Insight Through Misconceptions:

Helping Students Build Strong Geometric Understanding

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Keys to Building Geometric Proficiency

Use assessment and tasks that reveal and allow for analysis of student misconceptions.

Allow students to explore with a wide variety of types of geometric figures, both physical and visual.

Have students actively engaged in discourse about the properties of and relationships among various geometric figures.

Misconceptions vs. Mistakes

Misconception

a faulty belief

"set within deeper levels of knowledge"

MAY BE:

- misapplication of a rule
- over-generalization
- under-generalization

Mistake

an error or inaccuracy

"principally formed within surface levels of knowledge"

MAY BE DUE TO:

- carelessness
- misreading of problem
- leaving out a step
- lack of prior knowledge

Hansen, Children's Errors in Mathematics

Misconceptions in Geometry

Misconception

a faulty belief

"set within deeper levels of knowledge"

MAY BE:

- misapplication of a rule
- over-generalization
- under-generalization

Why do students develop misconceptions?

Concept Image & Concept Definition

Concept Image

"all the mental pictures and associated properties and processes"

Concept Definition

"a form of words used to specify a concept"

Vinner & Hershkowitz (1980)
Tall & Vinner (1981)

Concept Image & Concept Definition

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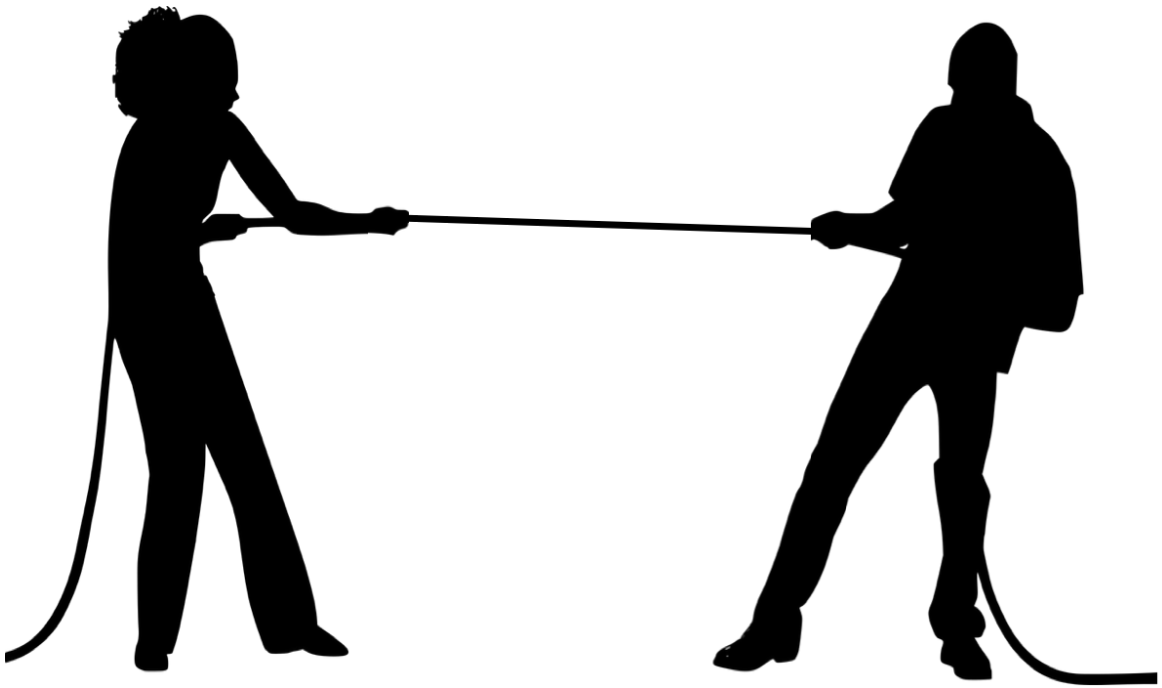

Vinner & Hershkowitz (1980)
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Concept Image & Concept Definition

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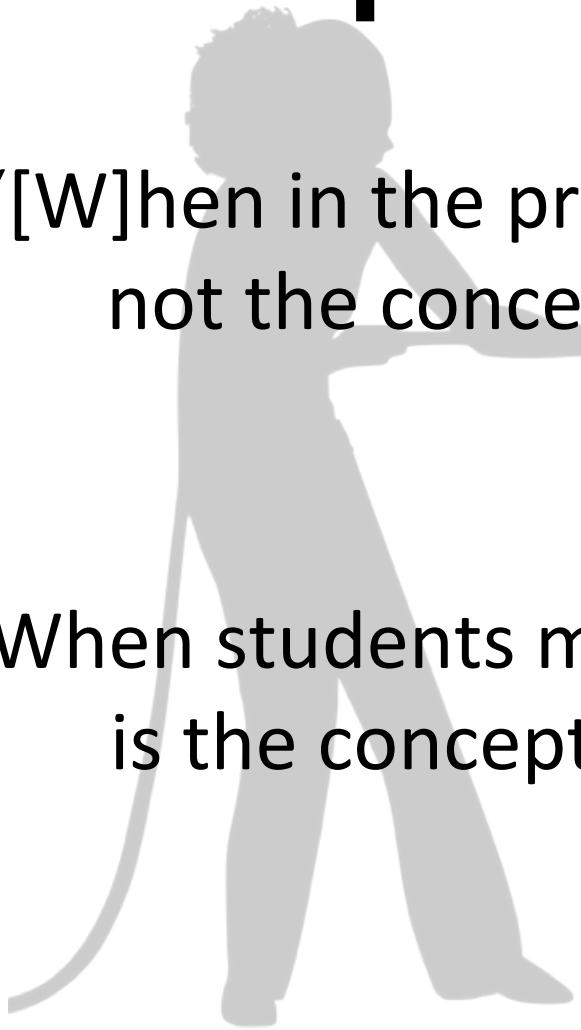
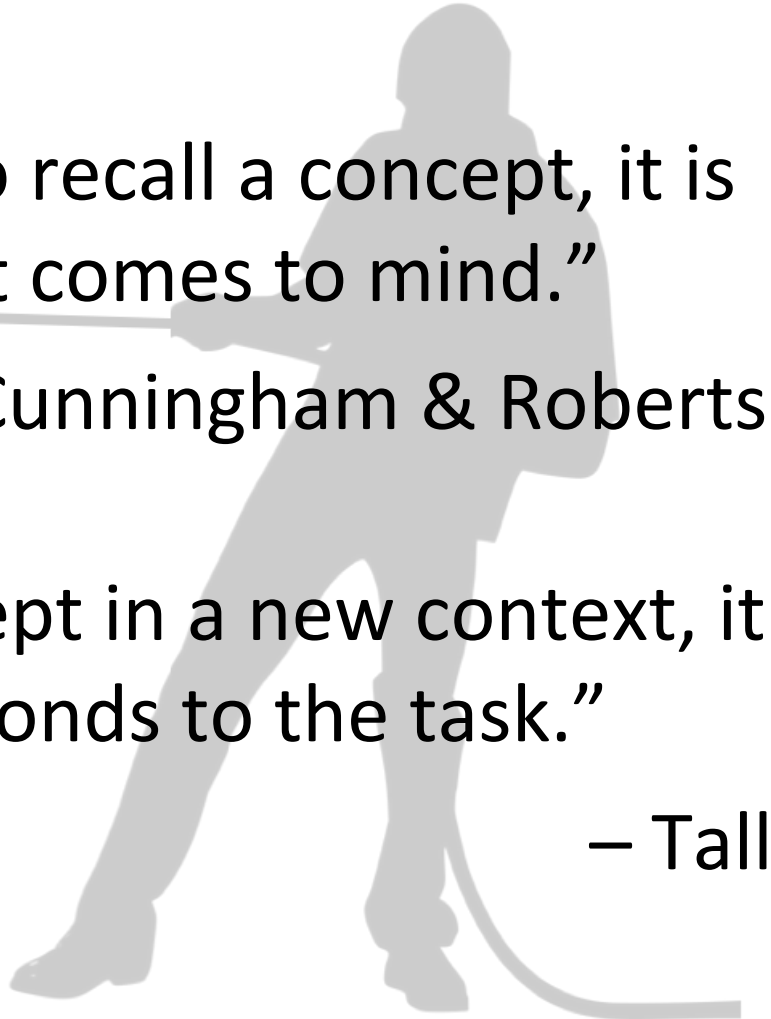




"[W]hen in the process of trying to recall a concept, it is not the concept definition that comes to mind."

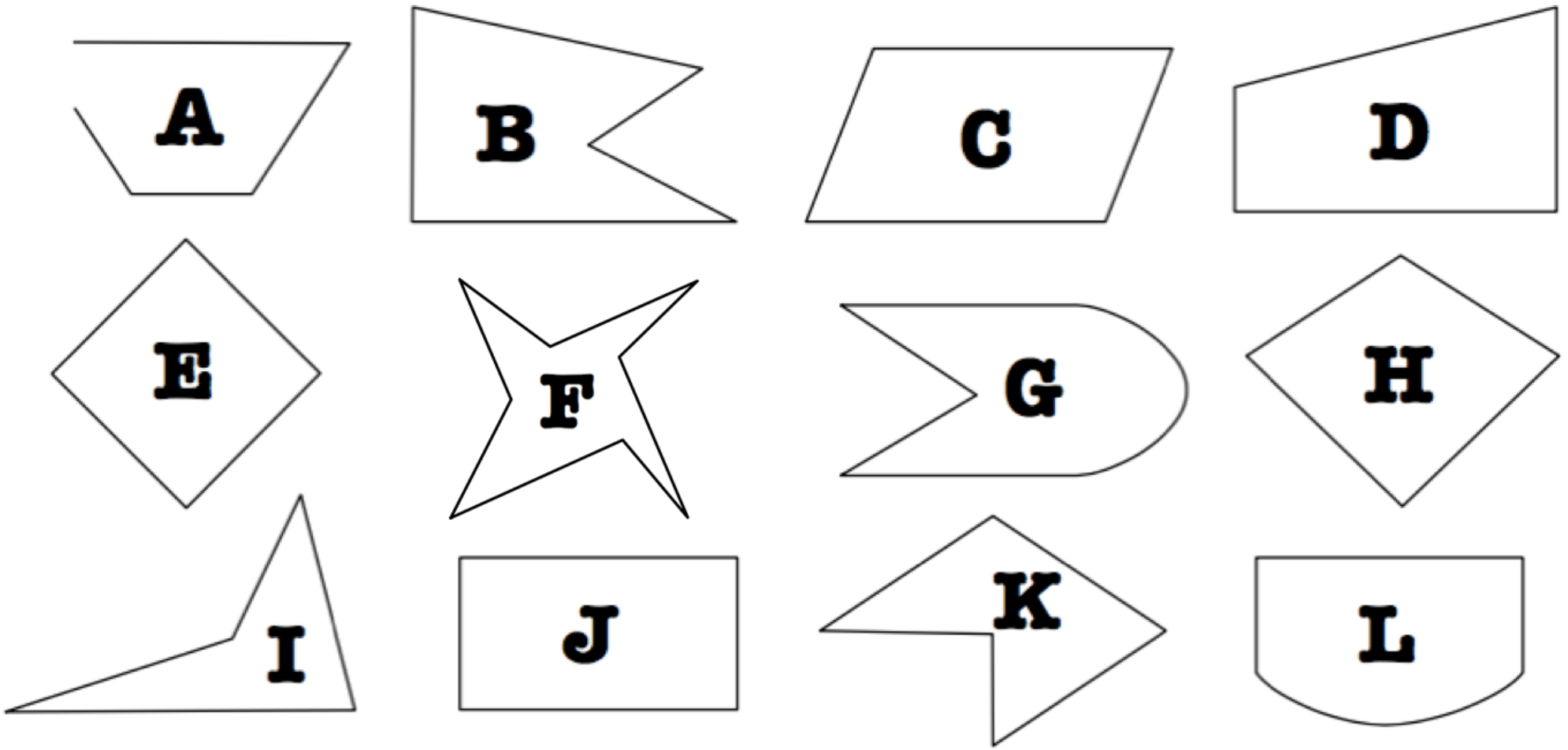
– Cunningham & Roberts

"When students meet an old concept in a new context, it is the concept image that responds to the task."

