Imagine That: The Unfortunate Naming of i

Pam Goodner

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Math Trivia

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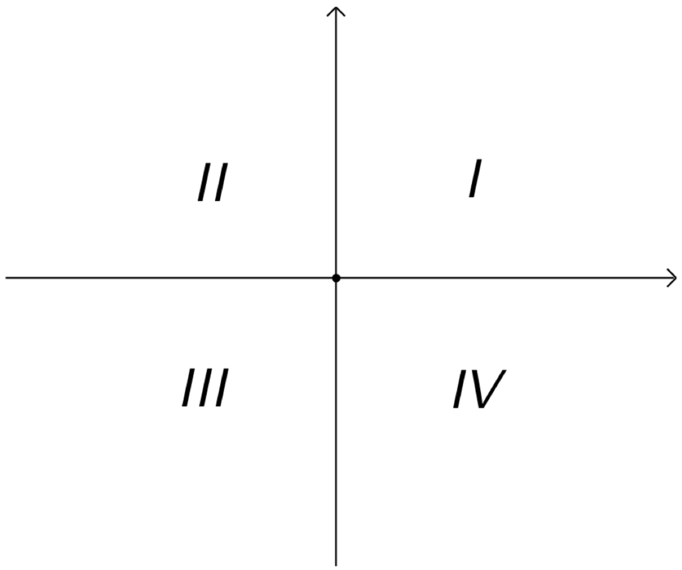
Why are the quadrants labeled as they are?



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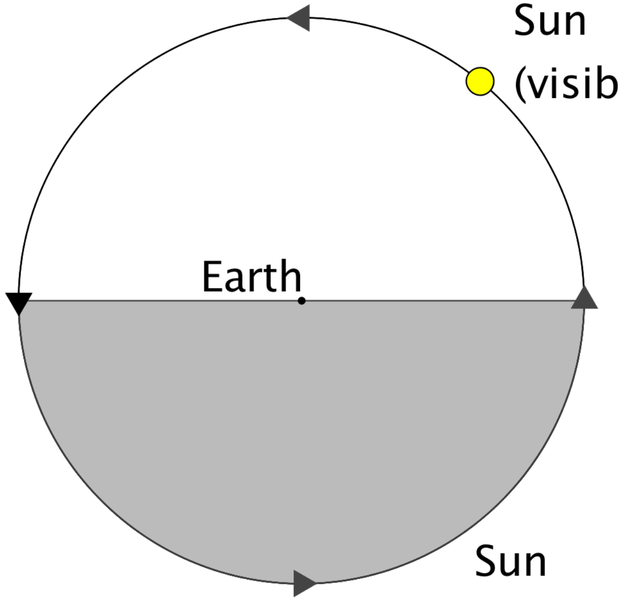

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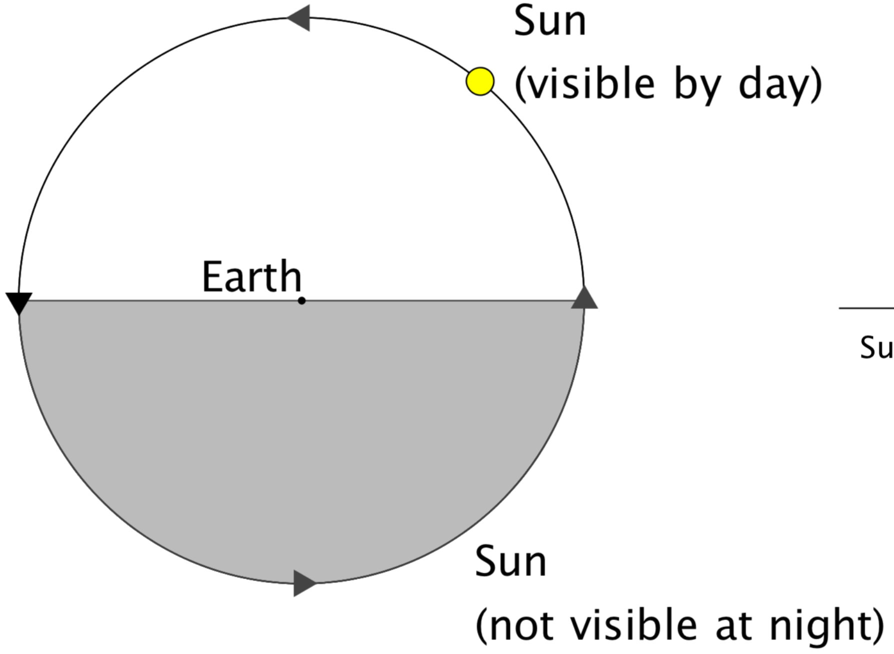

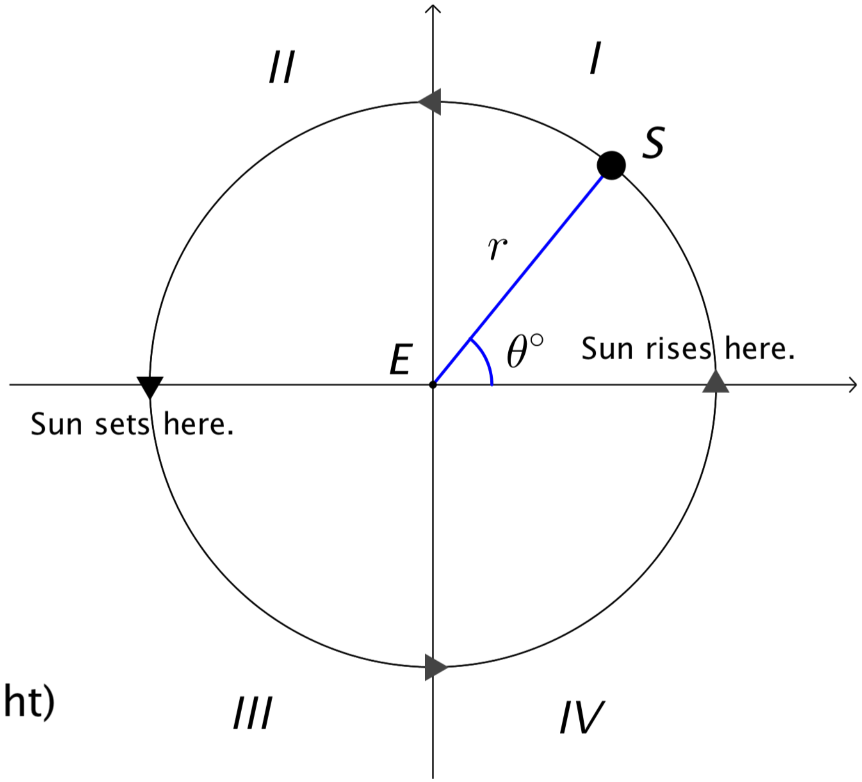

What is i?

What is i?

Why?

Evaluate

$\sqrt{9} \cdot \sqrt{4}$

Evaluate

$\sqrt{9} \cdot \sqrt{4} = 6$

Evaluate

$\sqrt{9} \cdot \sqrt{4} = 6$

$\sqrt{9} \cdot \sqrt{-4}$

Evaluate

$$\sqrt{9} \cdot \sqrt{4} = 6$$

$$\sqrt{9} \cdot \sqrt{-4} = 6i$$

Evaluate

$\sqrt{9} \cdot \sqrt{4} = 6$

$\sqrt{9} \cdot \sqrt{-4} = 6i$

$\sqrt{-9} \cdot \sqrt{4}$

Evaluate

$\sqrt{9} \cdot \sqrt{4} = 6$

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Evaluate

$\sqrt{9} \cdot \sqrt{4} = 6$

$\sqrt{9} \cdot \sqrt{-4} = 6i$

$\sqrt{-9} \cdot \sqrt{4} = 6i$

$\sqrt{-9} \cdot \sqrt{-4}$

Evaluate

$\sqrt{9} \cdot \sqrt{4} = 6$

$\sqrt{9} \cdot \sqrt{-4} = 6i$

$\sqrt{-9} \cdot \sqrt{4} = 6i$

$\sqrt{-9} \cdot \sqrt{-4} = -6$

Evaluate

$\sqrt{3} \cdot \sqrt{2}$

$\sqrt{3} \cdot \sqrt{-2}$

$\sqrt{-3} \cdot \sqrt{2}$

$\sqrt{-3} \cdot \sqrt{-2}$

Evaluate

$$\sqrt{3} \cdot \sqrt{2} = \sqrt{6}$$

$$\sqrt{3} \cdot \sqrt{-2} = \sqrt{6}i$$

$$\sqrt{-3} \cdot \sqrt{2} = \sqrt{6}i$$

$$\sqrt{-3} \cdot \sqrt{-2} = -\sqrt{6}$$

Using Geometry to Understand i

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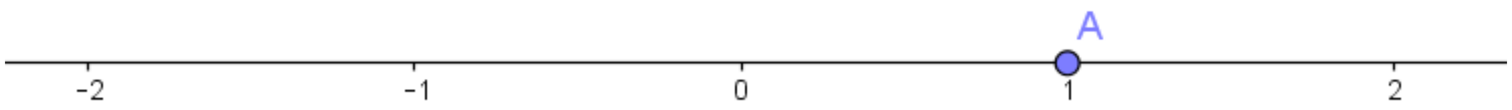
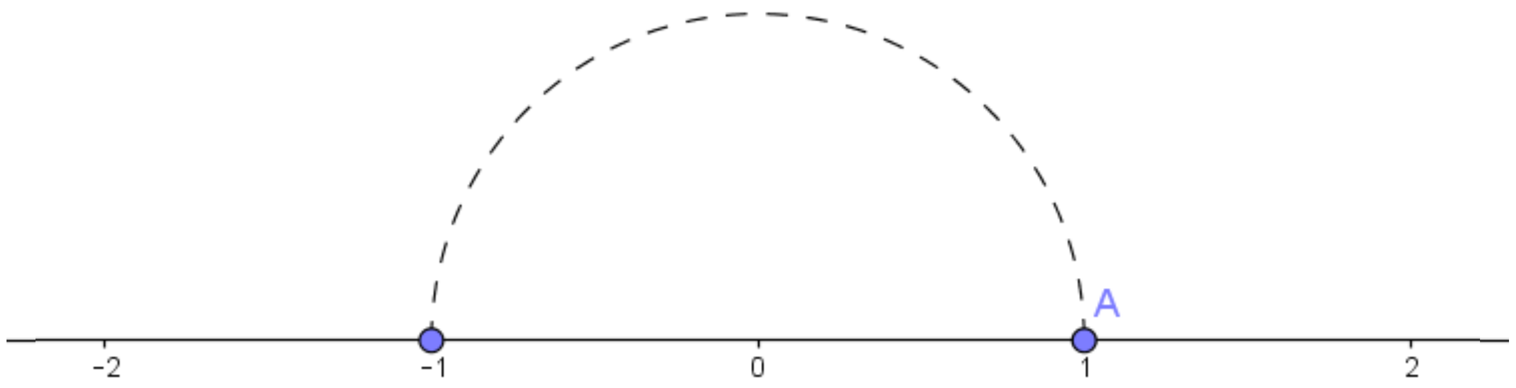
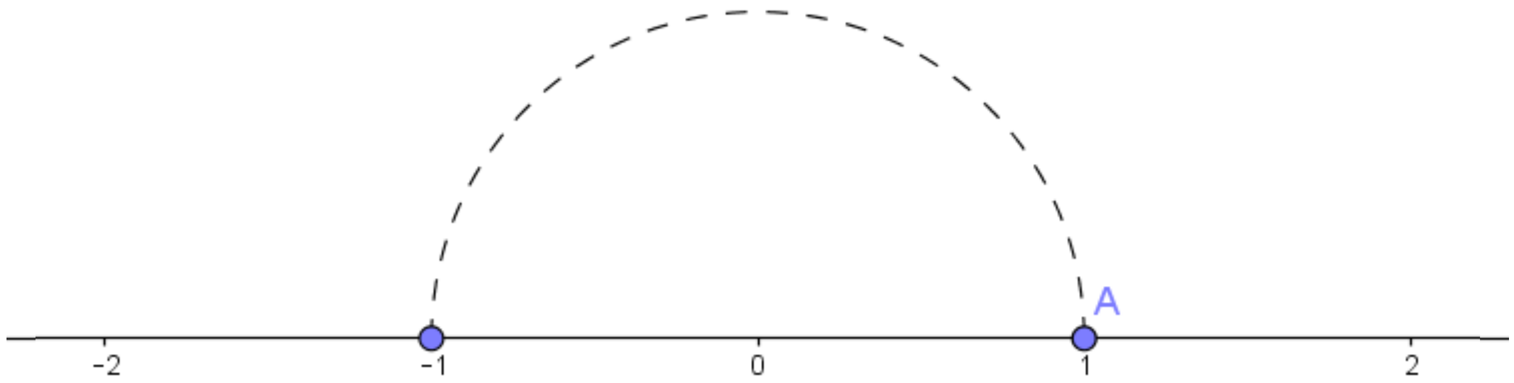


Using Geometry to Understand i



Using Geometry to Understand i

Rotate $180°$



Using Geometry to Understand i

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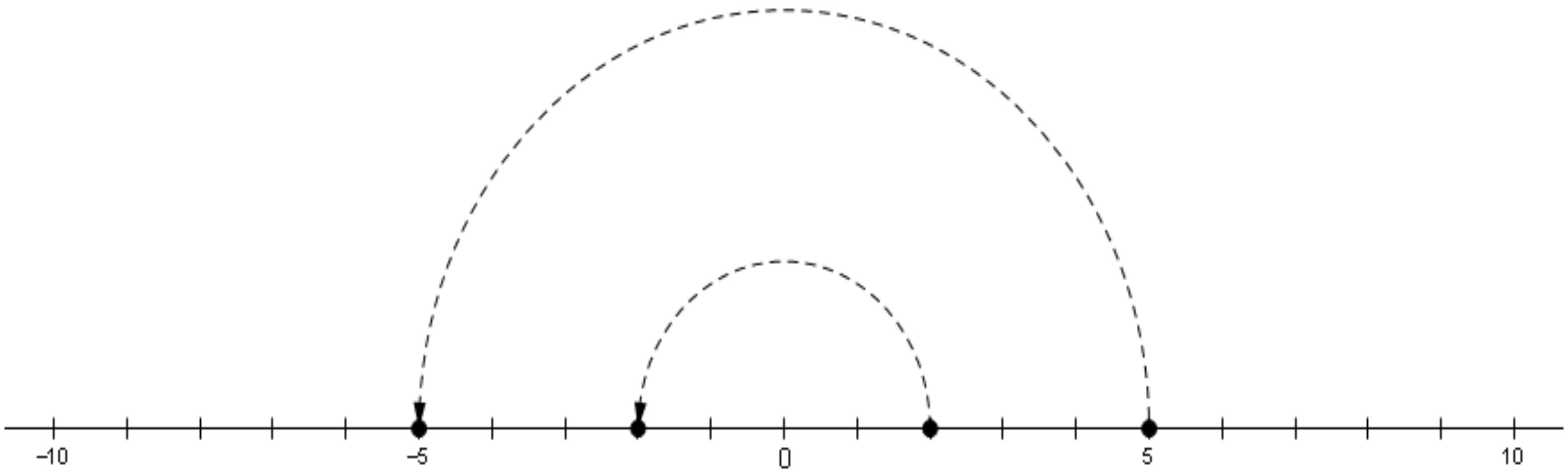

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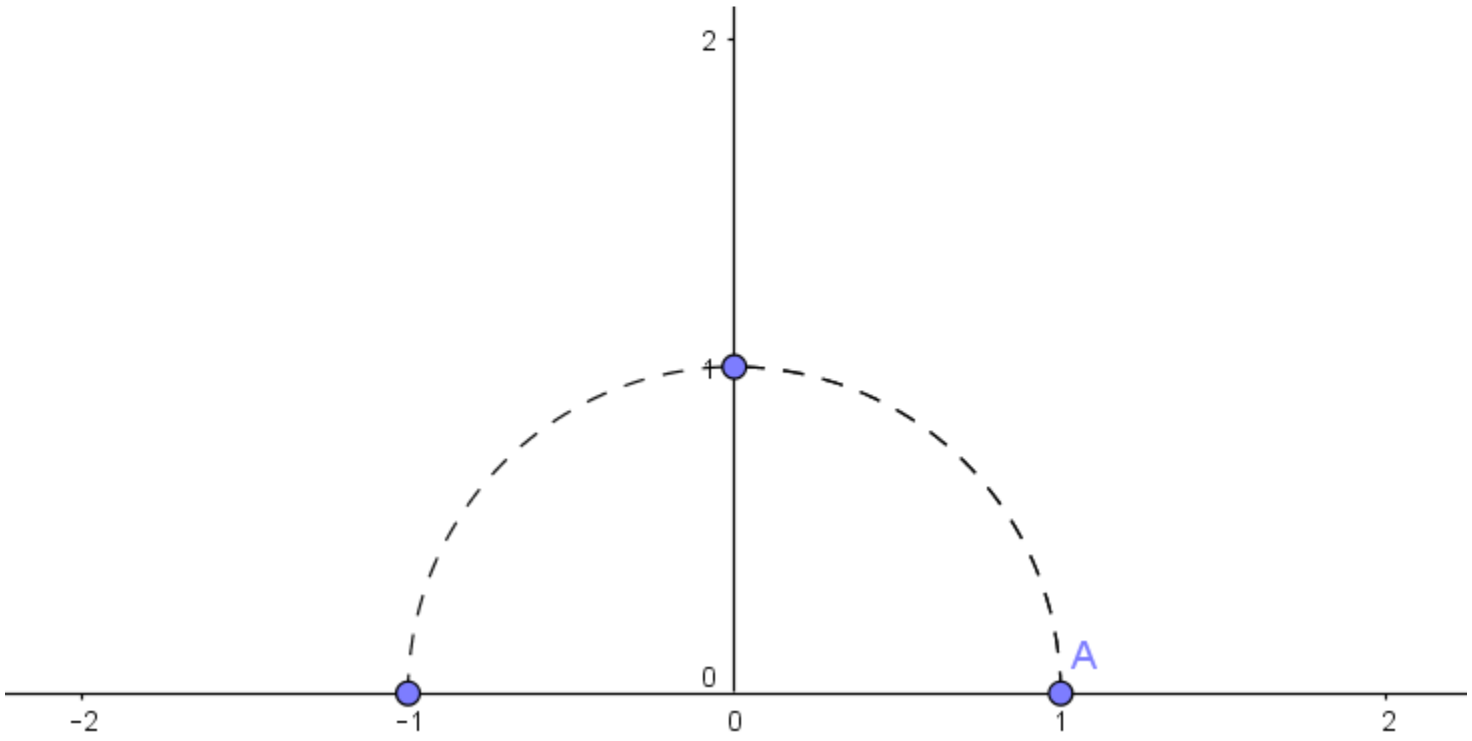

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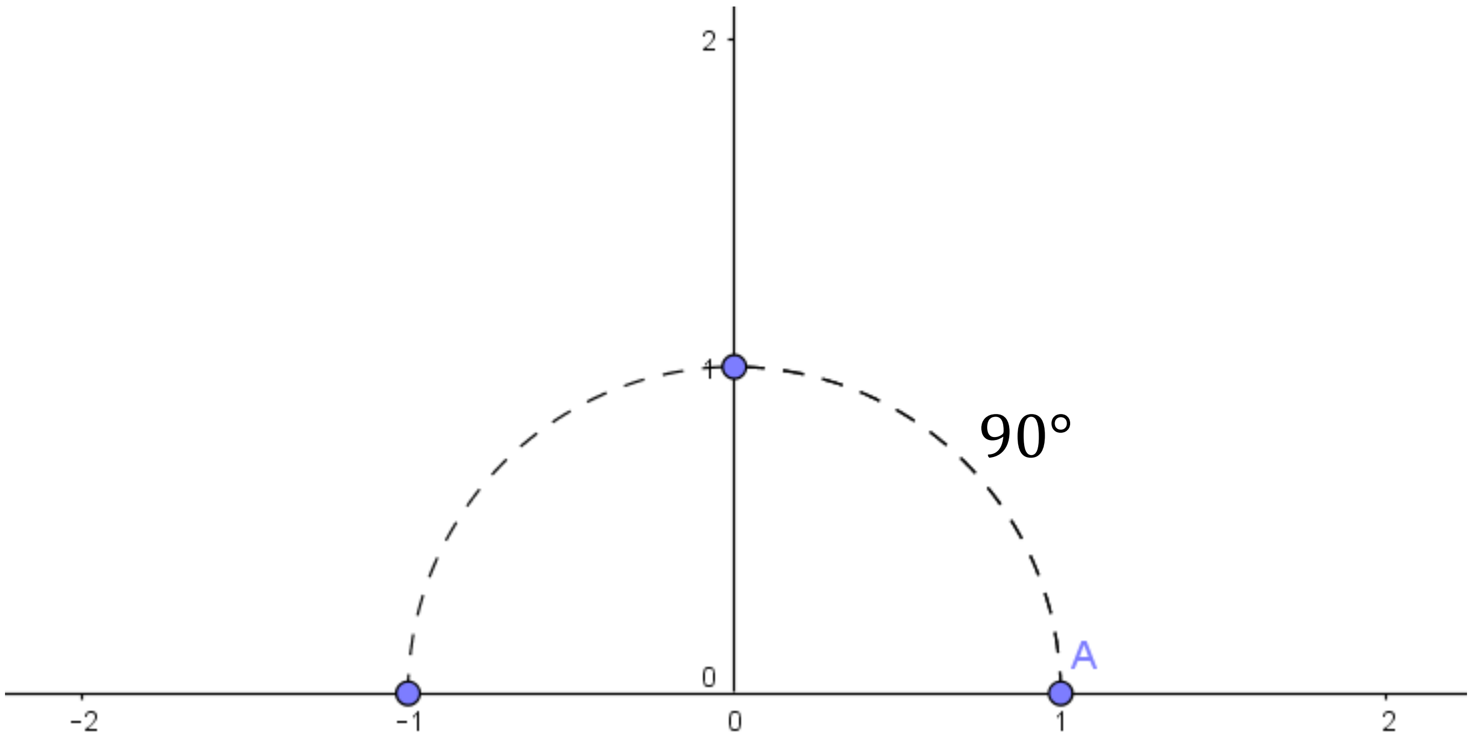

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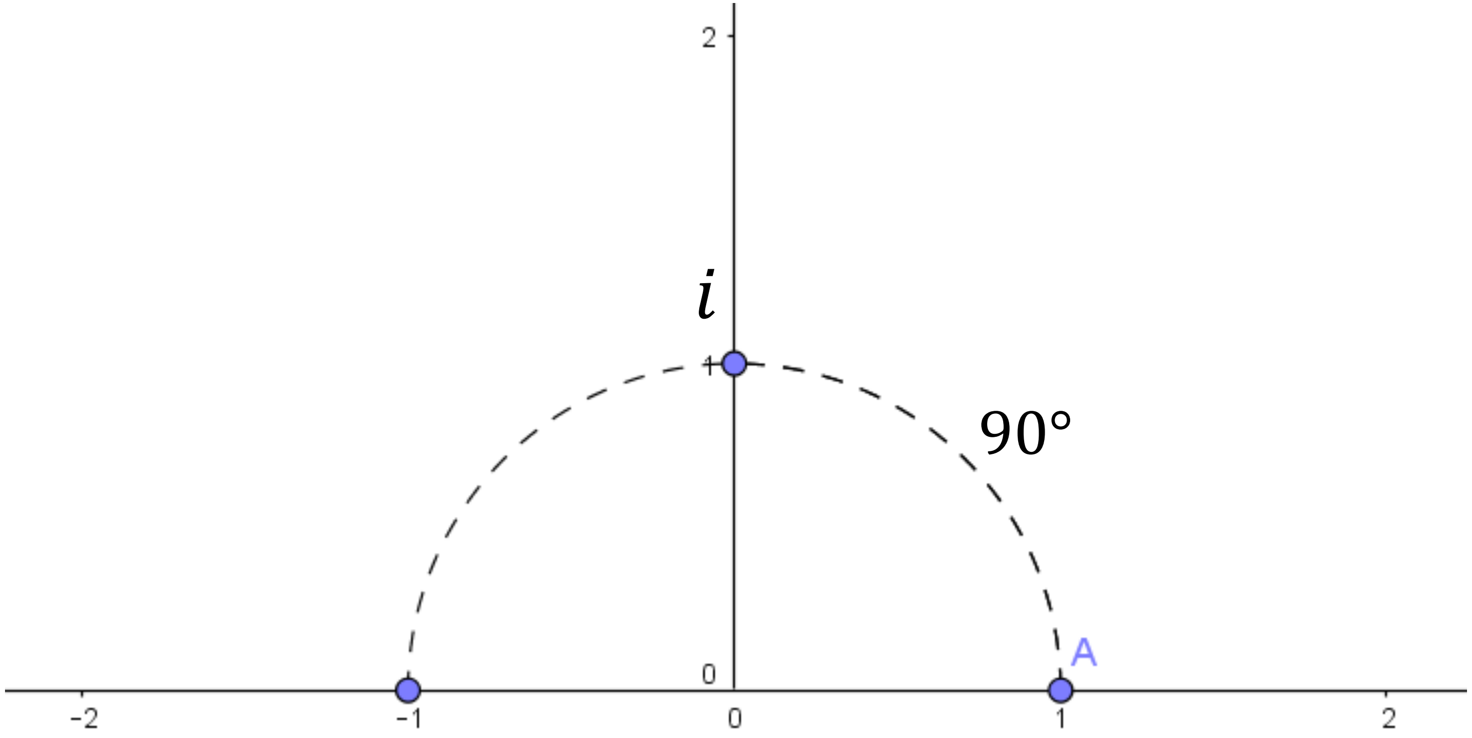

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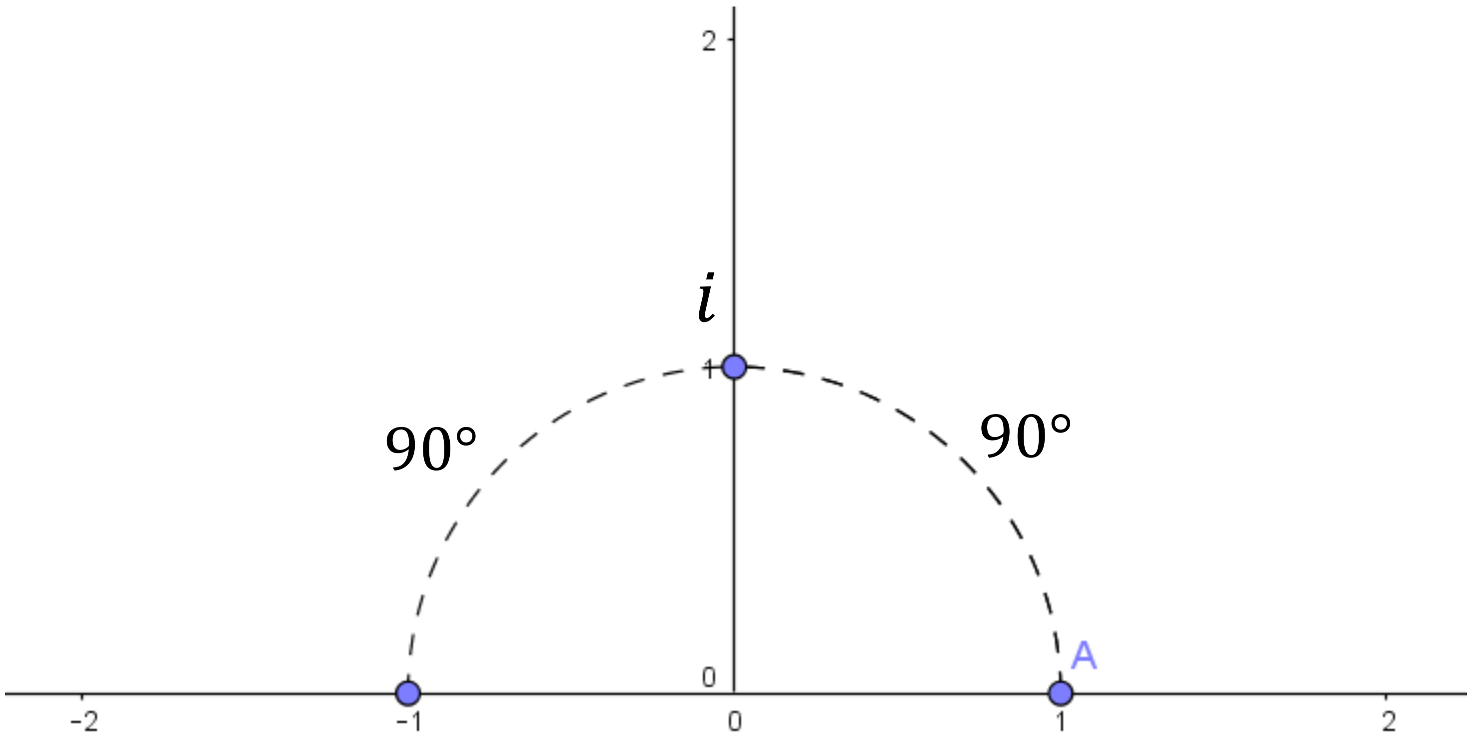
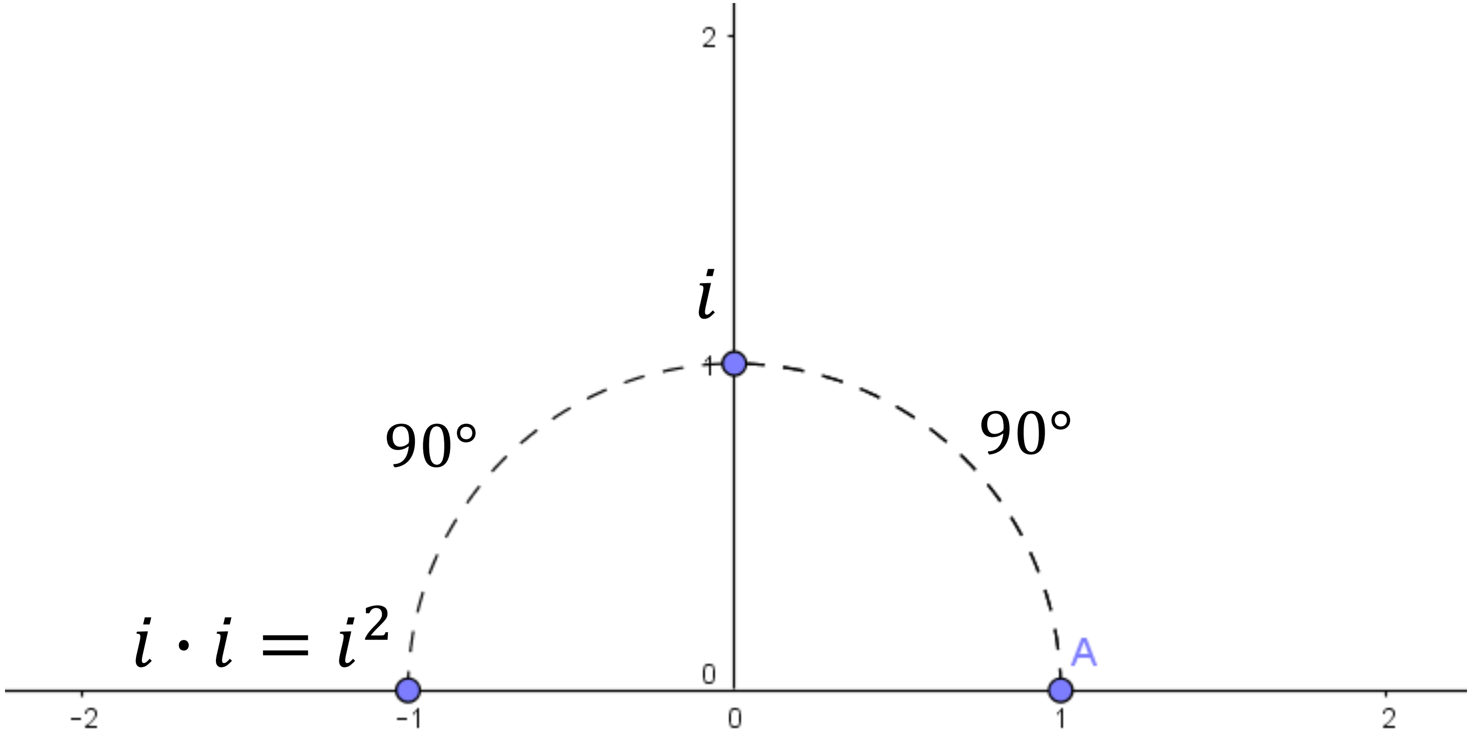


