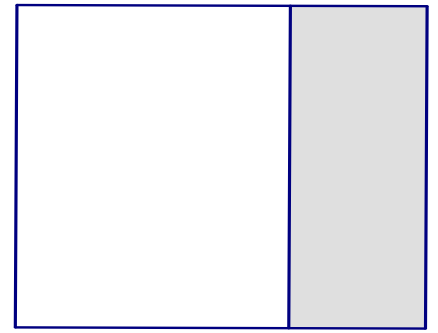
$$V(x) = \frac{11x^2}{2\sqrt{2}} - \frac{x^3}{2}, \quad x \leq \frac{6}{\sqrt{2}}$$



1



2



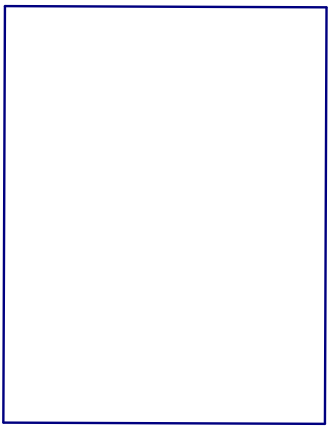
3

Step 1. Take the 11 inches by 8.5 inches rectangular measurement sheet.

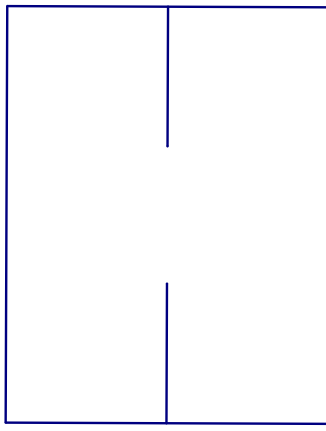
Place it on a hard surface so that the shorter sides are vertical, and the longer sides are horizontal.

Step 2. By folding and unfolding the shorter edges of the sheet onto each other, make a valley crease that is parallel to the shorter sides of the rectangular sheet.

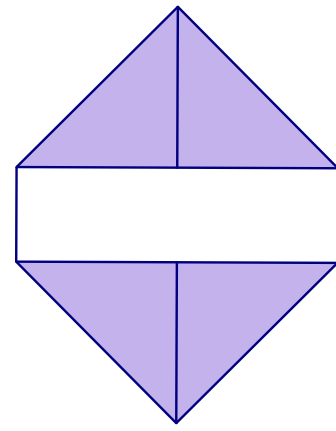
Step 3. Fold the right shorter edge of the rectangular sheet onto the crease created in step 2 to create another crease that is parallel to the crease created in step 2. Set aside the measurement sheet for now. We will use it later.



4



5

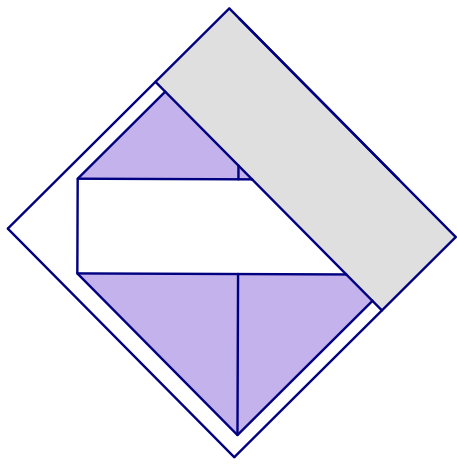


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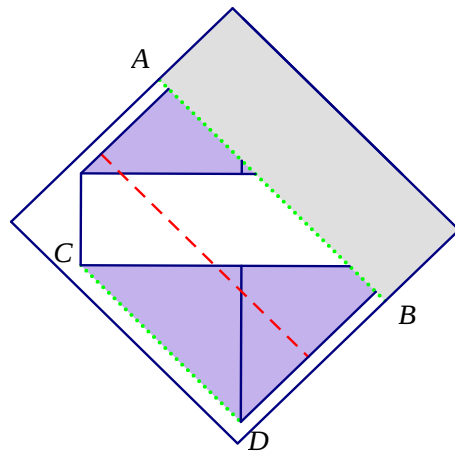
Step 4. Take the 11 inches by 8.5 inches origami sheet. Place it on a hard surface so that the longer sides are vertical, and the shorter sides are horizontal. Make sure the plain side is facing up.

Step 5. By folding and unfolding the longer edges of the sheet onto each other, make a valley crease that is parallel to the longer sides of the rectangular origami sheet. Leave the middle portion of the crease blank.

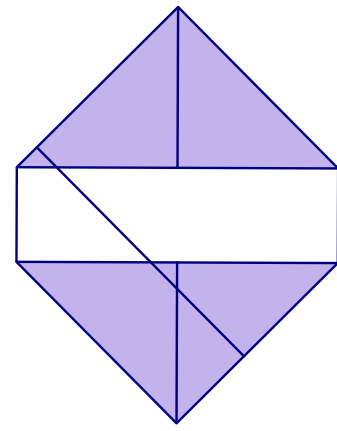
Step 6. Fold the left half and the right half of the top horizontal edge of the rectangular sheet over so that the two halves meet on the crease created in step 5. Fold the left half and the right half of the bottom horizontal edge of the rectangular sheet over so that the two halves meet on the crease created in step 5. The folded paper should be hexagonal in shape.



