Models for Teaching Fractions: The Case for Using Multiple Representations

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http://www.cehd.umn.edu/ci/rationalnumberproject/
http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html
http://www.cehd.umn.edu/ci/rationalnumberproject/rnp2.html
http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-13-grade3.html

Sample Fraction Circle Tasks

Name_____ Grade 3 Lesson 1 / Student Page A

Exploring with the Fraction Circles

1. _____ browns equal 1 whole circle.
2. 1 whole circle equals _____ pinks.
3. _____ reds equal 1 whole circle.
4. _____ pinks equal 1 brown.
5. 1 brown equals _____ reds.
6. 1 brown is (less than, equal to, greater than) 1 pink.
7. 1 red is (less than, equal to, greater than) 1 brown.
8. 1 yellow is (less than, equal to, greater than) 1 brown.
9. 1 yellow and 1 brown and 1 _____ equals 1 whole circle.
10. 1 yellow equals 1 brown and 2 _____ .
11. 3 pinks and 1 _____ equal 1 whole circle.
12. _____ grays and 1 blue and 1 yellow equals 1 whole circle.
13. 2 grays and _____ blue equals 1 yellow.
14. 1 pink equals _____ reds.
15. 4 _____ equal 1 yellow.

Name_____ Grade 3 Lesson 2 / Student Page A

Work with a partner to complete this activity. Use your fraction circles. Say the fraction name to each other when asked.

1. Take out one yellow piece. This is the unit or the whole you will divide into equal parts.

   How many blues cover one yellow piece? _____

   1 blue is _____ of one yellow.

   (Say the fraction name)

2. Take out one brown piece. This is the unit or the whole you will divide into equal parts.

   How many pinks cover one brown piece? _____

   1 pink is _____ of one brown.

   (Say the fraction name)

3. One yellow piece is the unit or whole you will divide into equal parts.

   How many pinks cover one yellow piece? _____

   1 pink is _____ of one yellow.

   (Say the fraction name)

4. What color piece is 1-half of one blue? _____

5. What color piece is 1-third of one yellow? _____

6. What color piece is 1-half of one black circle?_____

7. What color piece is 1-third of one black circle?_____

8. What color piece is 1-third of one orange piece?_____

9. What color piece is 1-half of one pink piece?_____

Naming Fraction Amounts Using Circles

Use fraction circles to find the names of the different fraction pieces.

I. The black circle is the unit. What fraction name can you give these pieces?

1 yellow 1-half 1 brown _____

1 blue _____ 1 gray _____

1 white _____ 1 green _____

1 red _____ 1 pink _____

II. Now make 1 yellow unit. What fraction name can you give these pieces?

1 blue _____ 1 gray _____

1 pink _____ 1 red _____

III. Change the unit to 1 blue. What fraction name can you give these pieces?

1 gray _____ 1 red _____

IV. Change the unit to 1 orange. What fraction name can you give these pieces?

1 purple _____ 1 green _____

Name_____

Directions: Use fraction circles to fill in the table.

Color	How many cover 1 whole circle?	Which color takes MORE pieces to cover 1 whole?	Which color has SMALLER pieces?
1. Brown	3		
Orange	5	√	√
2. Orange			
White			
3. Purple			
White			
4. Gray			
Green			
5. White			
Green			
6. Orange			
Purple			
7. Gray			
Brown			
8. Brown			
Green			

Name_____

Directions:
Use fraction circles to compare the two fractions. Circle the larger fraction.

$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{4}{5}$	$\frac{3}{5}$	$\frac{2}{7}$	$\frac{2}{6}$
$\frac{6}{7}$	$\frac{2}{7}$	$\frac{4}{12}$	$\frac{4}{15}$
$\frac{8}{12}$	$\frac{11}{12}$	$\frac{6}{7}$	$\frac{3}{7}$
$\frac{2}{7}$	$\frac{2}{9}$	$\frac{9}{10}$	$\frac{3}{10}$
$\frac{4}{8}$	$\frac{4}{6}$	Try these without manipulatives.	
$\frac{2}{3}$	$\frac{2}{8}$	$\frac{13}{100}$	$\frac{27}{100}$
$\frac{7}{10}$	$\frac{7}{9}$	$\frac{9}{10}$	$\frac{9}{100}$

Name_____

Red	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{12}{12}$
White												
Gray												
Pink												
Blue												
Brown												
Yellow												

Name_____

Practicing Fraction Equivalence with Our Fraction Circles

Show the Equal Fractions with your Fraction Circles	Record the Equivalent Fractions with Symbols
1 yellow = _____blues	$\frac{1}{2} = \frac{\quad}{\quad}$
1 yellow = _____pinks	$\frac{1}{2} = \frac{\quad}{\quad}$
1 yellow = _____grays	$\frac{1}{2} = \frac{\quad}{\quad}$
1 blue = _____grays	$\frac{1}{4} = \frac{\quad}{\quad}$
1 pink = _____grays	$\frac{1}{6} = \frac{\quad}{\quad}$
1 brown = _____pinks	$\frac{1}{3} = \frac{\quad}{\quad}$
3 blues = _____grays	$\frac{3}{4} = \frac{\quad}{\quad}$

Exploring $\frac{1}{2}$ With Fraction Circles

Use the whole circle as your unit. Make the fraction $\frac{2}{5}$ with the fraction circles.

Decide if $\frac{2}{5}$ is greater or less than $\frac{1}{2}$.