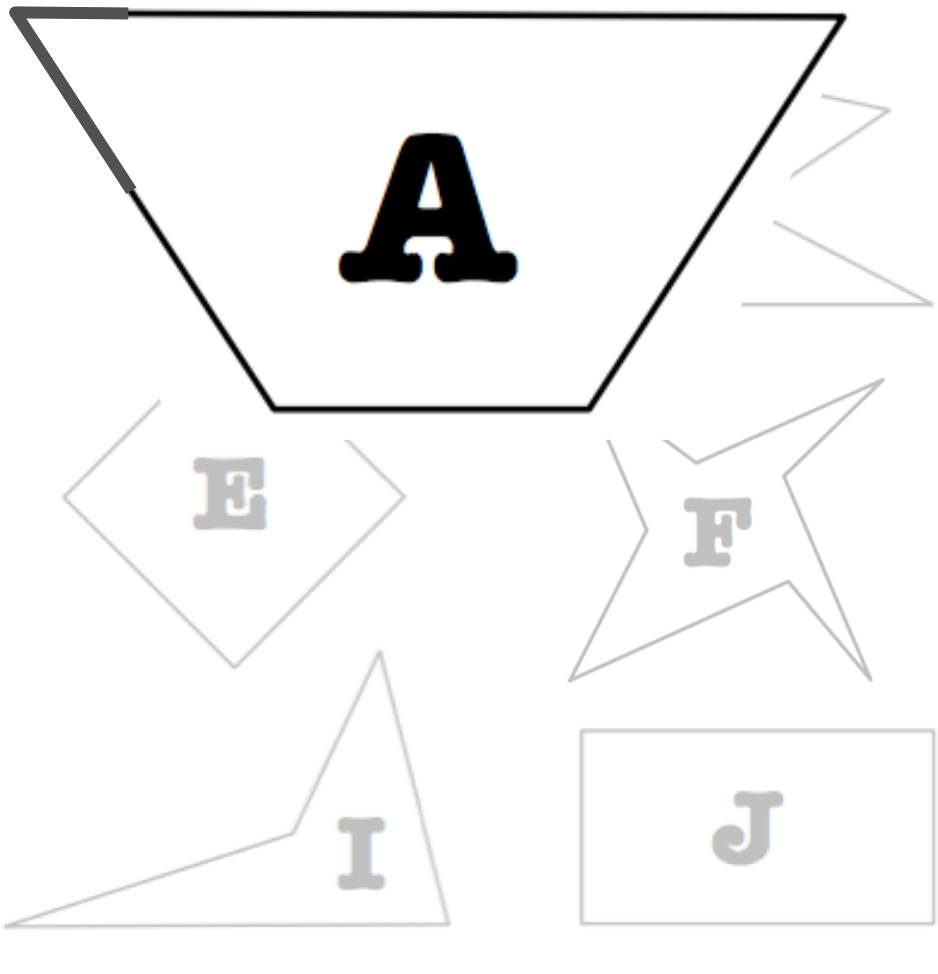
– Tall

Cunningham & Roberts (2010), Tall (1988)

The Power of the Concept Image



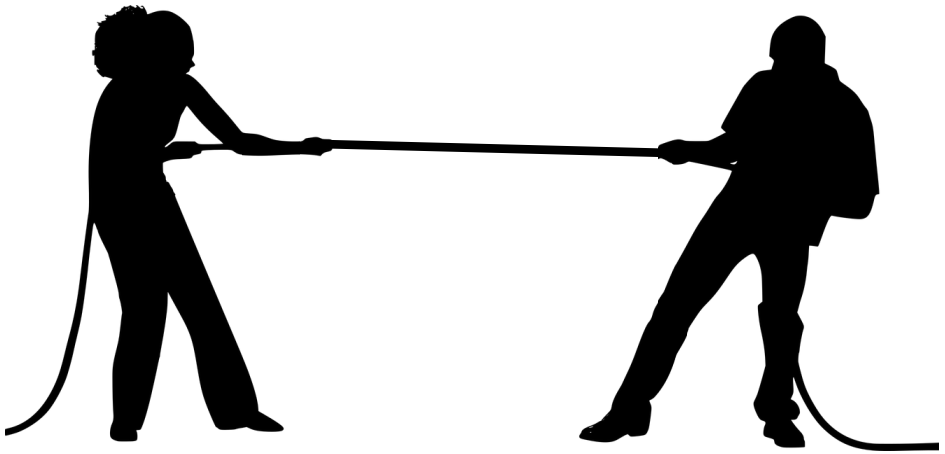
The Power of the Concept Image



You made a typo with this shape. Either that, or this paper didn't copy right on the copier. Should I just fix it?

The Power of the Concept Image

Concept Image



Concept Definition

"[W]hen in the process of trying to recall a concept, it is not the concept definition that comes to mind. A student typically remembers...the concept image, [which] can be limited to a single prototypical image, and an over-reliance on it can impact their understanding."

– Cunningham & Roberts

Cunningham & Roberts (2010)

Concept Image & Concept Definition

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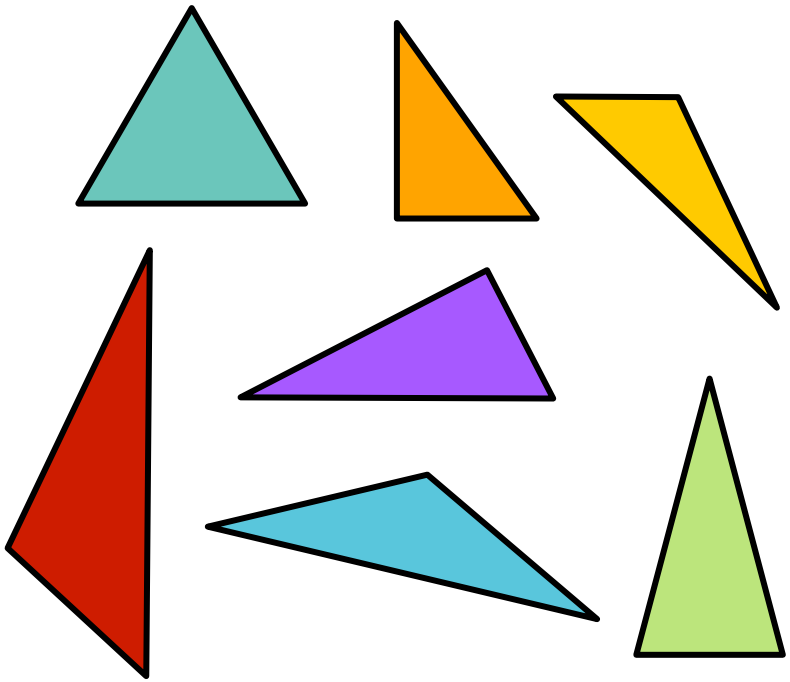
"all the mental pictures and associated properties and processes"



Concept Definition

"a form of words used to specify a concept"

A triangle is a closed figure with three straight sides and three angles.

Vinner & Hershkowitz (1980)
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Concept Image & Concept Definition

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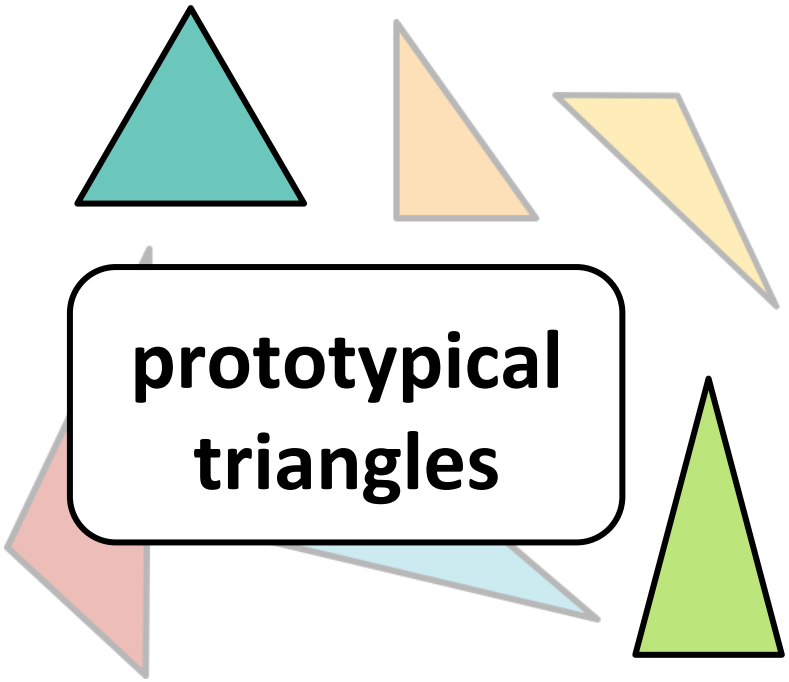

Concept Definition

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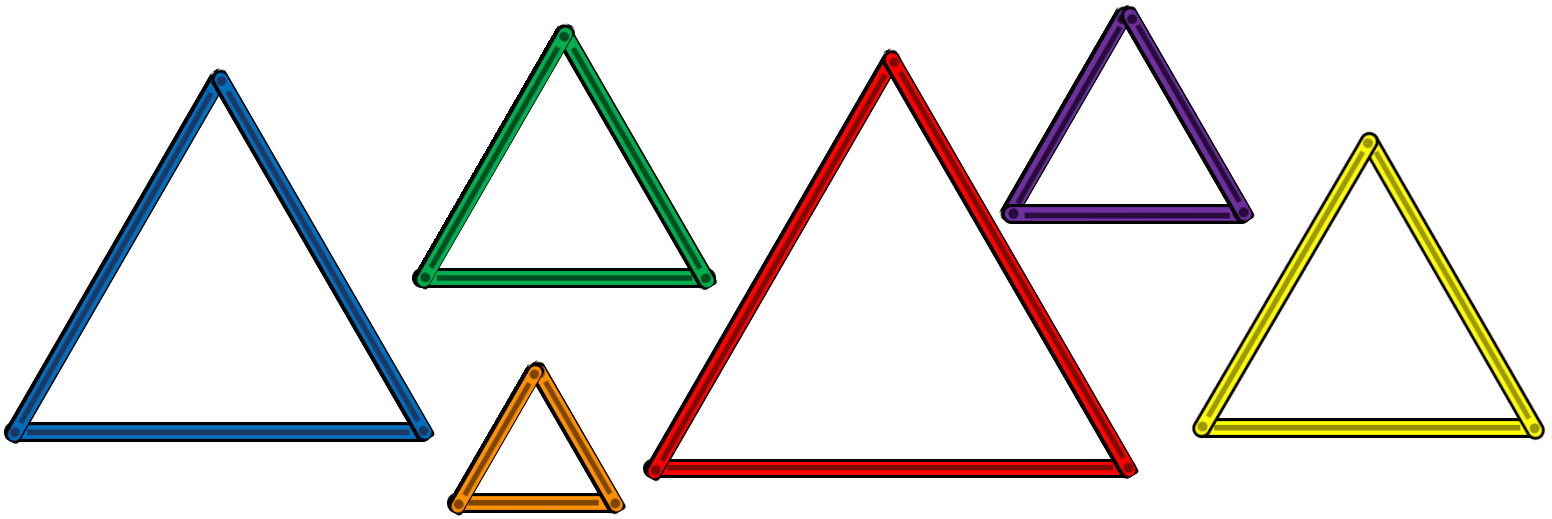
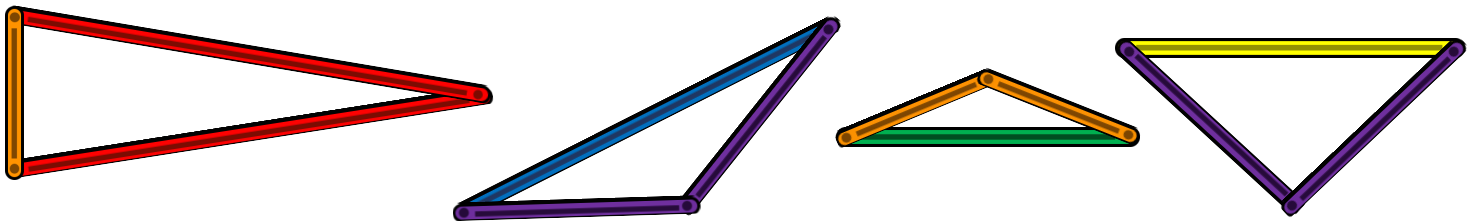
Building Triangles with AngLegs