7



8

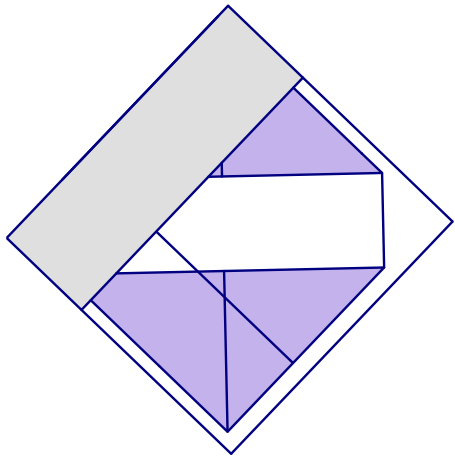


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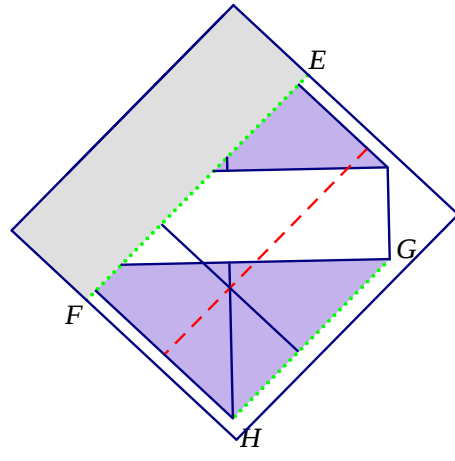
Step 7. Insert the top right slant edge of the origami sheet between the two layers of the measurement sheet. Make sure the entire top right slant edge of the origami sheet touches the crease in the measurement sheet that was created in step 3.

Step 8. Fold and unfold the bottom left edge *CD* of the origami sheet onto the edge *AB* of the measurement sheet to create a slant crease along the broken line.

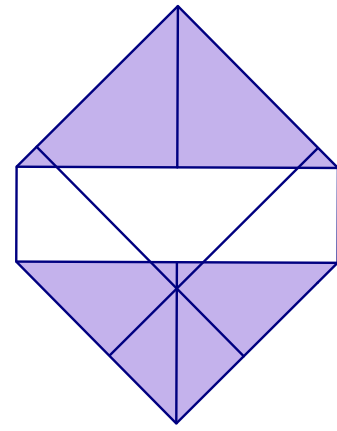
Step 9. Separate the origami sheet from the measurement sheet and observe the slant crease created in step 8.



10



11

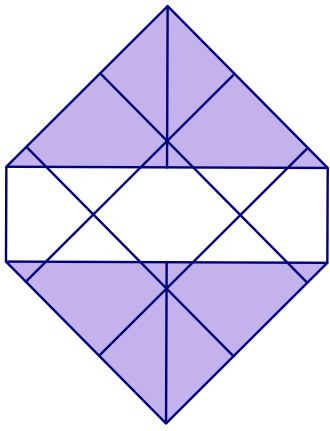


12

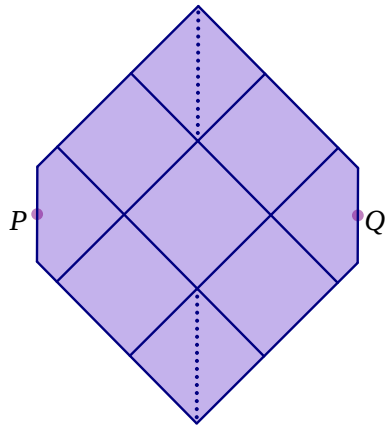
Step 10. Insert the top left slant edge of the origami sheet between the two layers of the measurement sheet. Make sure the entire top left slant edge of the origami sheet touches the crease in the measurement sheet that was created in step 3.

Step 11. Fold and unfold the bottom right edge  $GH$  of the origami sheet onto the edge  $EF$  of the measurement sheet to create a slant crease along the broken line.

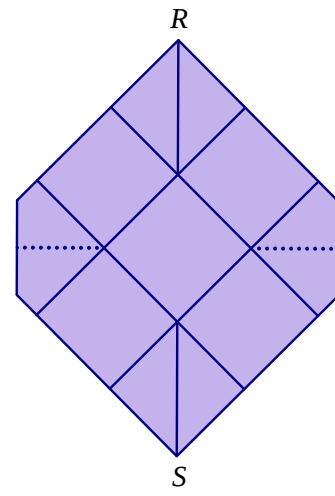
Step 12. Separate the origami sheet from the measurement sheet and observe the slant crease created in step 11.



