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Mathematical Modeling: Soap Bubbles - Spanning Geometry to Calculus

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The NC School of Science and Mathematics

NCTM Conference

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http://www.bit.ly/NCTMSoapBubbles/

@mathmodeling  @mrgibsonncssm

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What is Math Modeling?

"Whether the problem is huge or little, the process of 'interaction' between the mathematics and real world is the same . . . This entire process is what is called *mathematical modeling*."

Henry Pollack (2012). *Mathematical Modeling Handbook: Introduction*. COMAP.

Available at http://www.comap.com/modelingHB/

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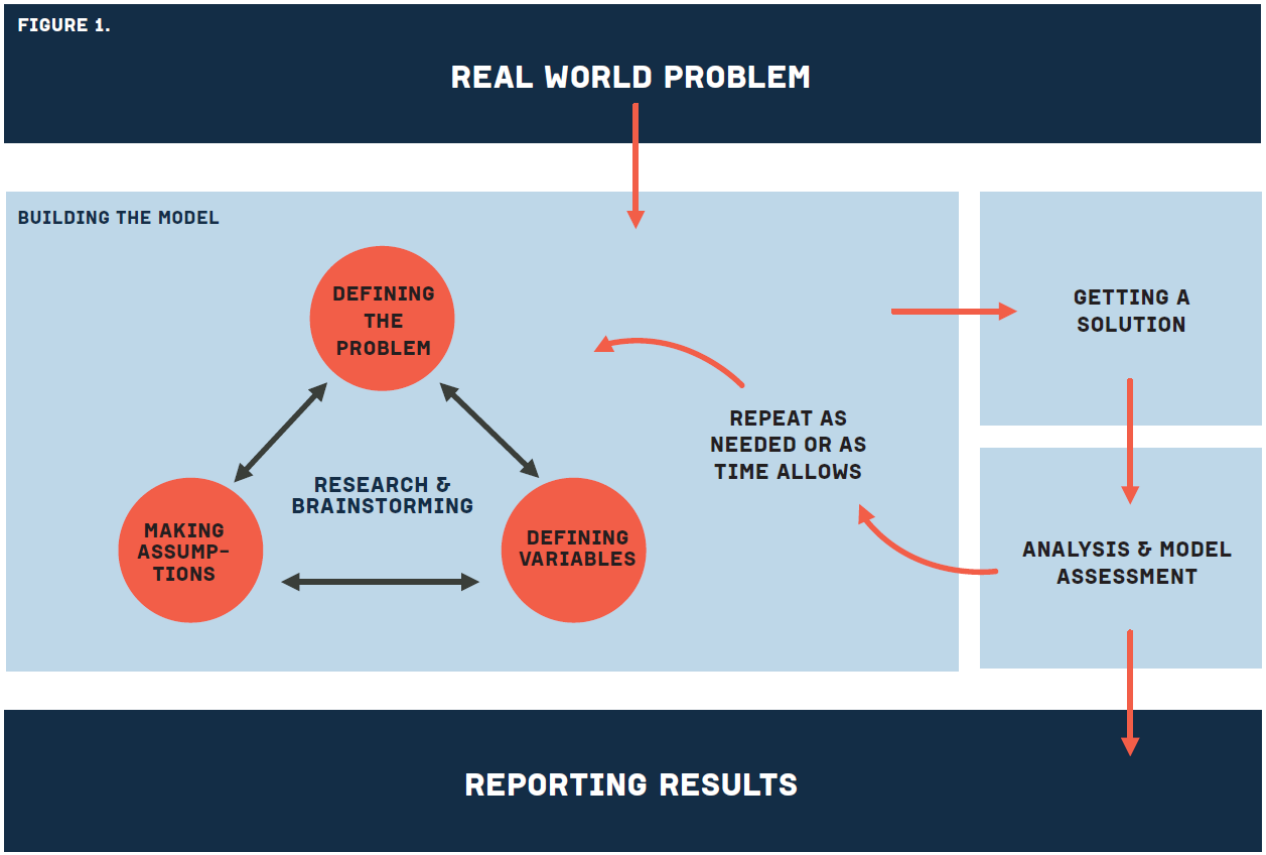
Modeling Cycle



From SIAM *Modeling Getting Started and Getting Solutions Handbook*

https://m3challenge.siam.org/resources/modeling-handbook

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Math Modeling Reference Cards

- Defining the problem statement
- Making assumptions
- Defining the variables
- Getting a solution
- Analysis and model assessment
- Reporting the results

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From Henry Pollack

"When you use mathematics to understand a situation in the real world, and then use it to take action or even to predict the future, both the real-world situation and the ensuing mathematics are taken seriously."