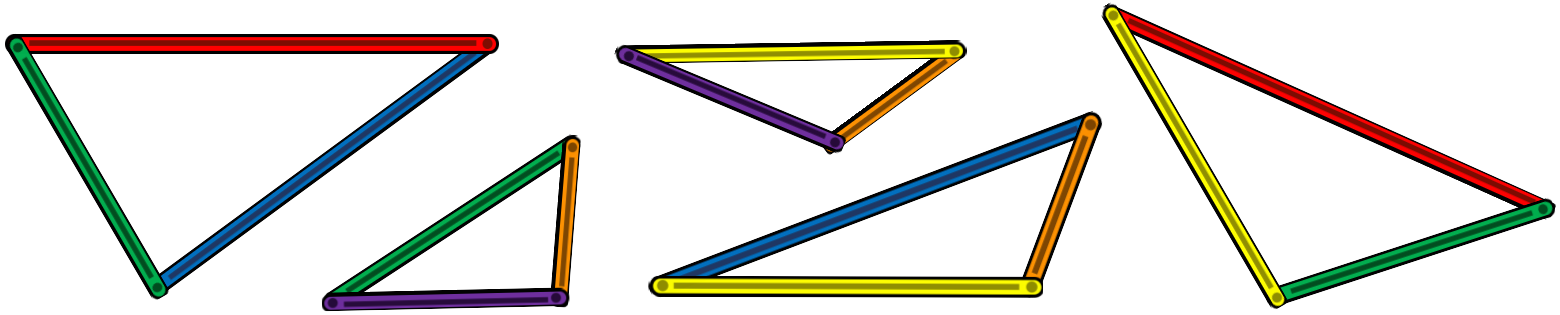
Equilateral Triangles



Isosceles Triangles



Scalene Triangles



Expanding Beyond Prototypes

prototypical – original or standard; that which comes immediately to mind

Choose two of the figures named and work with a partner to discuss and draw a picture of what you would consider to be the "prototype" of each figure, and then draw one or two non-prototypical examples of each.

Geometric Figures: Prototypical and Non-Prototypical Examples

Figure(s)	Prototypical Example	Non-Prototypical Examples
square		
trapezoid		
perpendicular segments		
pentagon		
pyramid		

created by Dennis McDonald

"Common" and "Rare" Examples

Geometric Figures: Common vs. Rare

Figure(s)	"Common" Examples	"Rare" Examples

created by Dennis McDonald

"Common" and "Rare" Examples

Geometric Figures: Common vs. Rare

Figure(s)	"Common" Examples	"Rare" Examples

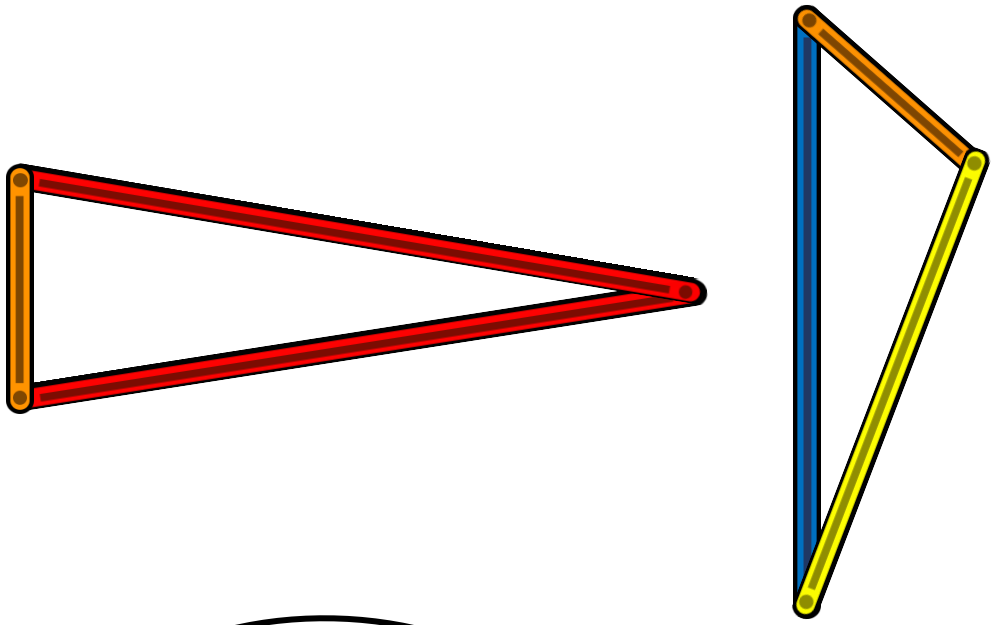
pre-assessment

post-assessment

created by Dennis McDonald

Why Might? — A Reasoning Routine

Shana's Figures



I made two triangles.

Amy

I don't think those are triangles.

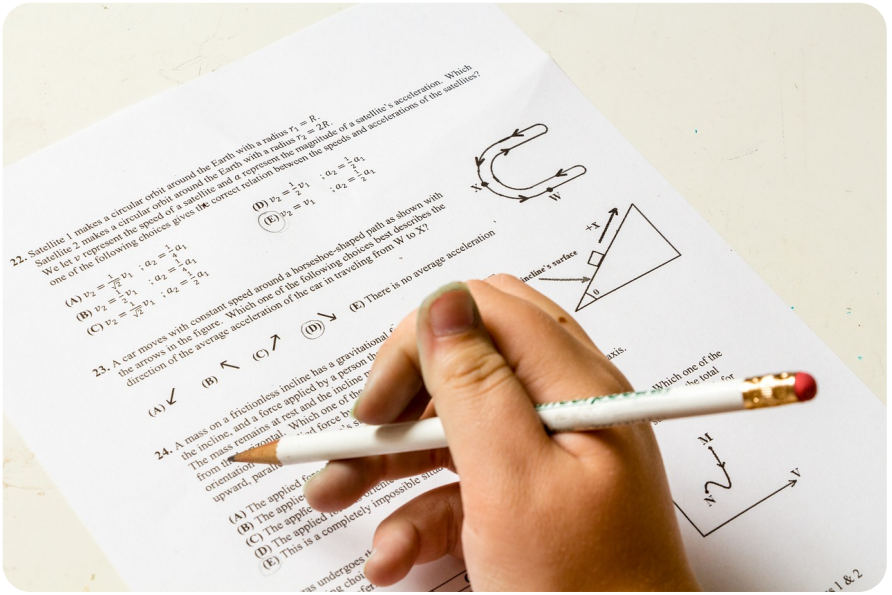
Why might Amy think that Shana's figures are not triangles?

How could we prove to Shana that they ***are*** actually triangles?

Why Might? & Worked Examples

What Are They?

- presents fully completed student work for analysis
- can show correct and incorrect work



Why Use Them?

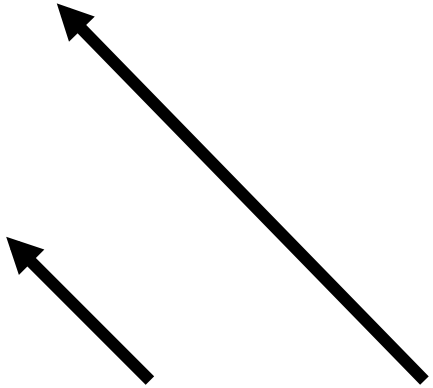
- improved conceptual and procedural understanding
- focuses students' attention on big mathematical ideas
- builds awareness of misconceptions
- results in fewer errors, increased work speed, and decreased need for teacher assistance

McGinn, Lange, and Booth (2015)

Why Might? — A Reasoning Routine

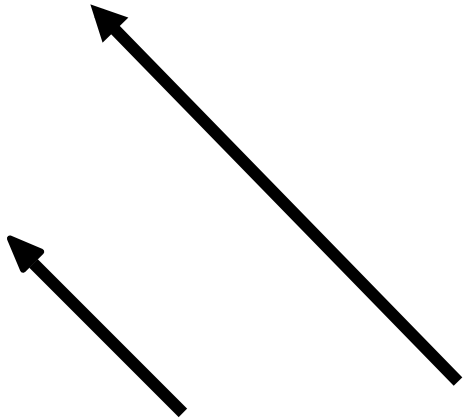
Mya

I drew a pair of parallel rays.



Kelly

Actually, they're not. But if you make the bottom one longer...like this...they *would* be parallel.



Why might Kelly have thought that Mya's rays were not parallel?

How could Mya prove to Kelly that her rays ***are*** actually parallel?

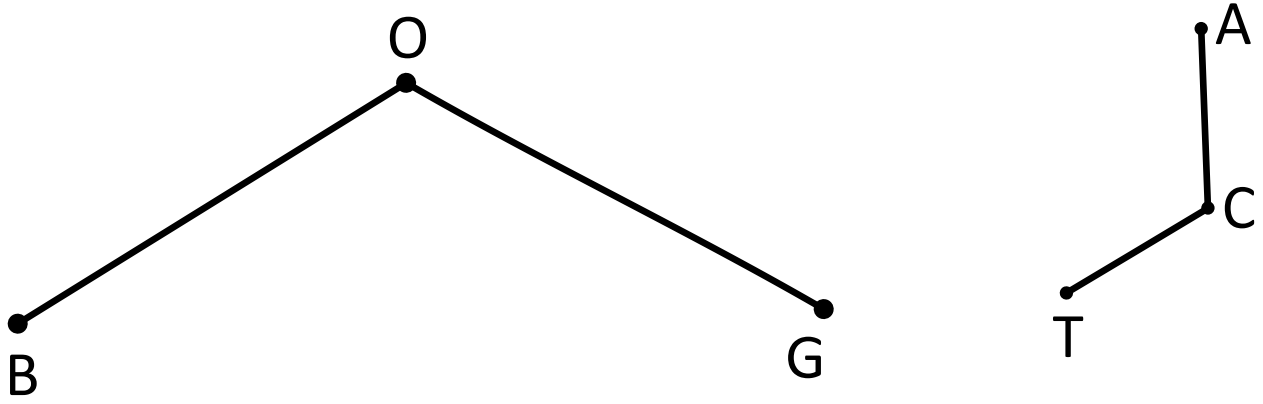
Why Might? — A Reasoning Routine

Kelvin

∠ACT is smaller than ∠BOG.

