



MATH TRANSFORMATIONS

Leading High Quality Math Lesson Study: What, Why, & How

NCTM Session 212

Thursday, April 26

3:15 PM - 4:30 PM

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www.mathtransformations.org

In this session we will . . .

Explore Lesson Study alongside a vertical team of elementary teachers.

Table Group Discussion

What experiences have you had with Lesson Study?



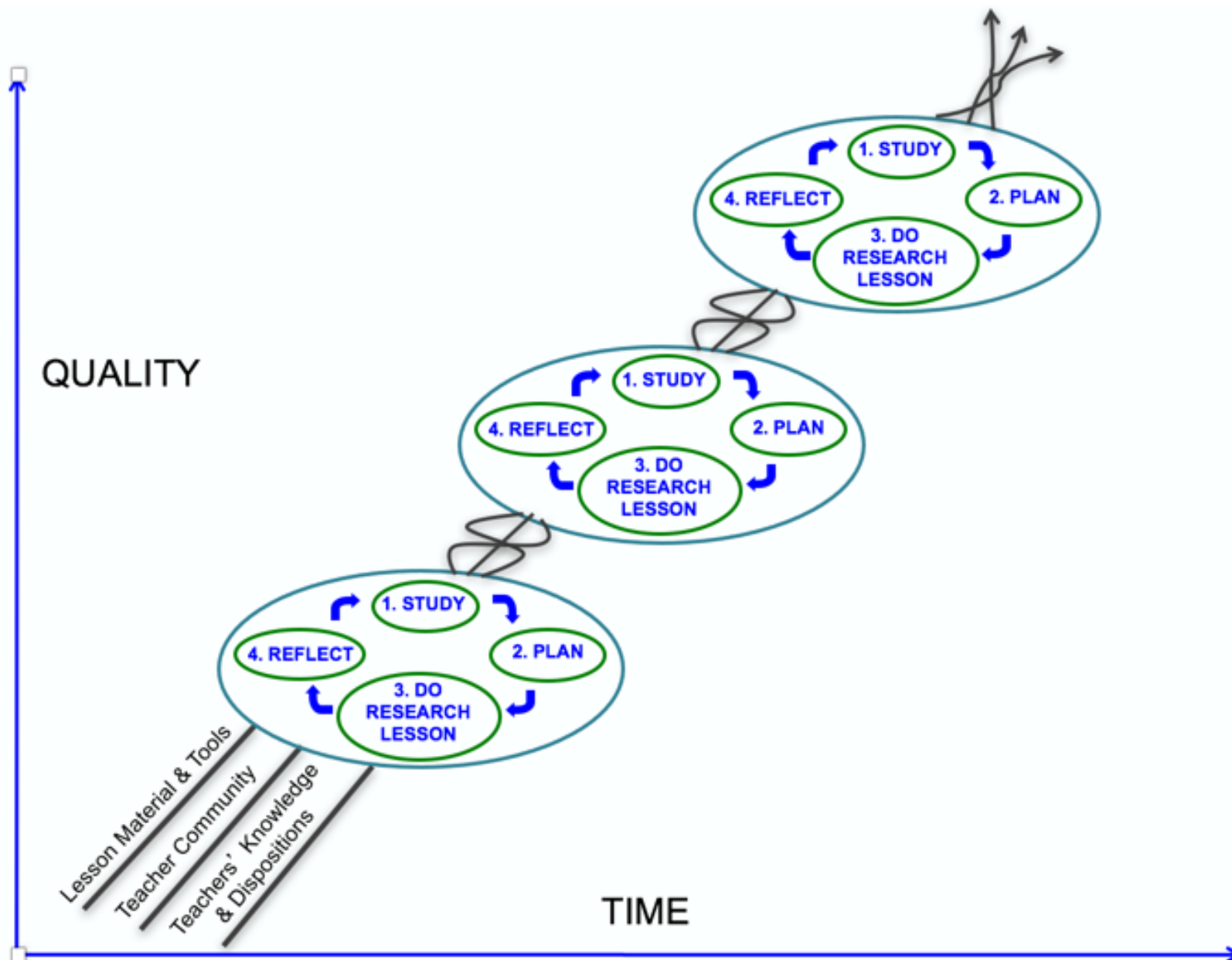
What is Lesson Study?