Using Geometry to Understand i



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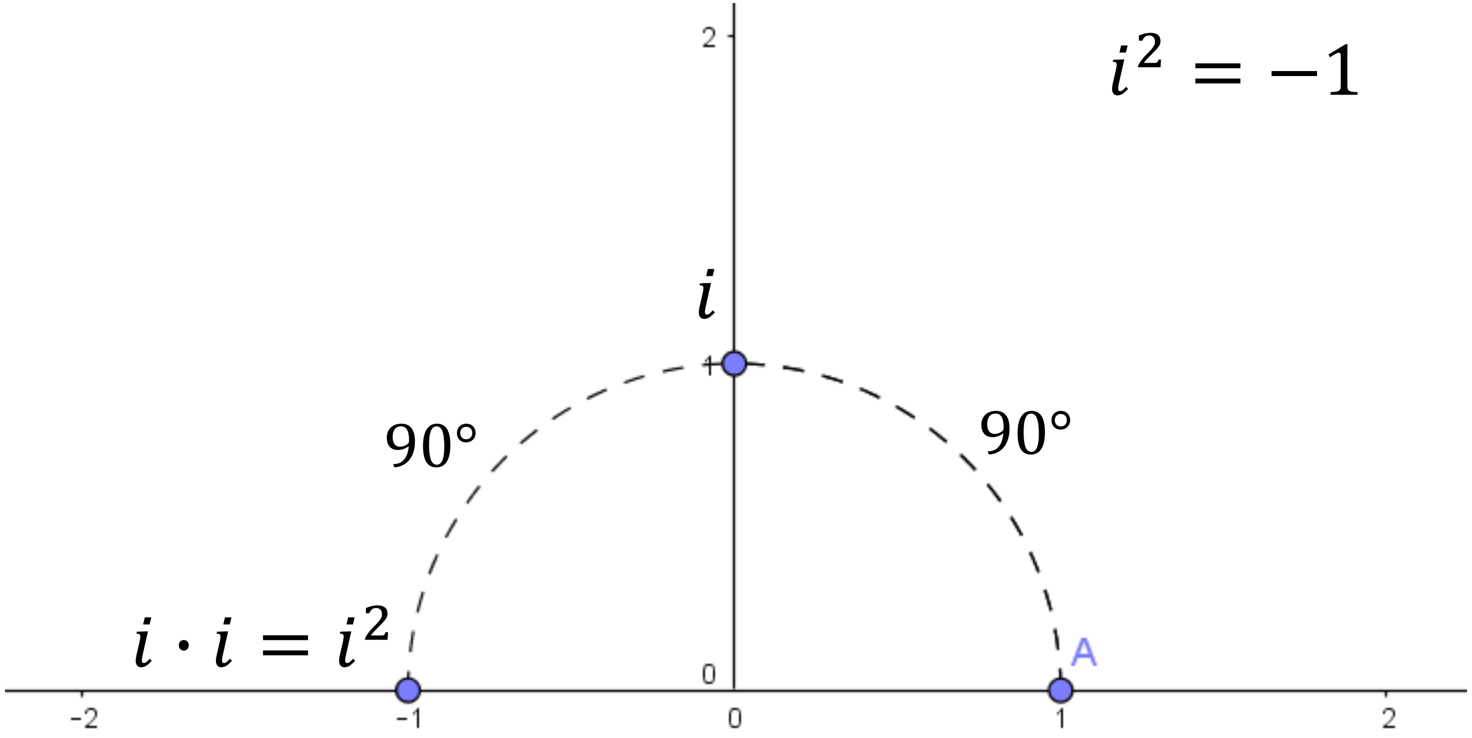
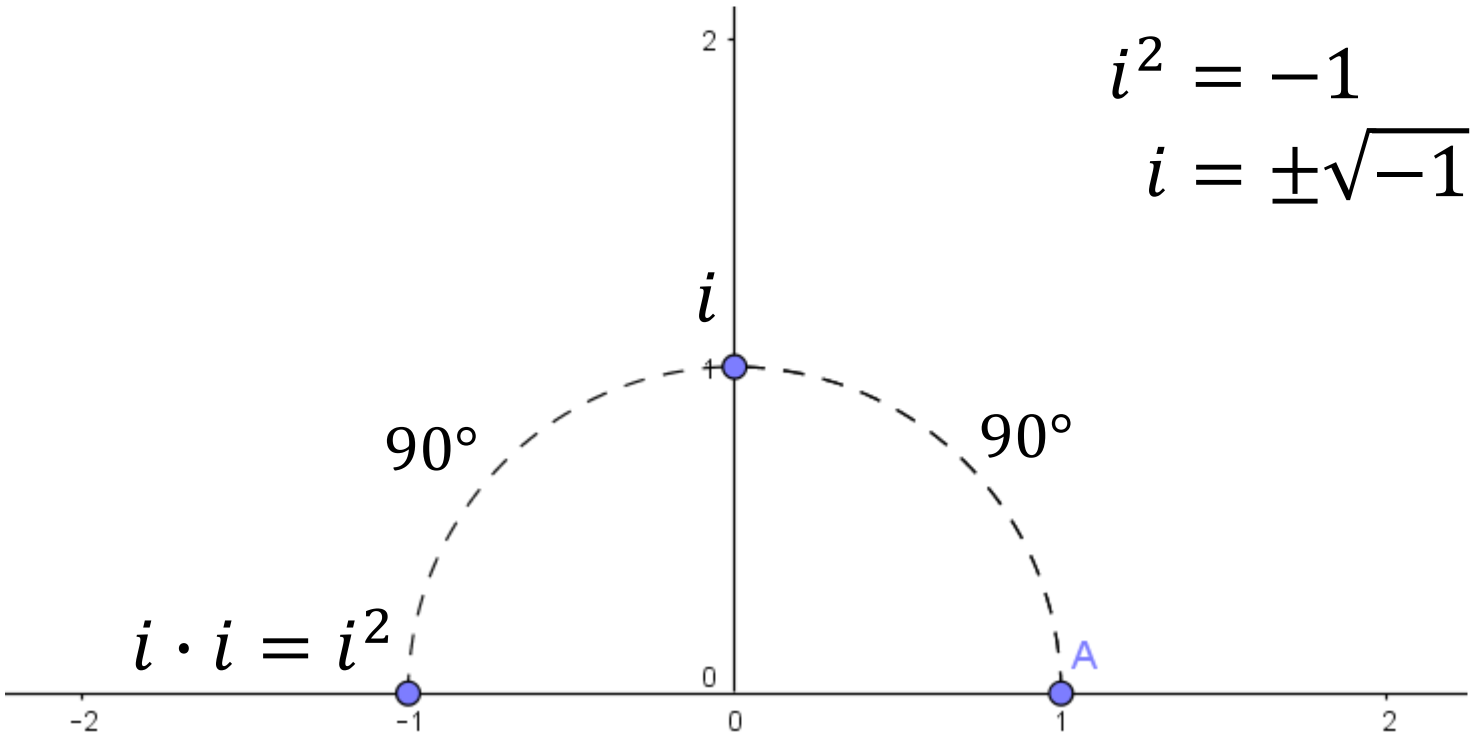
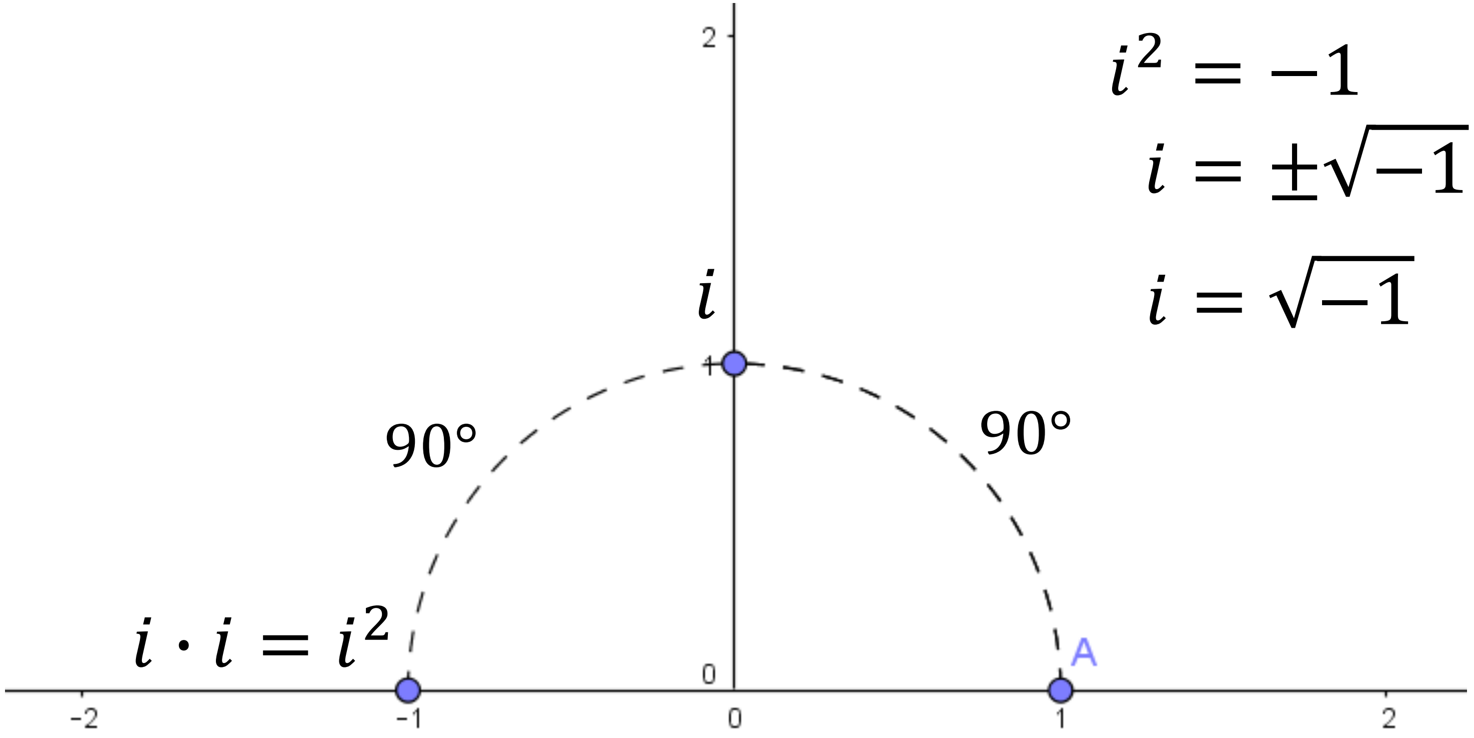