From *Introduction: What is Mathematical Modeling*

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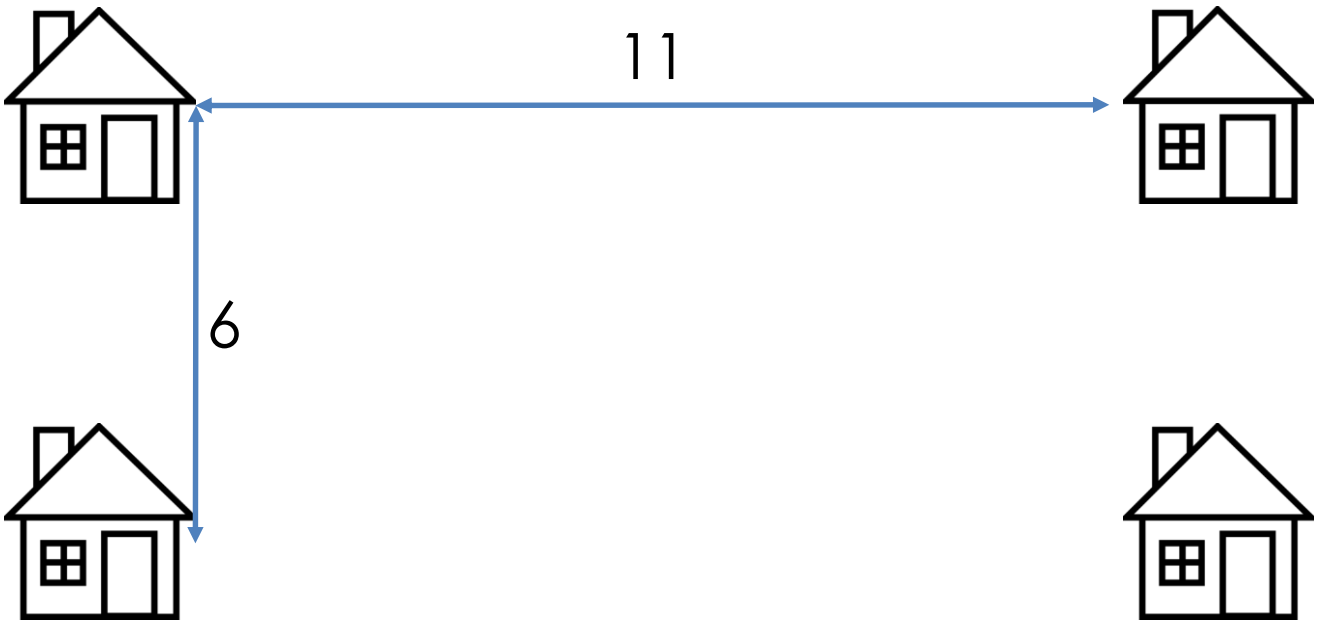
Our Problem Statement

Find the length of the shortest road to connect the houses.



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Pause and discuss with your partners

Come up and share some possible configurations at the doc camera.

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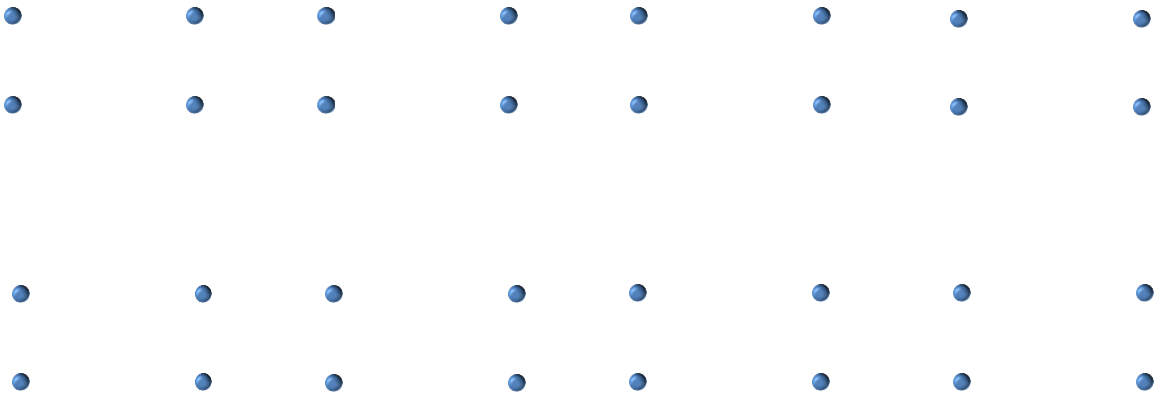
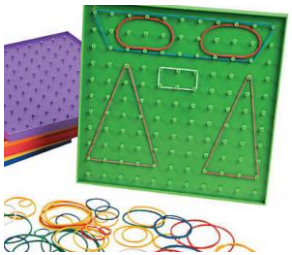
Some possible solutions

What would your students do?



Take a minute to calculate the total road lengths



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Exploring the Bubbles

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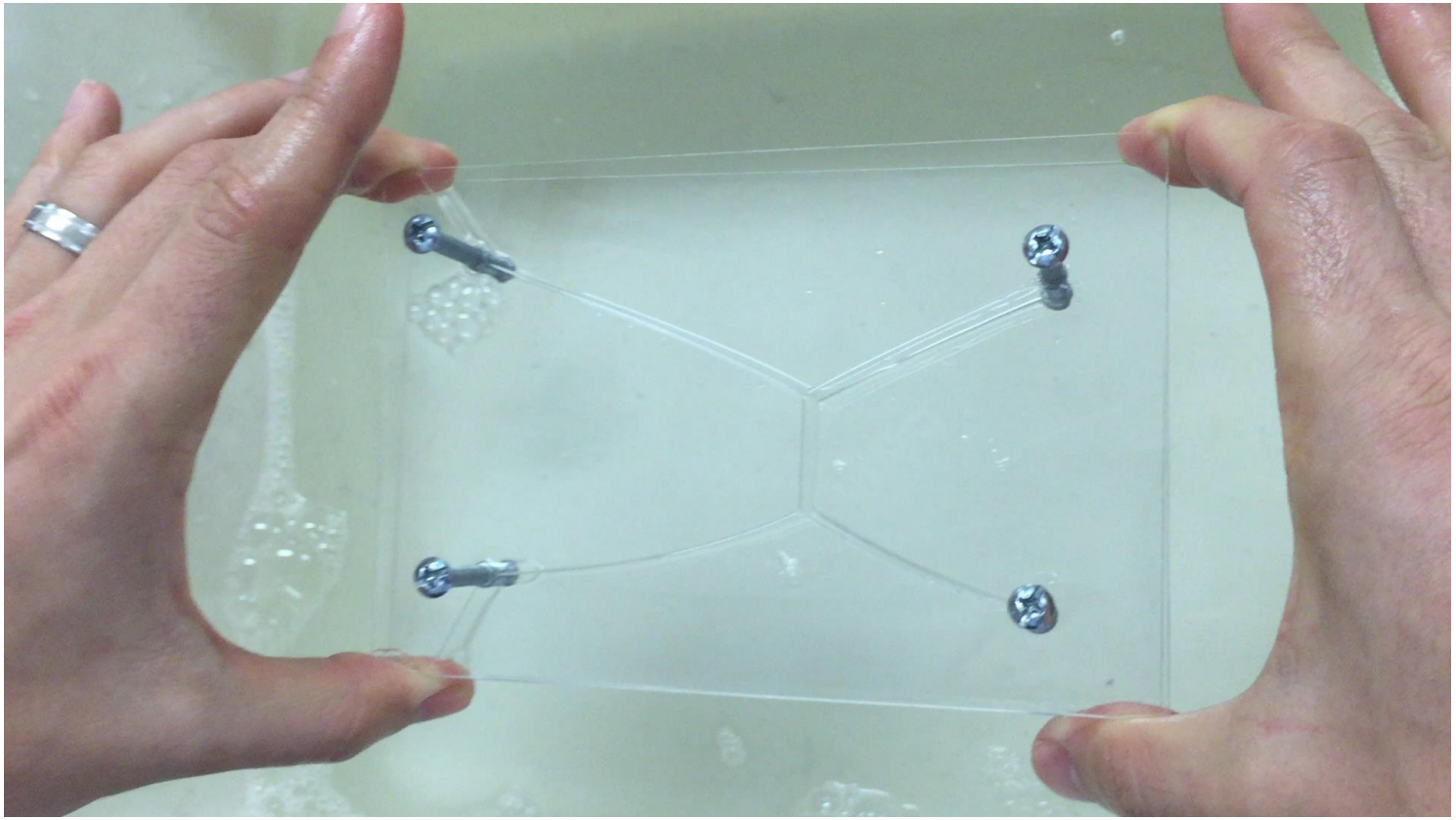
Video of Soap Bubbles