An iterative process of action research and teacher collaboration.

Cycles of Teacher Learning

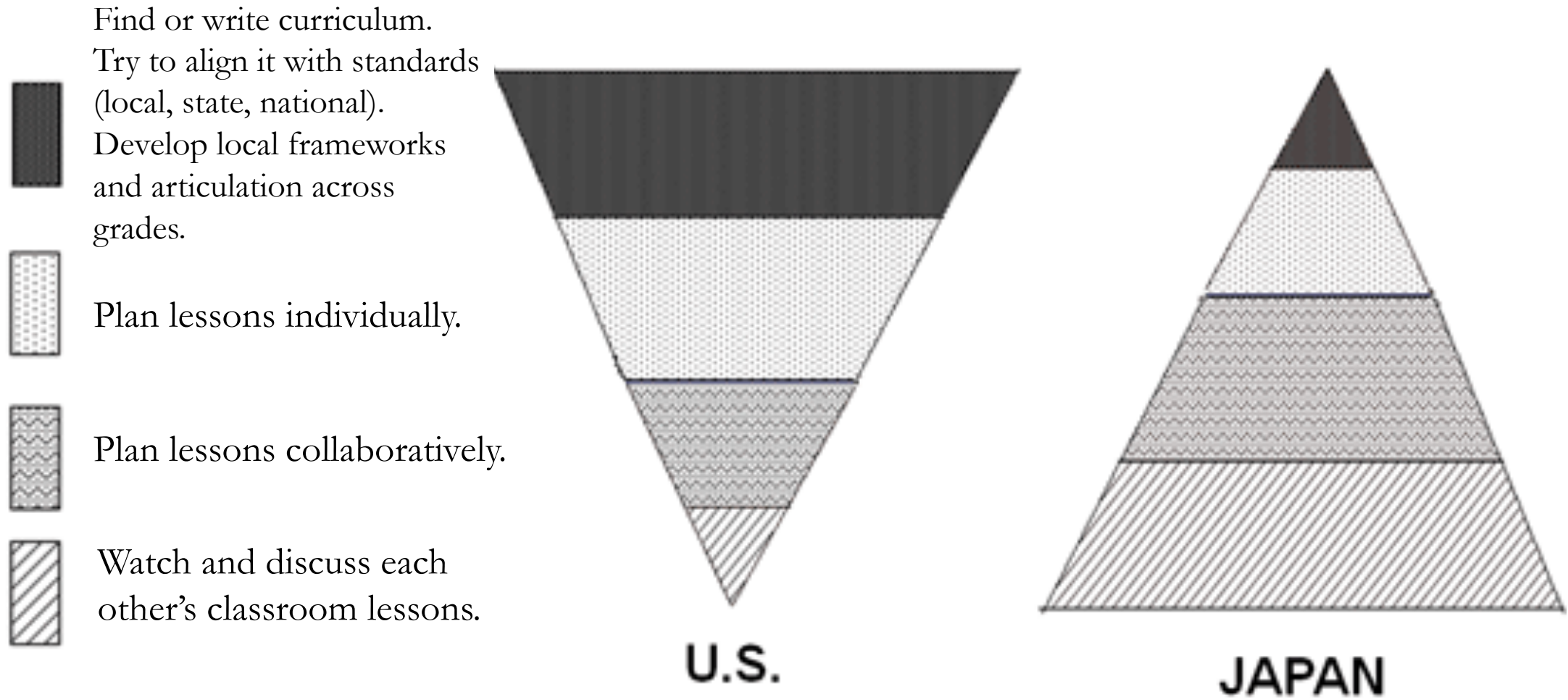
Research Lesson

Student Learning



Instructional Improvement Time in the United States and in Japan (Lewis & Hurd, 2011)