Using Geometry to Understand i



Using Geometry to Understand i

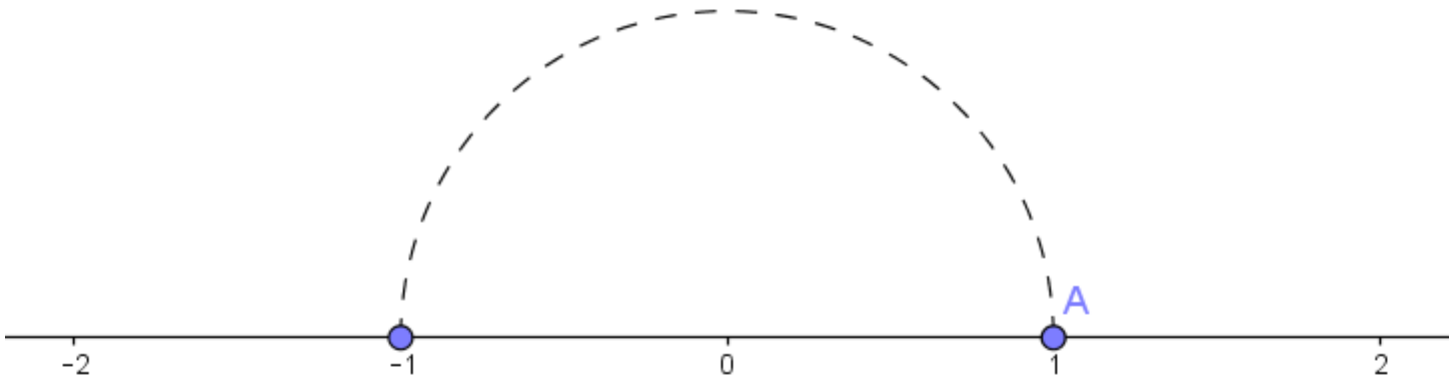


$$i^2 = -1$$

$$i = \pm\sqrt{-1}$$

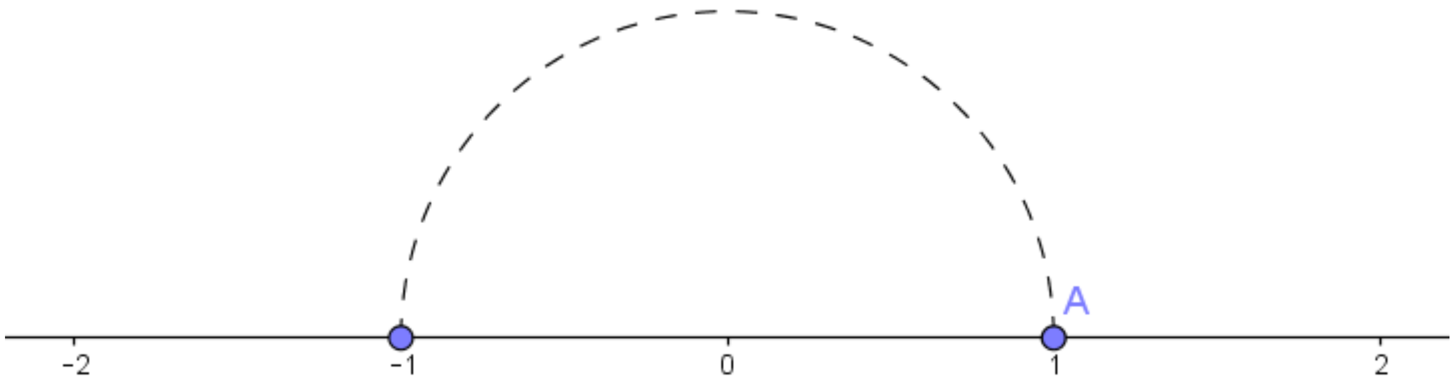
$$i = \sqrt{-1}$$

The Math Behind i



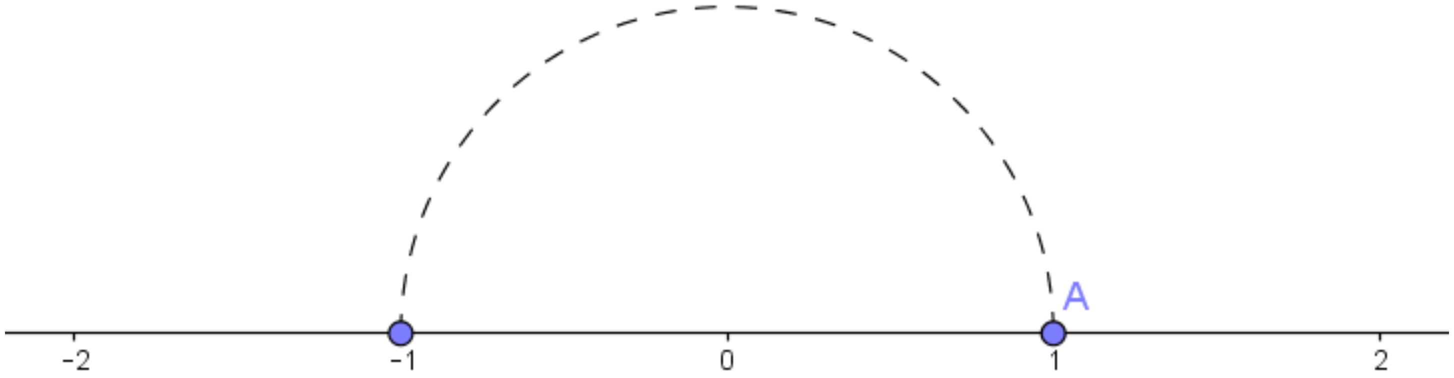
The Math Behind i

$$f(x) = kx$$



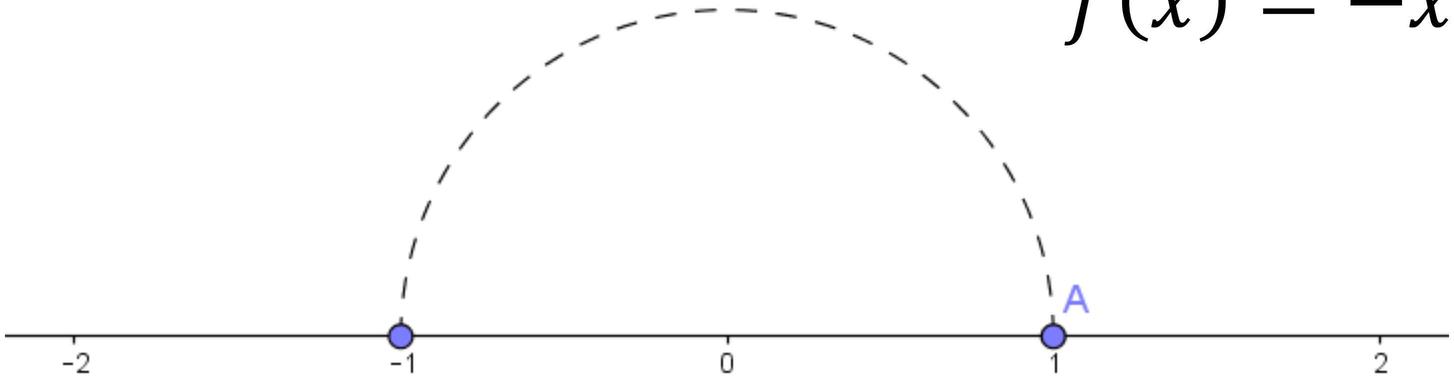
The Math Behind i

$$f(x) = kx$$
$$k = -1$$

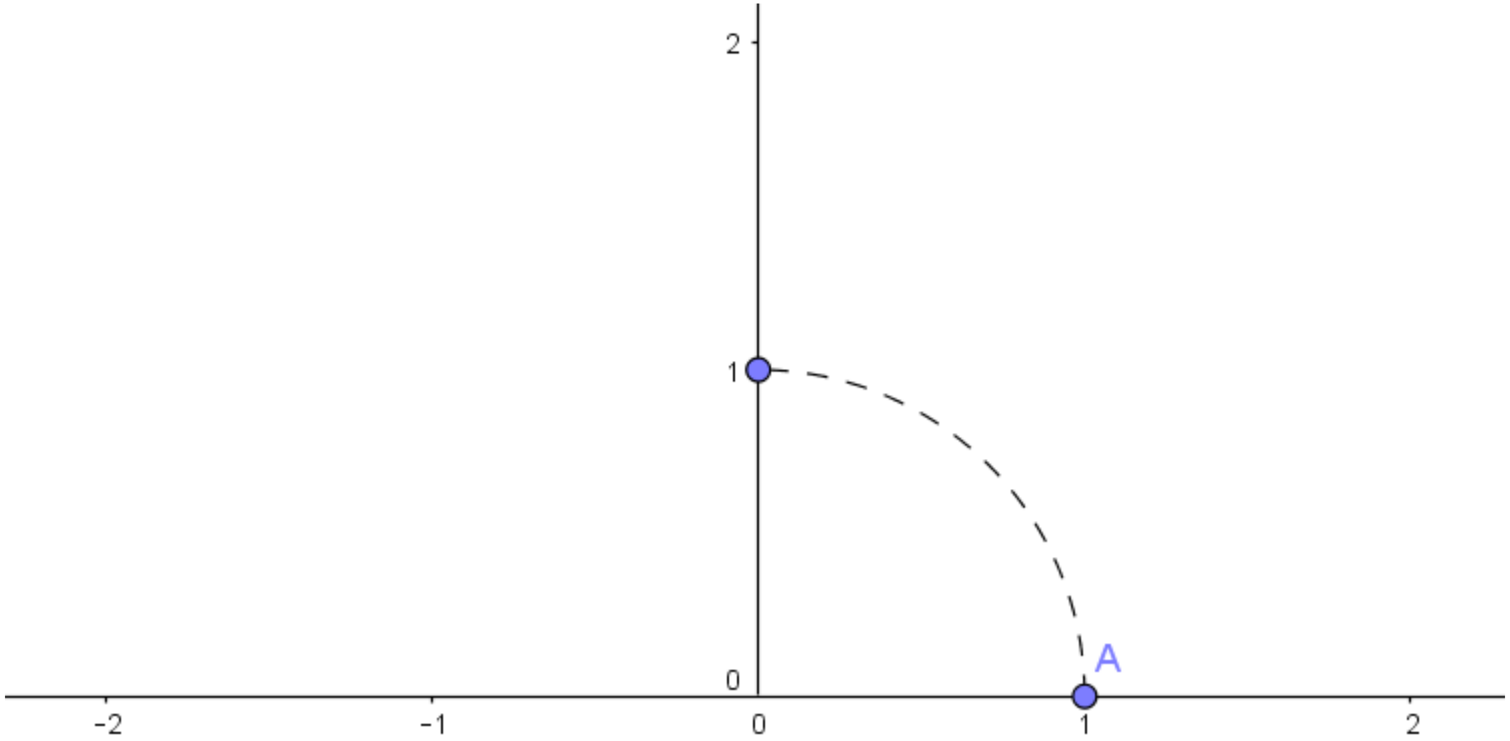


The Math Behind i

$$f(x) = kx$$
$$k = -1$$
$$f(x) = -x$$



The Math Behind i

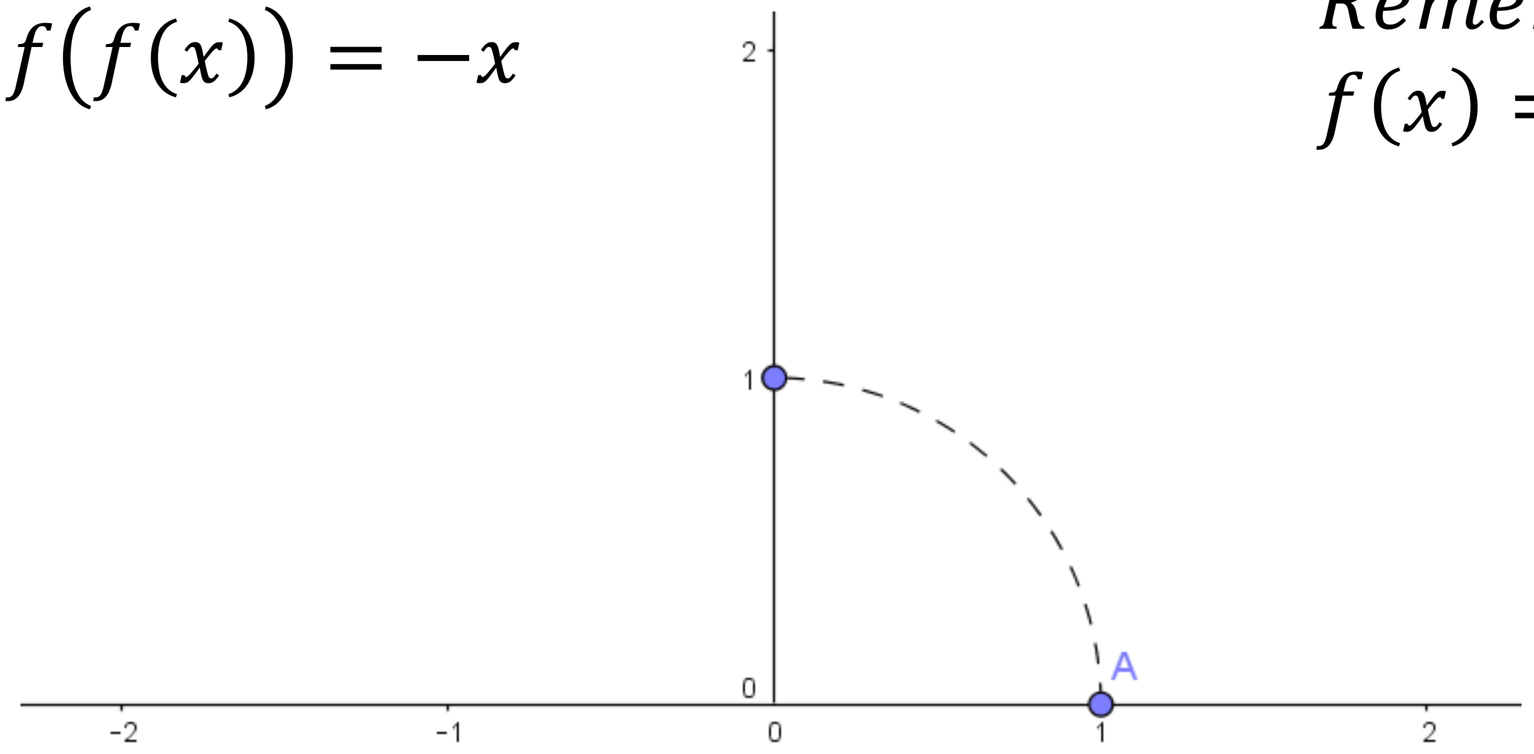


The Math Behind i

$$f(f(x)) = -x$$

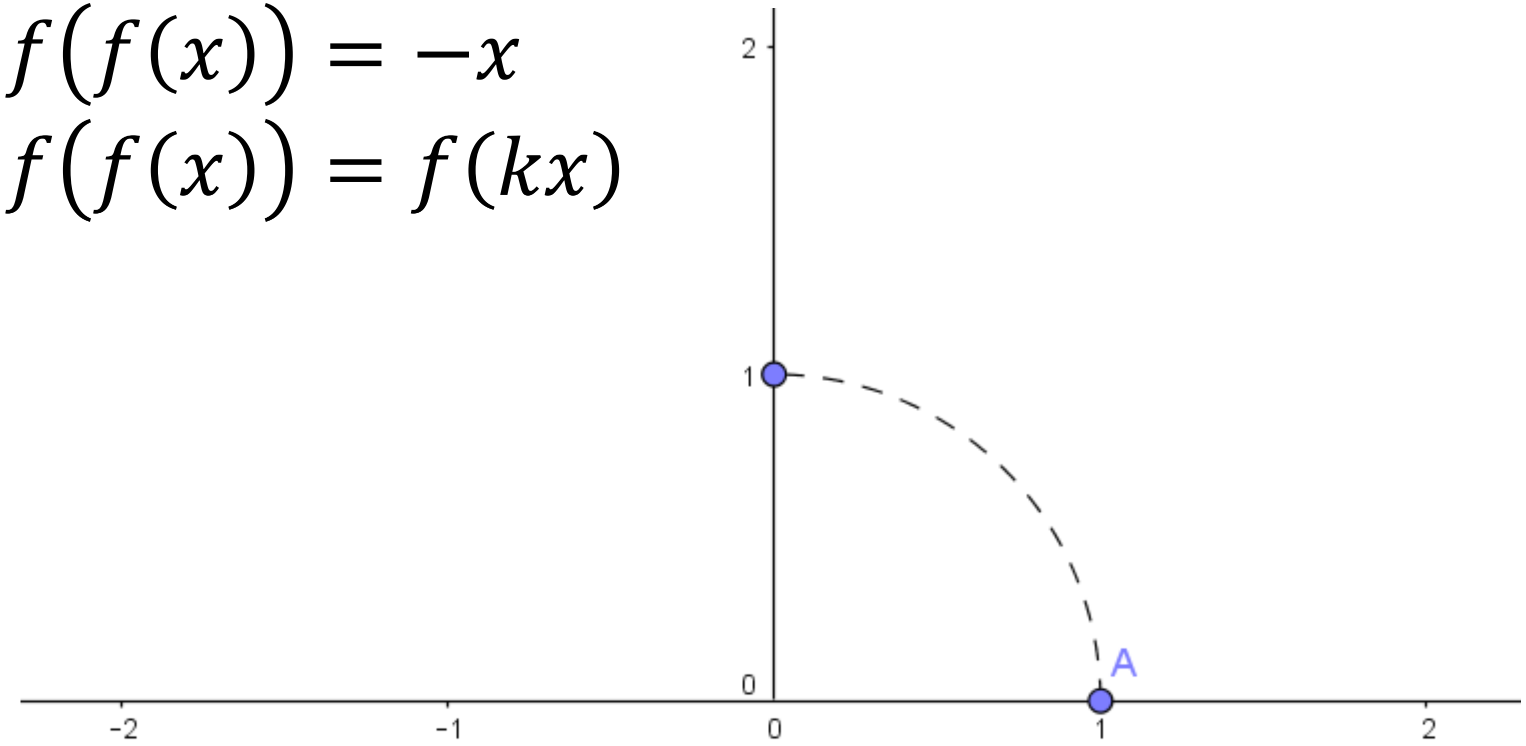
Remember

$$f(x) = kx$$



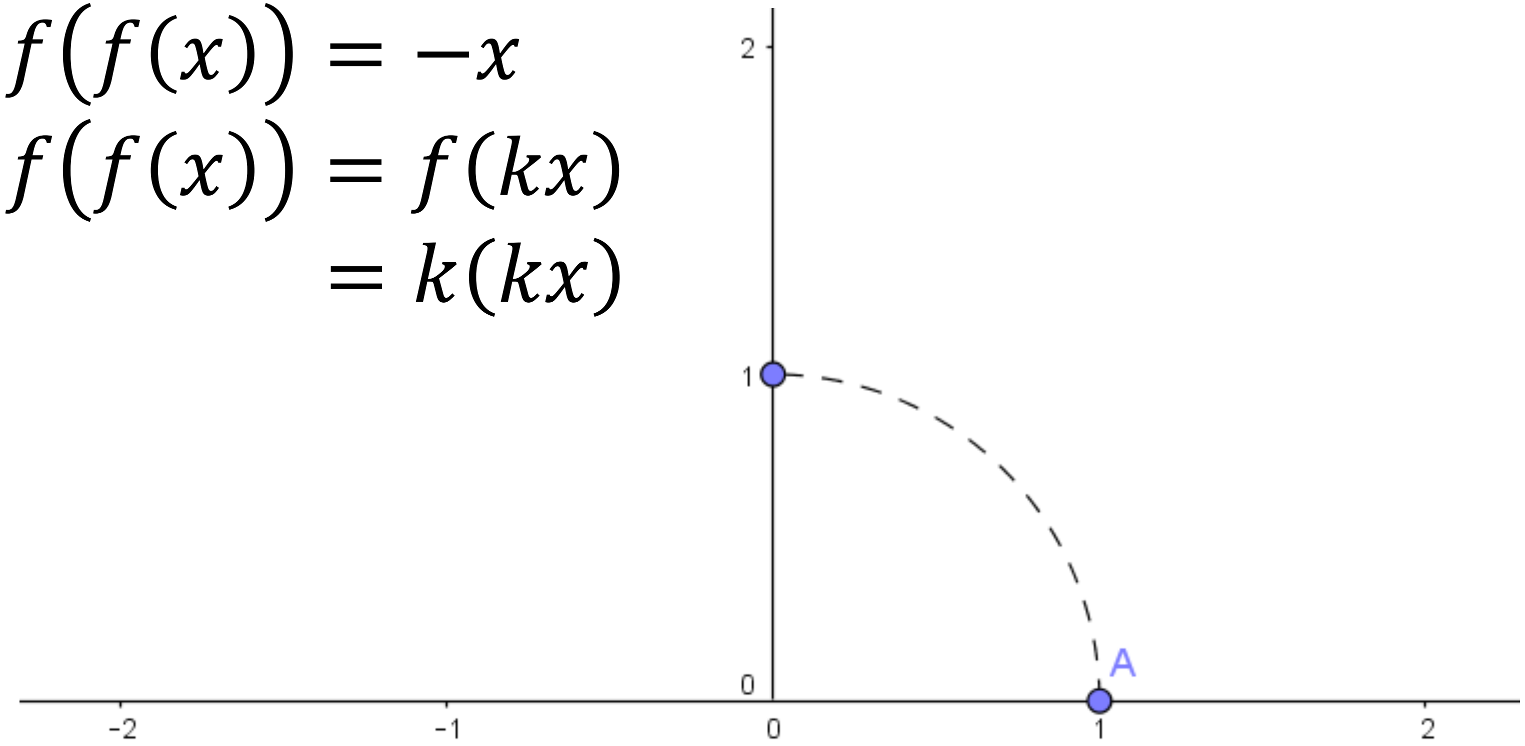
The Math Behind i

$$f\big(f(x)\big) = -x$$
$$f\big(f(x)\big) = f(kx)$$



The Math Behind i

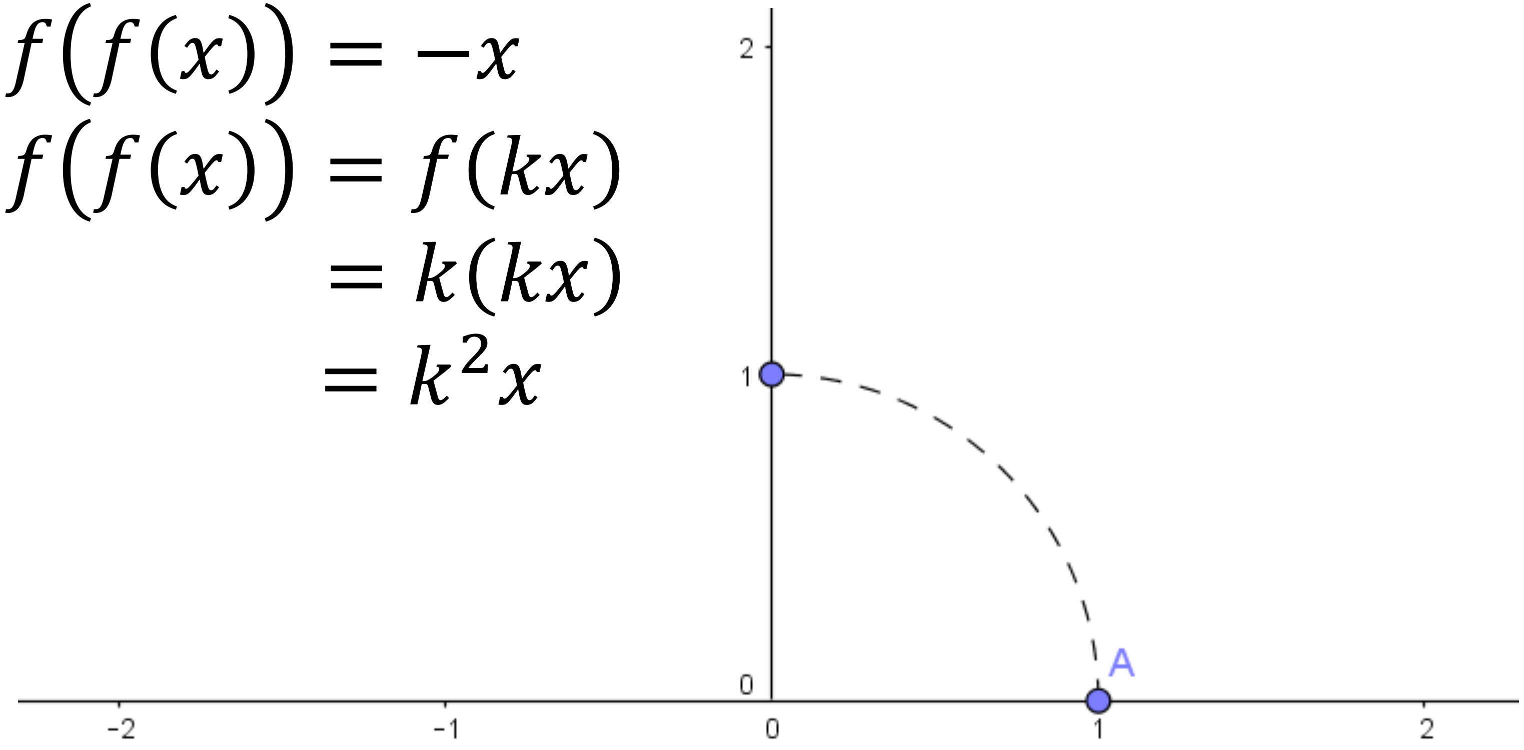
$$f\big(f(x)\big) = -x$$
$$f\big(f(x)\big) = f(kx)$$
$$= k(kx)$$



The Math Behind i

$$f\big(f(x)\big) = -x$$

$$\begin{aligned} f\big(f(x)\big) &= f(kx) \\ &= k(kx) \\ &= k^2x \end{aligned}$$



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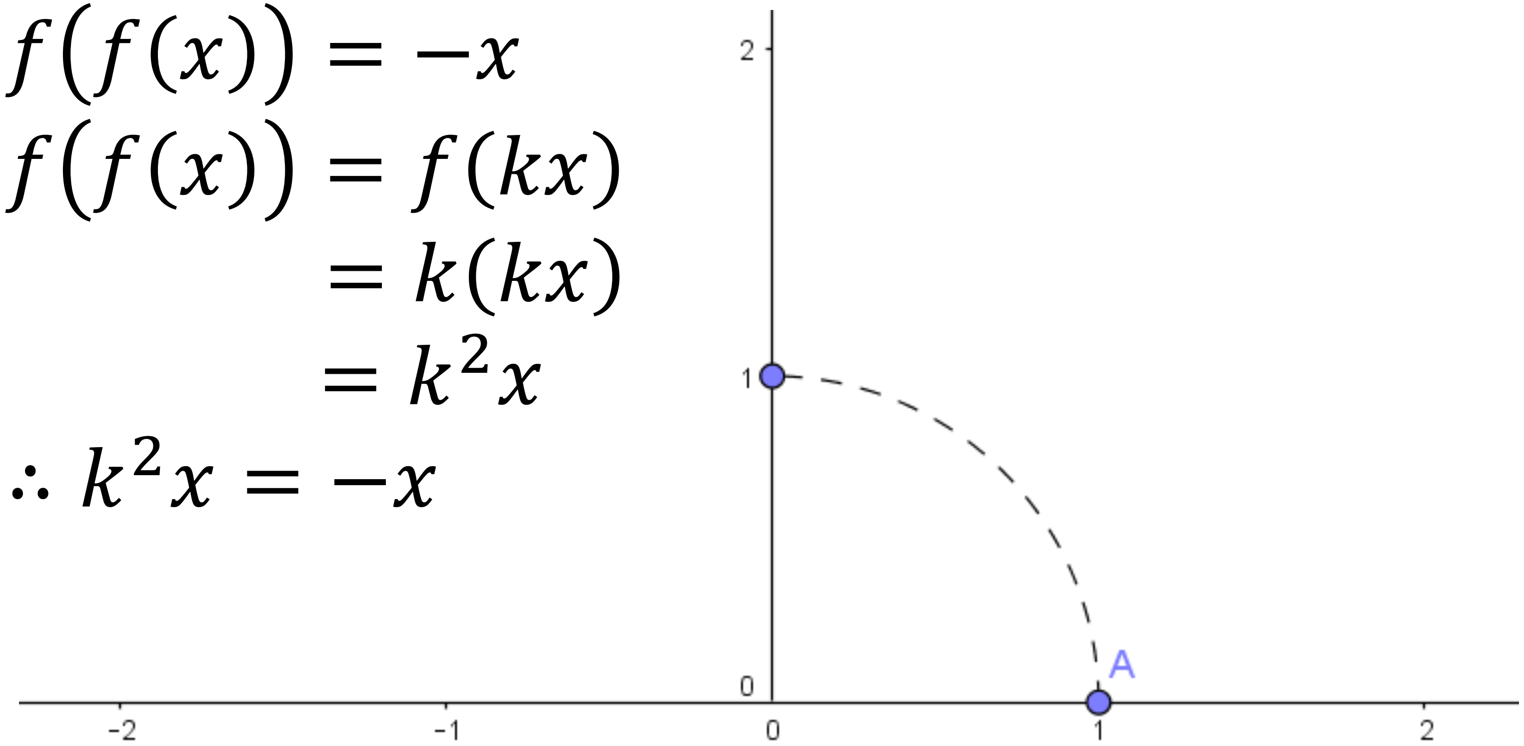
$$f\big(f(x)\big) = -x$$
$$\begin{aligned} f\big(f(x)\big) &= f(kx) \\ &= k(kx) \\ &= k^2x \end{aligned}$$
$$\therefore k^2x = -x$$



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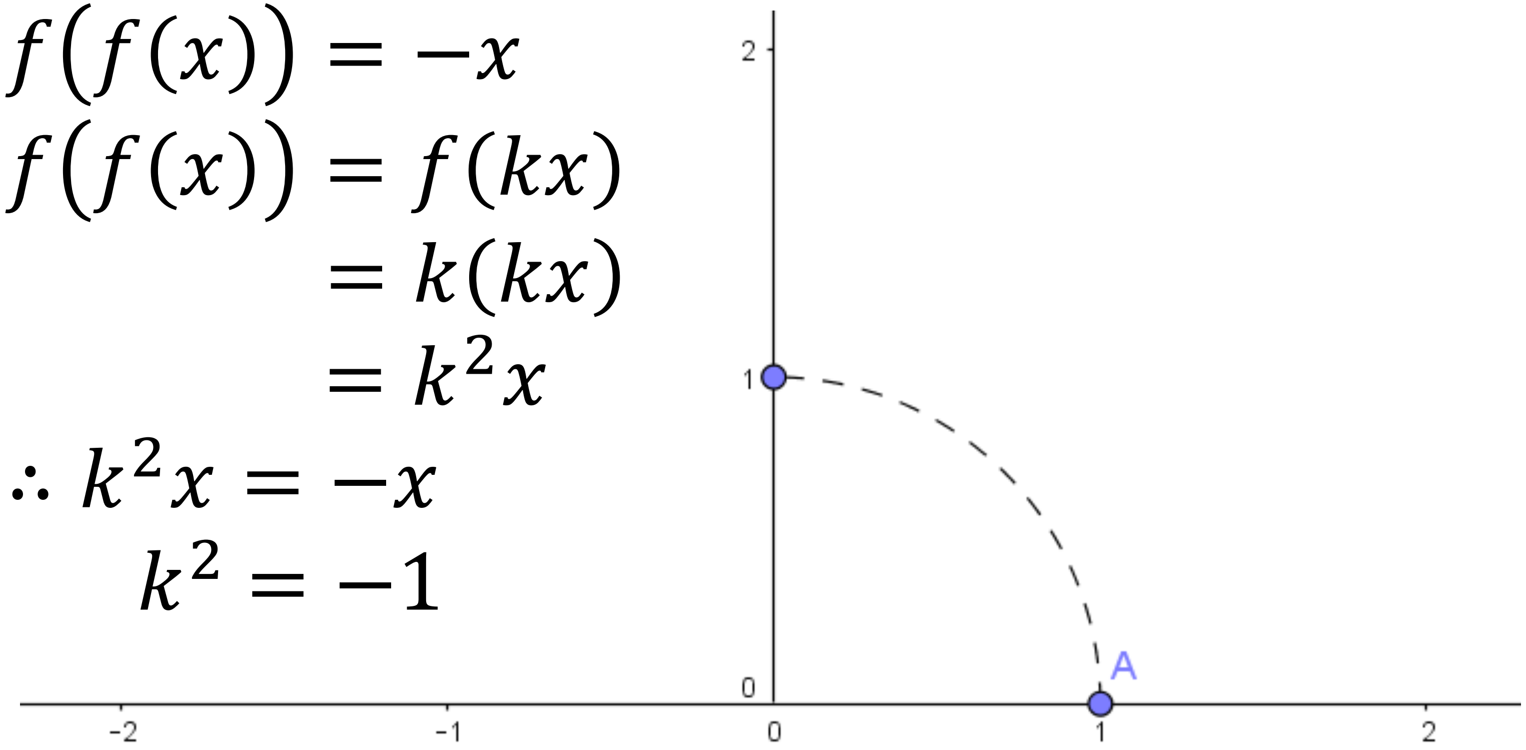
$$k^2 = -1$$



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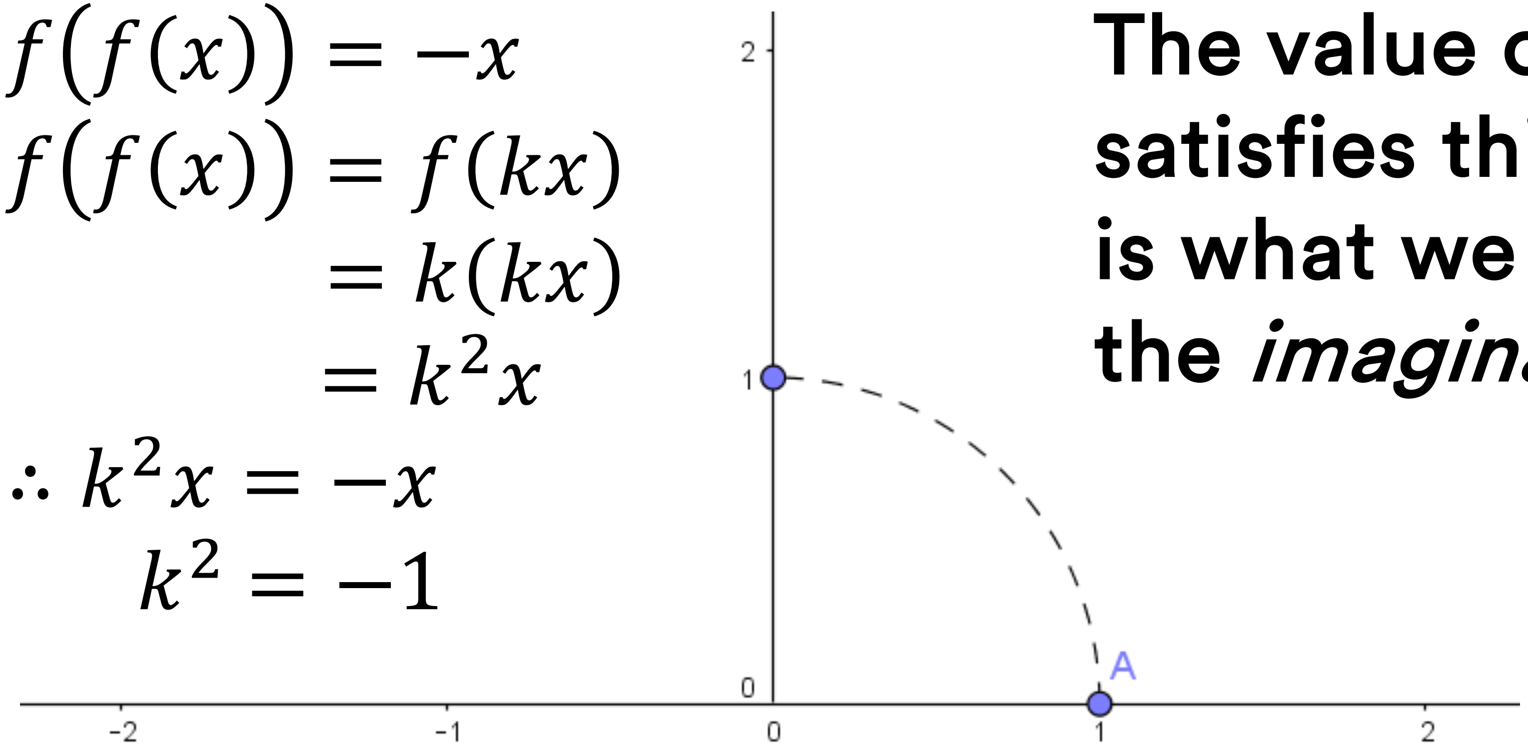
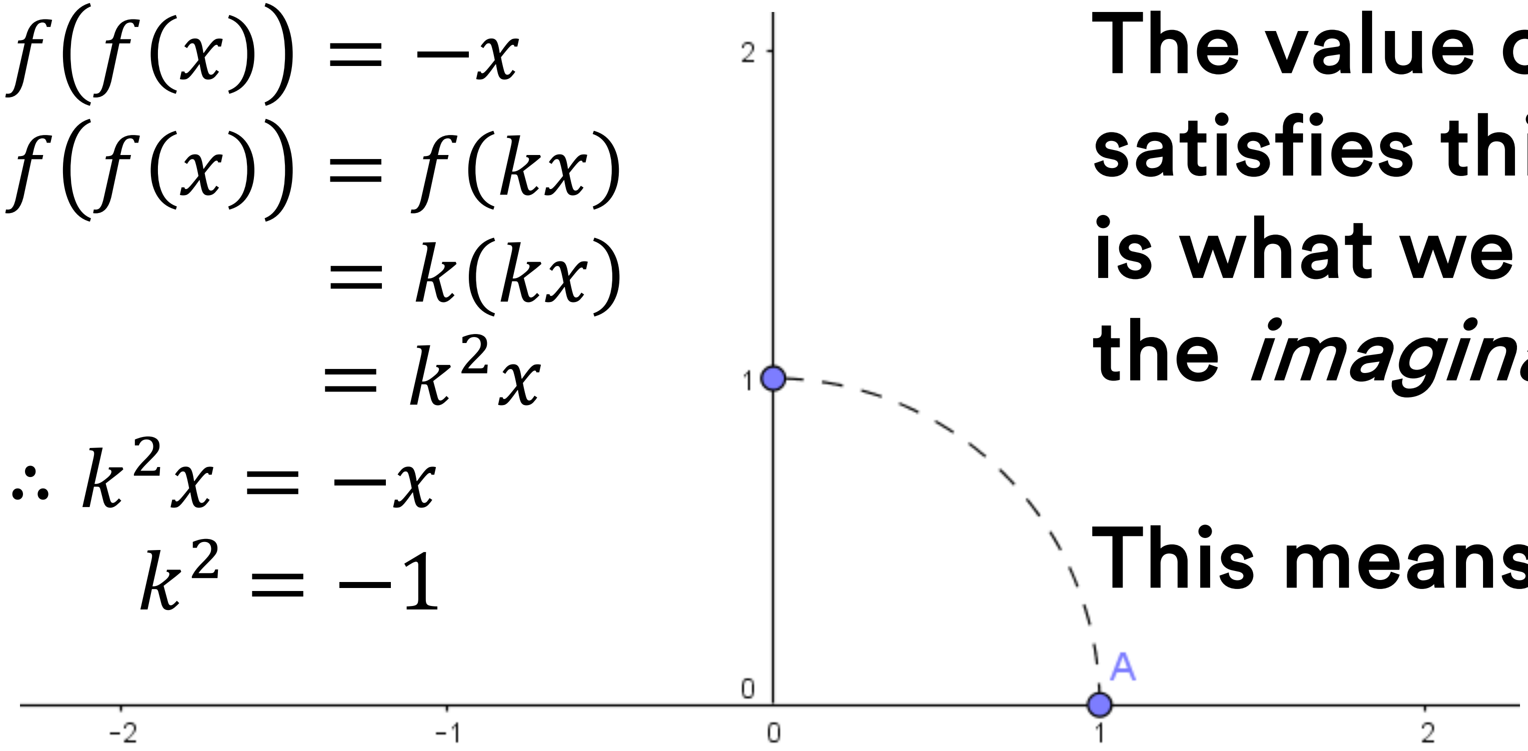


The value of k that satisfies this condition is what we call i, or the *imaginary unit*.

The Math Behind i

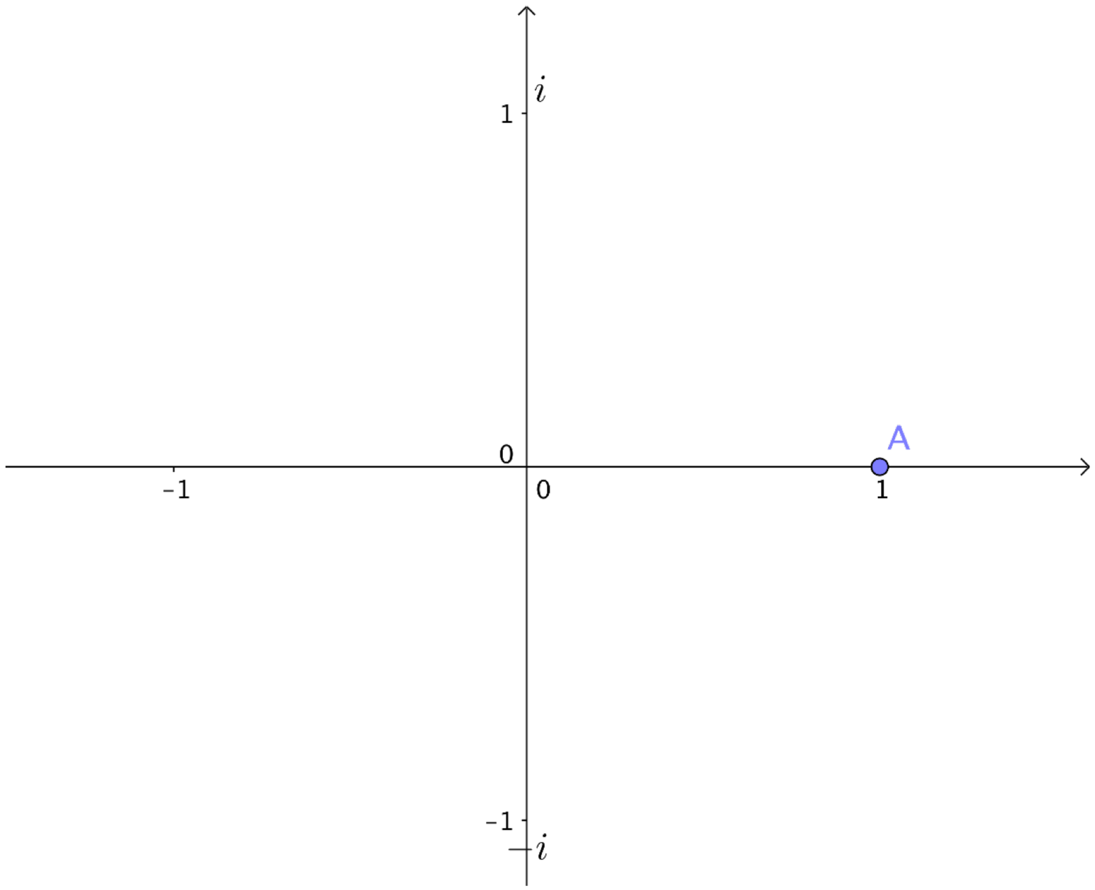
$$f\big(f(x)\big) = -x$$
$$f\big(f(x)\big) = f(kx)$$
$$= k(kx)$$
$$= k^2x$$
$$\therefore k^2x = -x$$
$$k^2 = -1$$



The value of k that satisfies this condition is what we call i, or the *imaginary unit*.

This means $i^2 = -1$.

