Writing Fair and Effective Scoring Rubrics for Free-Response Questions

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What is a scoring rubric?

According to Merriam Webster:

A guide listing specific criteria for grading or scoring academic papers, projects, or tests.

Working definition:

Directions to follow to attribute a fair and consistent score to a response of a test question.

What is a scoring rubric?

Why are rubrics important?

- To obtain a fair and accurate score for granting full or partial credit
- Consistency of scoring between responses
- To eliminate bias based on construct irrelevancies (handwriting, grammar)
- Reduce bias based upon knowing the respondent or bias based on responses to other questions

Why are rubrics important?

What types of exams can use scoring rubrics?

- National or state standardized exam
- District-wide exam
- School-wide exam
- Teacher across multiple classes
- Teacher for a single class

What types of exams can use scoring rubrics?

What are the types of scoring rubrics?

- HOLISTIC RUBRIC – Scored based on overall impression of response

- ANALYTIC RUBRIC – Scored based on whether or not very specific criteria are met

The presentation will focus on analytic rubrics.

What are the types of scoring rubrics?

How complex do the rubrics need to be?

- No more complex than they need to be.
- It depends upon the complexity of what is being asked.
- Multi-step questions lend themselves to less complex rubrics.
- The more open-ended the question, the more complex the rubric will tend to be.

How complex do rubrics need to be?

What are the characteristics of questions that are best for rubrics?

- A single best answer
- A limited number of reasonable correct solutions
- Answer characteristics that are clearly correct or incorrect
- Clearly stated expectations of the test taker

What are the characteristics of questions that are best suited for rubrics?

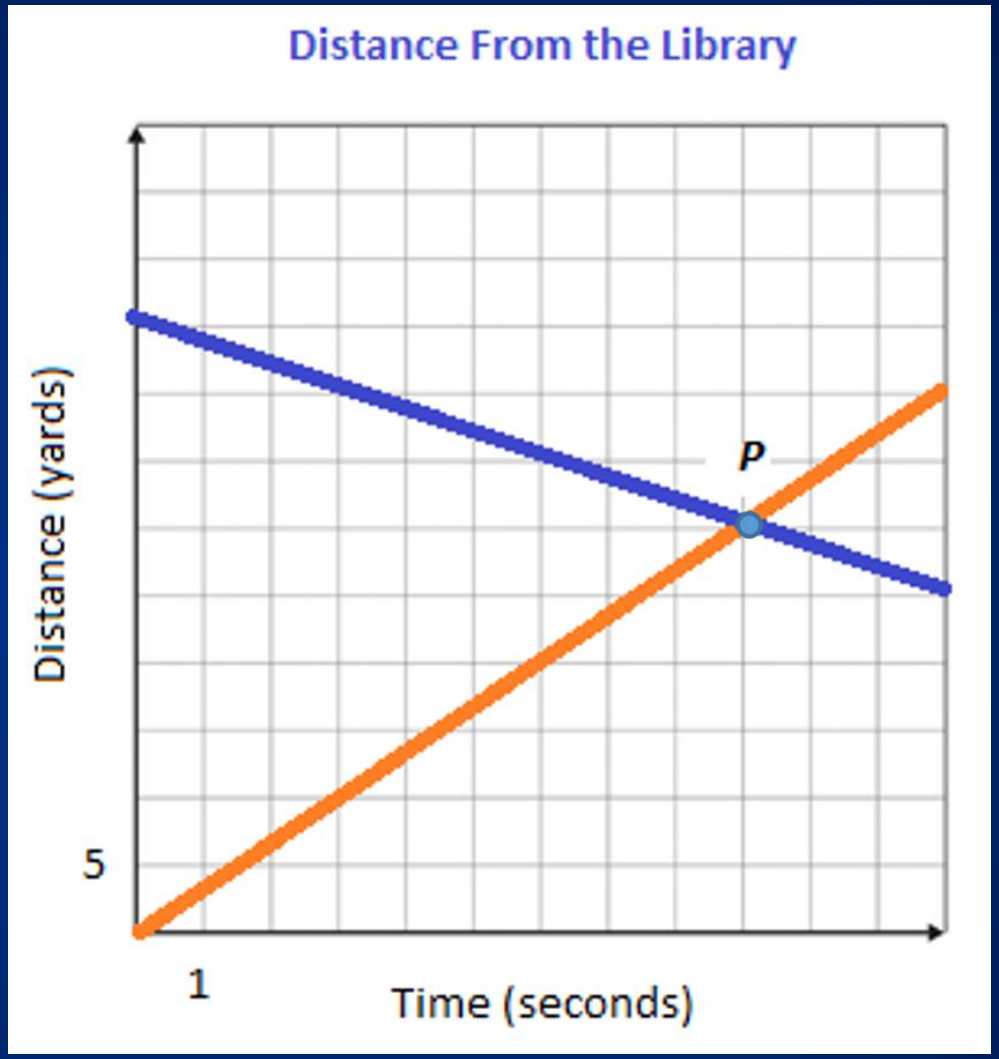
Example of a question that is NOT well-suited for a scoring rubric

Maya left the library and began walking along Main Street toward Mark's house.