Teachers' Activities to Improve Instruction



Interaction of Factors Affecting a Teacher's Instructional Decisions



Beliefs and Mathematical Knowledge for Teaching



Social interactions and professional networks



Policies and “sense-making” of them



Curriculum resources and use of them



Vertical Lesson Study

Valley Elementary: Kinder, 1st, 3rd, & 4th Grades

Elba

Veronica

Danielle



Artemisa

Maribel

Andrea



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Getting Started: Overarching Goal

As our students grow and develop as learners in our world, what do we want them to realize about mathematics?

Valley's Goal:

We want our students . . .

- To understand that math is a way to make sense of the world,
- To be curious about the math that is everywhere in their lives,
- To know they can solve any math problem by thinking critically, being resourceful, and persisting.



Valley's Vertical Lesson Study Wonderings

How does students' strategic thinking about whole number operations progress across the grades?

How do our questions affect students' strategic thinking? Are the most effective questions the same for each grade or do they vary?

How does the depth of student mathematical thinking vary across the grades?



Getting Started: Research Question

Based on evidence of student need . . .

How does _____ affect _____?

What are the _____ that will allow students to _____?

Valley's Math Problem Solving Research Question:

What are the key questions and timing of the questions that will allow students to be curious, to make sense of the problem, to persevere, and go beyond?

Are these questions and timing consistent across the grades or do they vary?



Choosing the Task



What do you
notice?

What do you
wonder?

What might
the math
problems be?



The Math Problem

Mr. Jones is stacking boxes of soda. He created a display out of the boxes. He wonders how many cans are in the display. Please help him figure it out.

What questions do you have?





Challenge



Now that you know how many boxes and cans are in this display . . .

If you kept building the display upward, how many boxes and cans would be in a display that is 10 boxes tall? 20 boxes tall? 100 boxes tall? Any number of boxes tall?

What tools might you use to organize your work?



Lesson Study Template

How might you design a problem solving lesson with this task at the center?

Lesson Study

PART I: INTRODUCTION

Lesson Topic:

Lesson Study Overarching Goal:

Lesson Study Research Question:

What are some of the instructional strategies you are interested in exploring?

Relevant Standards:

What will a successful student be able to do as a result of this lesson.

PART II RESEARCH:

What will you do to learn more about the strategies you are interested in? What resources will you use? Who will you talk to?

PART III: SITUATING THE LESSON:

What unit is this lesson part of? Where does it fall within the unit?

What prior knowledge do students have?



Part IV: Lesson Plan

Student Learning Goal:

Timing	Lesson Parts	Activity Description	Teaching Roles
	Launch		
	Explore		
	Summary		

What evidence of student learning will we look for?

Differentiation:

Part V: Reflections



Planning the Research Lesson

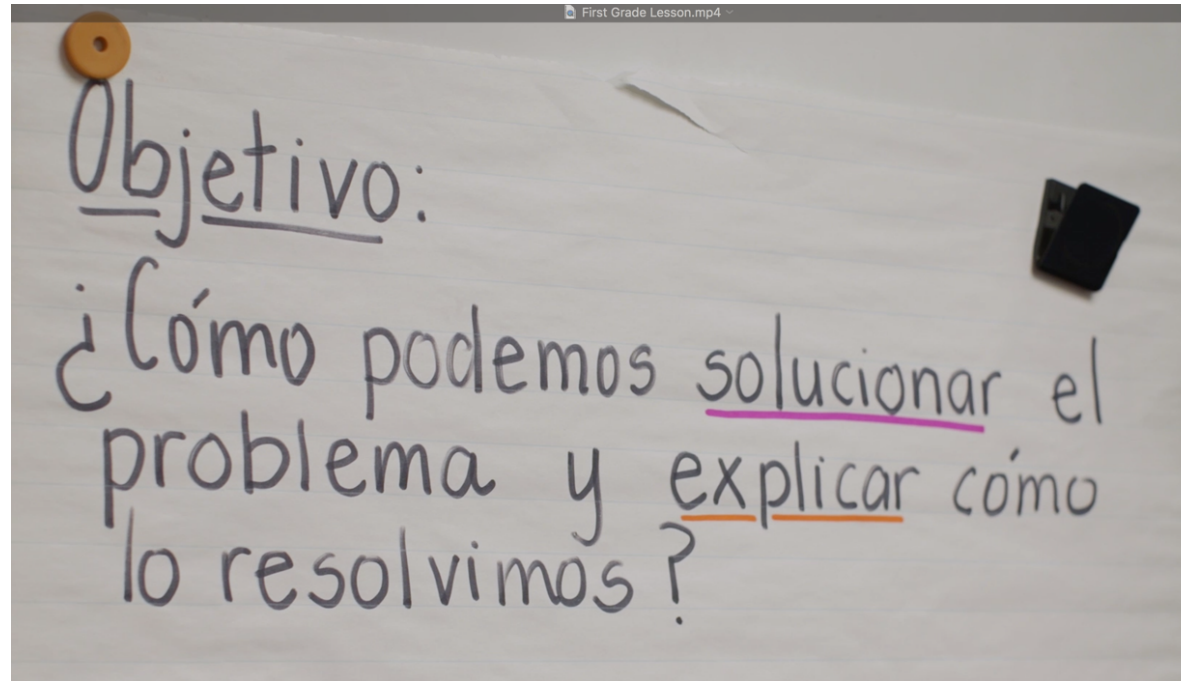
What do you notice about their collaboration?



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First Grade Implementation



How can we solve the problem and explain how we solved it?



Aa Bb Cc Ch ch Dd Ee Ff

¿Qué ves?



Pregunta:
¿Cuántas cajas hay apladas?
¿Cómo lo sabes?

Objetivo:

¿Cómo podemos solucionar el problema y explicar cómo lo resolvimos?

las formas

círculo

rectángulo

Todo cuerpo de mí

La pila

What do you notice? What do you wonder?



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MATEMÁTICAS

Vocabulario

$3+5+5=9$
Total
todo junto

$2+1=3$
sumar
juntar

quitar
 $6-5=1$
restar

más pequeño
menor <

más grande
mayor >

$6=3+3$
= es igual a


$6 \neq 3+2$
≠ no es igual a

grupos de diez
decenas

unas sueltas
unidades
 $23=11$

$5=3+\square$
número desconocido

Herramientas para las matemáticas

cubitos manos 

bloques geométricos

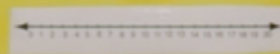
Rekenrek

fichas

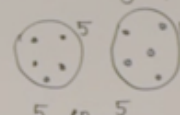
marcos de diez


gráfica de 120

recta numérica



Maneras de ORGANIZAR mi trabajo

formar grupos

5 en 5

en filas


en orden
1, 2, 3, 4, 5

rotular
 $3+2=5$

$0+1=1$
 $1+1=2$
 $2+1=3$
 $3+1=4$
 $4+1=5$
 $5+1=6$
 $6+1=7$
 $7+1=8$
 $8+1=9$
 $9+1=10$

tabla

desayuno	almuerzo	cena
Alberto Adrian	Alvaro Edu Isabel	Rene Ses. Shasha Derrick

Estrategias

1 en 1 2 en 2
Contar de... 5 en 5

$1+9=10$ $5+5=10$ $8+2=10$
formar un grupo de $3+7=10$

3, 4, 5, 6...
contar hacia adelante

$10-8=2$ $8+2=10$
sumas ↔ restas



How are you solving the problem?
How is that helping you solve the problem?

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What is the problem you are solving?
How is that helping you solve the problem?

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What do these numbers mean?