13



14

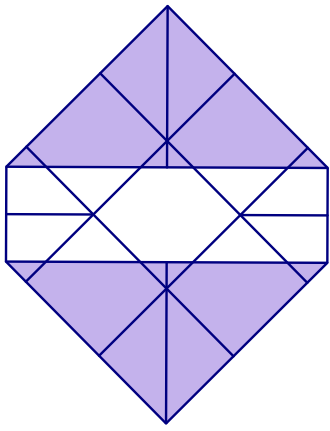


15

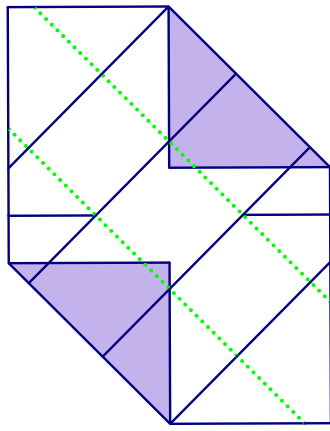
Step 13. Rotate the origami sheet (in the shape of a hexagon) around its center by an angle of  $180^\circ$ , and repeat steps 7 through 12 to create two more slant creases. Discard the measurement sheet; we do not need the measurement sheet anymore.

Step 14. Flip the hexagonal shape over. By dropping point *P* onto point *Q*, make two valley creases along the dotted line segments. Unfold the paper back to its hexagonal shape.

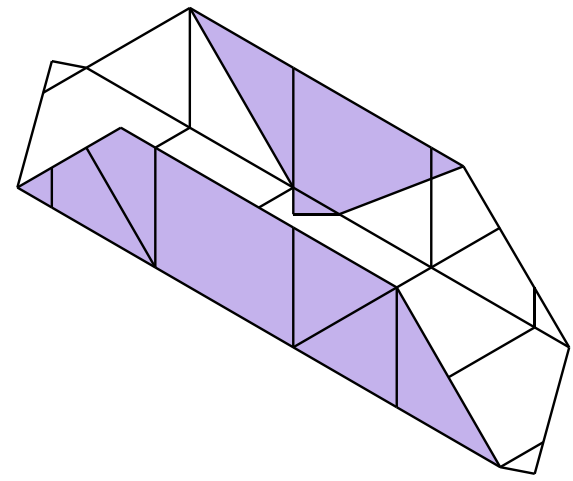
Step 15. By dropping point *R* onto point *S*, make two valley creases along the dotted line segments. Unfold the paper back to its hexagonal shape.



16



17

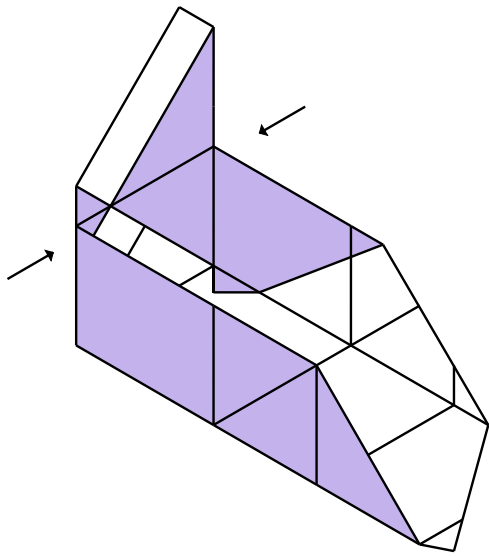


18

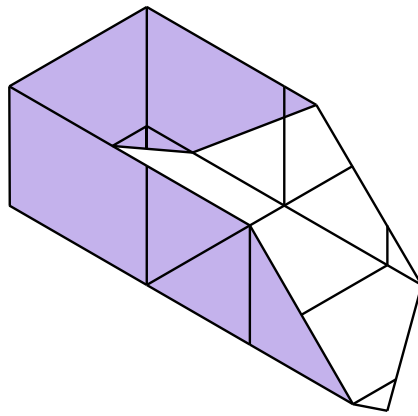
Step 16. Flip the shape over.

Step 17. Unfold the top left corner and the bottom right corner of the original rectangular origami sheet. By folding and then unfolding appropriately, make two valley creases along line segments marked with dotted lines.

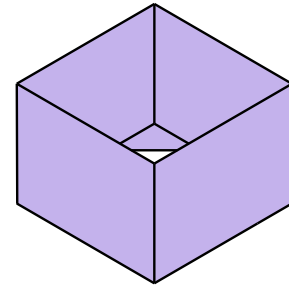
Step 18. Lift the outer parts of the hexagon up along the creases created in step 17 to form two of the walls of the box that will be constructed.



19



20



21

Step 19. By pushing the two short creases created in steps 14 and 15 inward (as shown by the arrows), lift one of the flat ends of the structure up to form the next wall of the box that will be constructed.

Step 20. Tuck the excess paper inside.

Step 21. Repeat steps 19 and 20 with the other flat end to complete the box.

Figures for steps 1-3 show the measurement sheet (this sheet is discarded).

Figures for steps 4-6, 9, and 12-21 show the origami sheet (this sheet is folded into the box).

Figures for steps 4-8, and 10-11 show the origami sheet slid between the layers of the measurement sheet.

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Link to the video:

<https://youtu.be/zyAYHDeY0PE>