At the same time, Mark left his house and began walking along Main Street toward the library.

In the graph, the orange line represents Maya's distance, in yards, from the library, and the blue line represents Mark's distance, in yards, from the library.

What does the marked point represent?



Question that is not well-suited for a rubric

Intended Response:

- **After 9 seconds, Maya and Mark were at the same location on Main Street.**

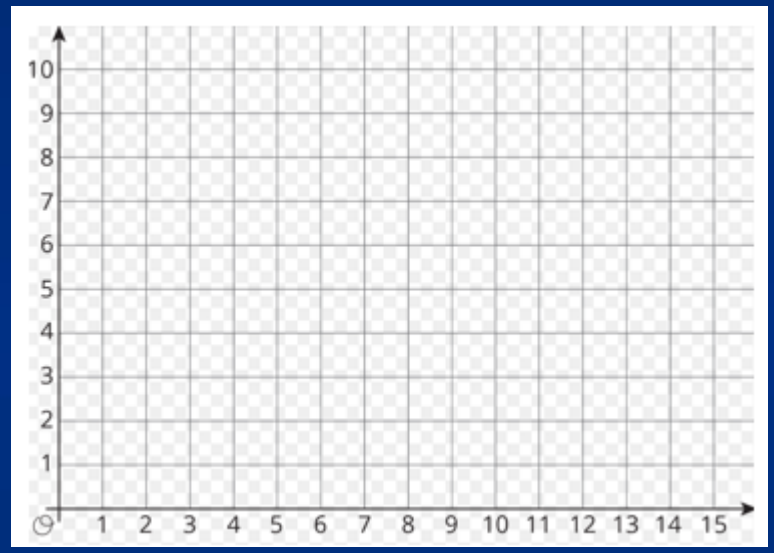
Unintended Responses:

- **It is the point where the two lines intersect.**
- **It is 30 yards from Mark's house.**
- **It is 15 yards from the library.**

Question that is not well-suited for a rubric

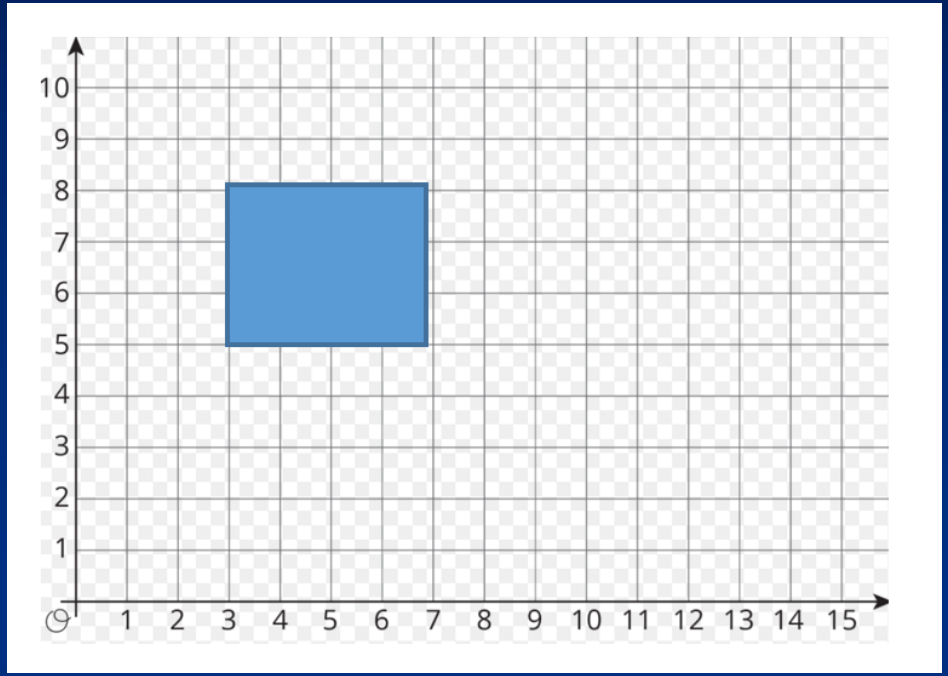
Simple Scoring Rubric

In the coordinate plane shown, each small grid square has an area of 1 square unit.