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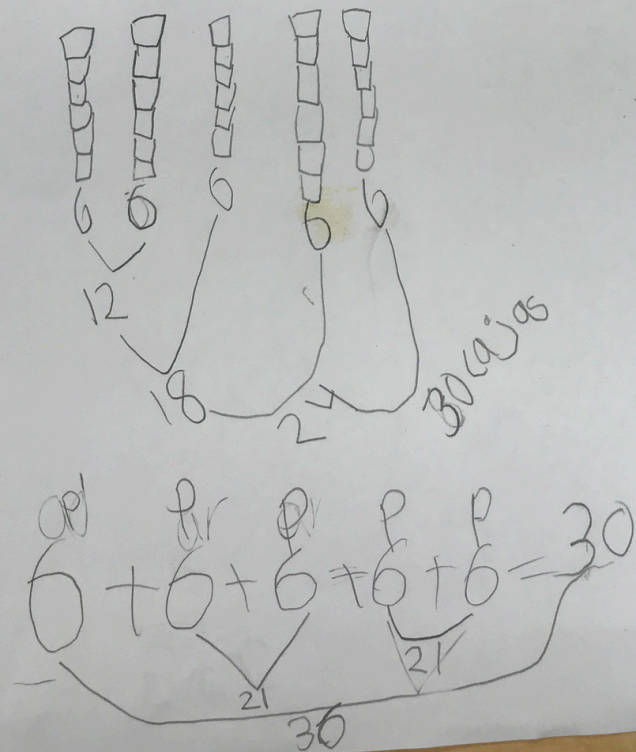


How did they solve the problem?

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ma Kie y Malakai



Estrategias • Formar grupos Carlos y Maya
• Cubitos

Usamos los cubitos para resolver este
Problema con grupos de 6.

$$6 + 6 = 12$$

$$12 + 6 = 18$$

$$18 + 6 = 24$$

$$24 + 6 = 30$$

Treinta cajas de soda es la respuesta.



First Grade Analyze Data, Reflect and Revise

What do you notice about how they process the lesson?



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Fourth Grade Implementation

Research Question: What are the key questions and timing of them that will allow students to be curious, to make sense of the problem, to persevere, and go beyond?

As you watch . . .

What do you observe that can help us answer our research question?

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What changes do you notice from the first lesson?
How is student learning affected?



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Processing

Research Question: What are the key questions and timing of them that will allow students to be curious, to make sense of the problem, to persevere, and go beyond?

- What did you observe that can help us answer our research question?
- What changes did you notice from the first lesson? How was student learning affected?

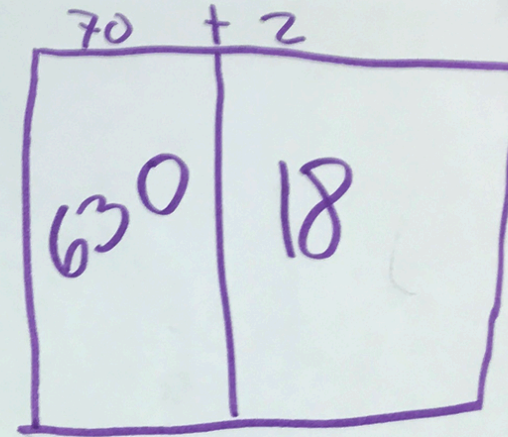


$$\begin{aligned}
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72 \\
 12 \times 6 &= 72
 \end{aligned}$$

$$\begin{array}{r}
 648 \\
 9 \times 70 = 630 \\
 9 \times 2 = 18 \\
 \hline
 \end{array}$$

total de
latas de

El 12
se representa
las latas y
el 72 representa
las latas en
cada línea. El
número 9 representa
cada capa. El 6
representa cada
caja.



$$\begin{aligned}
 9 \times 70 &= 630 \\
 9 \times 2 &= 18 \\
 \hline
 9 \times 72 &= 648
 \end{aligned}$$

Por: Sarah y
Perla.