Bryce

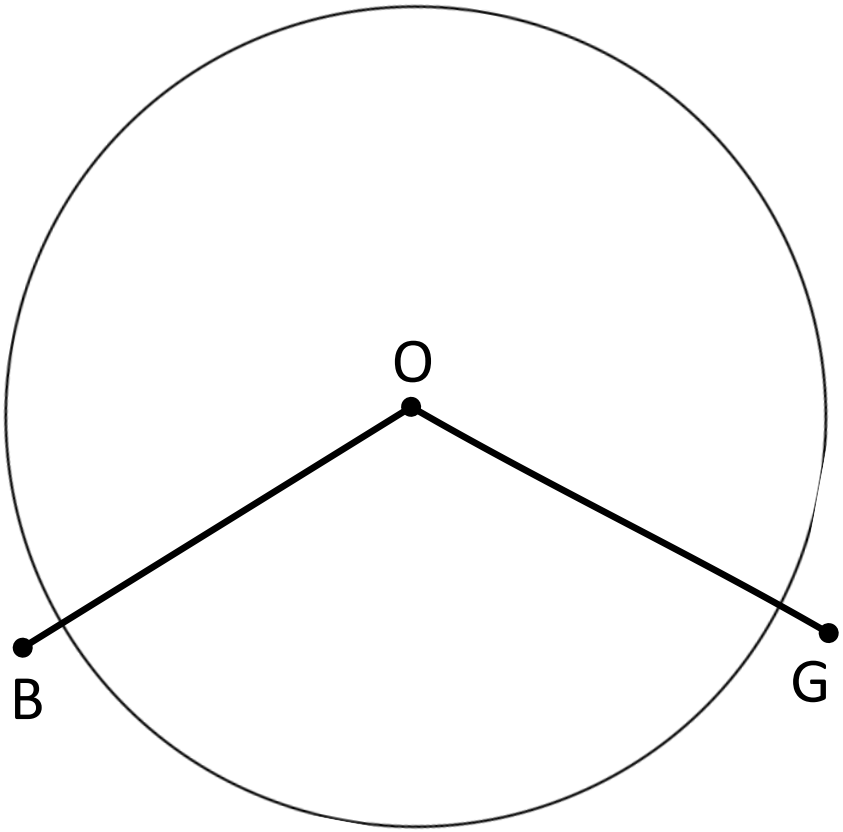
I think the angles are the same size or congruent to each other.

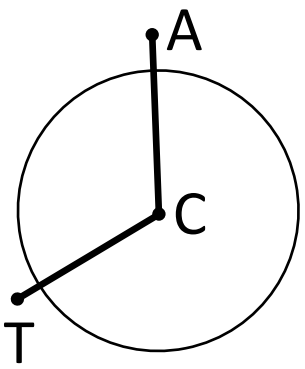


Why might Kelvin think that ∠ACT is smaller than ∠BOG?

How could Bryce prove to Kelvin that the two angles ***are*** congruent to each other?

Why Might? — A Reasoning Routine





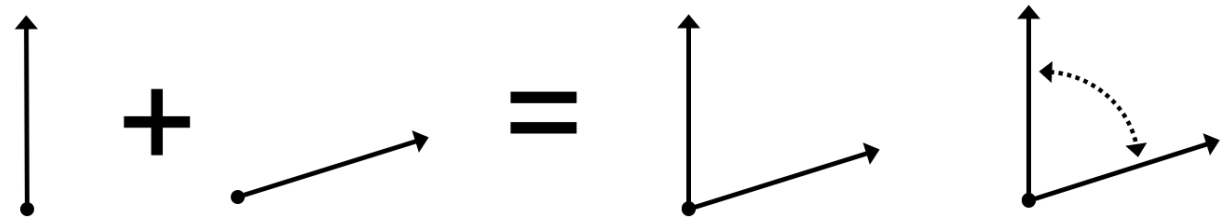
Why might Kelvin think that ∠ACT is smaller than ∠BOG?

How could Bryce prove to Kelvin that the two angles ***are*** congruent to each other?

Developing the Concept of 'Angle'

ANGLES AS STATIC FIGURES

puts the focus on difference in direction of rays



Angles, Andrew Harris, 2000

Developing the Concept of 'Angle'

An angle is:

The area between two lines.

The gap where two lines meet.

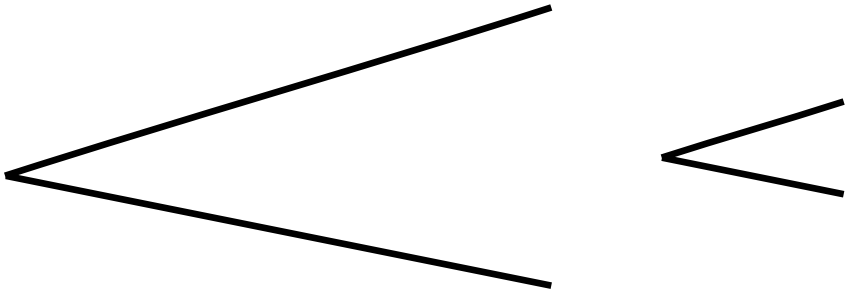
The spacing between two lines which meet at a point.

The distance measured between two lines.

Children Learning Mathematics, Dickson, Brown & Gibson (1984)

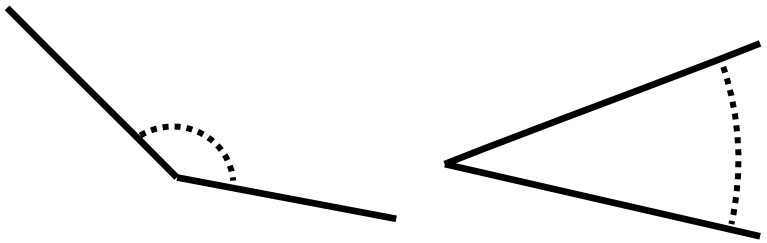
Common Angle Misconceptions

Angle size is dependent on length of legs.



The left angle is larger.

Angle size is dependent on placement or size of arc.



The right angle is larger.

Angle size is related to orientation of angle.

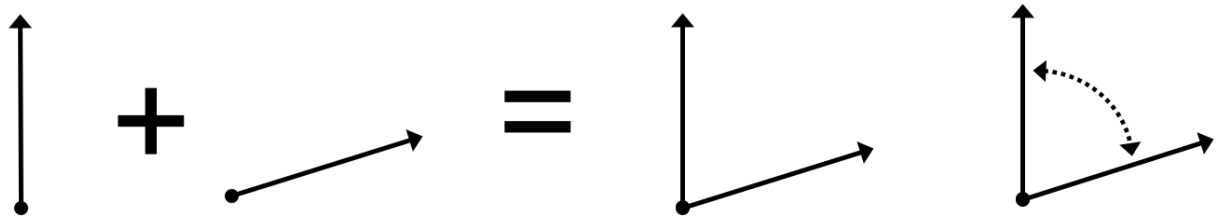
The angles are different sizes.

Angles, Andrew Harris, 2000

Developing the Concept of 'Angle'

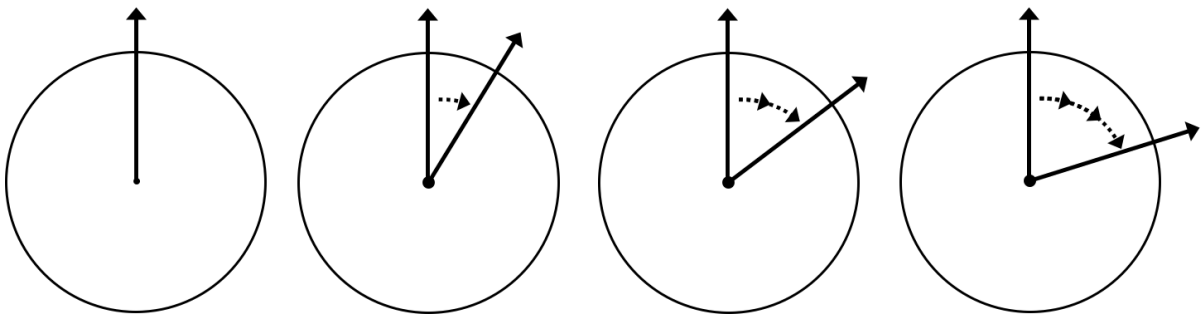
ANGLES AS STATIC FIGURES

puts the focus on difference in direction of rays



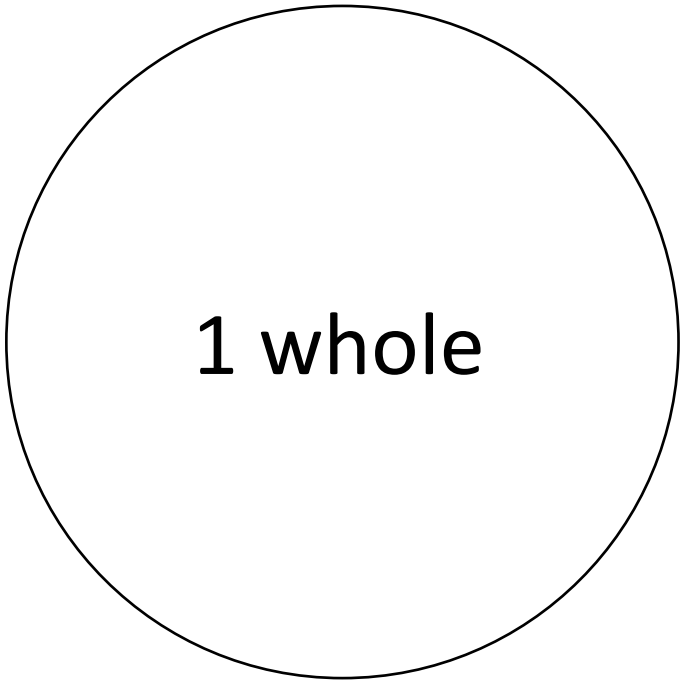
ANGLES AS DYNAMIC FIGURES

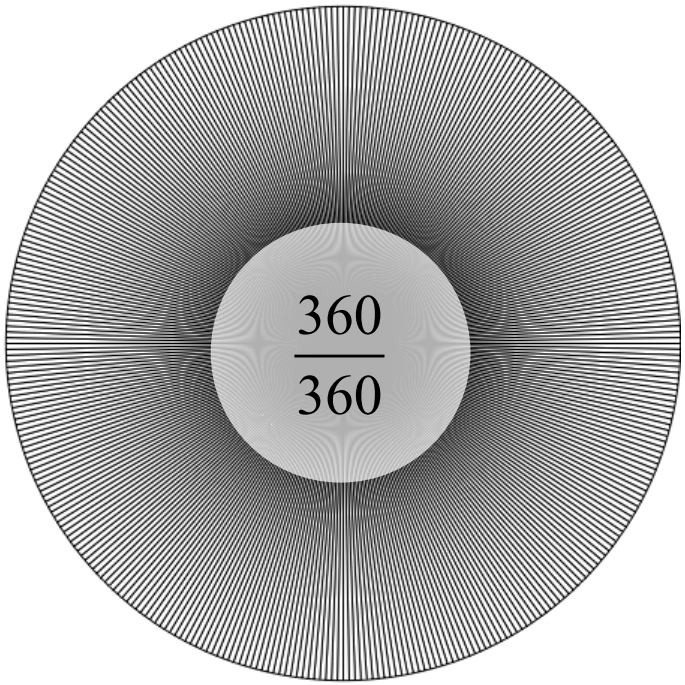
focuses on the rotation of one figure in relation to the other