In the plane, shade a rectangle with sides along the grid lines that has an area of 12 square units.

Simple Scoring Rubric (Possible Score of 0 or 1)



- 1: Rectangle with area 12
- 0: Otherwise

Simple Scoring Rubric (Sample Solution)

Moderate Scoring Rubric

Consider the system of inequalities: $2y < x + 3$

$y > x$

The following statement is FALSE: If $(x, y) = (a, b)$ is a solution to the system of inequalities AND $0 < a < 3$, then $0 < b < 1.5$

Show that $(a, b) = (1.5, 1.6)$ is a counterexample that proves the statement to be false. Show all steps that justify your answer.

Moderate Scoring Rubric (Possible Score of 0 to 3)

- Show that the ordered pair (1.5, 1.6) meets the hypothesis:
  - There are two parts to the hypothesis: 1) $0 < a < 3$ AND 2) $(a, b) =$ (1.5, 1.6) is a solution to the system of inequalities.
  - 1) $0 < 1.5 < 3$, so $0 < a < 3$.
  - 2) $2b < a + 3$: $2(1.6) < 1.5 + 3$ yields $3.2 < 4.5$ AND $b > a$: $1.6 > 1.5$. Therefore, (1.5, 1.6) is a solution to the system.
- Show that the conclusion is false for the counterexample:
  - Since $b = 1.6$, $0 < b < 1.5$ is false.
- Therefore, the counterexample proves the statement is false.

Moderate Scoring Rubric (Sample Solution)

What are the required components of the solution?

- Show that the hypothesis is true for the counterexample.
- Show that the conclusion is false for the counterexample.

Moderate Scoring Rubric (The Components)

The question will be scored from 0 to 3.

- 3: Both components are present and correct. (the hypothesis and the conclusion)
- 2: The correct conclusion is stated AND one of the following errors was made in the hypothesis:
  - Not stated that $0 < 1.5 < 3$
  - An arithmetic or algebraic error was made in the substitution and/or solving of one of the two inequalities in the system
  - Not stated that (1.5, 1.6) satisfies the system
- 1: Demonstrate that $0 < b < 1.5$ is false, but missing or multiple errors with the hypothesis.
- 1: Verified that the hypothesis is satisfied, but did not check the conclusion.

Moderate Scoring Rubric (Sample Rubric)

Complex Scoring Rubric

Kyle sold two sizes of candy bars for a school fundraiser: small bars for $3 each and large bars for $5 each.

She sold a total of 57 candy bars and collected a total of $236.

When Kyle turned in the money, she was told that she must have made a mistake with her records.

Is there enough information to indicate whether Kyle made a mistake?

Use a system of equations to justify your answer.

Complex Scoring Rubric (Possible Score of 0 to 4)

- Let x represent the number of small candy bars sold and let y represent the number of large candy bars sold.
- Since she sold a total of 57 candy bars: $x + y = 57$
- The total amount of money collected is represented by: $3x + 5y = 236$
- Solving for x and y yields $x = 24.5$ and $y = 32.5$
- Since Kyle did not sell a fraction of a candy bar, she must have made a mistake.
- There is sufficient information to determine that Kyle did made a mistake.

Complex Scoring Rubric (Sample Solution)

What are the required components of the solution?

- Each equation is given AND the variables are correctly defined.
- The system of equations is correctly solved.
- A correct conclusion is made based on the solution of the system and the situation.

Complex Scoring Rubric (The Components)

The question will be scored from 0 to 4.

- 4: All three components are present.
- 3: All three components are present EXCEPT for one of the following:
  - The variables are not defined.
  - The variables were interchanged based on their definition.
  - The conclusion was not made.
  - One Algebra error carried through solution.
- 2: One of the two equations contains a logic error. The error is carried through and all other components are correct based on the error.
- 1: Attempt to set up equations is made, but system is incorrect. Attempt to draw conclusion must be somewhat reasonable based upon equations.

Complex Scoring Rubric (Sample Rubric)

Sample Question 1

Charles drove the first m miles of a trip at an average speed of 50 miles per hour. He made a one-hour stop for lunch and gas. Then he drove the remaining k miles of the trip at an average speed of 55 miles per hour.

What is the average speed for the entire trip (including the stop), in terms of m and k. State the units of all variables. Justify your answer.