“The 12 represents the cans and the 72 represents the cans in each line. The number 9 represents each layer. The 6 represents each box.”



So

Answer:

We think that there is 648 cans in all of those boxes. We counted 9 boxes in 1 row and 5 around. We multiplied those and that was 45. We added the boxes in the middle and we have 54 in total. There is 12 cans in each box so when we multiply 54×12 that made 648.

Boxes in a row
↓

$$9 \times 5 = 45 + 9 = 54$$

all around

Boxes in middle

all together

54 divided / 54 from our equation
12 divided / 12 from cans in each box

$$50 \times 10 = 500$$

$$50 \times 2 = 100$$

$$4 \times 10 = 40$$

$$4 \times 2 = 8$$

From our other equation
From our other equation
From our other equation
From our other equation
whole answer

$$500 + 100 + 40 + 8 = 648$$

Mikki & Seonna





9 Boxes
(at the moment)

Key Information:
12 cans per box.

HOW MANY CANS?

We know that there are five boxes in each row.

We can multiply nine times five, which gets us forty-five.

We know that there are nine boxes in the middle because in the first pictures they stack one row and one box in the middle.

Then, we add nine to the forty-five, which lets us finish with fifty-four. Finally, we multiply 54 times 12.

x	50	+ 4
10	500	40
+ 2	100	8

$$\begin{array}{r}
 500 \\
 + 100 \\
 \hline
 600 \\
 + 40 \\
 \hline
 640 \\
 + 8 \\
 \hline
 648
 \end{array}$$

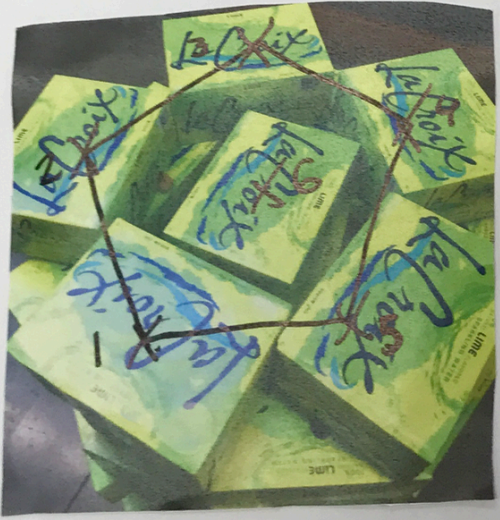
648 cans in total!

By: Sofia and EMM★



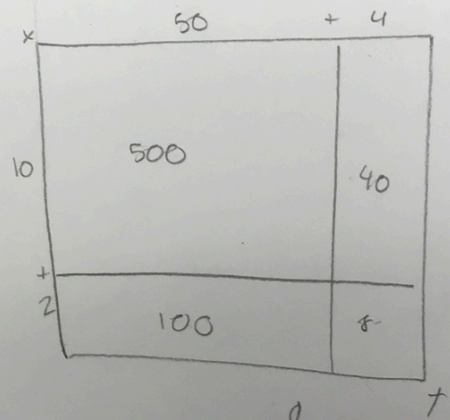
estimation:
500

First we multiplied 9×6 , 6 boxes in each row and 9 boxes in each layer counting the middle and we came up with 54. Then, we knew that since there are 12 cans in each box, 54×12 must be 648. So the answer is 648 cans in the whole display.



6 boxes in each row.

boxes cans
 $\underline{54 \times 12}$



$$\begin{array}{r} 8 \\ + 40 \\ + 100 \\ + 500 \\ \hline 648 \end{array}$$



9 boxes in each.