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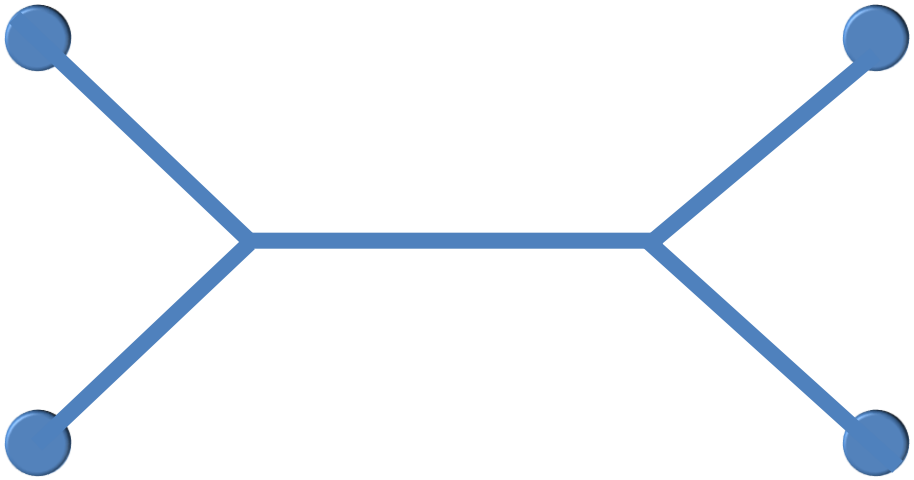
A New Conjecture



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How can we find the bubbles' solution using math?

- Create a Function as a Model
- Use Calculus or Precalculus
- Use Geometry

Getting a solution #2 – Look into your personal toolkit for a math technique.

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Creating a model for distance

Share your models and discussion how you will find the minimum.

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Pause to consider the teacher's role in the modeling process.

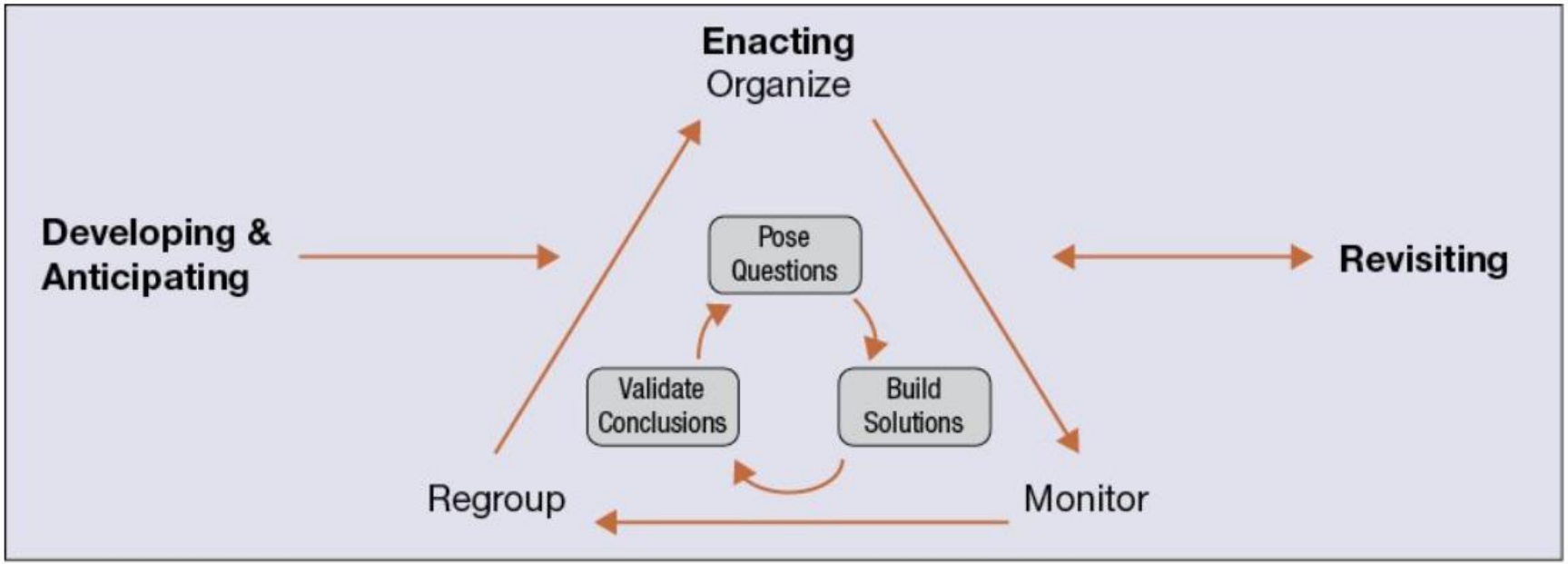

Fig. 4 The diagram illustrates the framework for teachers' roles in modeling (Carlson et al. 2016, p. 122).