Understanding the Powers of i



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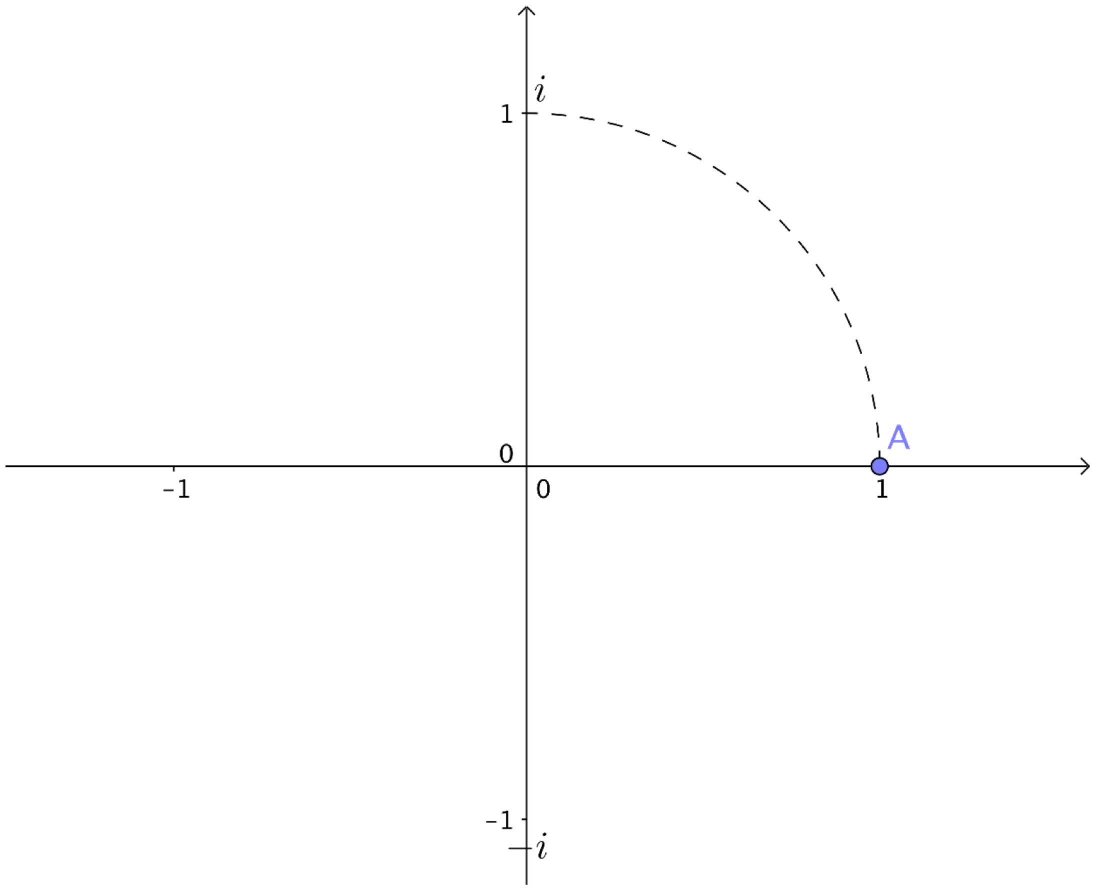

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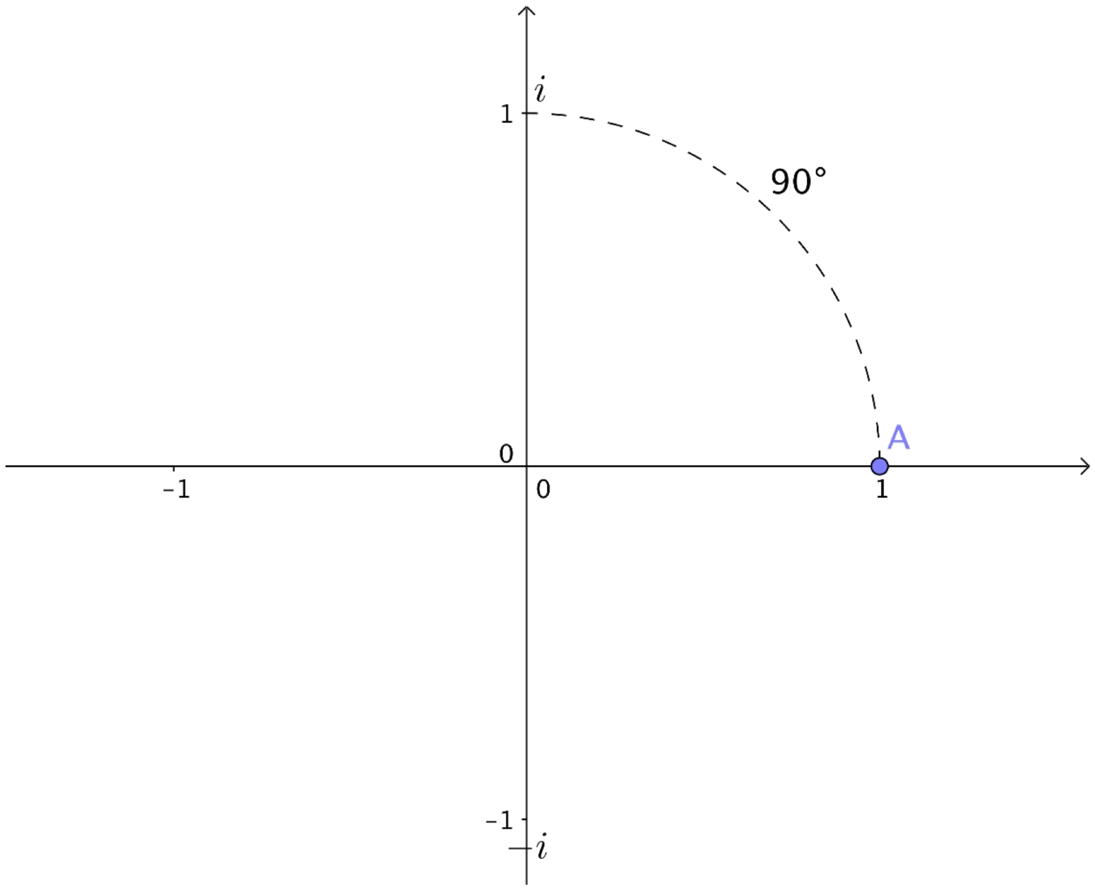

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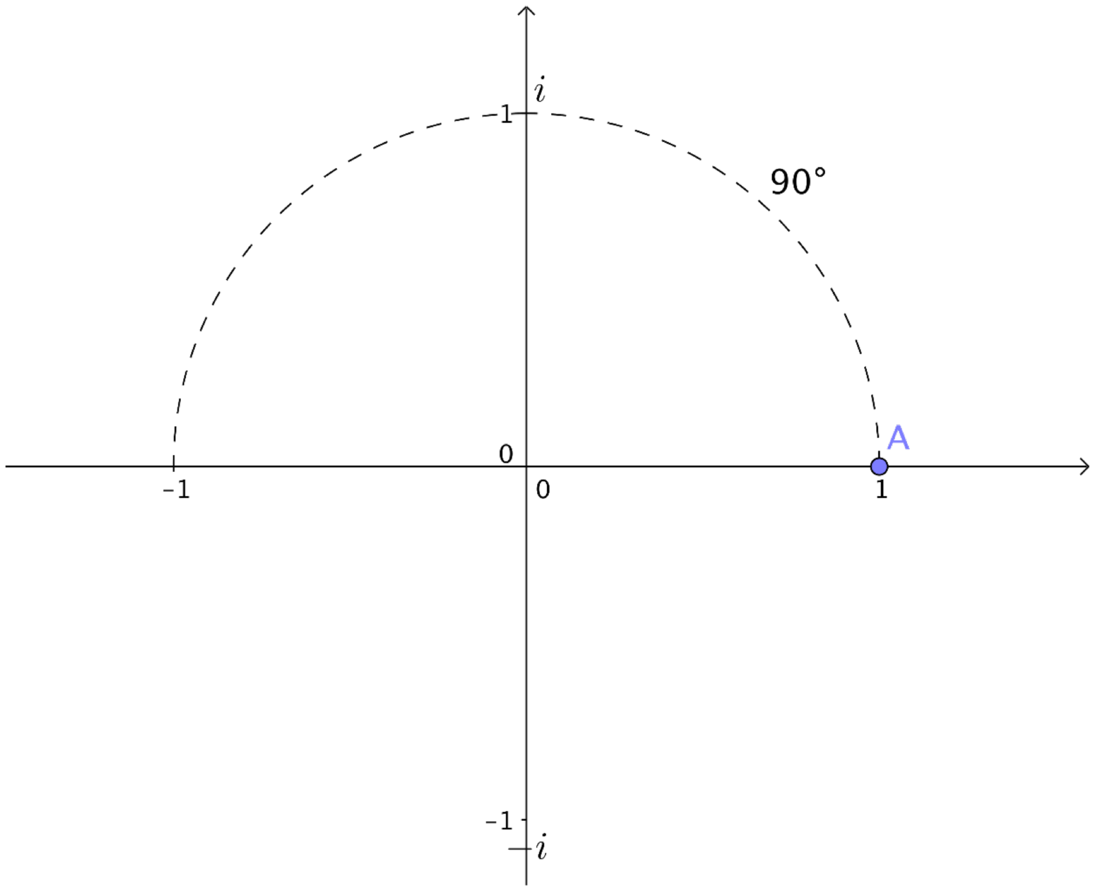
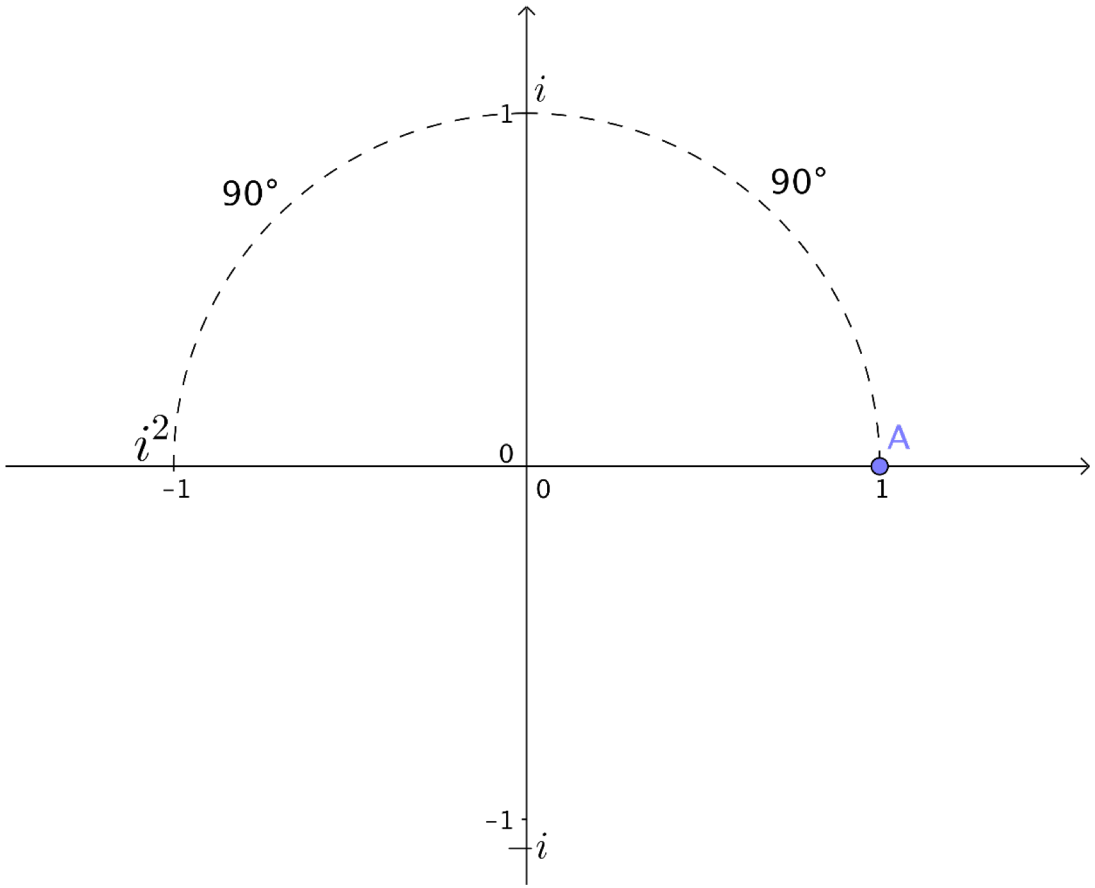


Understanding the Powers of i



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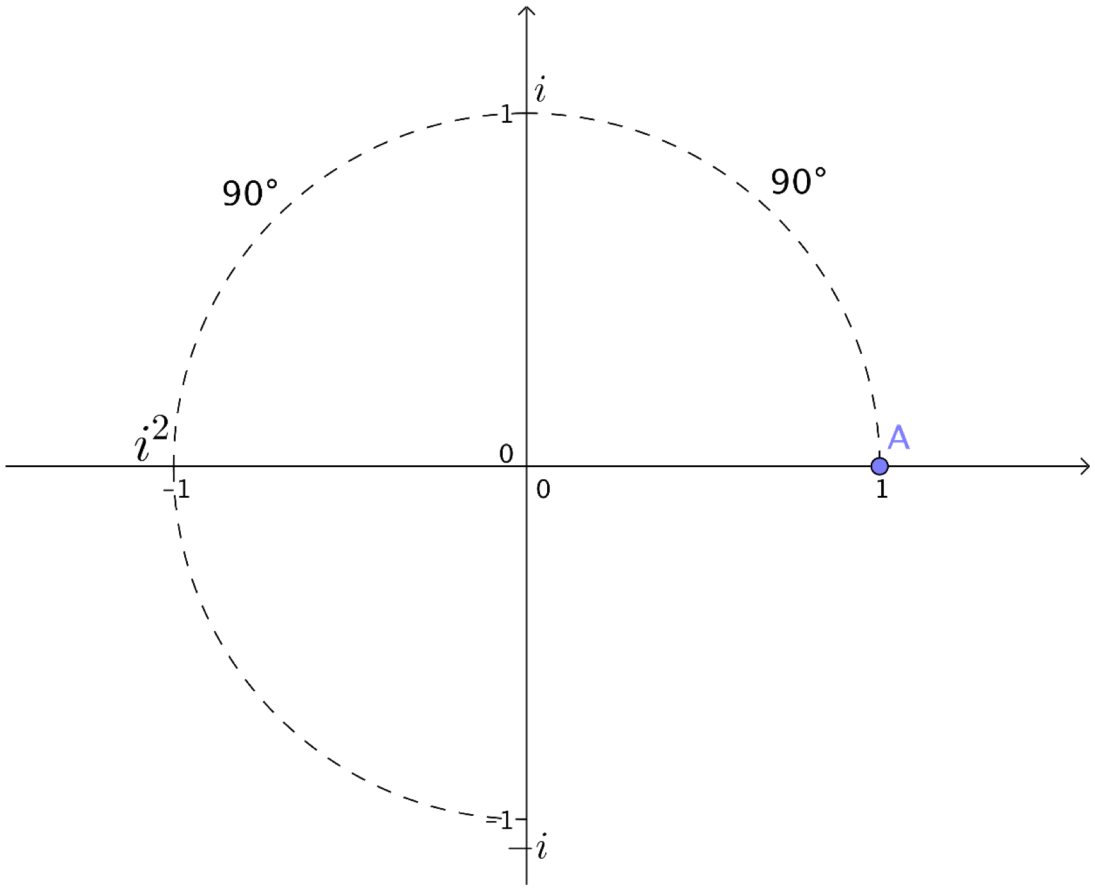

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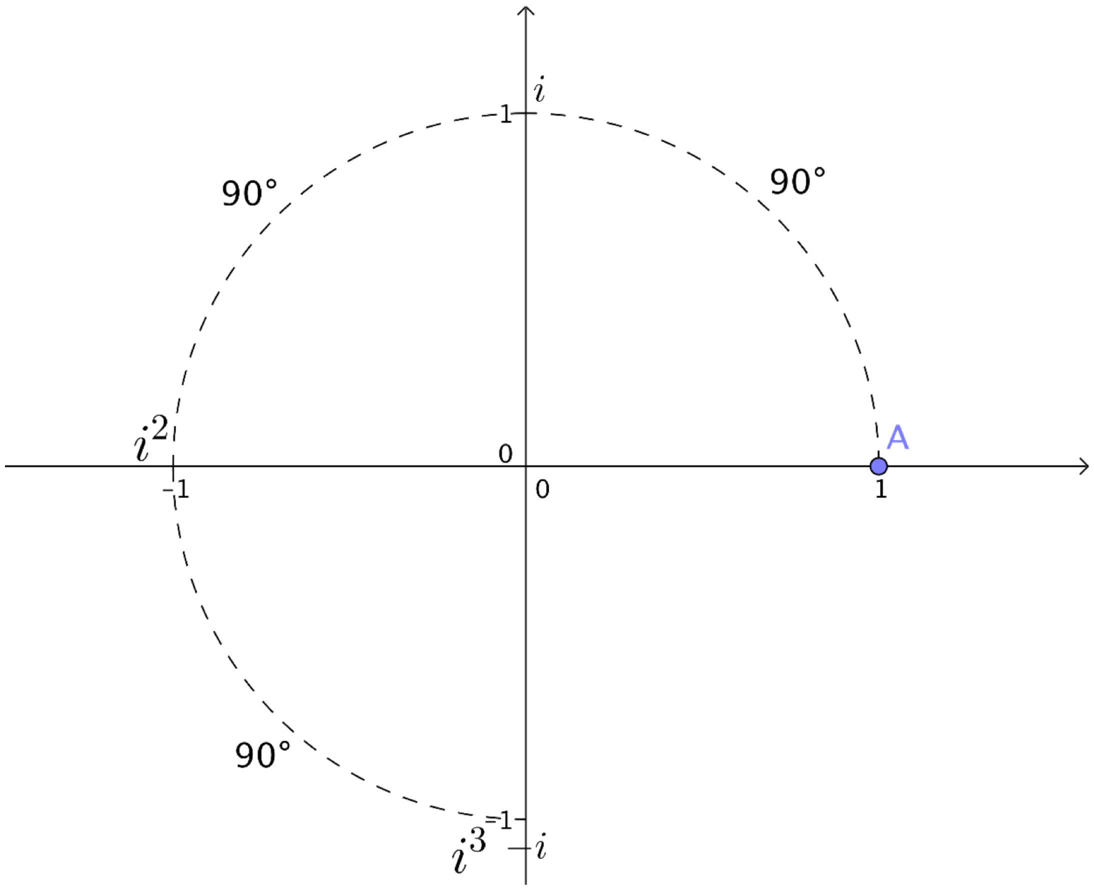

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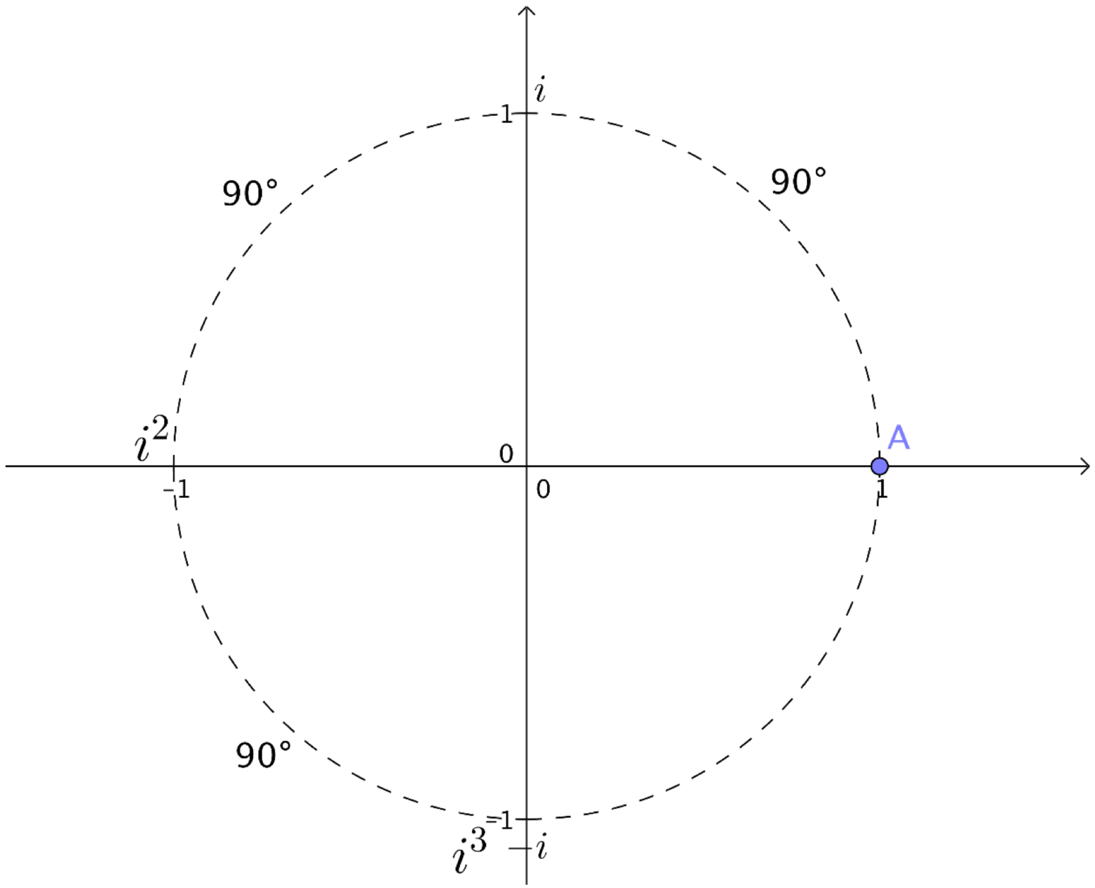

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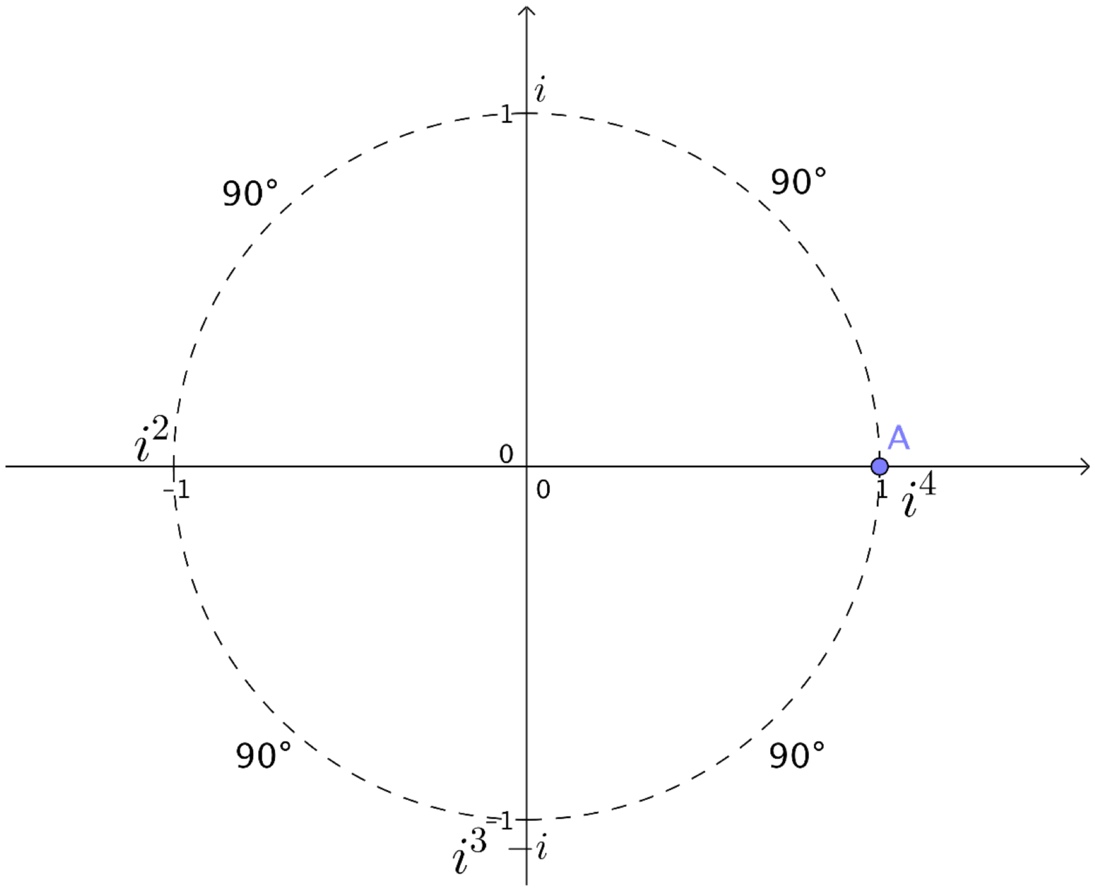

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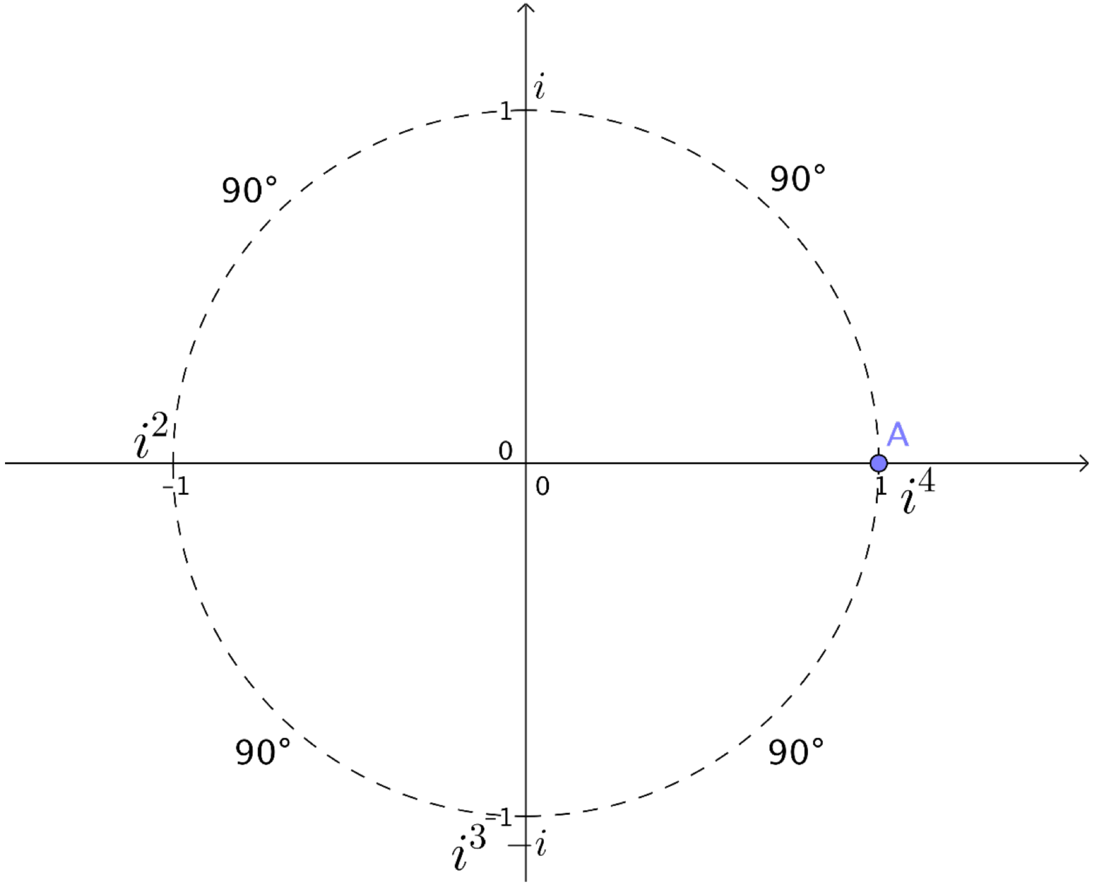


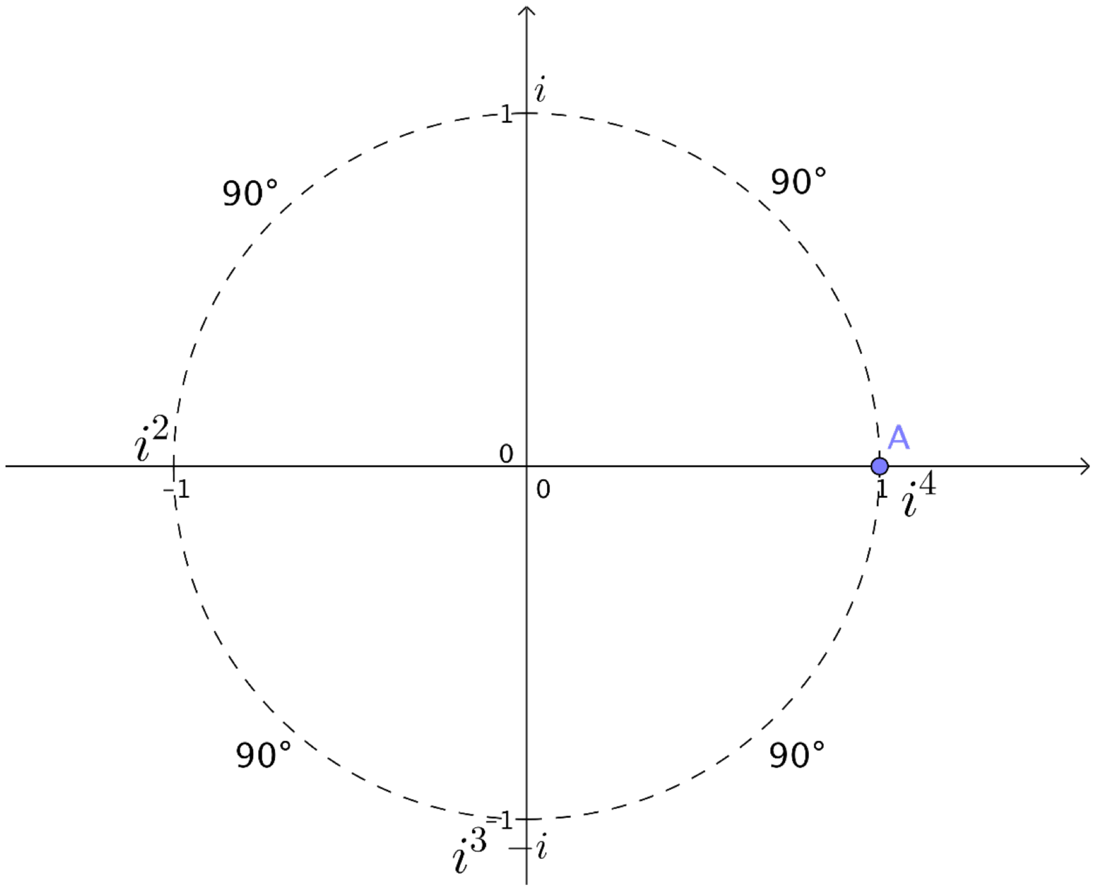
i^3

i^4

i^5

i^6

Understanding the Powers of i



i^3 $\quad i^2 \cdot i^1 = -1 \cdot i = -i$

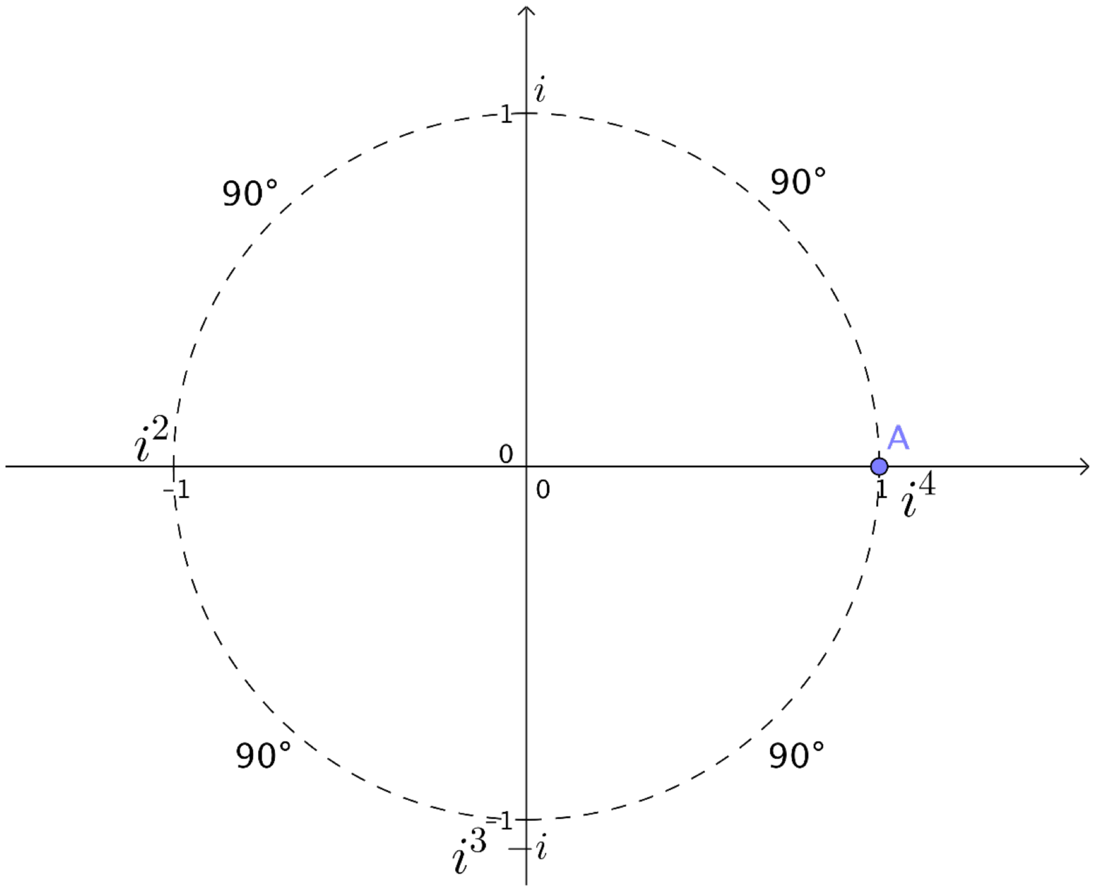
Rotation: $180° + 90° = 270°$

i^4

i^5

i^6

Understanding the Powers of i



i^3 $\quad i^2 \cdot i^1 = -1 \cdot i = -i$

Rotation: $180° + 90° = 270°$

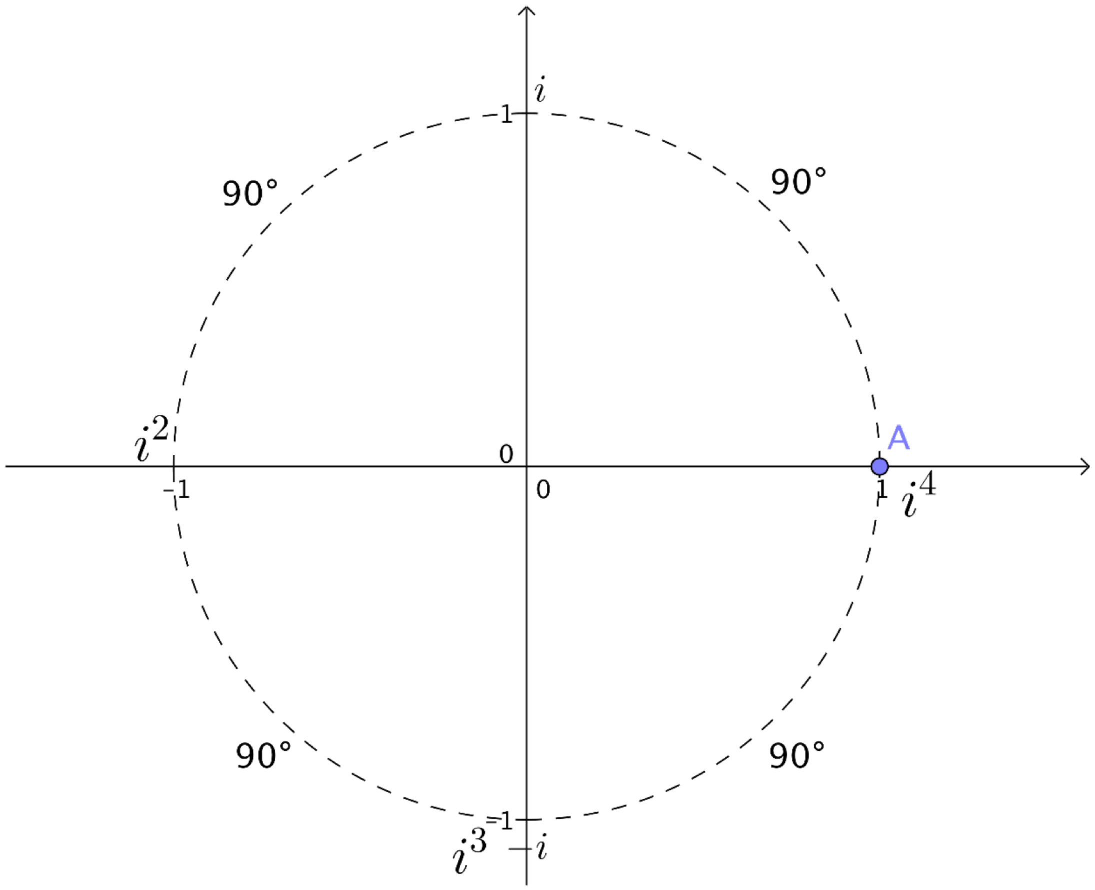
i^4 $\quad i^2 \cdot i^2 = -1 \cdot -1 = 1$