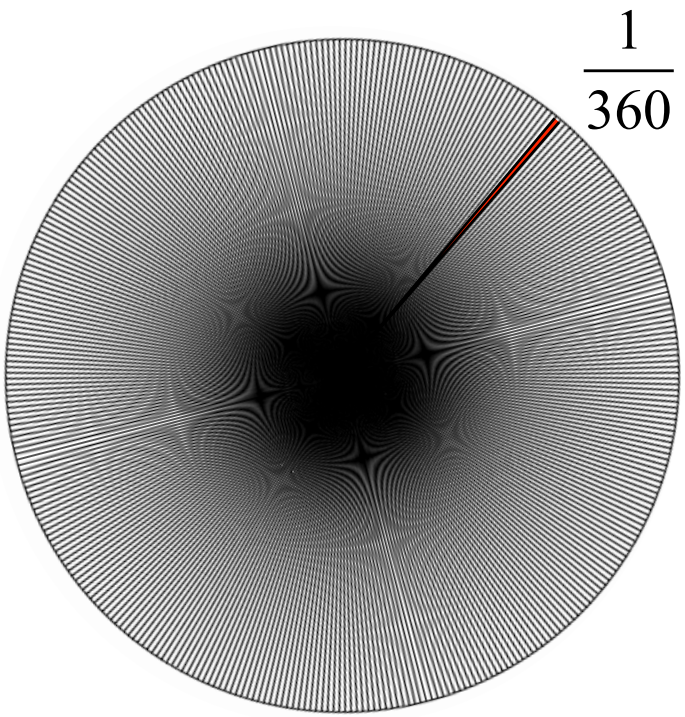


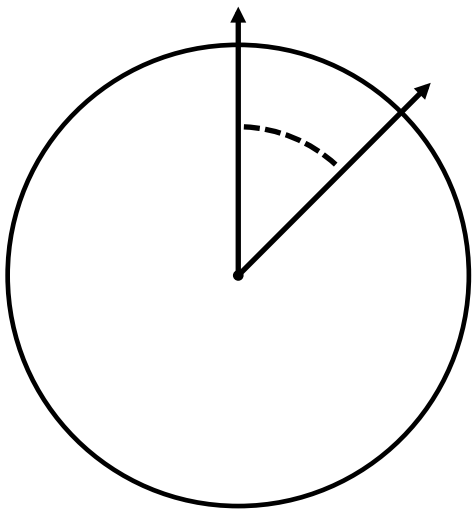
Angles, Andrew Harris, 2000

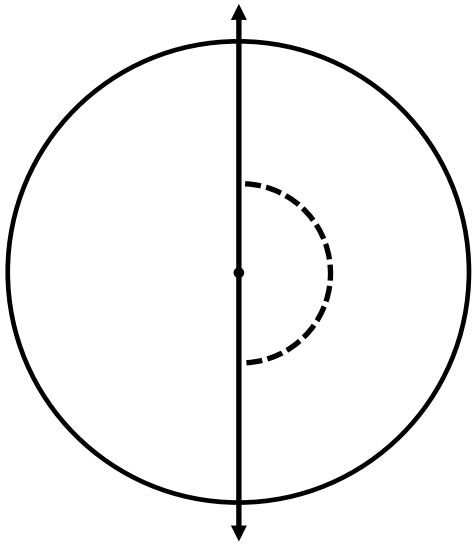
Angles as Fractions of a Circle

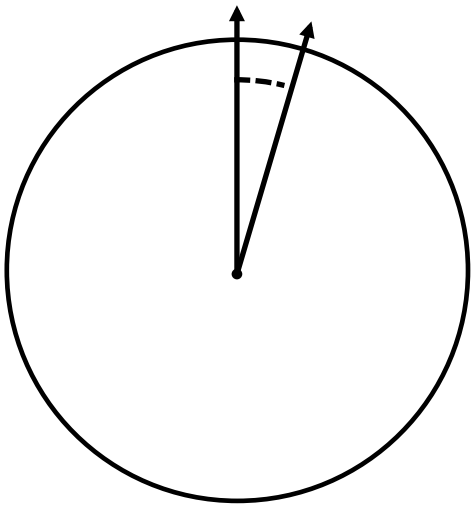


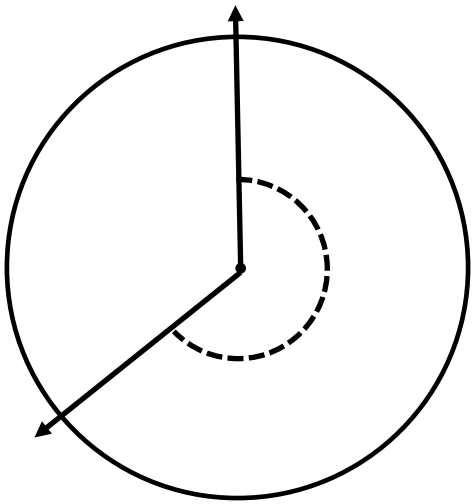








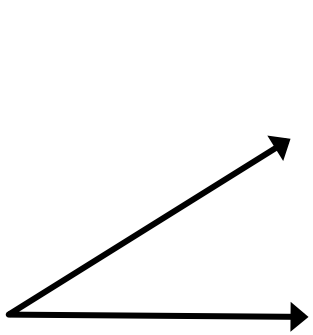


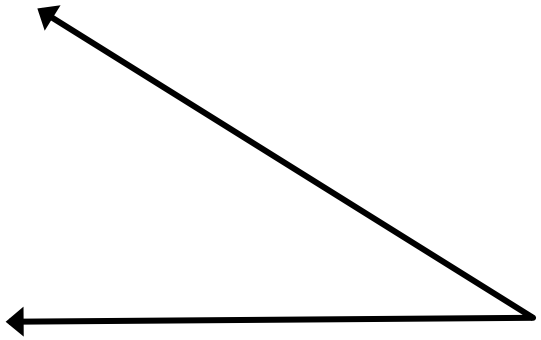


Addressing Angle Misconceptions

INSTEAD OF JUST:

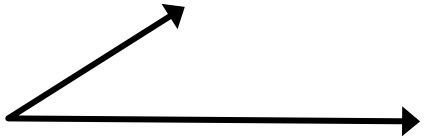
angles with horizontal bases and two rays of equal length



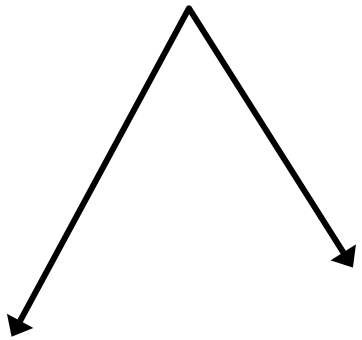


TRY INCORPORTATING:

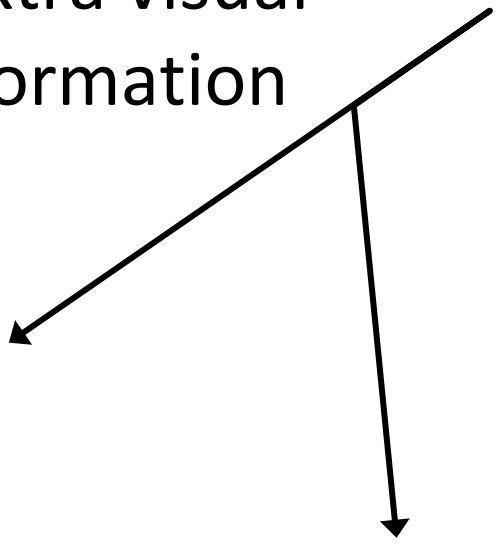
- varied ray lengths



- varied orientations

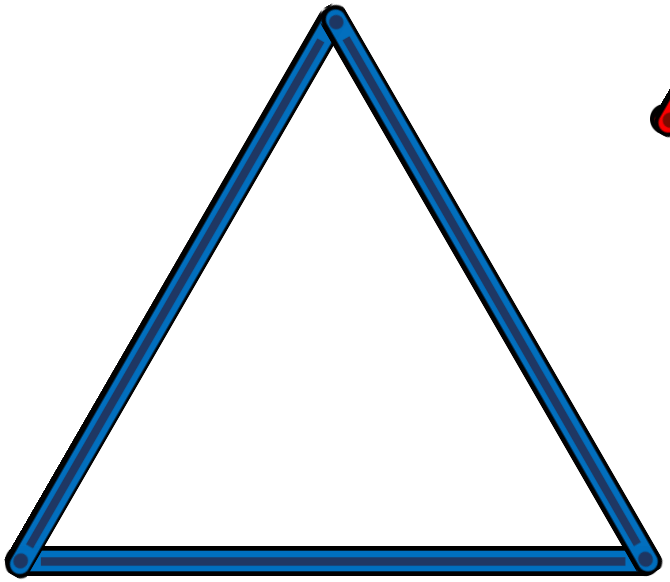


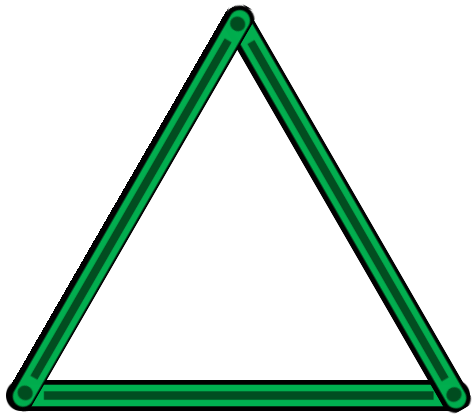
- extra visual information

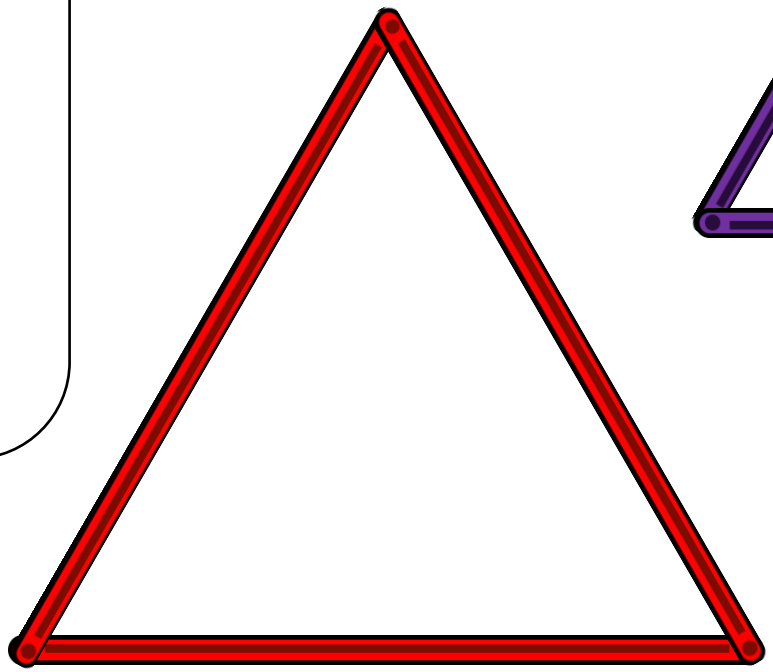


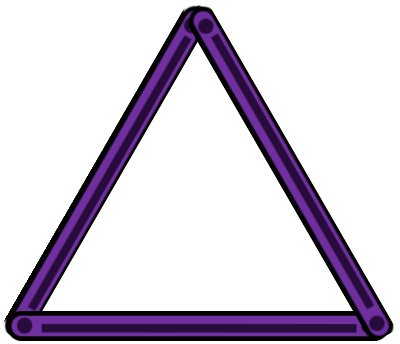
Addressing Angle Misconceptions

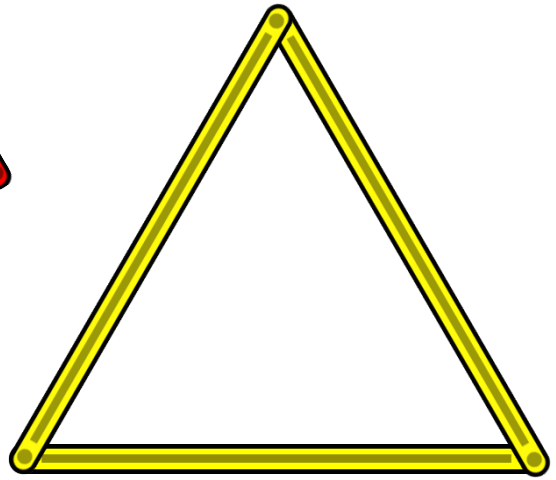
Which triangle has the largest angles? Why do you think so?





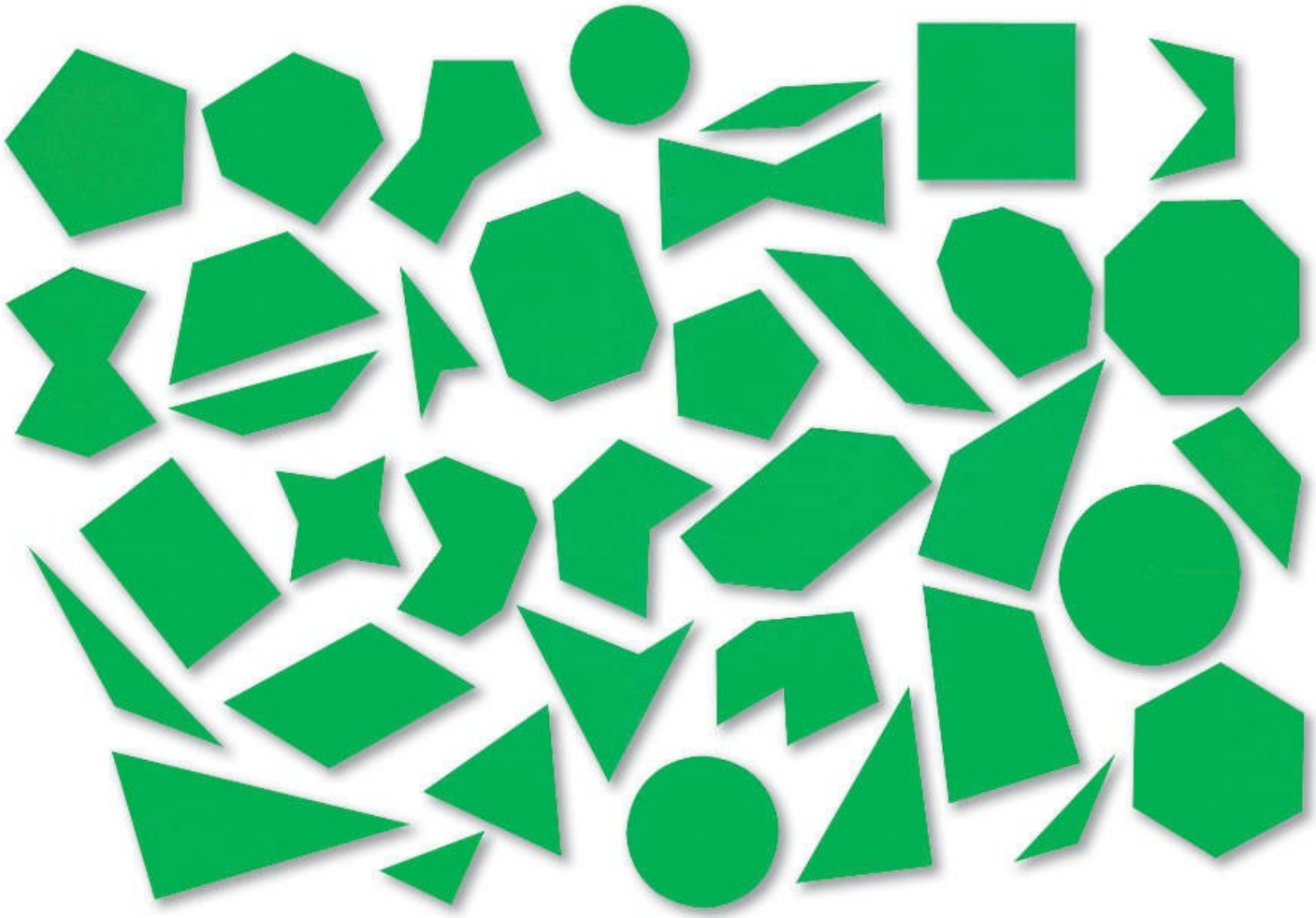






Broadening the Concept Image

Polygons+ Power™ Pack



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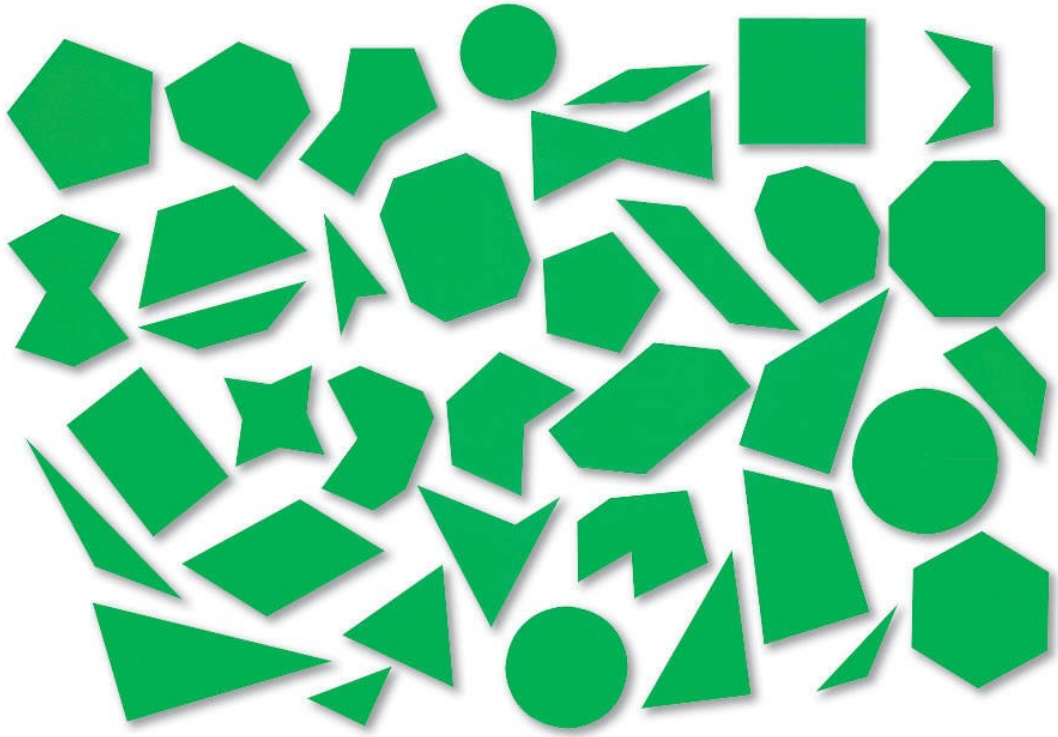