Record your response in the box:

$\frac{2}{5}$ | *is less than* | $\frac{1}{2}$

Complete the problems below. Use these choices:

is less than
is greater than or
is equal to

$\frac{2}{3}$		$\frac{1}{2}$	$\frac{1}{4}$		$\frac{1}{2}$
$\frac{5}{8}$		$\frac{1}{2}$	$\frac{8}{10}$		$\frac{1}{2}$
$\frac{3}{4}$		$\frac{1}{2}$	$\frac{1}{5}$		$\frac{1}{2}$
$\frac{2}{8}$		$\frac{1}{2}$	$\frac{4}{6}$		$\frac{1}{2}$
$\frac{7}{12}$		$\frac{1}{2}$	$\frac{3}{4}$		$\frac{1}{2}$

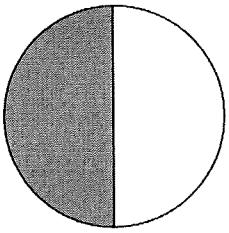
Sample Paper Folding Tasks

Name_____

Directions:

You'll need paper strips for folding. For any four of the figures shown below, fold paper strips to model the fraction that the figure models. After you have folded and shaded your paper, write on it the fraction you have shown (use words, not symbols).

1.



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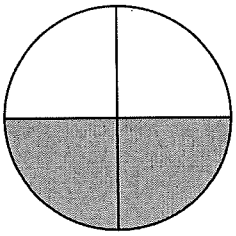

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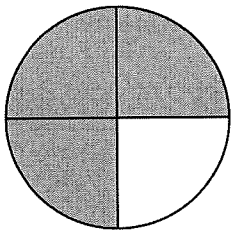

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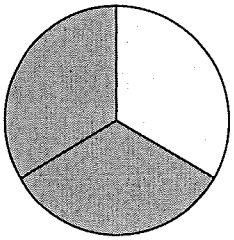
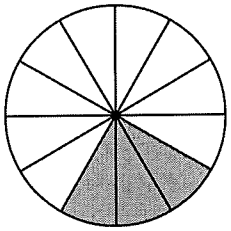
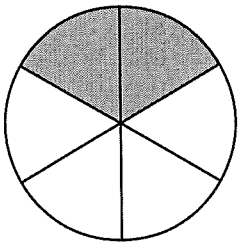


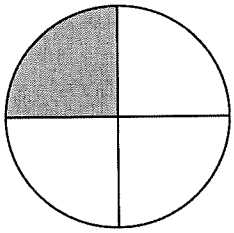
5.



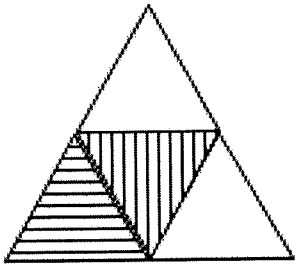
6.



7.



8.



Name_____

Directions:
Circle the larger fraction. Use your paper strips to determine the answers.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{2}{12}$
$\frac{2}{4}$	$\frac{2}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{9}$	$\frac{3}{3}$
$\frac{3}{8}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{4}{6}$
$\frac{1}{3}$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{8}$
$\frac{1}{2}$	$\frac{1}{12}$		
$\frac{5}{12}$	$\frac{8}{12}$		

Without your paper strips, circle the larger fractions.

$\frac{1}{10(}$	$\frac{1}{99}$
$\frac{3}{40}$	$\frac{3}{5C}$
$\frac{2}{1C}$	$\frac{4}{1C}$

Paper Folding and Equivalent Fractions

1) Show 3 fractions equal to 1/2. (Hint: you will need to start with 3 sheets of paper folded into 2 equal parts with one part shaded. Draw pictures to show your answers.

2) Use paper folding to find out which of these are true statements. Circle the number sentences that are true.

$\frac{1}{3}=\frac{2}{6}$ $\frac{2}{4}=\frac{4}{8}$

$\frac{1}{4}=\frac{3}{8}$ $\frac{1}{2}=\frac{6}{8}$

3) Use paper folding to find these equivalences.

$\frac{1}{2}=\frac{}{8}$ $\frac{1}{3}=\frac{}{6}$

$\frac{1}{4}=\frac{}{8}$ $\frac{1}{2}=\frac{}{4}$

Sample Number line Tasks

Sample Student Tasks

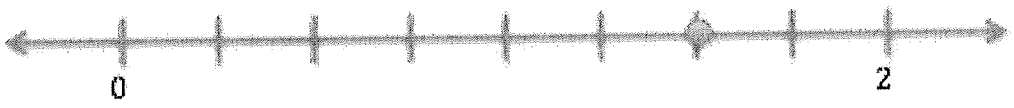
What strategies might students use to **successfully** solve each number line task? How are students reinterpreting key fraction ideas to do this?

1. Look at this number line. Where is the number $\frac{2}{3}$ on the number line? How do you know?

2. Construct the fraction $1\frac{2}{8}$ on the number line. Explain your strategy for showing $1\frac{2}{8}$ on the number line?

3. Show where $1\frac{1}{5}$ is on the number line. Explain what you are doing.

4. Name this point in two ways. Explain your thinking.



5. Does $\frac{2}{3} = \frac{6}{9}$? Use the number line to show if this is true or not. Explain your thinking.

6. Which is bigger $\frac{2}{3}$ or $\frac{11}{12}$ or are they equal? What do you picture in your mind as you think about these two fractions?

Show how to use the number line to justify your decision.