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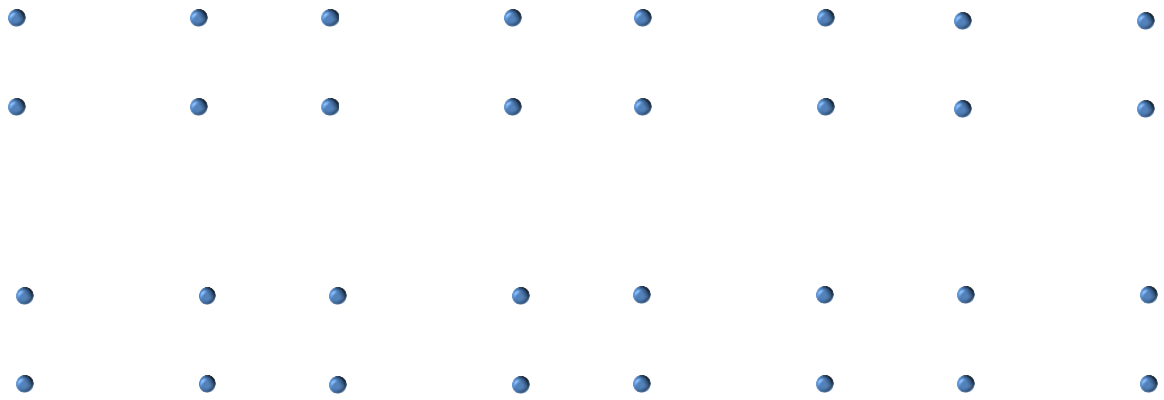
Some possible solutions – How do these connect to your functional model?

These were some of our first attempts at a model:



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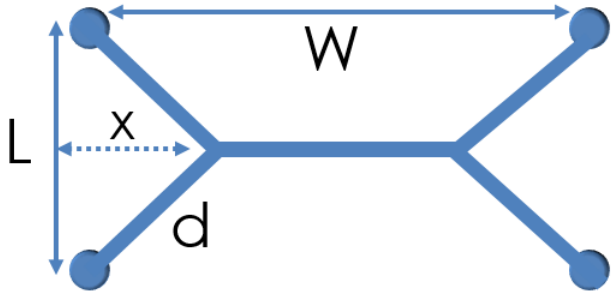
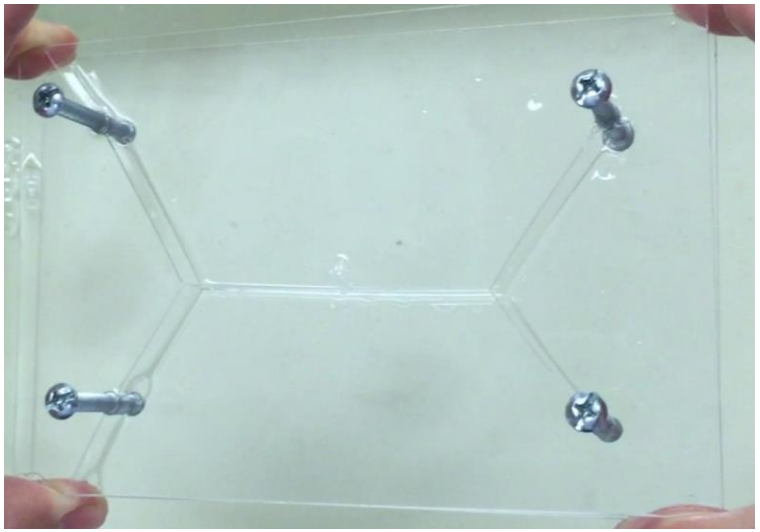
Assessing a Model





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Geometric Approaches

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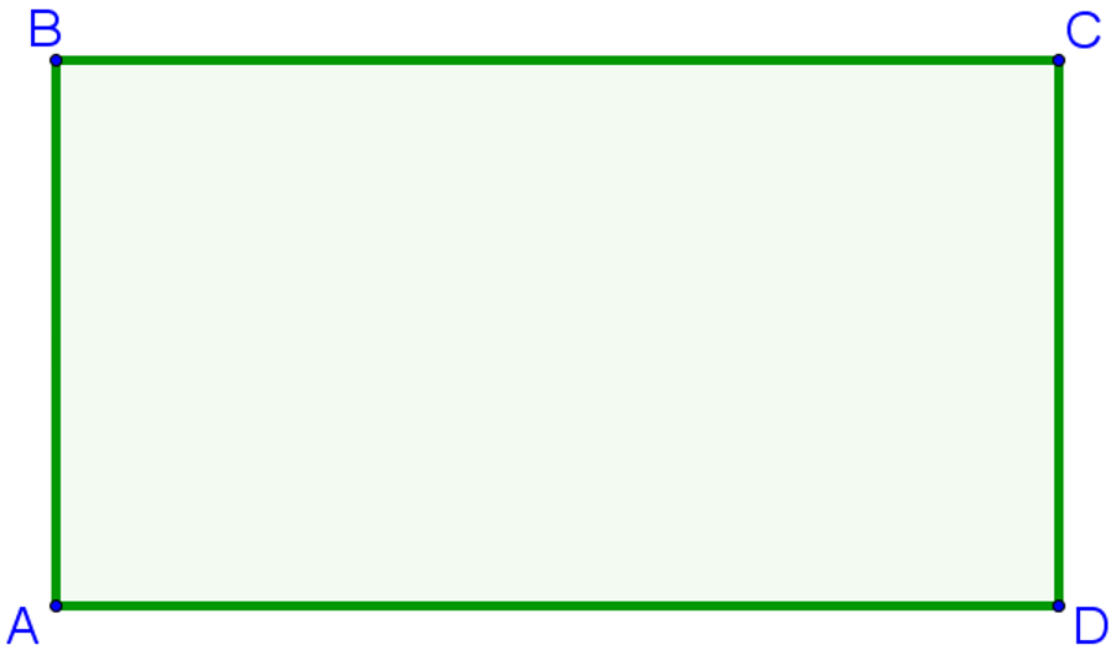
Geometry