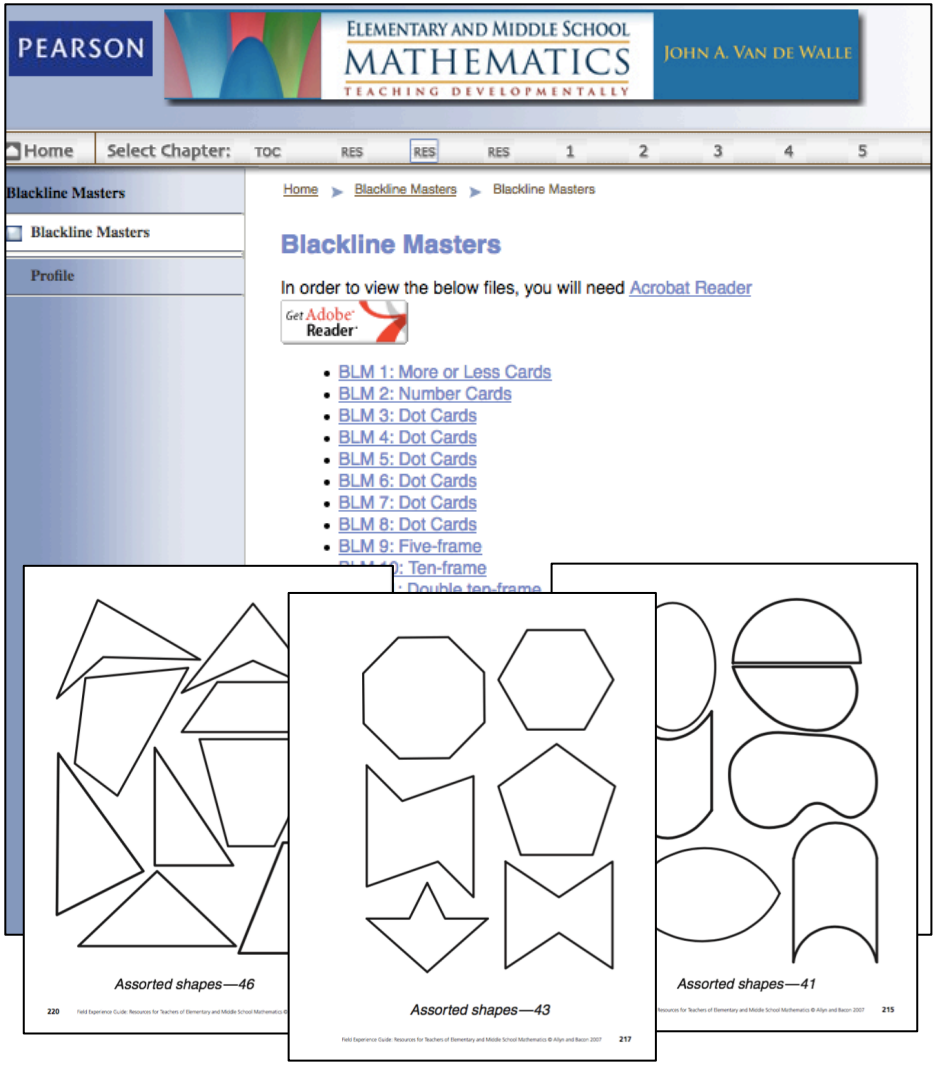

http://wps.ablongman.com/ab_vandewalle_math_6/

Broadening the Concept Image

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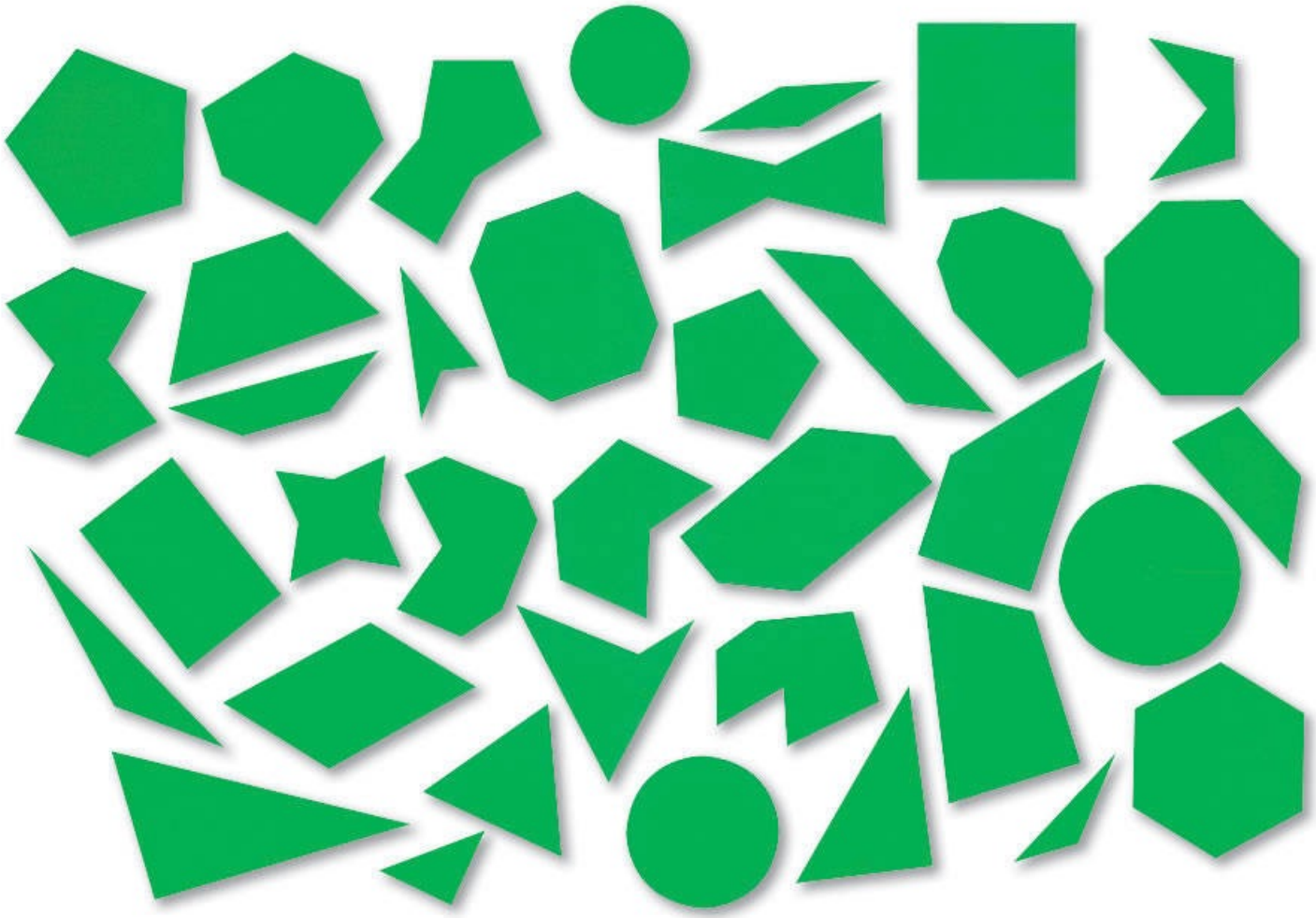

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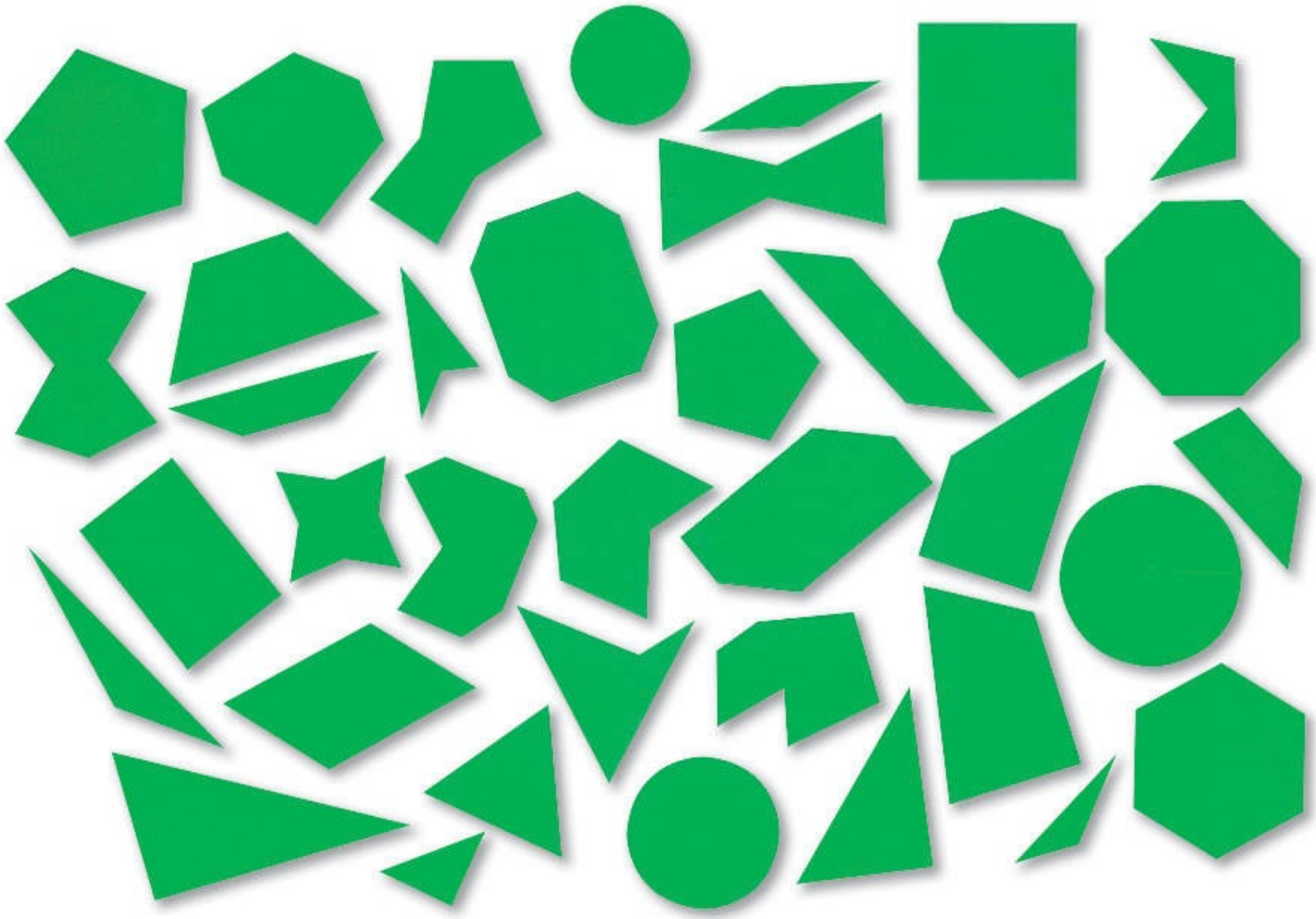
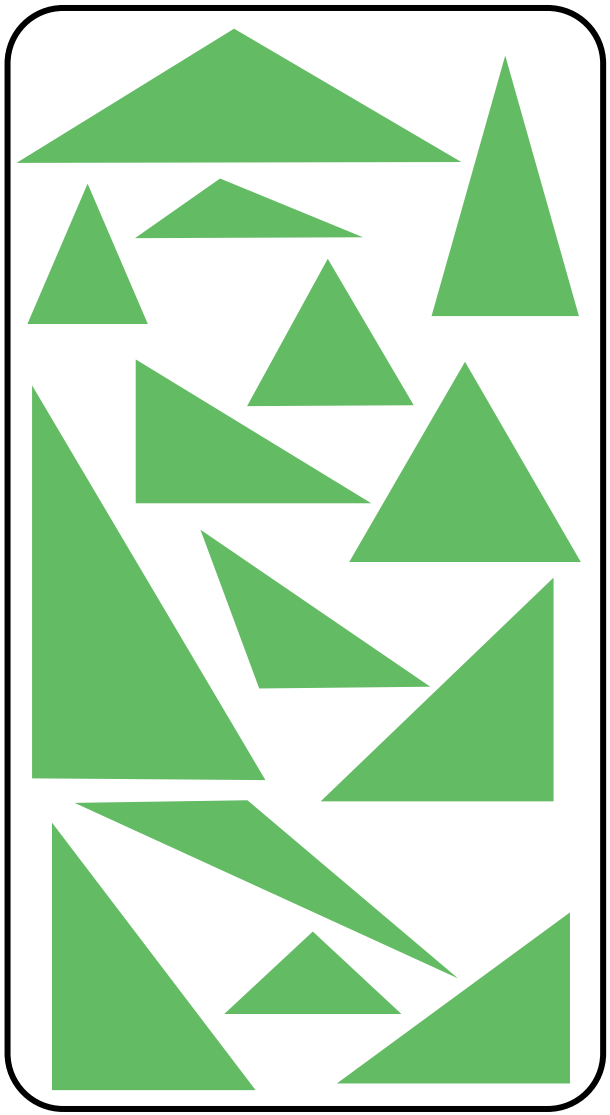


TRIANGLES



Broadening the Concept Image

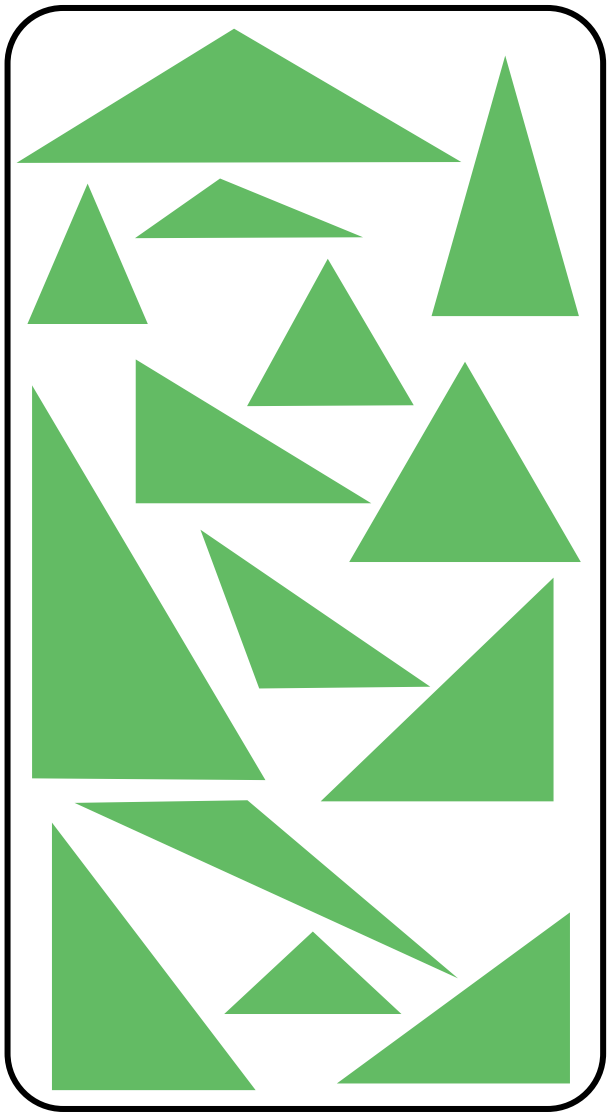
Identify the two triangles in the set that you think are the MOST SIMILAR to each other.