Rotation: $180° + 180° = 360°$

i^5

i^6

Understanding the Powers of i



i^3 $\quad i^2 \cdot i^1 = -1 \cdot i = -i$

Rotation: $180° + 90° = 270°$

i^4 $\quad i^2 \cdot i^2 = -1 \cdot -1 = 1$

Rotation: $180° + 180° = 360°$

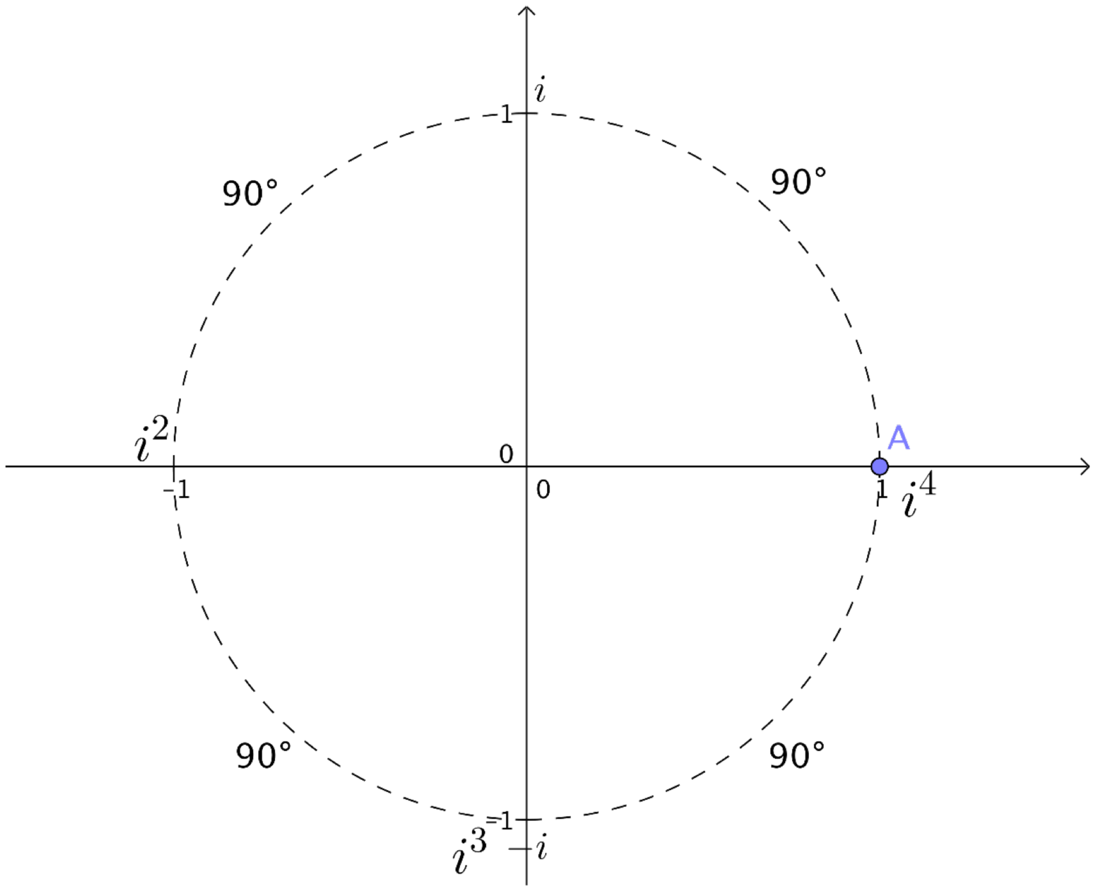
i^5 $\quad i^2 \cdot i^2 \cdot i^1 = -1 \cdot -1 \cdot i = i$

Rotation: $180° + 180° + 90°$

$= 90°$

i^6

Understanding the Powers of i



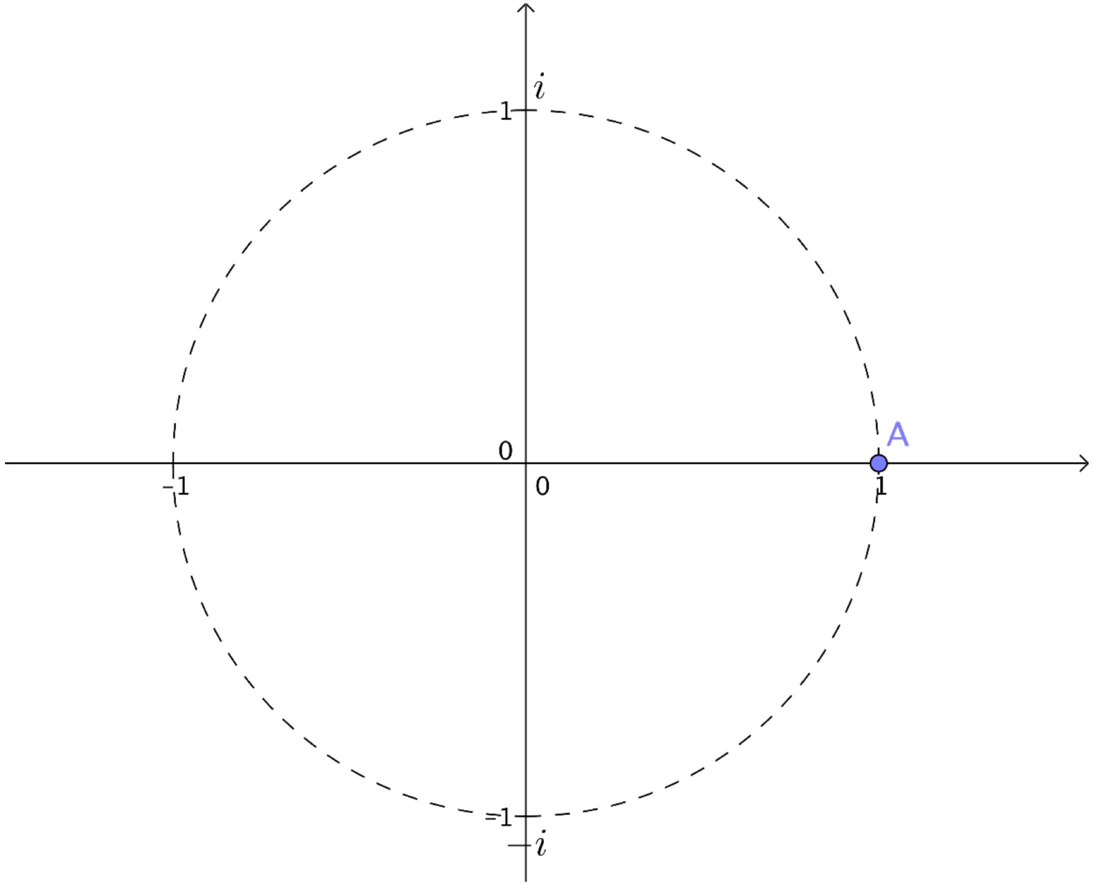
i^3 $i^2 \cdot i^1 = -1 \cdot i = -i$
Rotation: $180° + 90° = 270°$

i^4 $i^2 \cdot i^2 = -1 \cdot -1 = 1$
Rotation: $180° + 180° = 360°$

i^5 $i^2 \cdot i^2 \cdot i^1 = -1 \cdot -1 \cdot i = i$
Rotation: $180° + 180° + 90°$
$= 90°$

i^6 $i^2 \cdot i^2 \cdot i^2 = -1 \cdot -1 \cdot -1 = -1$
Rotation: $180° + 180° + 180°$
$= 180°$

Explain the following geometrically:



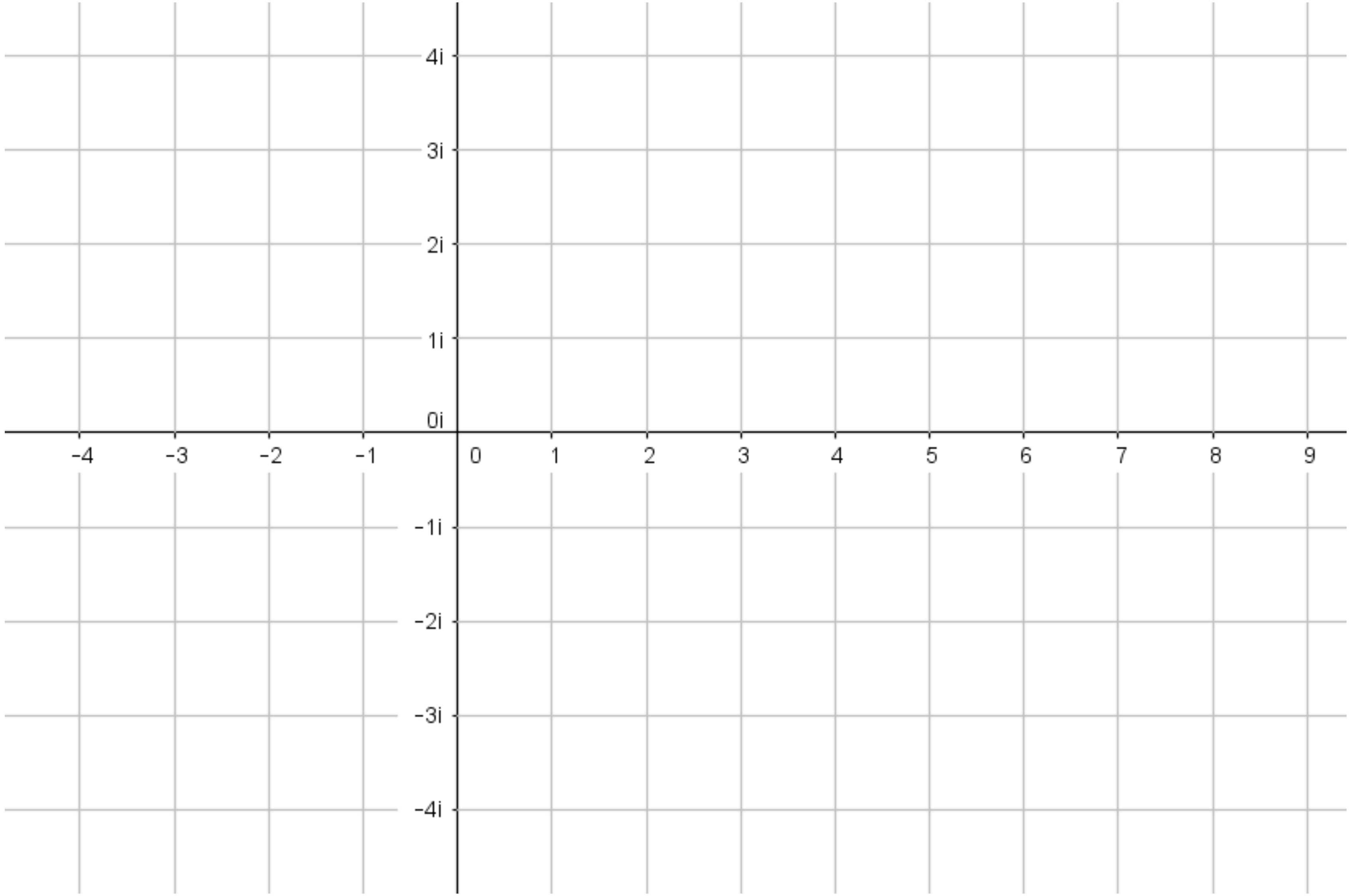
i^7

i^8

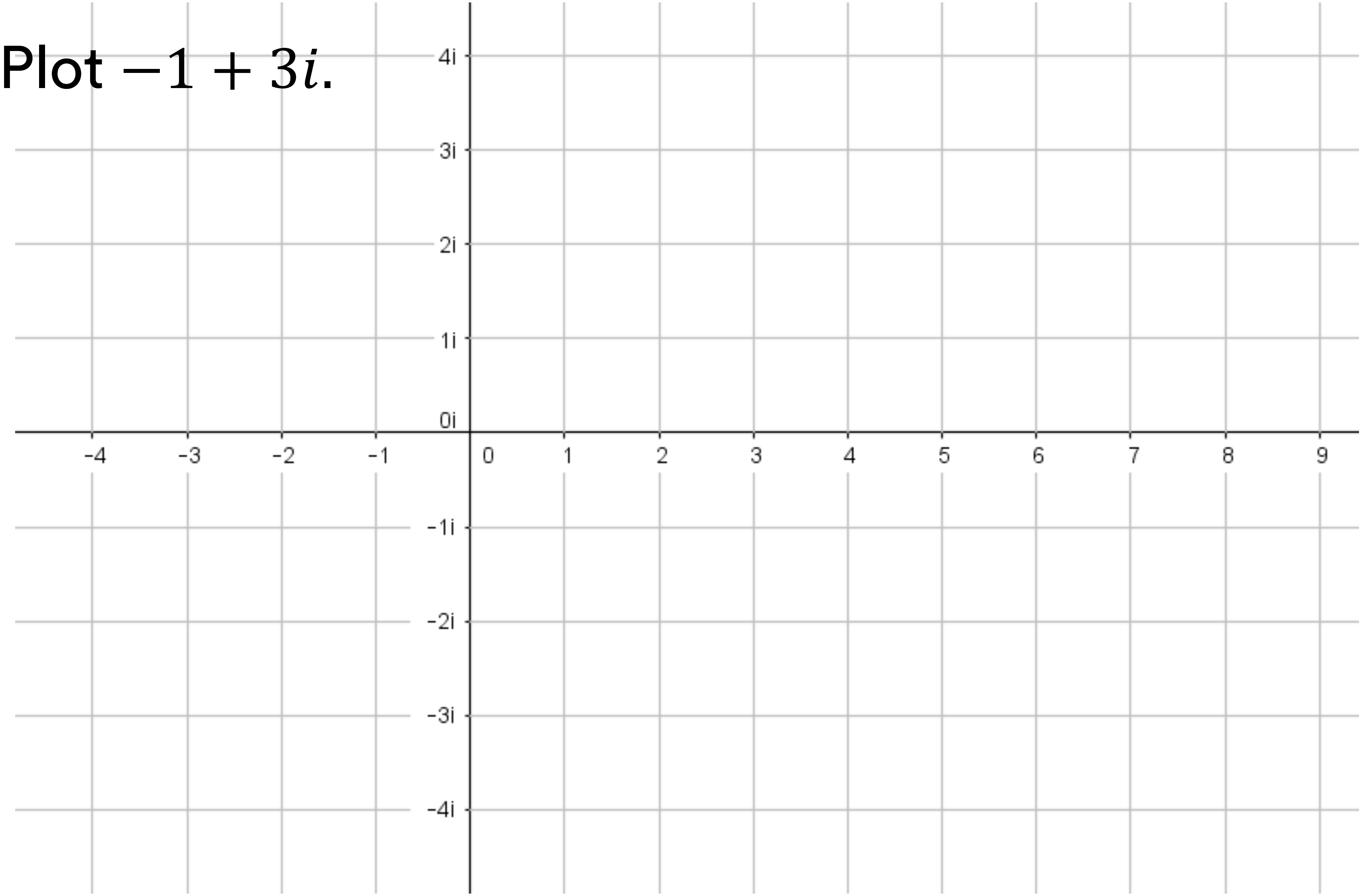
i^{21}

i^{102}

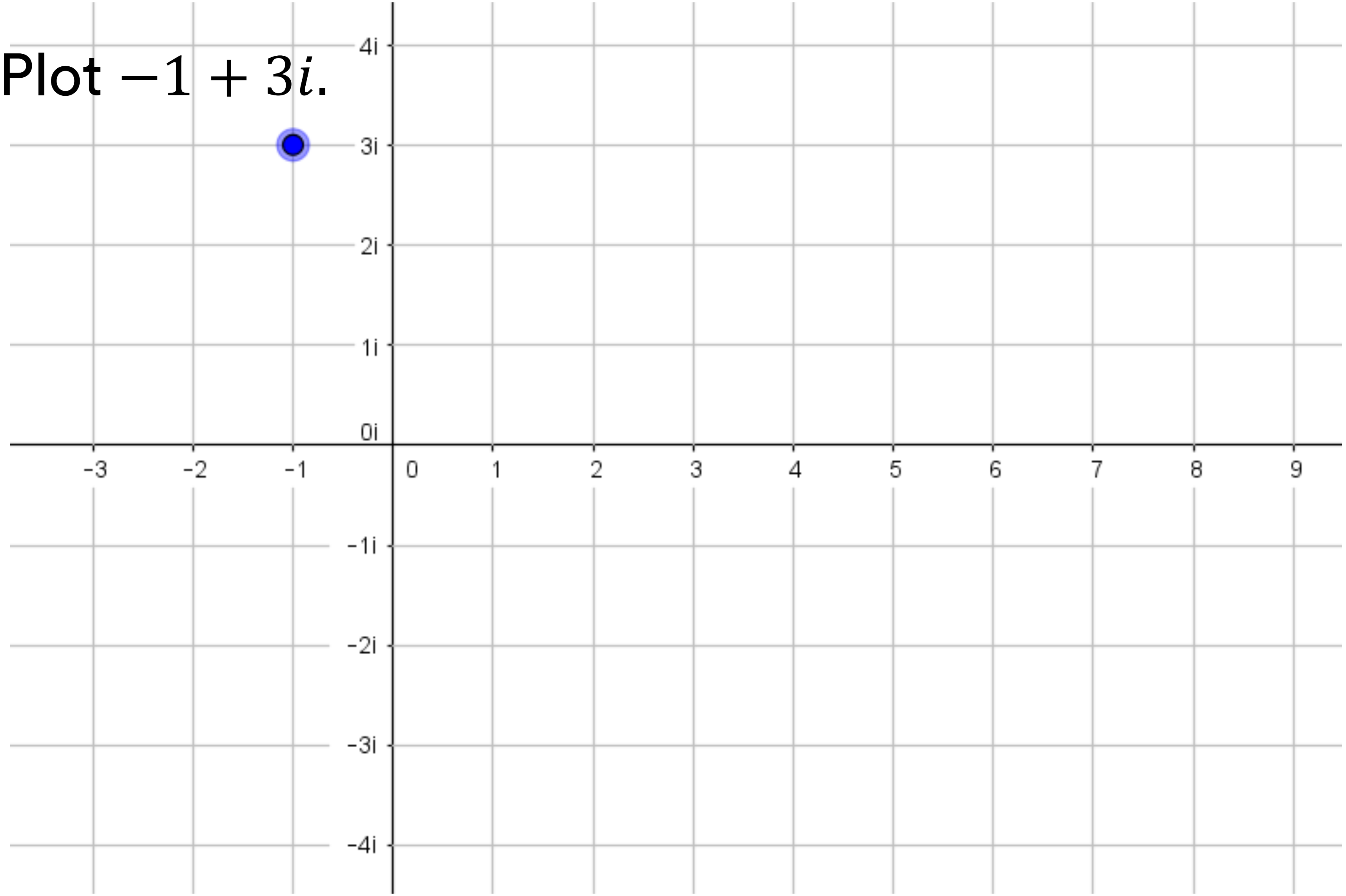
Using Geometry to Teach Complex Number Operations (Addition and Subtraction)


4i
3i
2i
1i
0i
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1i
-2i
-3i
-4i

Plot $-1 + 3i$.

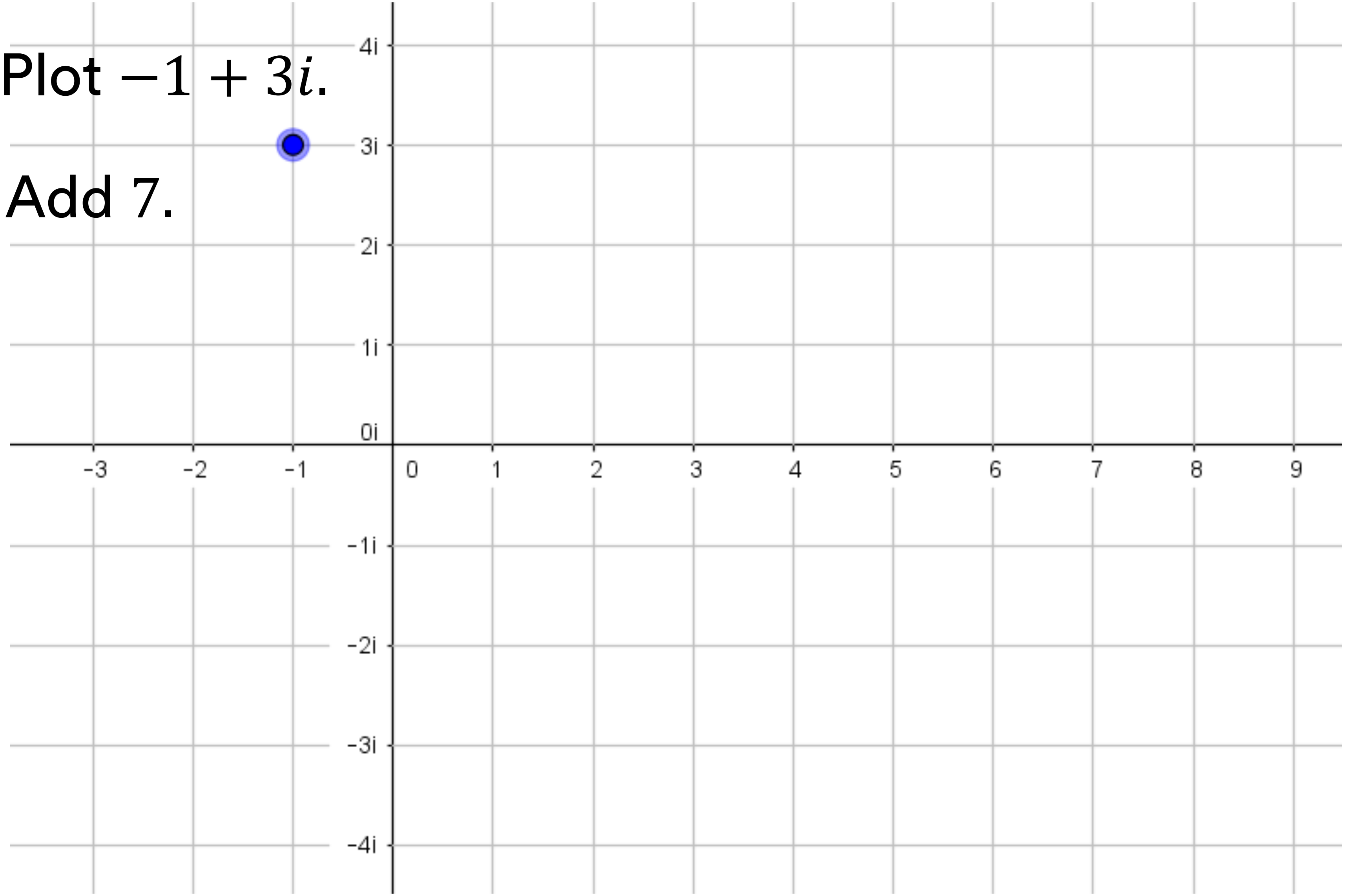


Plot $-1+3i$.



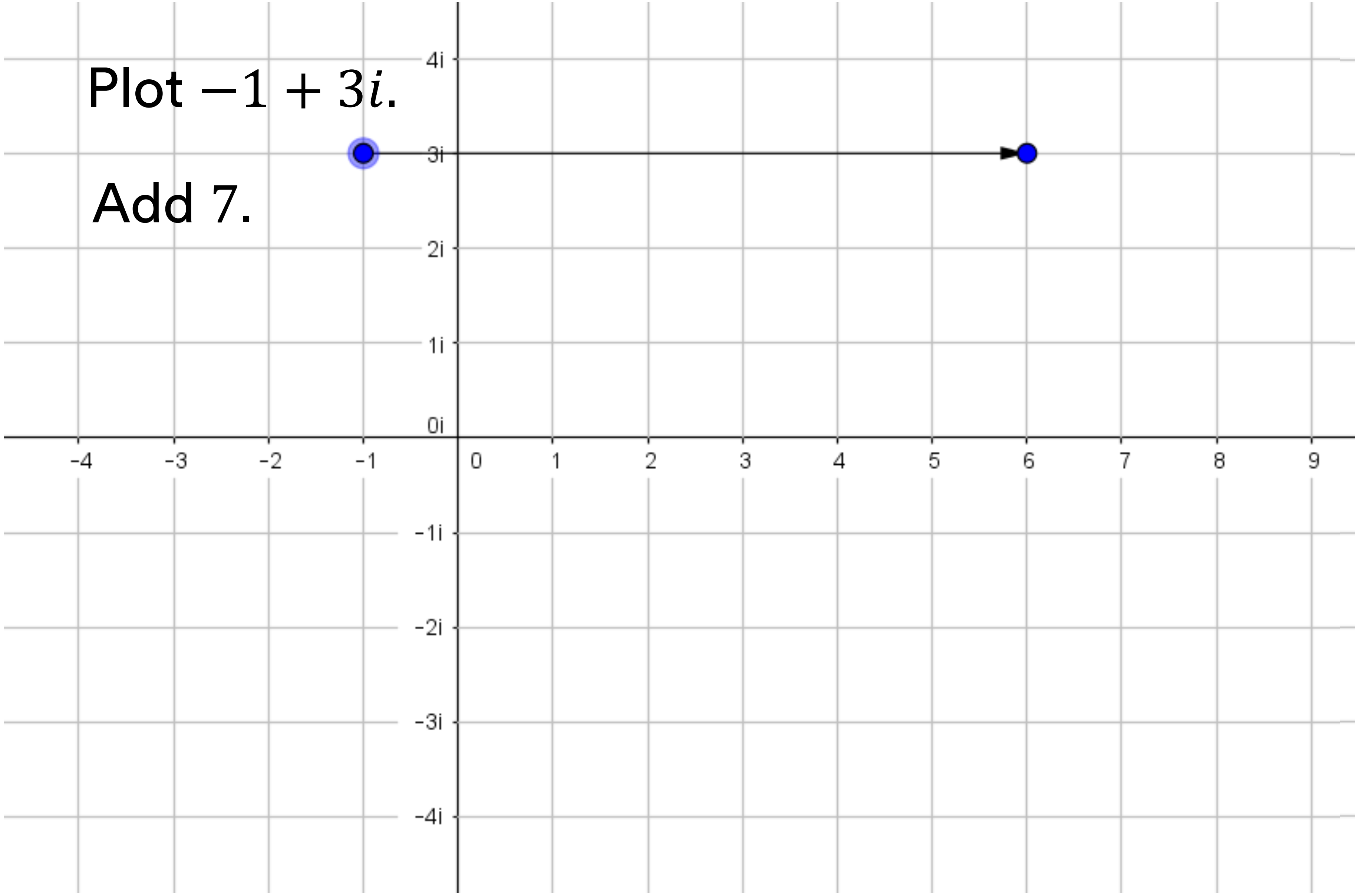
Plot $-1 + 3i$.

Add 7.



Plot $-1 + 3i$.

Add 7.