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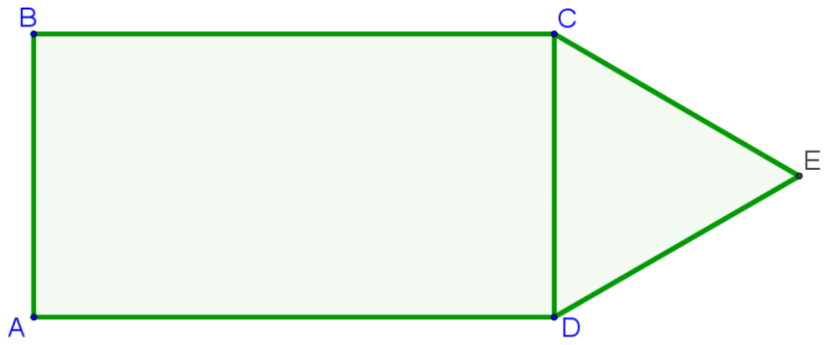
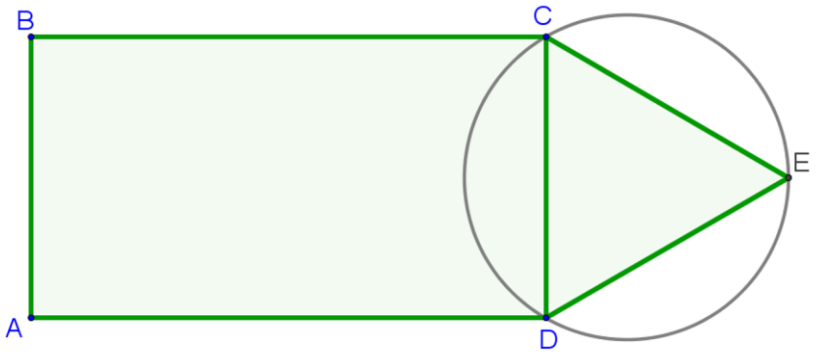
Geometry



Construct Equilateral Triangle

Construct Circle
Through 3 points



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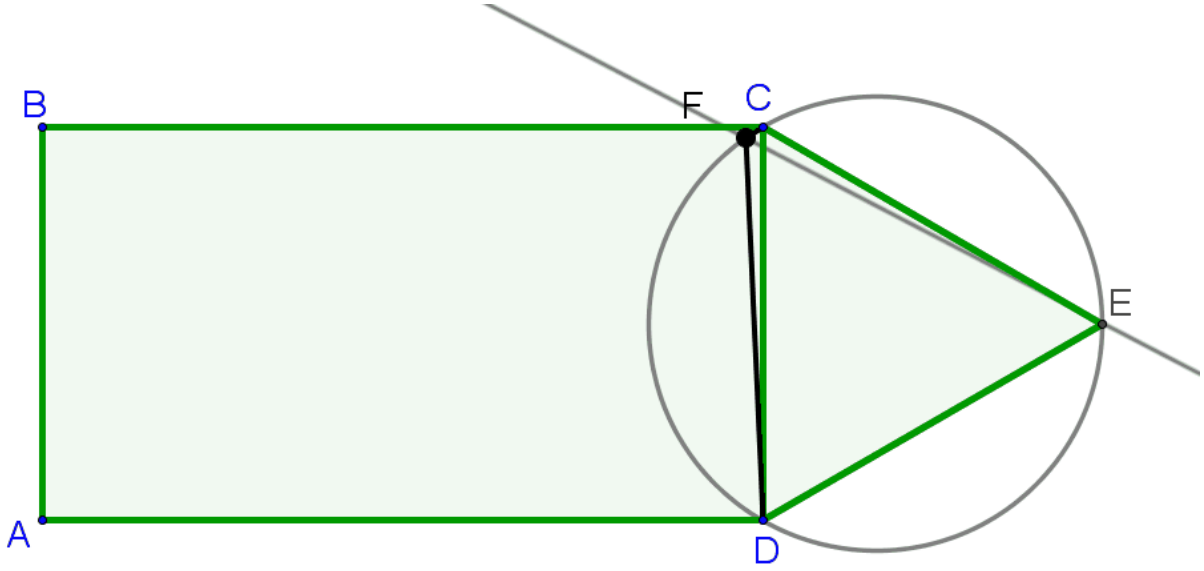