By: Scott F, Andre F 00



CHALLENGE PROBLEM

6 boxes per layer and 10 layers and 12 cans per box and

$$6_{\text{boxes}} \times 12_{\text{cans}} = 72_{\text{Per layer}} \text{ so } 72_{\text{Per layer}} \times 10_{\text{layers}} = 720$$

$$\begin{array}{r}
 70 \times 10 = 700 \\
 + 2 \times 10 = 20 \\
 \hline
 720 \\
 \text{cans total}
 \end{array}$$

6 boxes per layer and 20 layers and 12 cans per box and

$$\begin{array}{r}
 6_{\text{boxes}} \times 12_{\text{cans}} = 72_{\text{Per layer}} \text{ so } 72 \times 20 = 1,440 \\
 70 \times 20 = 1,400 \\
 + 2 \times 20 = 40 \\
 \hline
 1,440 \\
 \text{cans total}
 \end{array}$$

see back for
more amazing
math →

Teagan + Kendall



Fourth Grade *Analyze Data, Reflect, Revise*



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“This whole experience has brought back a sense of control, ownership, creativity, and trust in our ability as teachers to discover what works when teaching our students. It brings back a level of professionalism to our careers that had been stripped away by such an intense focus on testing, curriculum pacing, and evaluative feedback.”

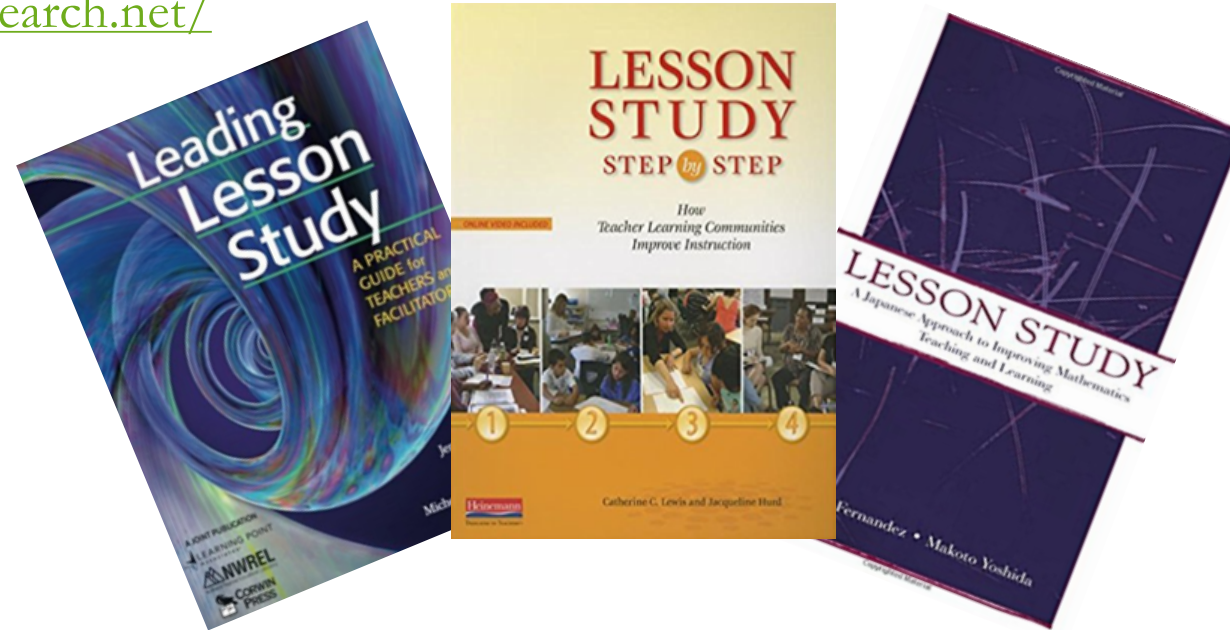
Artemisa Perucho
1st Grade Teacher
San Diego, CA



Key Resources

Lesson Study at Mills College

<http://www.lessonresearch.net/>



Lesson Study Template

Uploaded to NCTM

Lesson Study

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Lesson Topic:

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Questions?



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Thank you!

Andrea Barraugh
Math Transformations
mathtransformations.com