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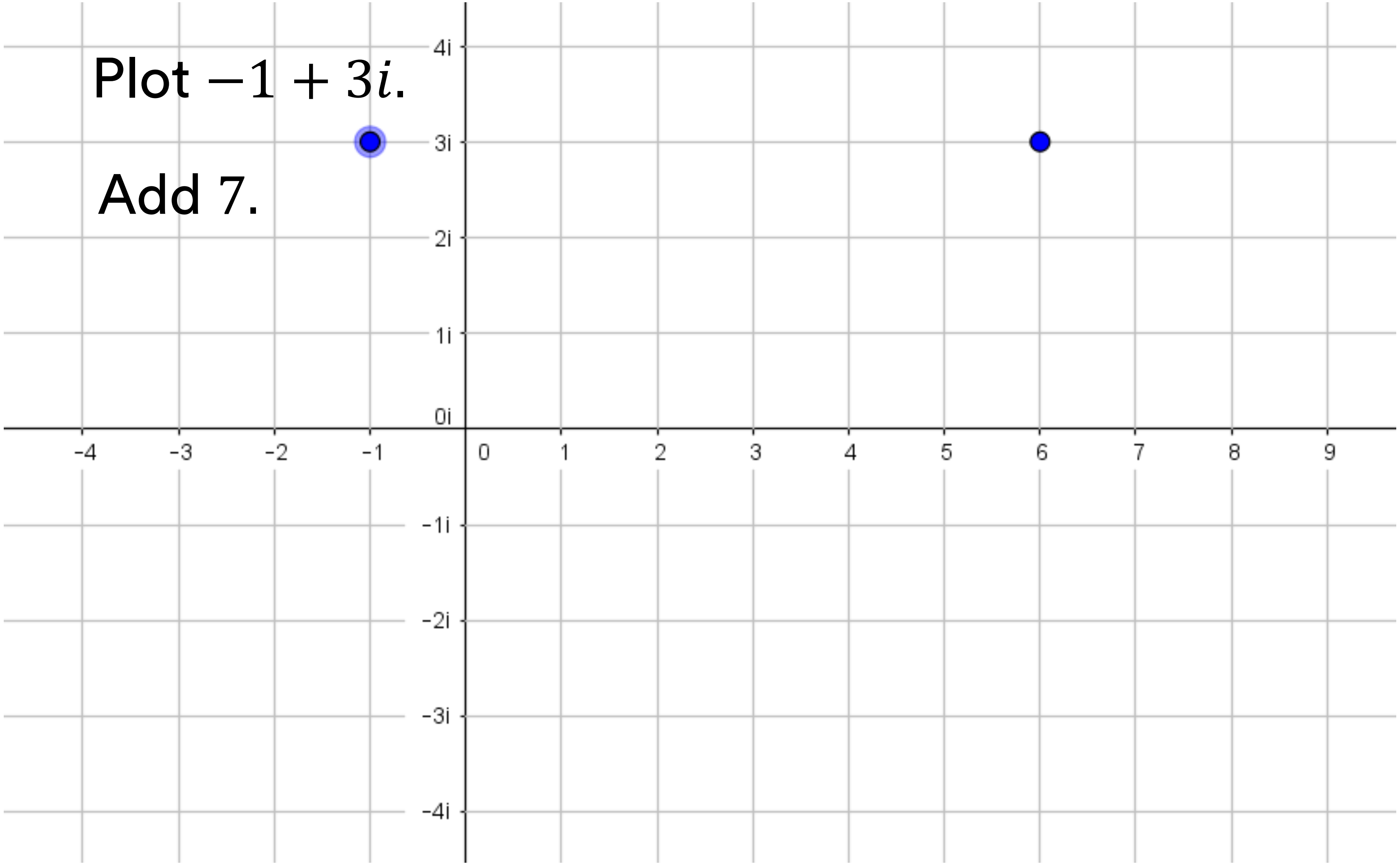
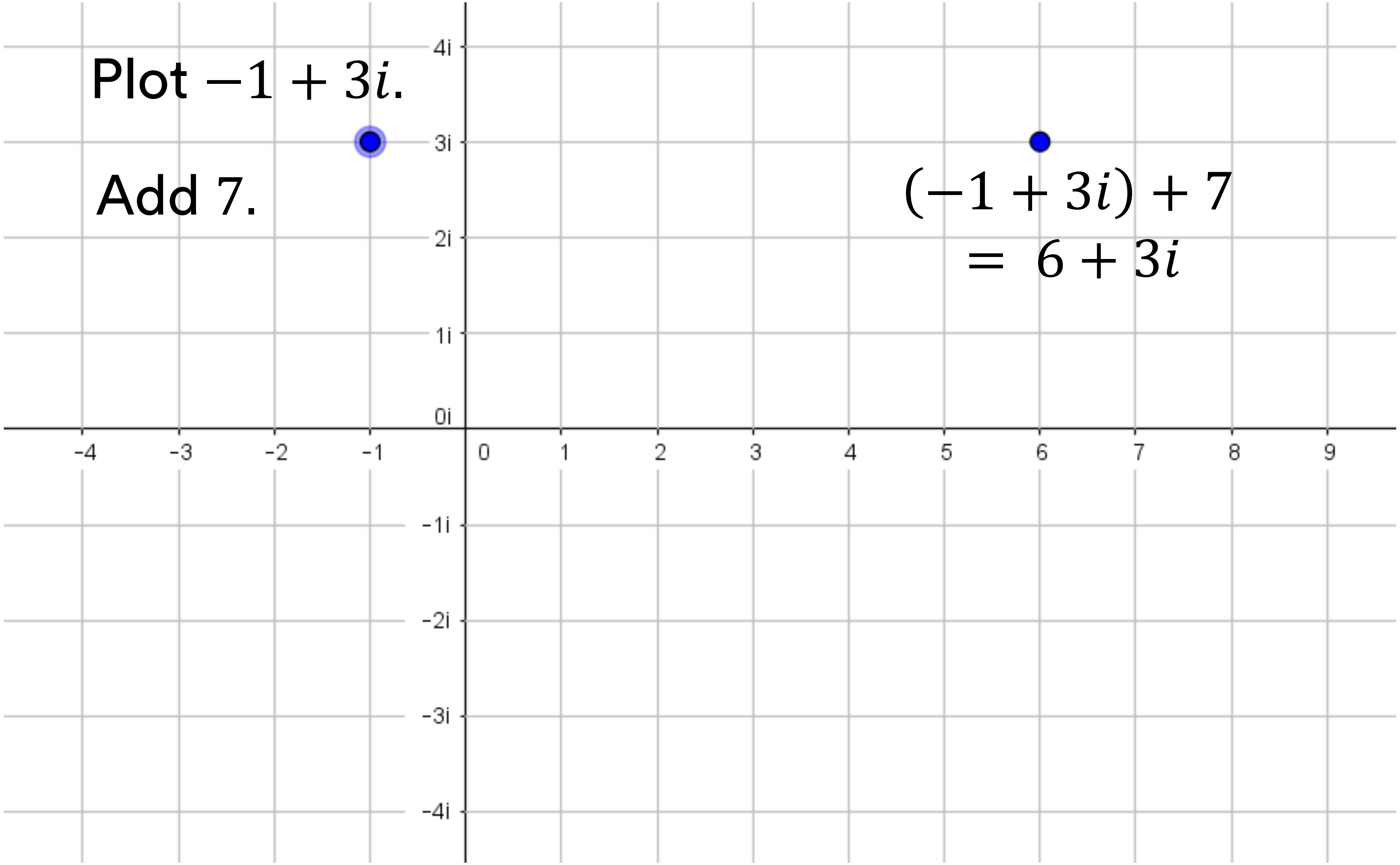
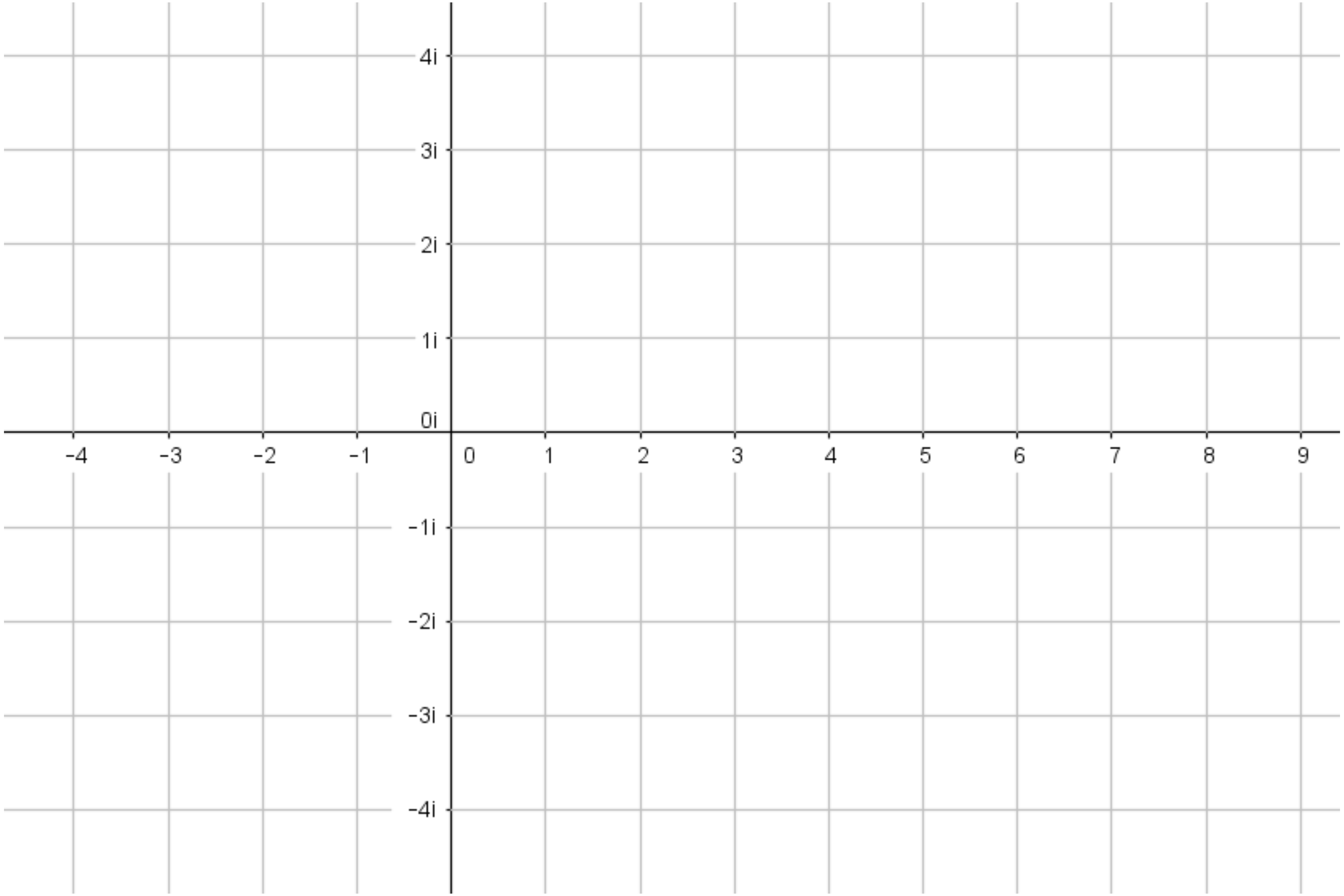


Plot $-1 + 3i$.

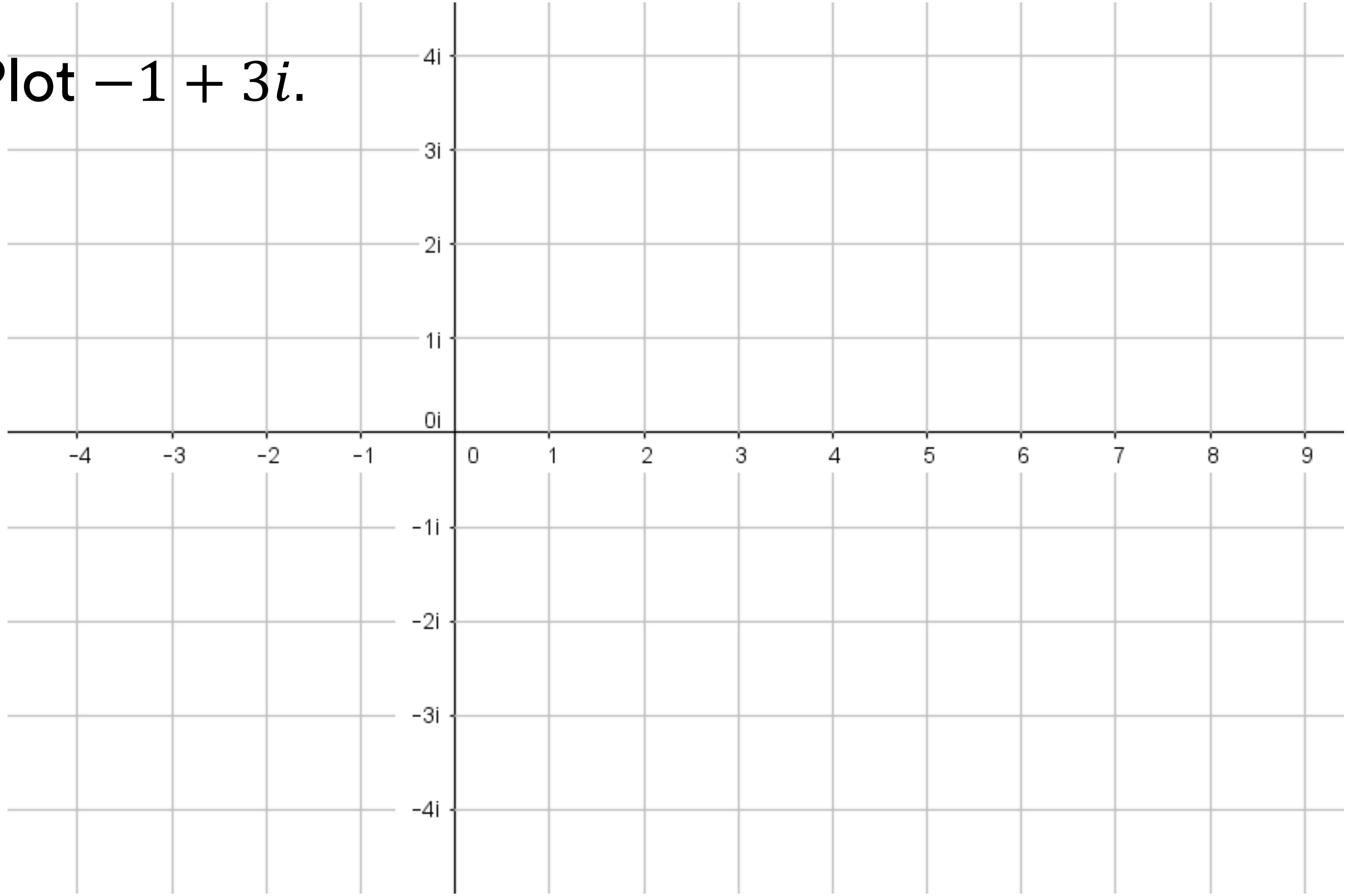
Add 7.



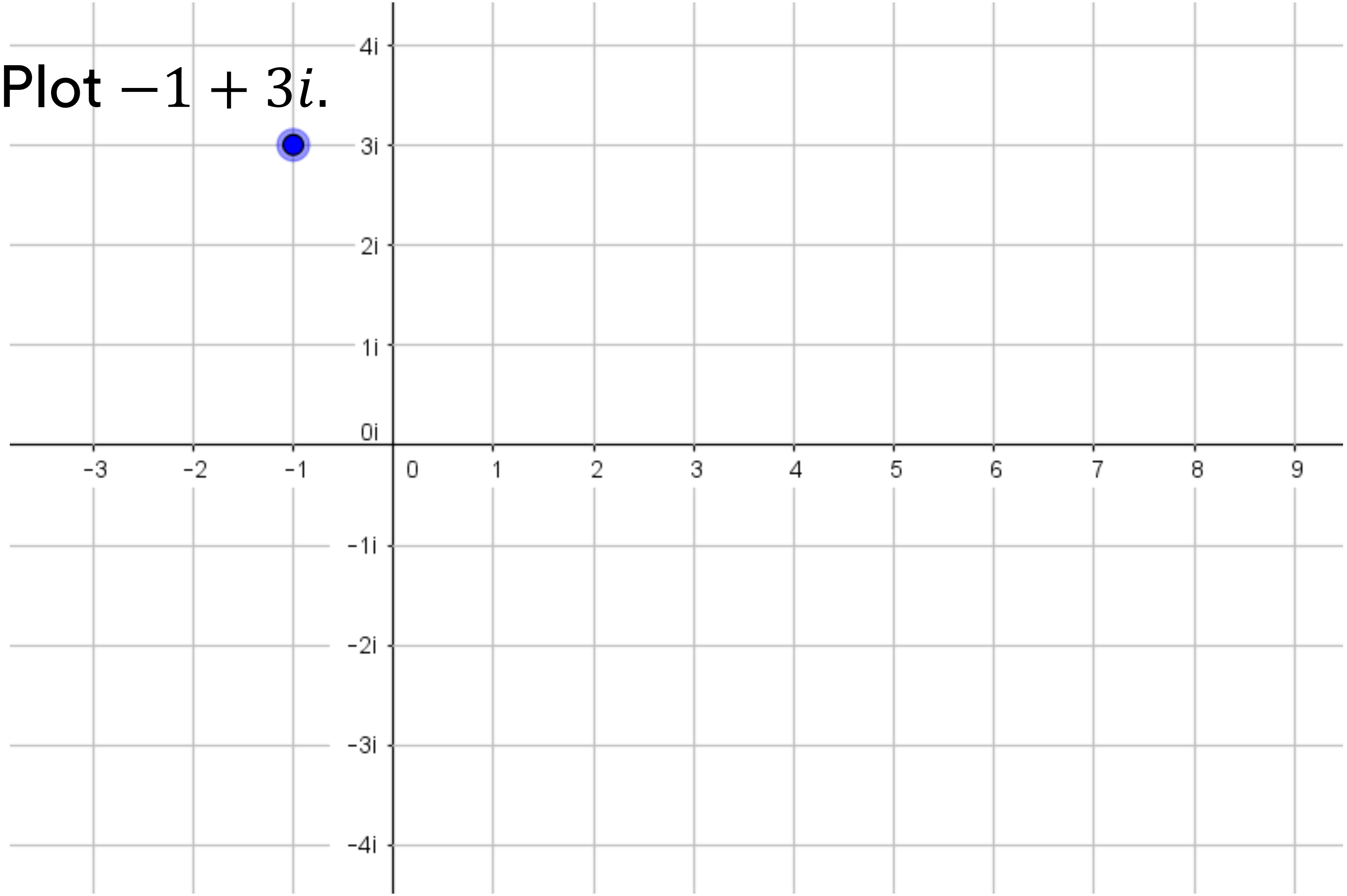
$$(-1 + 3i) + 7$$
$$= 6 + 3i$$


4i
3i
2i
1i
0i
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1i
-2i
-3i
-4i

Plot $-1 + 3i$.

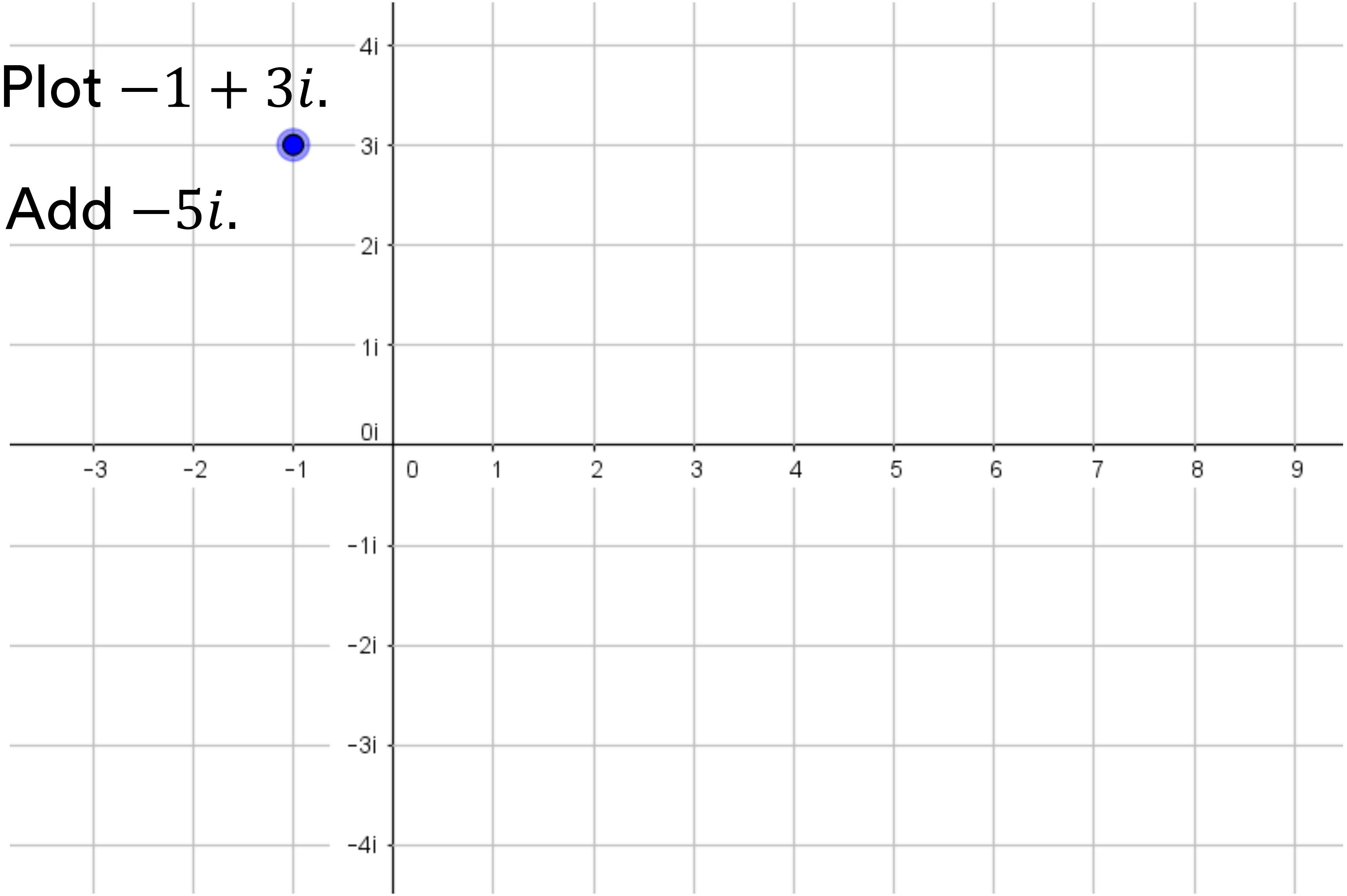


Plot $-1 + 3i$.



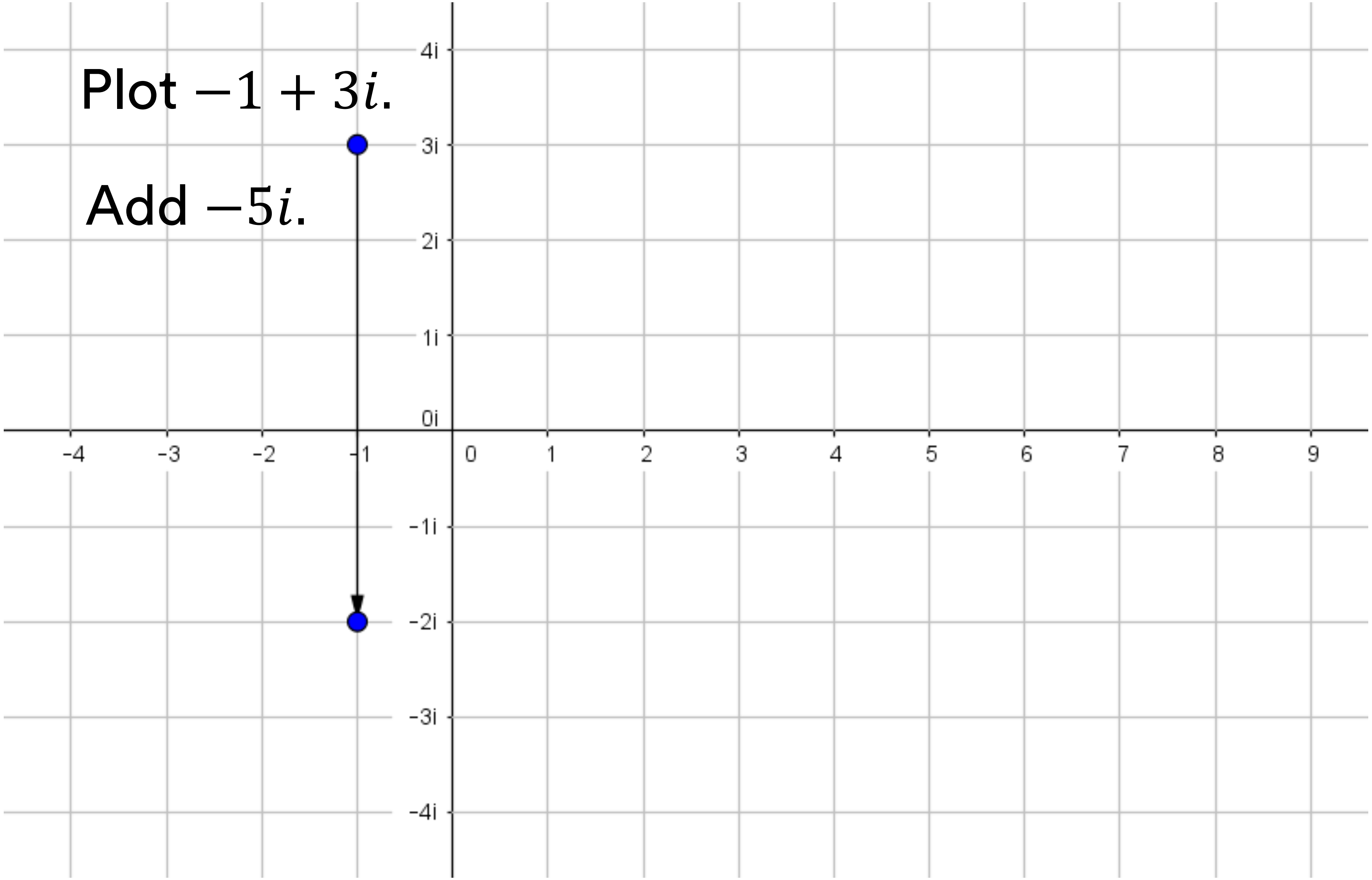
Plot $-1 + 3i$.

Add $-5i$.



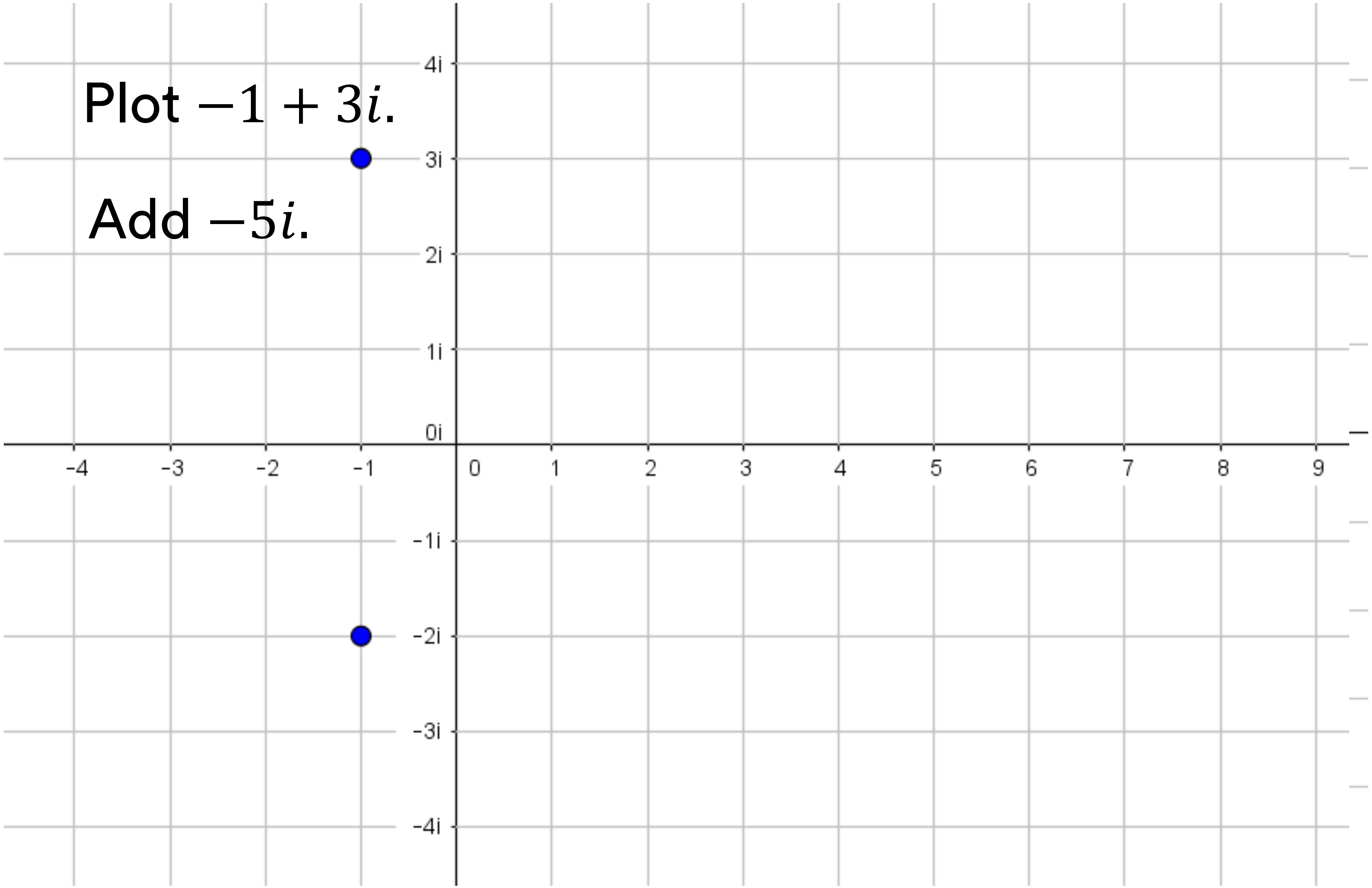
Plot $-1 + 3i$.

Add $-5i$.