Construct Angle Bisector of $\angle CFD$

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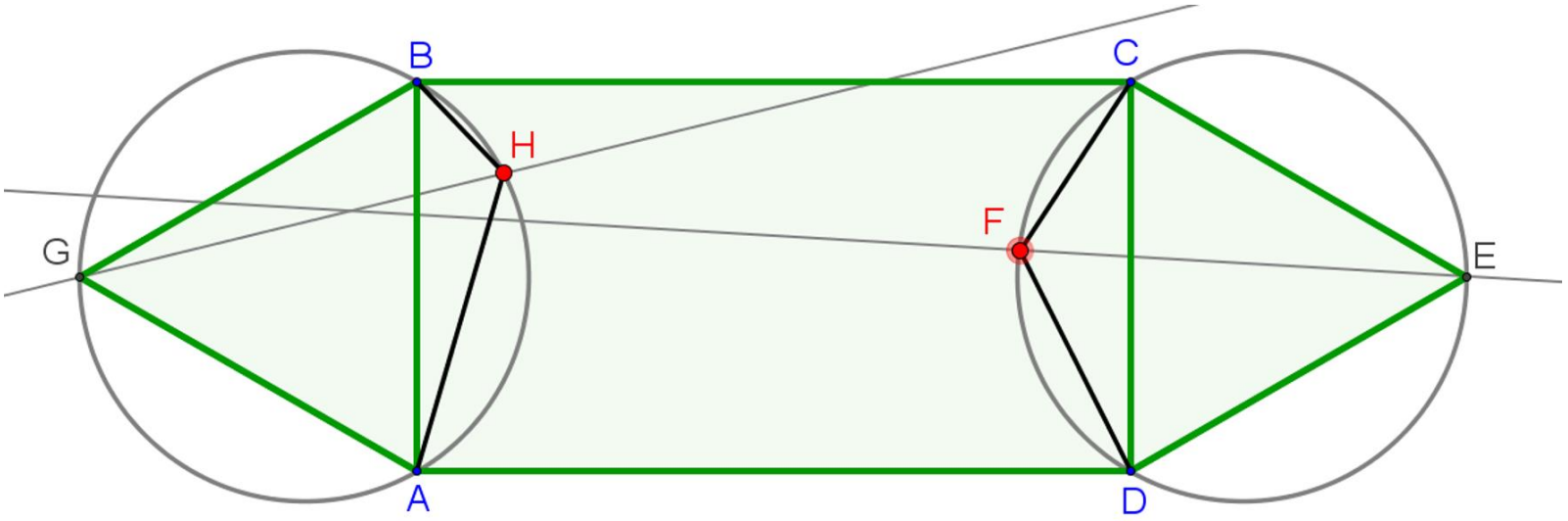

Repeat Construction on Other Side

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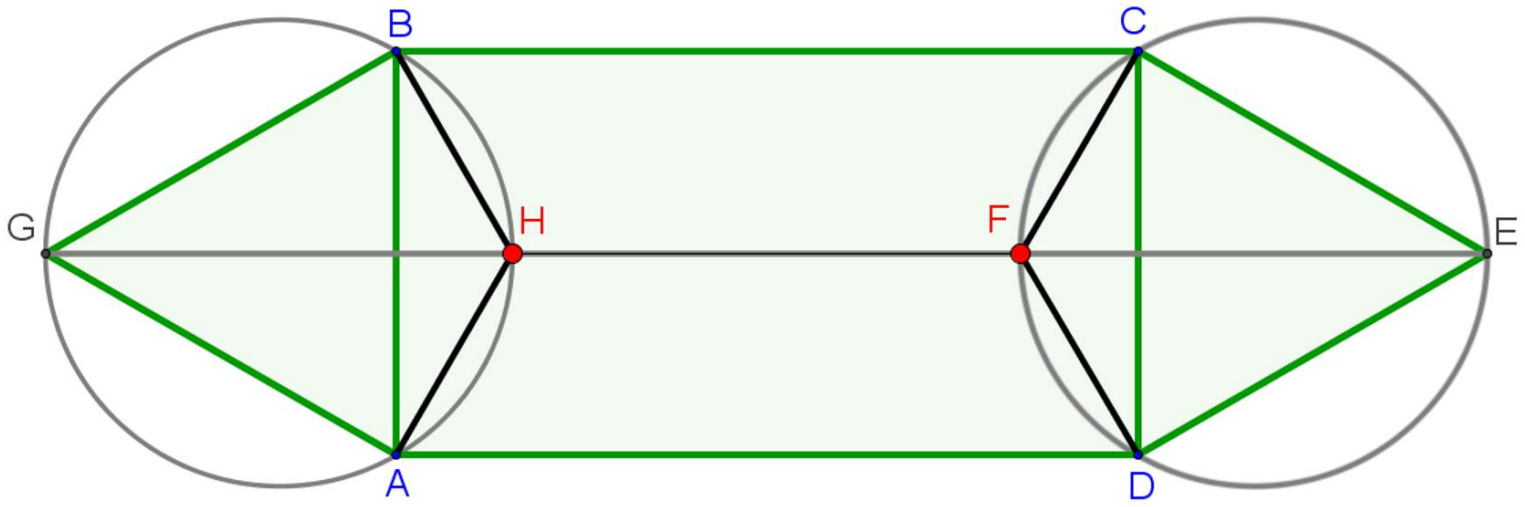

Points H and F must lie on $\overline{GE}$

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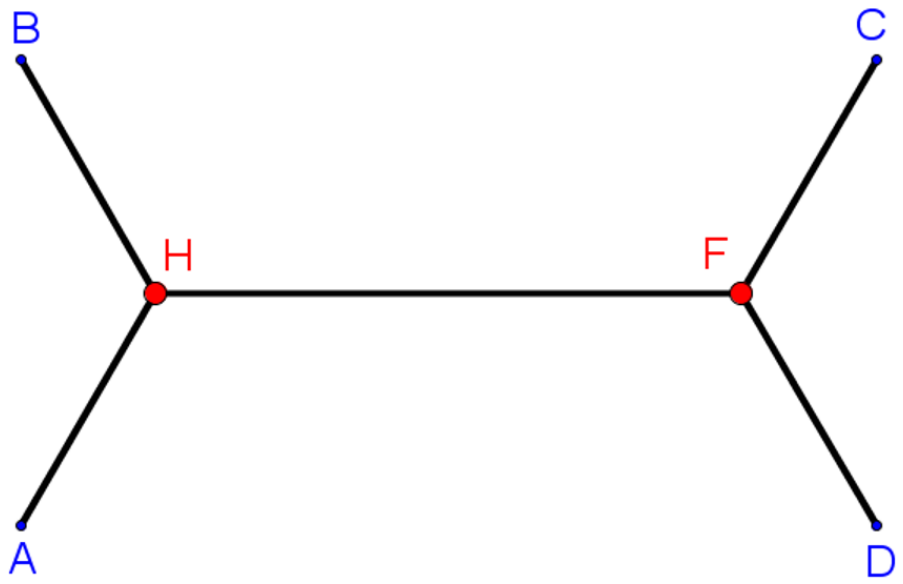