Question 1: Scoring Rubric (Possible Score of 0 to 3)

- State the units of the variables: m is in miles. t is in hours. Intermediate variable d is in miles.
- Time of first part of trip: $m = 50 \times t1$ or $t1 = m/50$ hours.
- Time of last part of trip: $k = 55 \times t2$ or $t2 = k/55$ hours.
- The total time of trip: $[(m/50) + (k/55) + 1]$ hours.
- The total distance of the trip: $d = m + k$ miles
- The average speed is the total distance divided by the total time:

$$\frac{m+k}{\frac{m}{50}+\frac{k}{50}+1}$$

Question 1: Scoring Rubric (Sample Solution)

What are the required components of the solution?

- The time for each part of the trip is correctly expressed.
- The total time of the trip is correctly expressed.
- The total distance of the trip is correctly expressed.
- The average speed is found by dividing the total distance by the total time.

Question 1: Scoring Rubric (The Components)

The question will be scored from 0 to 3.

- 3: Units of all variables are stated correctly. Correct distance, time, and average speed are given.
- 2: All components are correct and present EXCEPT for one of the following:
  - One Algebra error (not a logic error) carried through solution.
  - Did not include the one-hour break.
  - Units were not stated.
  - Variables *m* and *k* are interchanged.
- 2: Correct answer given, including the units, but no intermediate steps given.
- 1: Total distance given, and some understanding of distance is equal to rate times time is present.
- 0: All other responses.

Question 1: Scoring Rubric (Sample Rubric)

Sample Question 2
(Scientific Calculator Permitted)

Samples of substance A and substance B begin decomposing at the same time.

Sample A initially consists of 25 grams, and 20% of the substance is converted to another substance every 3 weeks.

Sample B initially consists of 32 grams, and 25% of the substance is converted to another substance every 3 weeks.

Write expressions that show how many grams of samples A and B remain *t* weeks after the samples begin to decompose.

After approximately what amount of time will the amounts remaining of samples A and B be the same? Justify your answer.

Question 2: Scoring Rubric (Possible Score of 0 to 4)

- For every 3-week period, 80% of substance A present at the beginning of the period remains at the end of the period. So $A = 25\ (0.8)\ ^\wedge\ (t/3)$, where A is the amount, in grams, of substance A remaining t weeks after the sample begins to decompose.
- For every 3-week period, 75% of substance B present at the beginning of the period remains at the end of the period. So $B = 32\ (0.75)\ ^\wedge\ (t/3)$, where B is the amount, in grams of substance B remaining t weeks after the sample begins to decompose.

- To determine after how many weeks the amounts remaining will be the same, you need to set A equal to B: $25\ (0.8)\ ^\wedge\ (t/3) = 32\ (0.75)\ ^\wedge\ (t/3)$
- Because the question asked for an approximation, there are a number of potential solution strategies: Here are three:
  1. **Logarithmic**
  2. **Tabular**
  3. **Graphical**

Question 2: Scoring Rubric (Sample Solution)

- Logarithmic:
  25 (0.8) ^ (t/3) = 32 (0.75) ^ (t/3), (16/15) ^ (t/3) = 32/25, log [(16/15) ^ (t/3)] = log (32/25), (t/3) log (16/15) = log (32/25), t/3 (0.028) = 0.1072. Solving for t yields t = 11.5.
- Tabular:

Week	Substance A Remaining	Substance B Remaining
0	25	32
3	20	24
6	16	18
9	12.8	13.5
12	10.24	10.125
15	8.192	7.6

Based on the values in the table, it appears that a few days before t = 12 (a little less than 12 weeks) the amount of the substances will be equal.

- Graphical: Using data points from the table to draw a similar conclusion.

Question 2: Scoring Rubric (Sample Solution)

What are the required components of the solution?

- Correct equations for A and B are given.
- The need to find the value of *t* for which *A* equals *B* is recognized.
- The approximate number of weeks after which the amounts of the samples will be the same is found.
- Correct units are stated.

Question 2: Scoring Rubric (The Components)

The question will be scored from 0 to 4. Break into two parts of 2 points each

Part 1:
- 2: Both equations are given correctly.
- 1: The variables or proportions are interchanged. (Only error)
- 1: Used t instead of $t/3$. (Only error)
- 1: Used 0.2 and 0.25 instead of 0.8 and 0.75. (Only error)
- 0: All other responses.

Part 2:
- 2: Uses a correct solution strategy to state answer is between 11 and 12 weeks.
- 2: Uses incorrect information from part 1, but draws an incorrect solution only because of the incorrect values from part 1. (No other errors)
- 1: Recognizes that the values of A and B should equal one another from part 1 but does not correctly determine an acceptable approximation.
- 1: Does not state the unit of weeks. (Only error)
- 0: All other responses.

Complex Scoring Rubric (Sample Rubric)

Any Questions or To Obtain a Copy of the PowerPoint Presentation

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