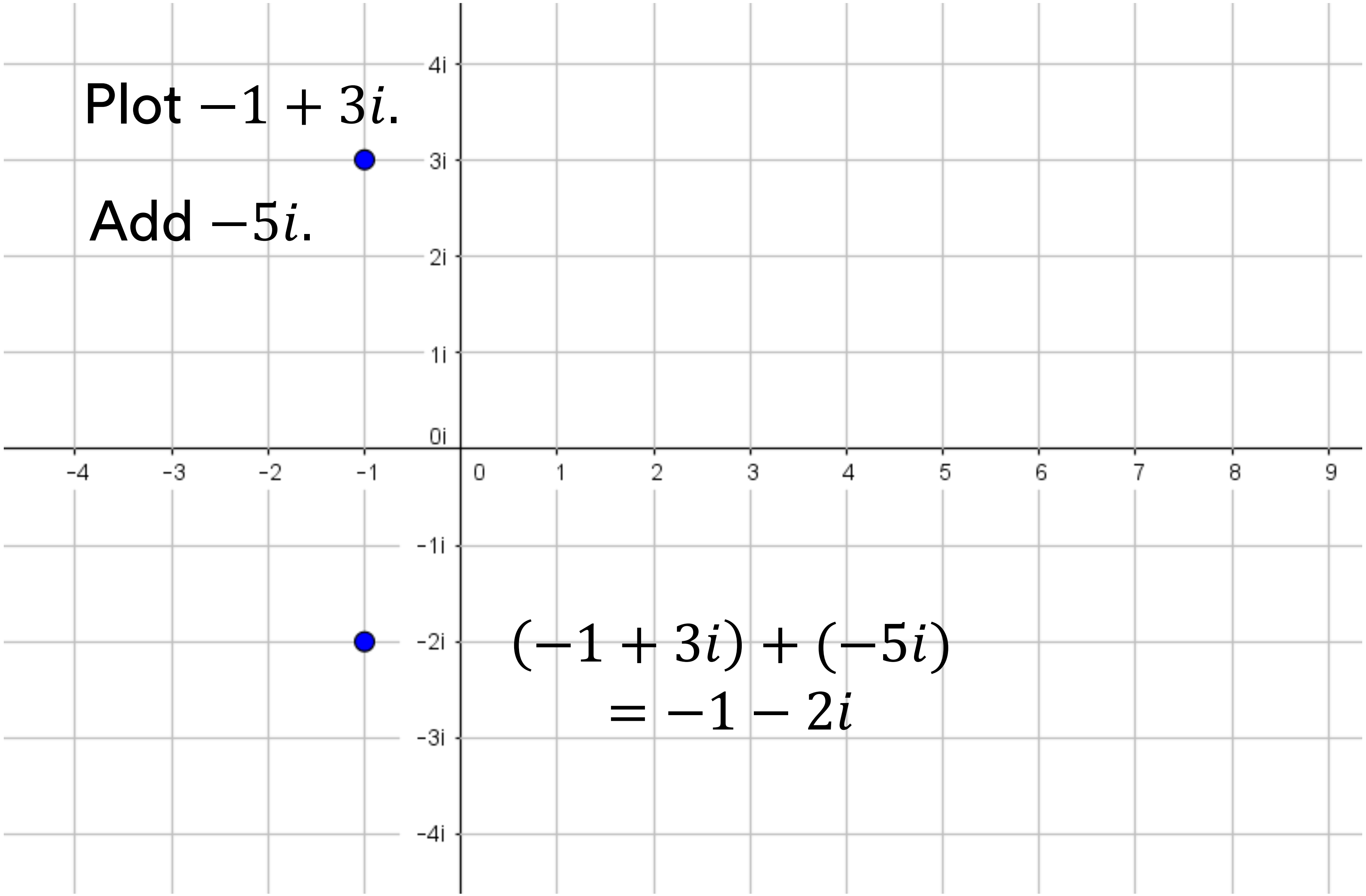
Plot $-1 + 3i$.

Add $-5i$.

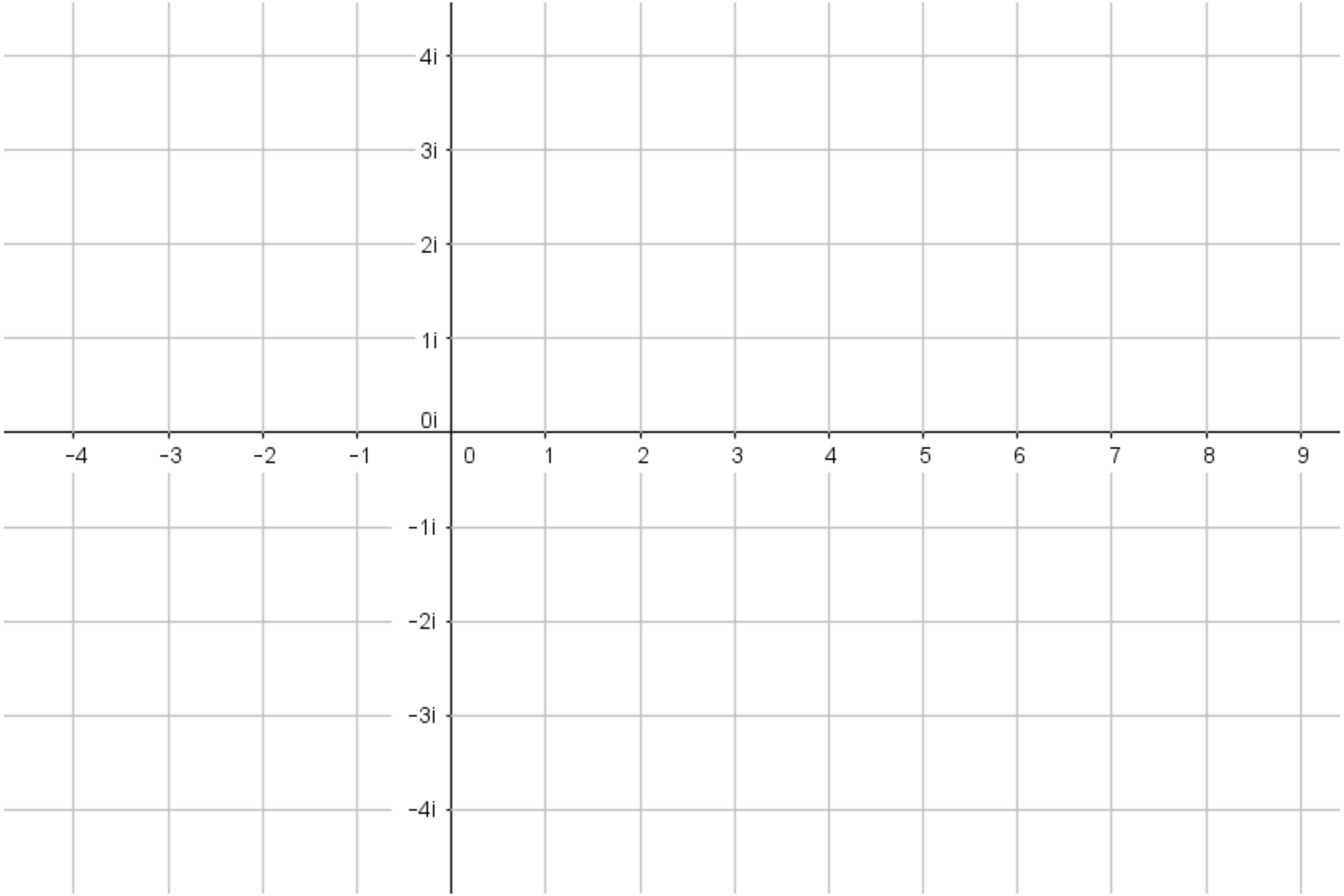


Plot $-1 + 3i$.

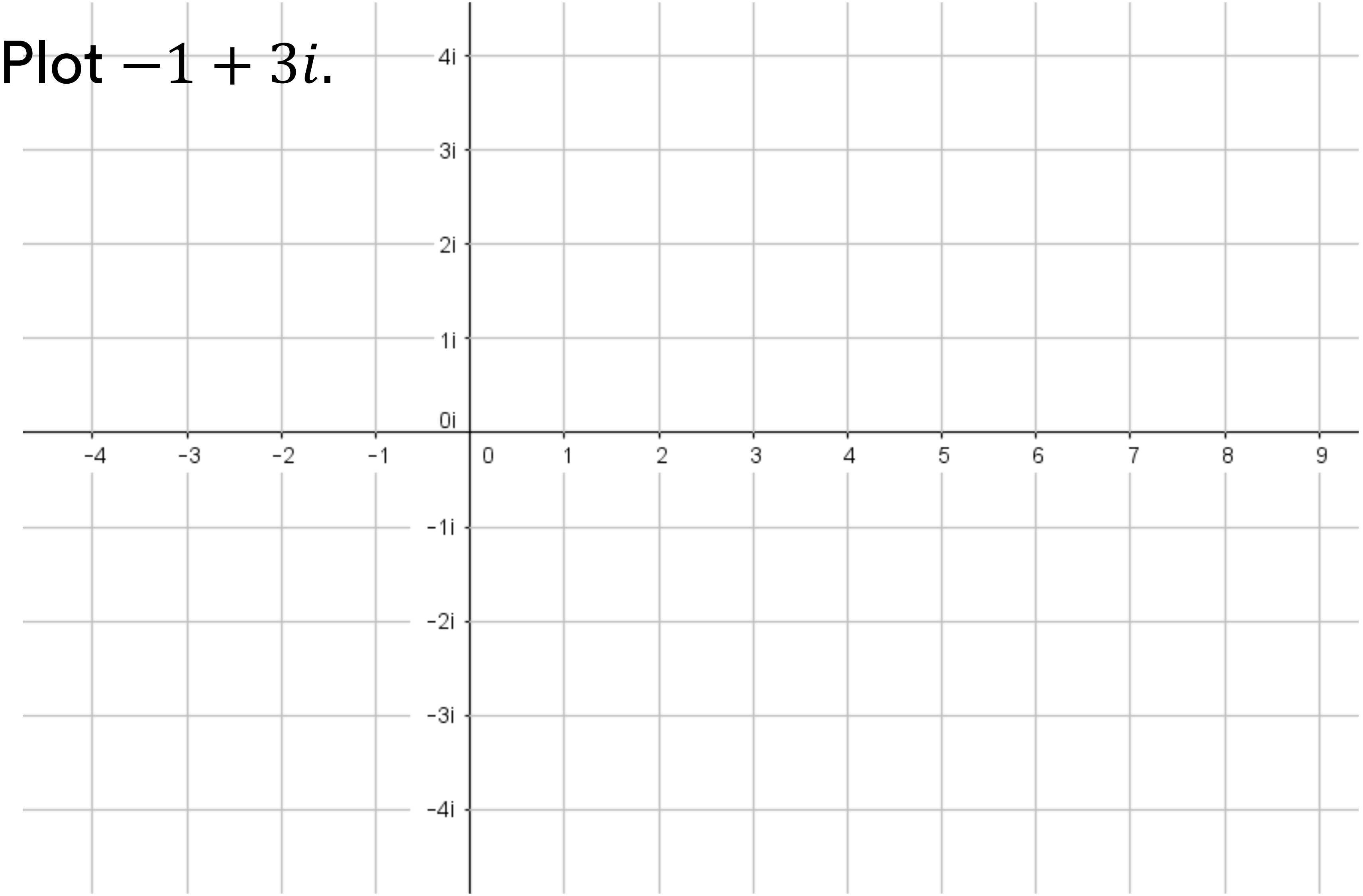
Add $-5i$.



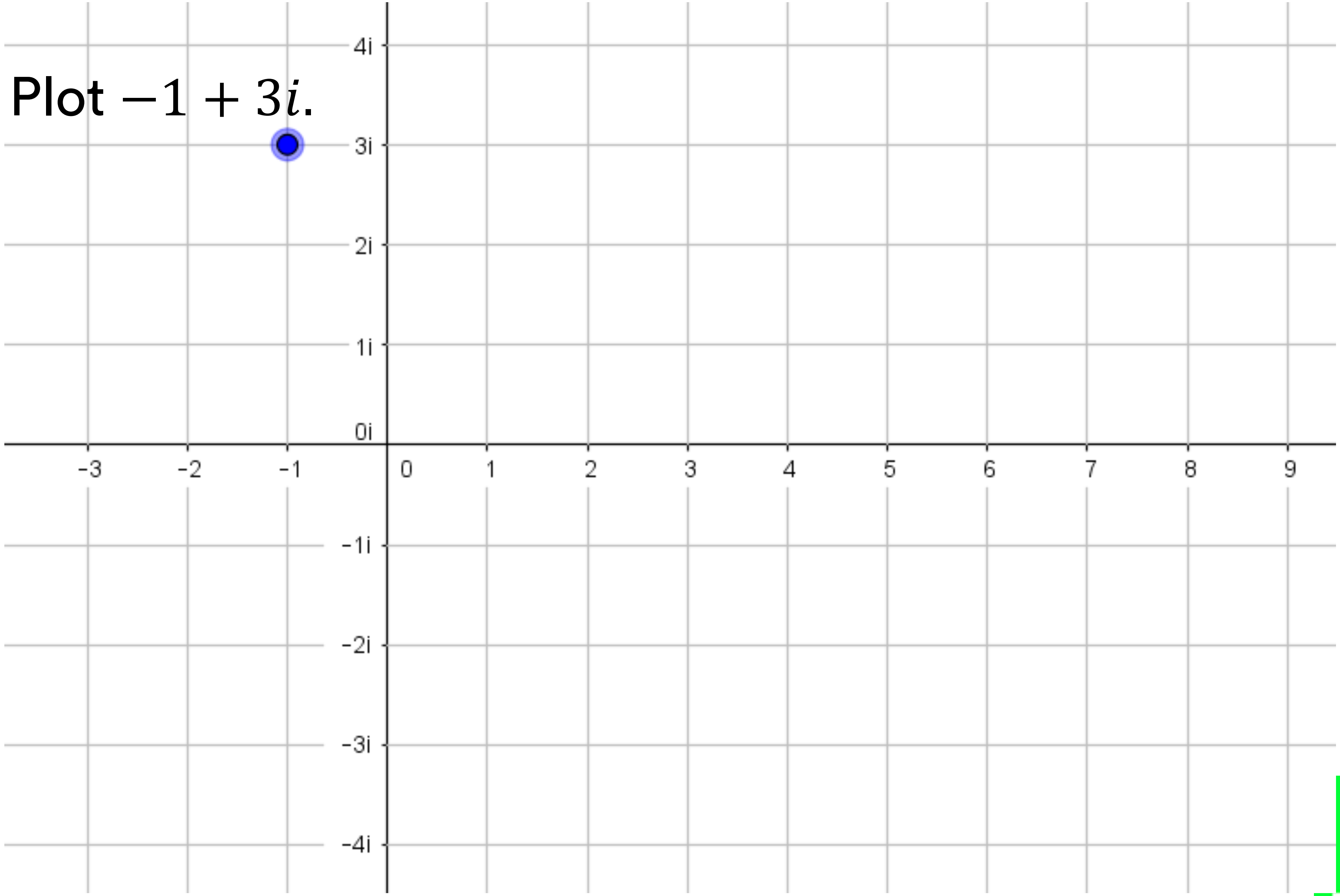
$$(-1 + 3i) + (-5i)$$
$$= -1 - 2i$$


4i
3i
2i
1i
0i
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1i
-2i
-3i
-4i

Plot $-1 + 3i$.

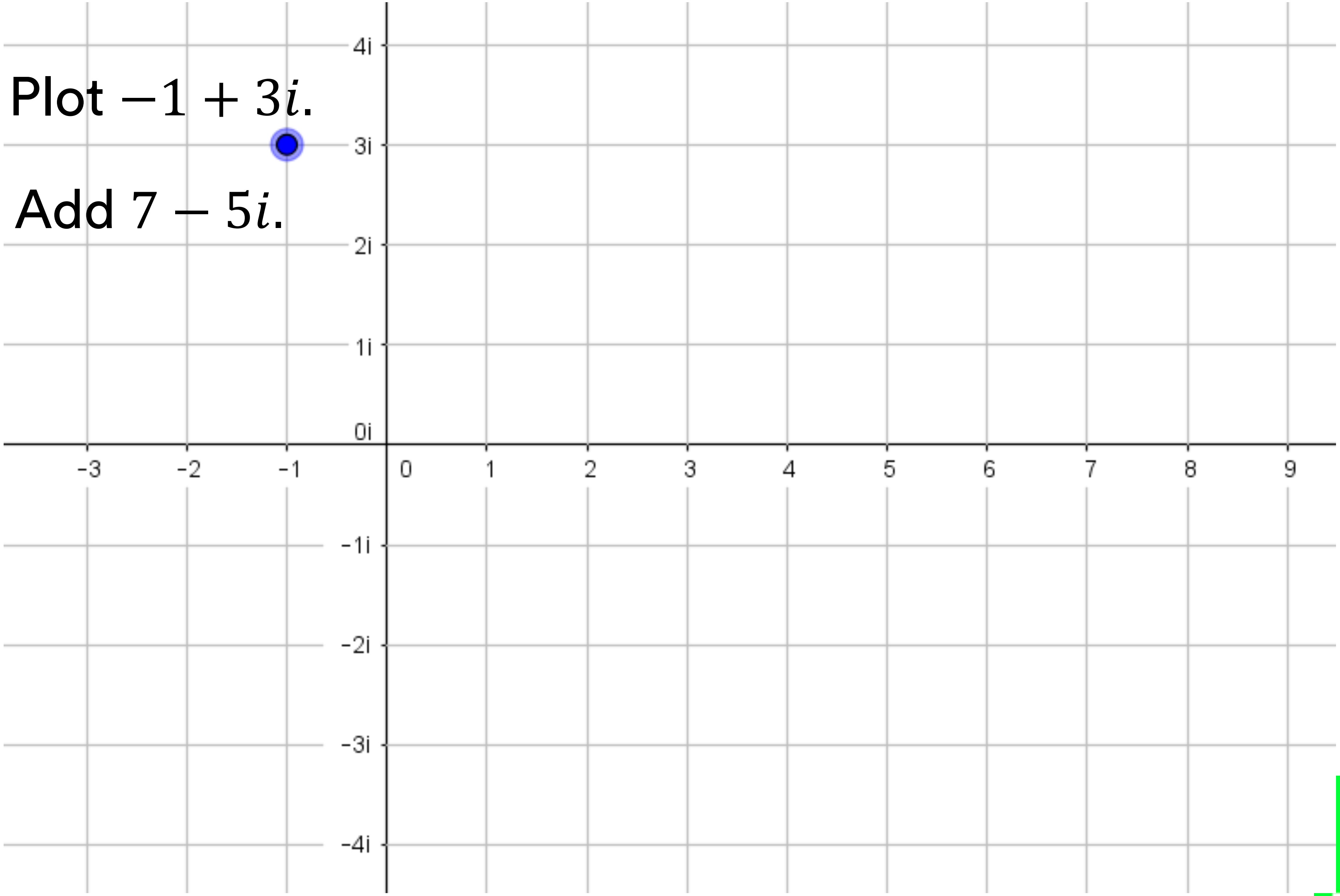


Plot $-1 + 3i$.



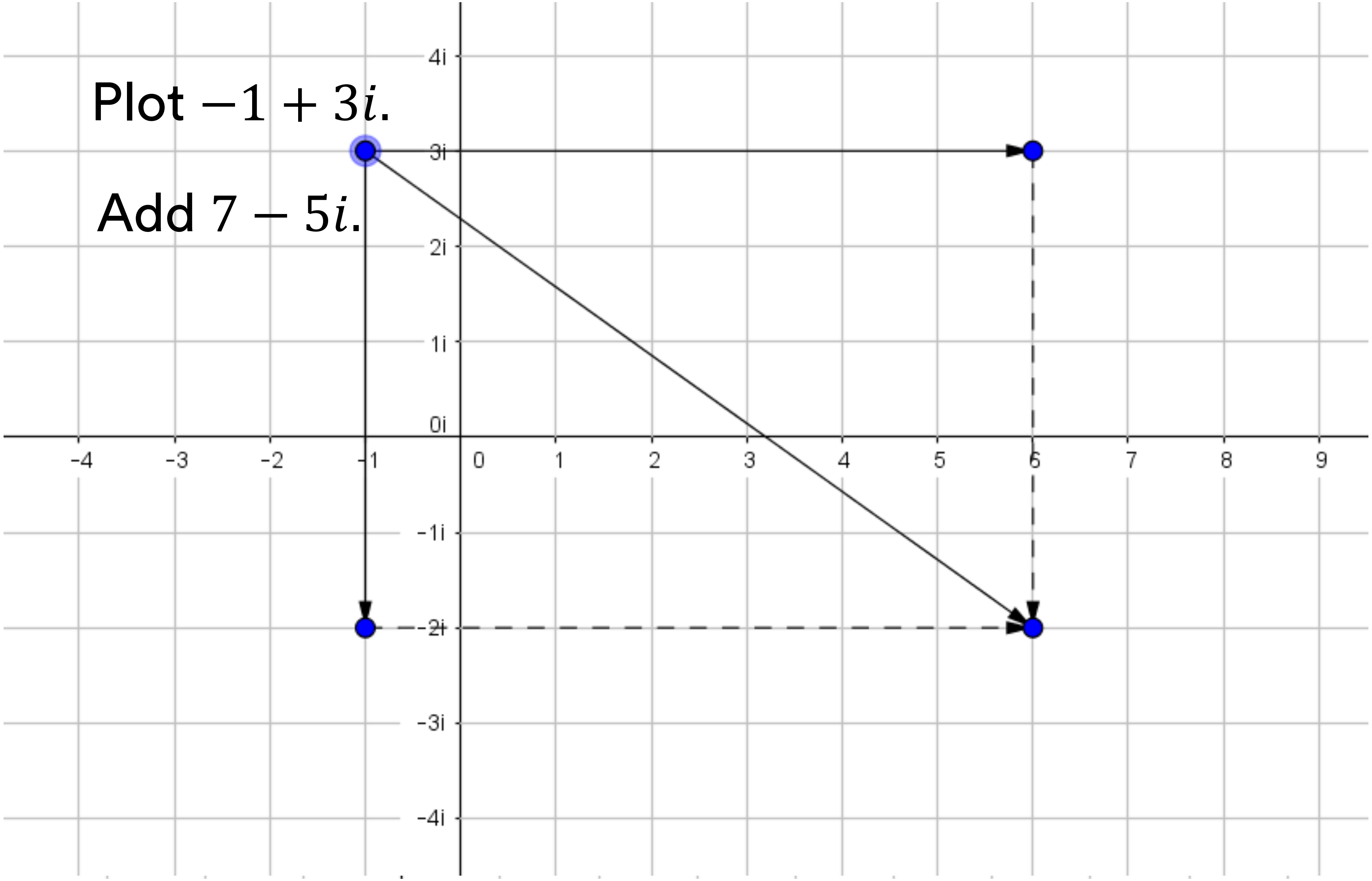
Plot $-1 + 3i$.

Add $7 - 5i$.



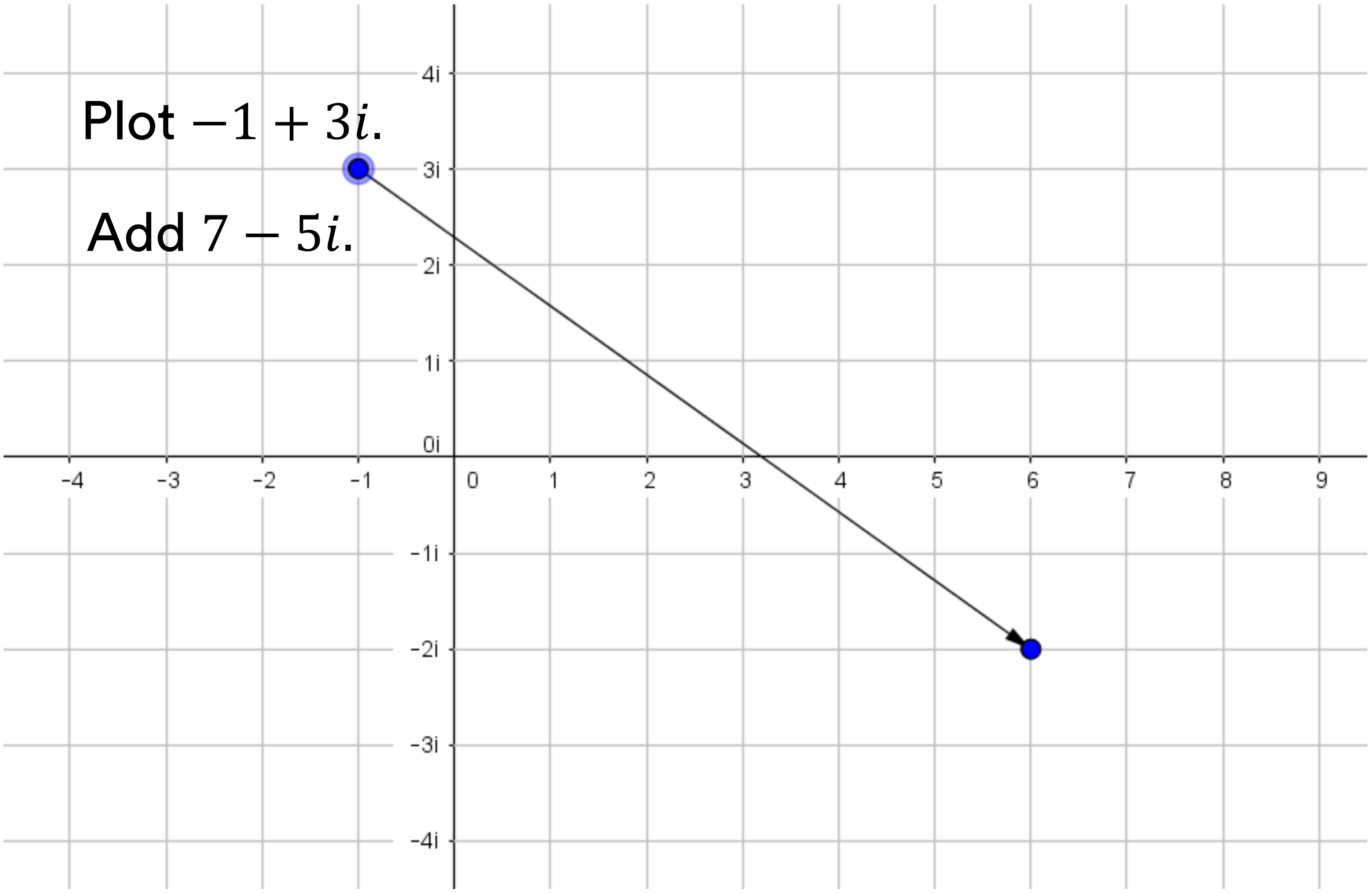
Plot $-1 + 3i$.

Add $7 - 5i$.



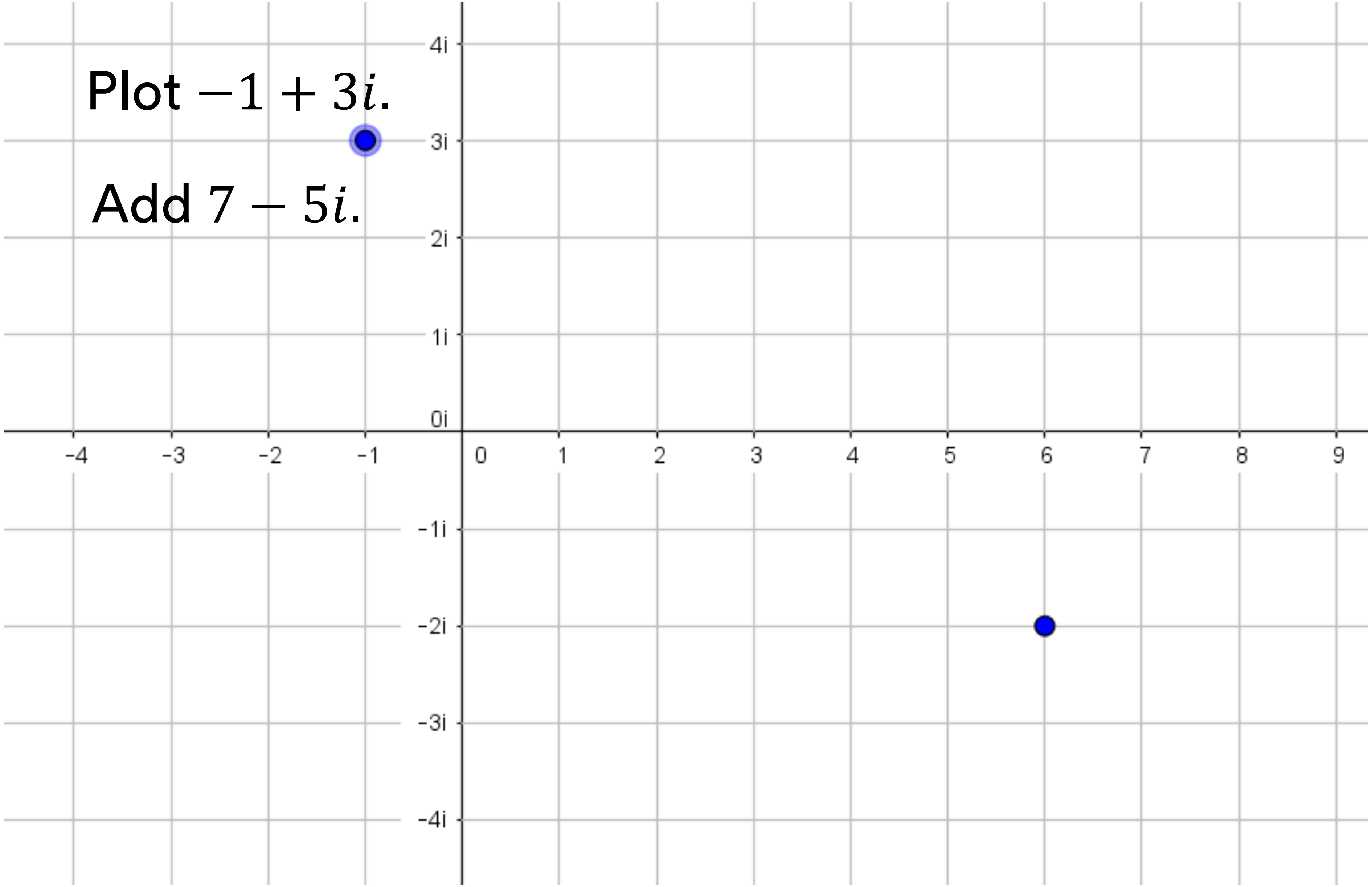
Plot $-1 + 3i$.