Minimum Spanning Tree Diagram

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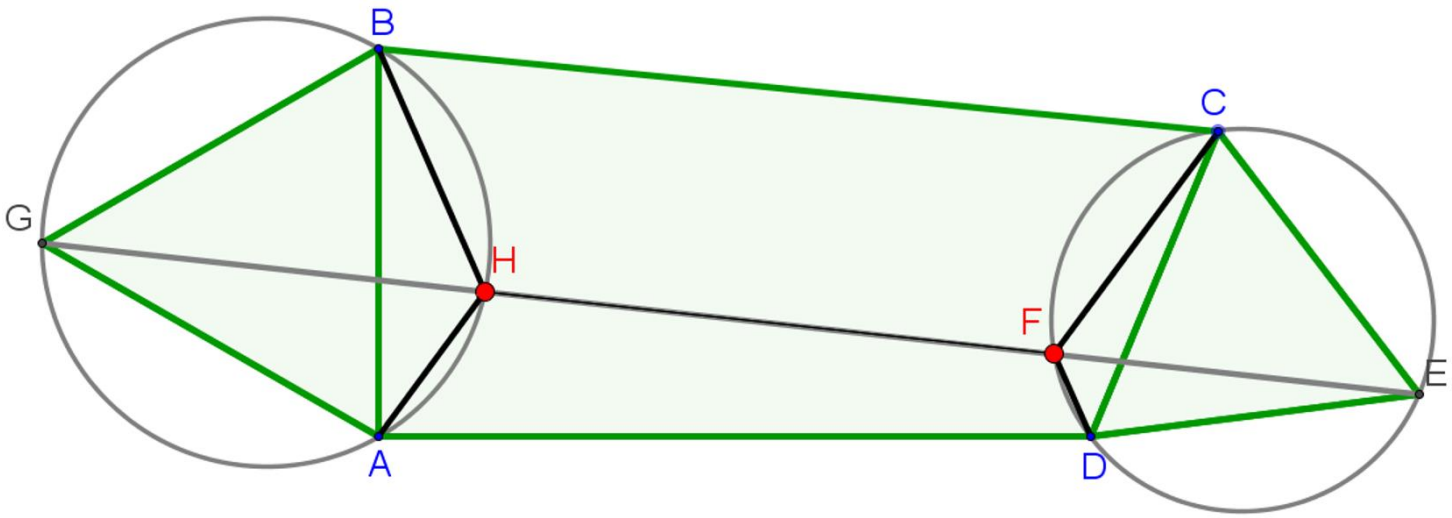

Construction Still Works for Non-Rectangles

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Highlights

- Fun optimization problem
- Application of The Extreme Value Theorem
- A chance for students to use parameters in a Calculus optimization problem – Why does one of those parameters "disappear"?
- Useful application of geometry properties and constructions

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Possible Extensions

Explore Different Number of Vertices – Steiner Points

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Student Conclusion Excerpt

"As neither of us is experienced in writing in math, we both found this assignment to have helped us to **better understand** the calculus concepts by **applying them in a real-world context**, and fully explaining the process of doing so. The value of the problem was proven through the demonstration where the bubbles between the two plastic sheets condensed to form a Steiner tree. This demonstration was **both fun to watch, but even more interesting when through calculus**, we were able to prove that this was in fact the shortest path."

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Conclusion continued…

"Additionally, by writing out our in-depth explanation of the process of our math, we were **forced to make the connections between all of the major concepts of calculus** that we learned so far in order to solve this modeling problem.

In this problem alone, we had to apply our knowledge of derivatives, power rule, chain rule, first derivative test, and applied maximums and minimums in one big problem, **helping to cement the connections between all of these big ideas."**

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Student Term Reflection Excerpt

"I typically like to work by myself, but after working in groups all year I've grown to like working with others. Working with **others helps me become a better learner because I have to explain my answers**. It helps me **listen** to others point of views, especially when we are working through **a hard word** problem.

I found this skill especially useful when we did the optimization road problem… Throughout solving this complex problem, my partner and I had to talk through each step, **admit when our answer was wrong**, and agree on our solution. Through this problem, I not only developed my optimization skills but I improved my group working skills as well. After this group assignment I understood how valuable it is to work in a group **because each member brings unique ideas that help form a collective answer.**"

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One more…

"I especially liked the motorway or minimum road problem. When we started out by brainstorming all the different configurations of roads between four houses, it was interesting to see what other students came up with; Jill's idea of a roundabout, in particular, stood out to me, as it factored traffic into the problem (although **we eventually did make some simplifying assumptions** that ruled out the traffic circle).

After using soap and rules of physics to confirm the shortest route between the houses, and after solving the problem, we **went back to some possible real-world contexts** that might change our configuration, such as if one of the houses was a hospital or fire station instead**. Solving real-world problems, acknowledging the simplifying assumptions we made, and understanding the contexts and more complicating possibilities for those kinds of problems was one of my favorite parts of class this term."**

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Take Away's

- Problem is accessible – Everyone can do something
- The modeling cycle comes to life
- Students get a chance to build functions as models
- The bubbles demo gets students interested in the problem
- There are beautiful connections across mathematical topics
- There are extensions for differentiated instruction

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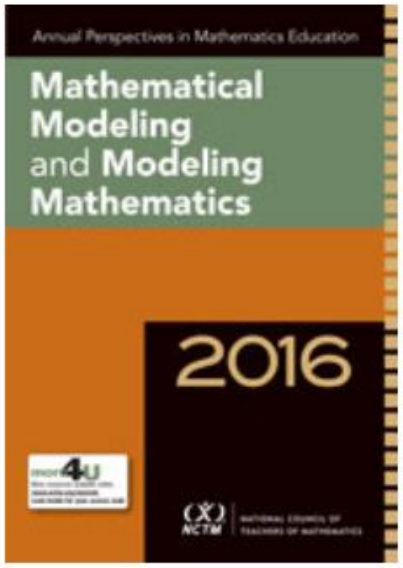
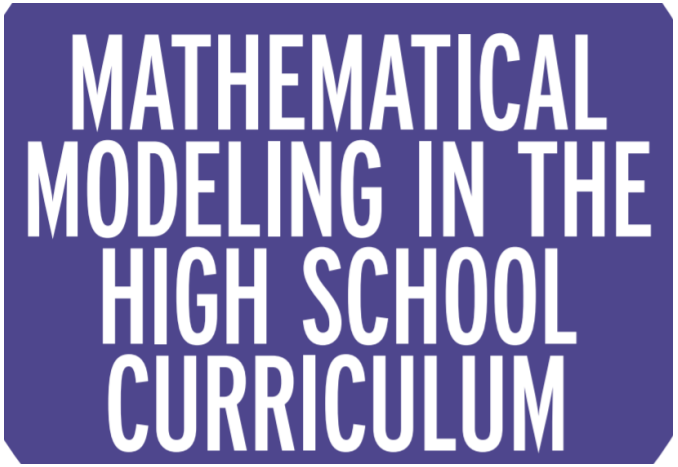
Resources and References