Then choose the two triangles in the set that you think are the MOST DIFFERENT from each other.

Discuss the reasons for your choices of triangle pairs.

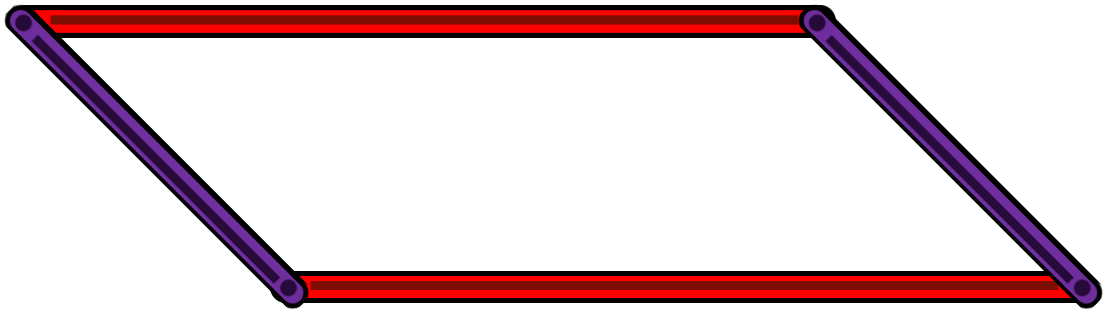
angles & **sides**

TRIANGLES

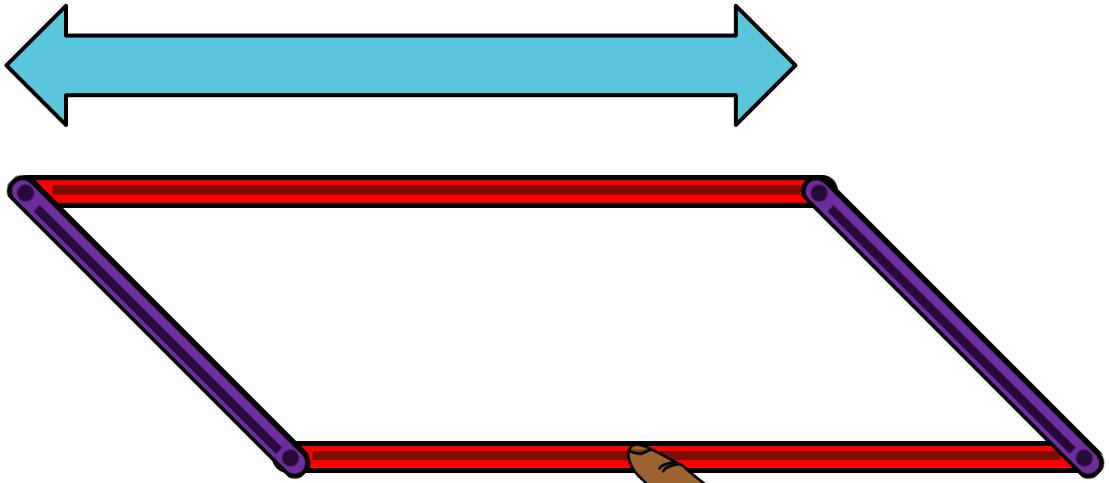


Broadening the Concept Image

Build a parallelogram using two red and two purple AngLegs.



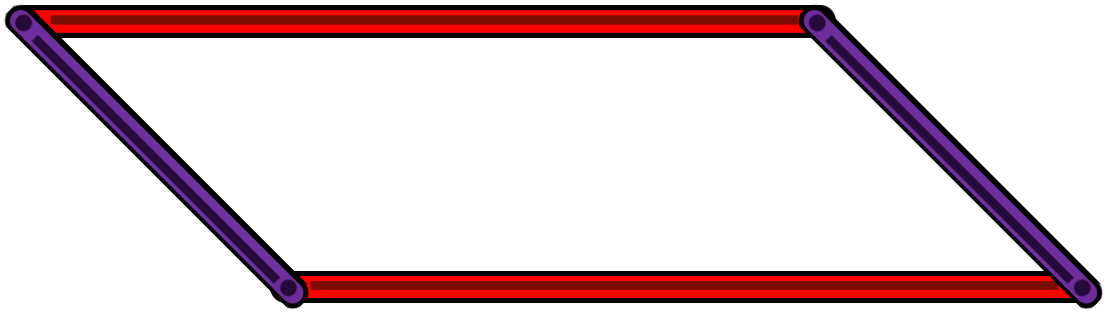
Hold down the bottom side and slide the top side to the left and right.



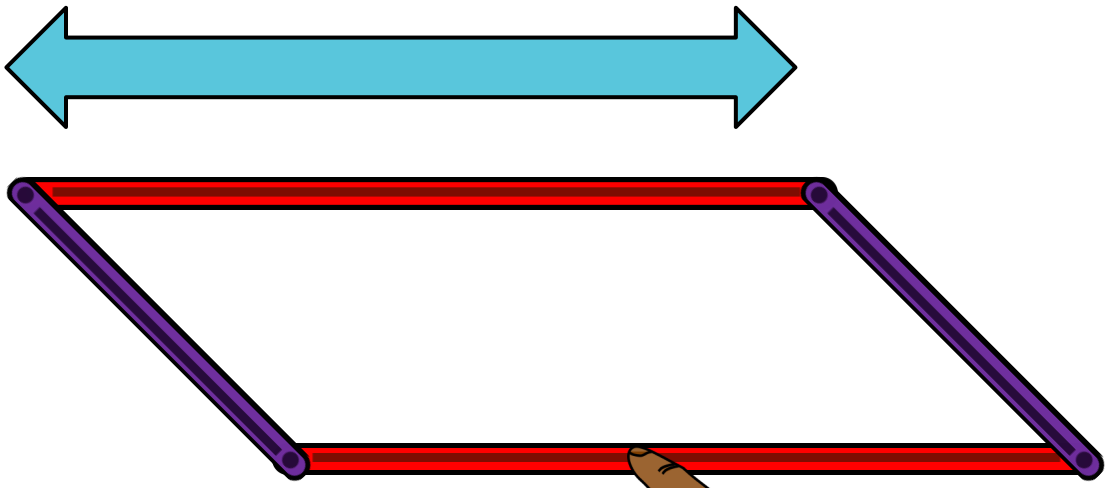
Which of the shape's attributes change? Which attributes do not change?

Broadening the Concept Image

Build a parallelogram using two red and two purple AngLegs.



Hold down the bottom side and slide the top side to the left and right.



Focus on one angle in the figure. As you shift the shape, how does *that* angle change?

Broadening the Concept Image

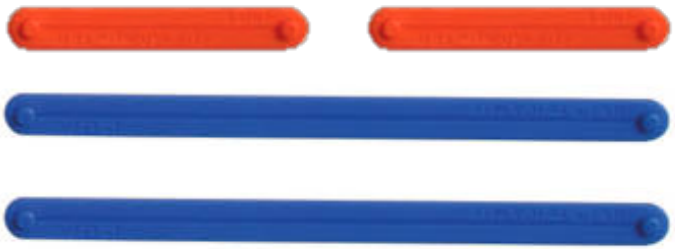
Take out shapes one at a time, discuss them with your partner, and place them in the chart if you would say...

YES	NO	MAYBE
"Yes! I definitely know the name of this figure."	"No! I definitely do NOT know the name of this figure."	"I think maybe I might know the name for this figure, but I'm not sure."

 YES	 NO	 MAYBE

Constructing Shapes with AngLegs

Imagine you had two orange AngLegs and two blue AngLegs and connected them to construct a shape.



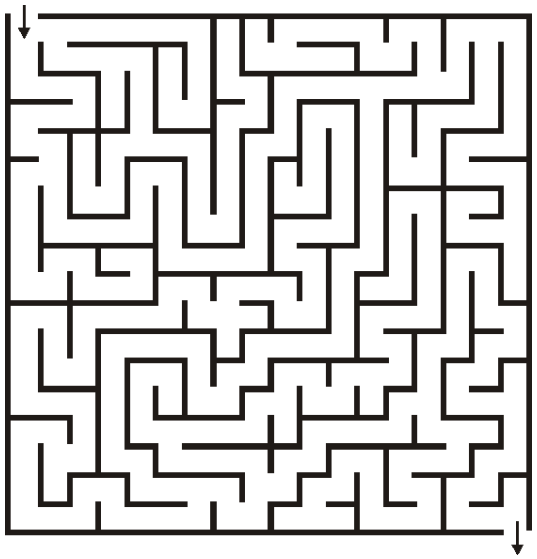
YES – NO – MAYBE

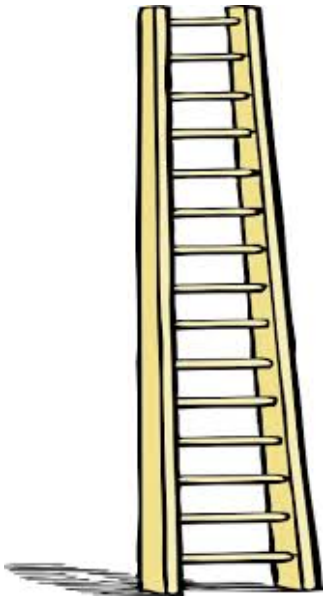
What figures do you **know** you definitely can make?

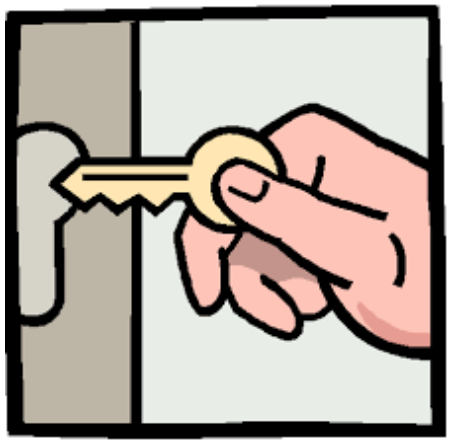
What figures do you **know** you definitely cannot make?

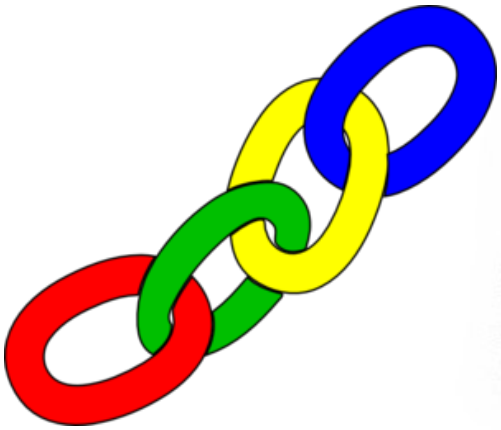
What figures do you **think** you might be able make but are not sure?

Closure: Images & Ideas









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