- James Grime Video https://www.youtube.com/watch?v=dAyDi1aa40E
- *Mathematical Modeling in the HS Curriculum*, MT article, December 2016
- *APME Mathematical Modeling*, 2016





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TCM Conference NCSSM

Teaching Contemporary Mathematics Conference

January 25 – 26, 2019

Durham, NC

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More Conferences and Information

Anja Greer Math, Science and Technology Conference
Phillips Exeter Academy, June 24 – June 29, 2018
Weeklong Mini-Courses

Knowles TEACHER INITIATIVE
Transforming Mathematics & Science Education

TEACHERS 2 TEACHERS GLOBAL
t2tglobal.org

SIAM Conference on Applied Mathematics Education (ED18)
July 9-11, 2018
Oregon Convention Center
Portland, Oregon, USA

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More Conferences

PBL
PROBLEM-BASED LEARNING
MATH TEACHING SUMMIT

JUNE 18-22, 2018

AVENUES: THE WORLD SCHOOL, NY, NY

"Using Status and Positioning Theory in the PBL Classroom"
Teresa K. Dunleavy, Ph. D. Vanderbilt University

"Mathematical Modeling and Problem-Based Learning: An Intersection of Student-Centered Learning"
Maria Hernandez, North Carolina School of Science & Math

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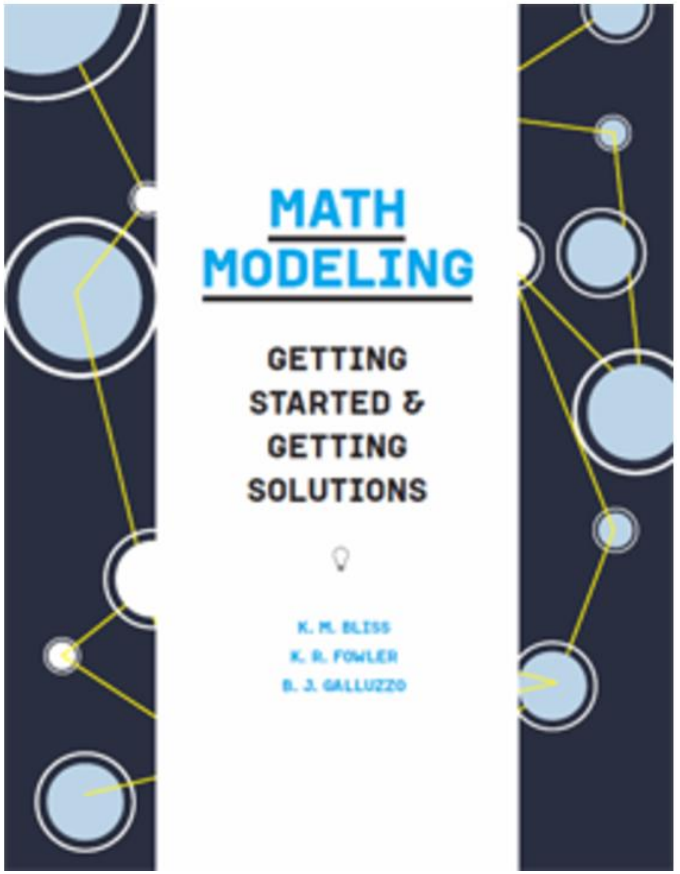
Free Downloads from the SIAM site



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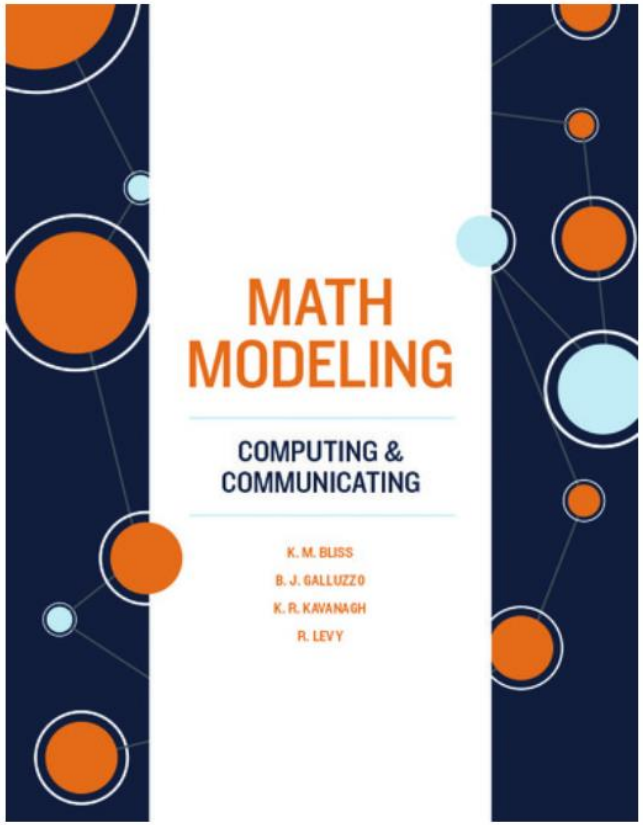
New Resource – Hot off the press…



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Thank you!

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