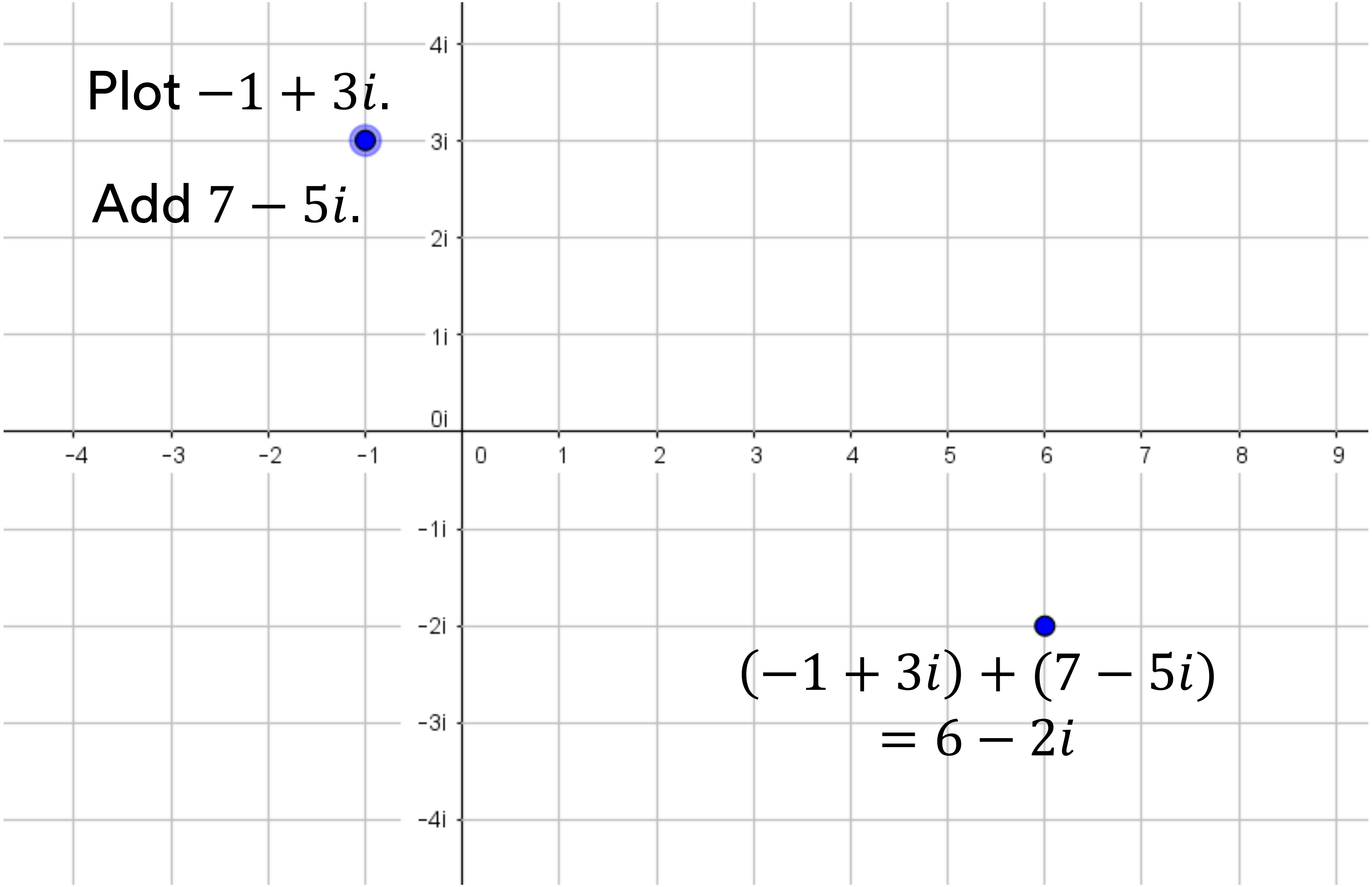
Add $7 - 5i$.



Plot $-1 + 3i$.

Add $7 - 5i$.



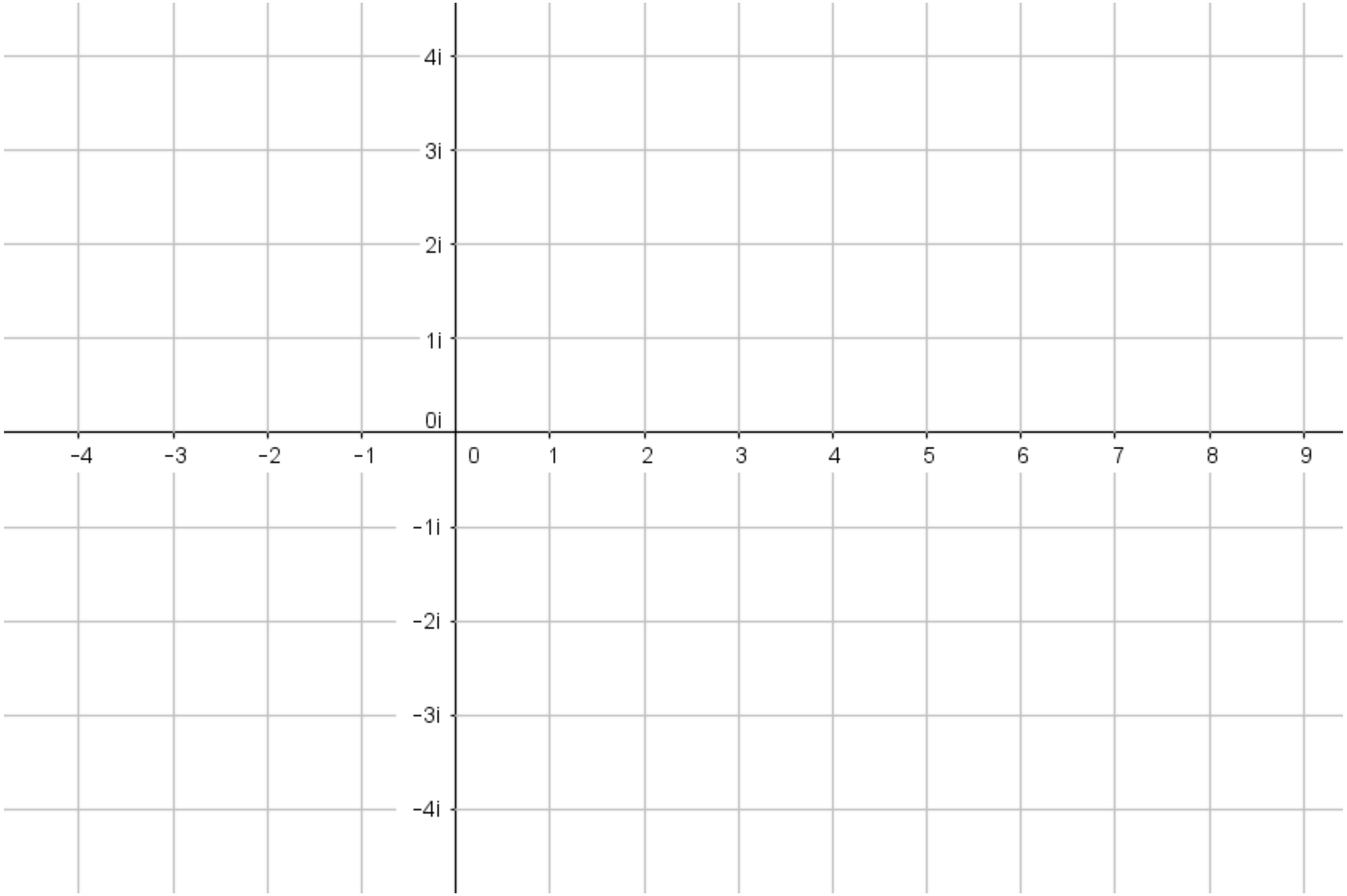


Plot $-1 + 3i$.

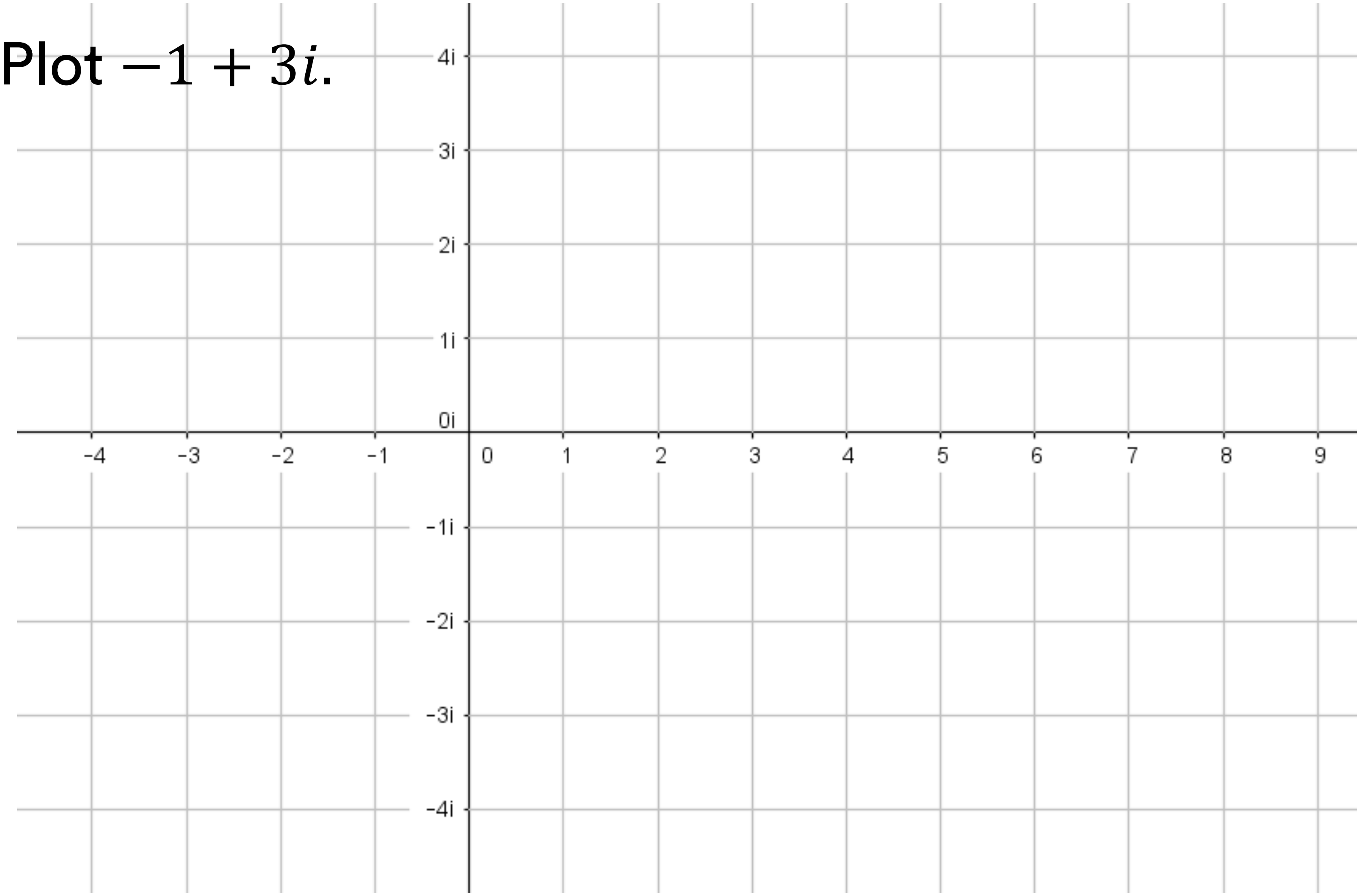
Add $7 - 5i$.

$$(-1 + 3i) + (7 - 5i)$$
$$= 6 - 2i$$

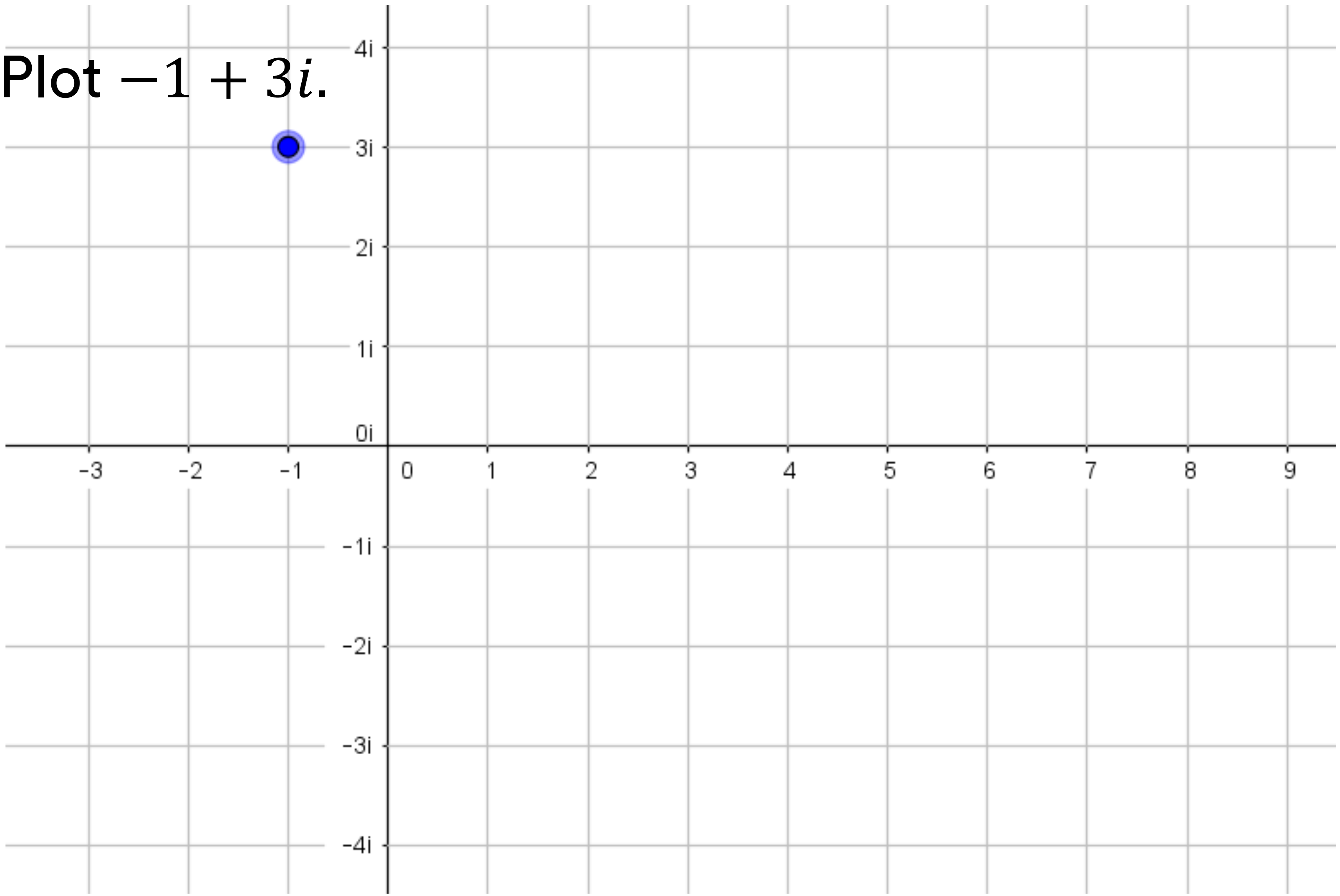
Using Geometry to Teach Complex Number Operations (Multiplication)


4i
3i
2i
1i
0i
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
9
−1i
−2i
−3i
−4i

Plot $-1+3i$.

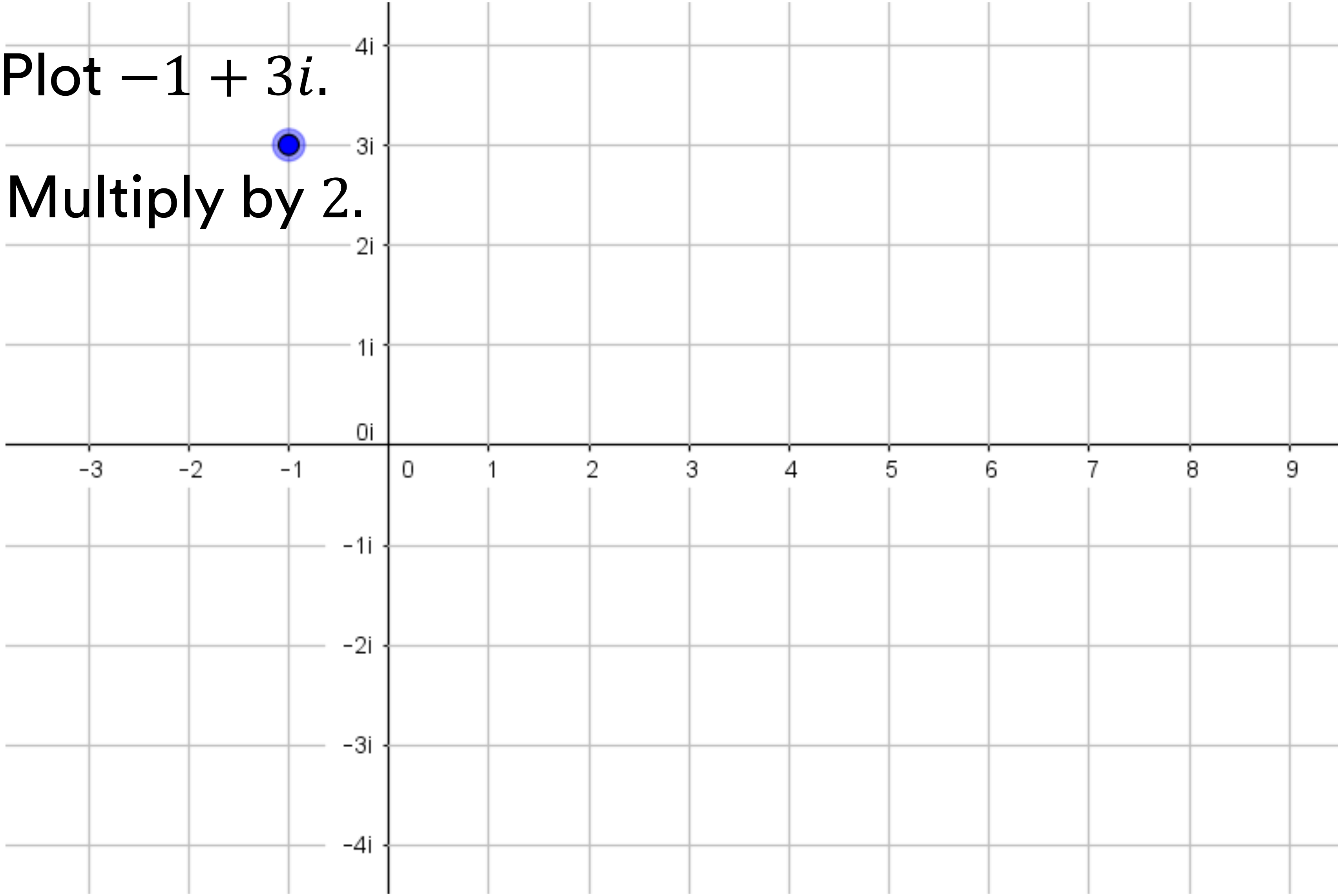


Plot $-1+3i$.



Plot $-1 + 3i$.

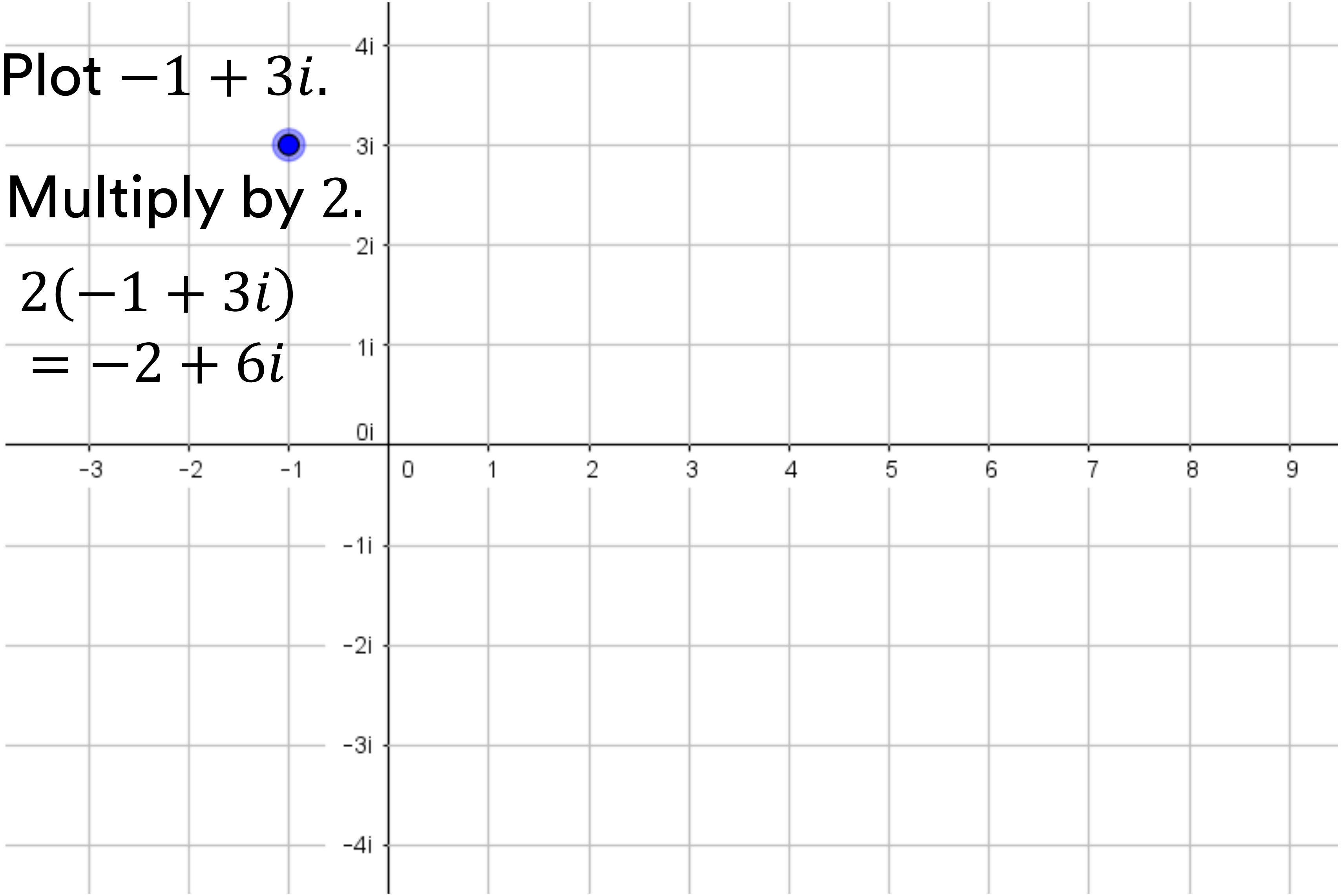
Multiply by 2.



Plot $-1 + 3i$.

Multiply by 2.

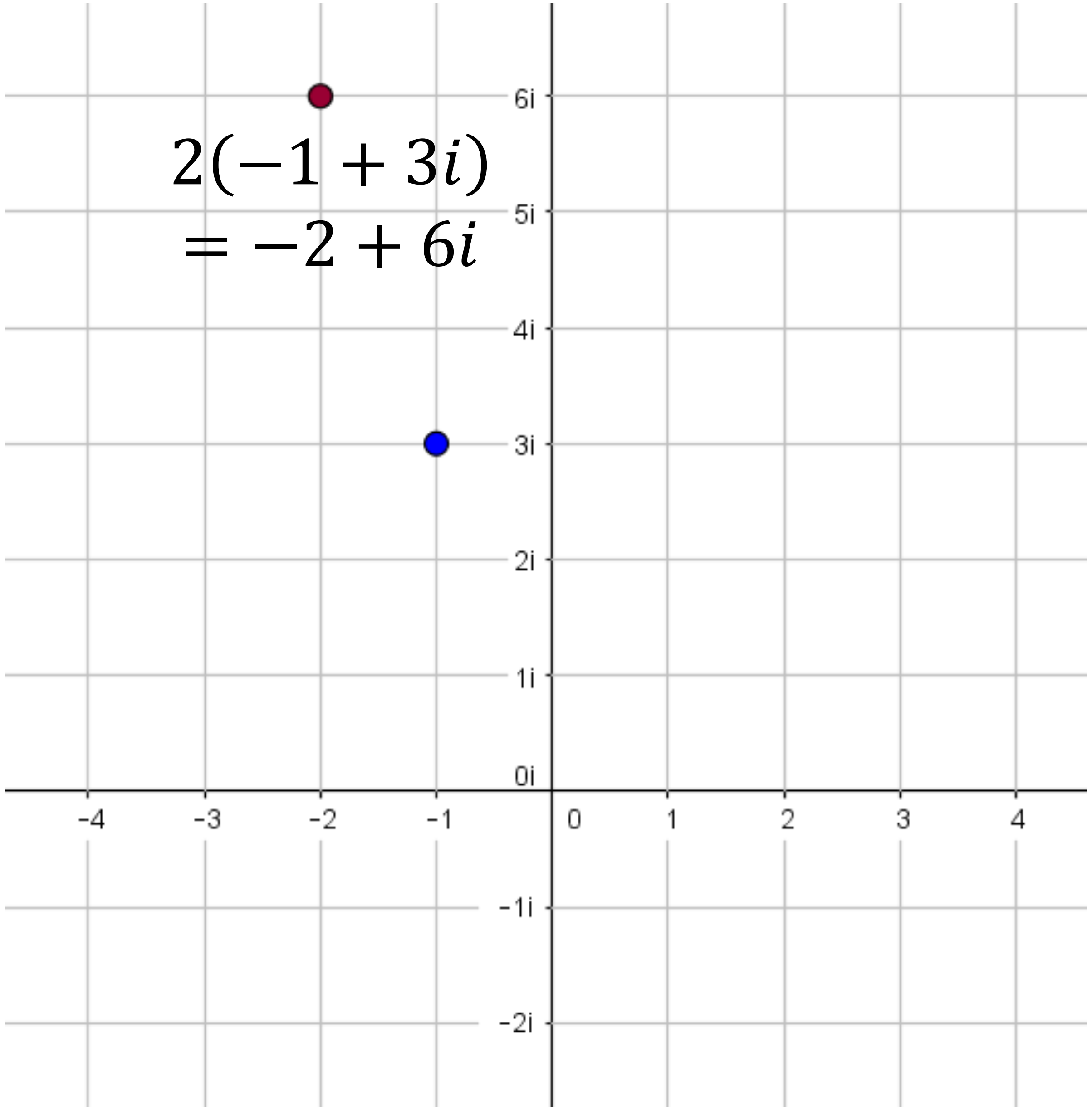
$$2(-1 + 3i)$$
$$= -2 + 6i$$



Plot $-1 + 3i$.

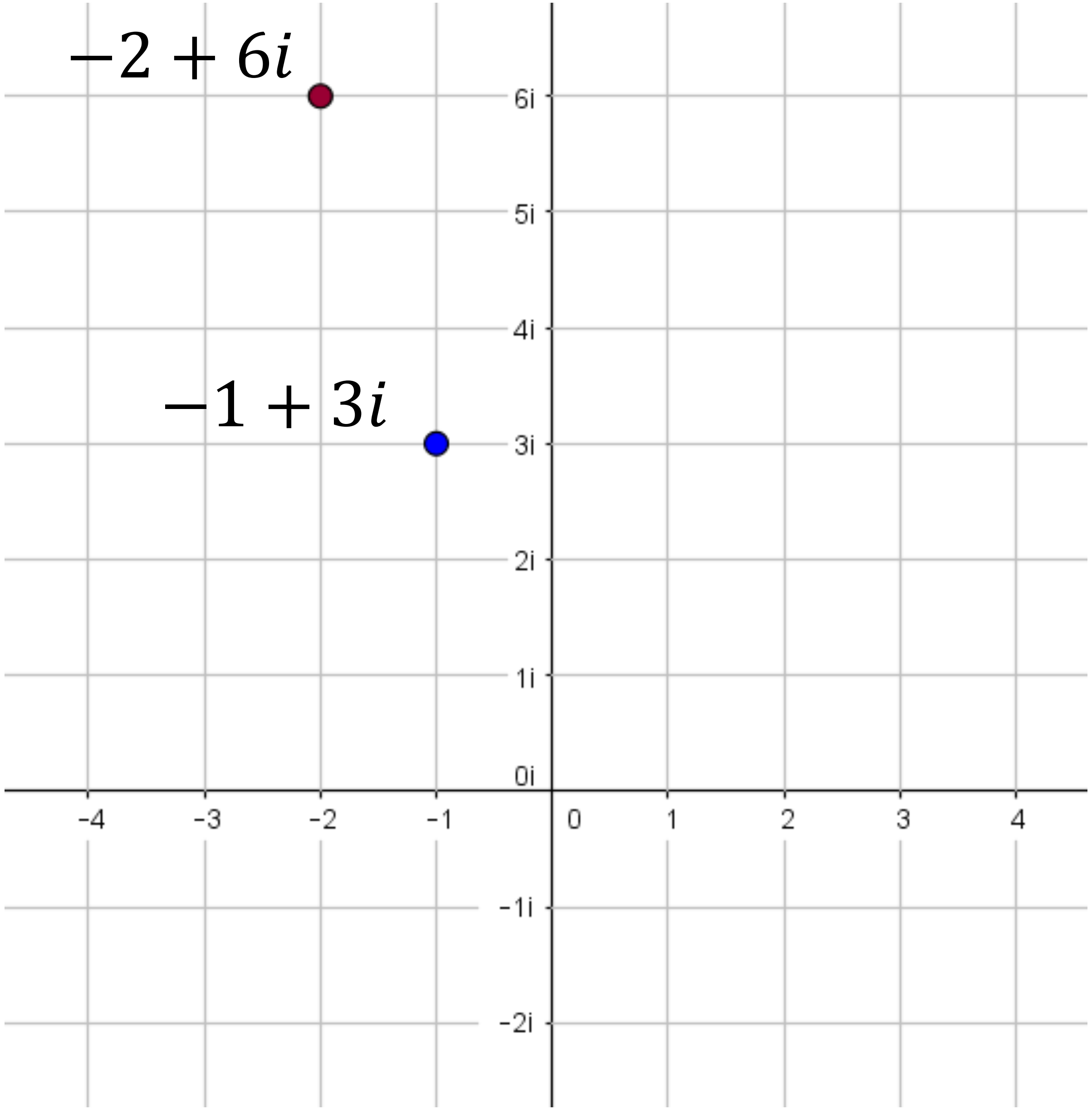
Multiply by 2.

$$2(-1+3i)$$
$$= -2+6i$$



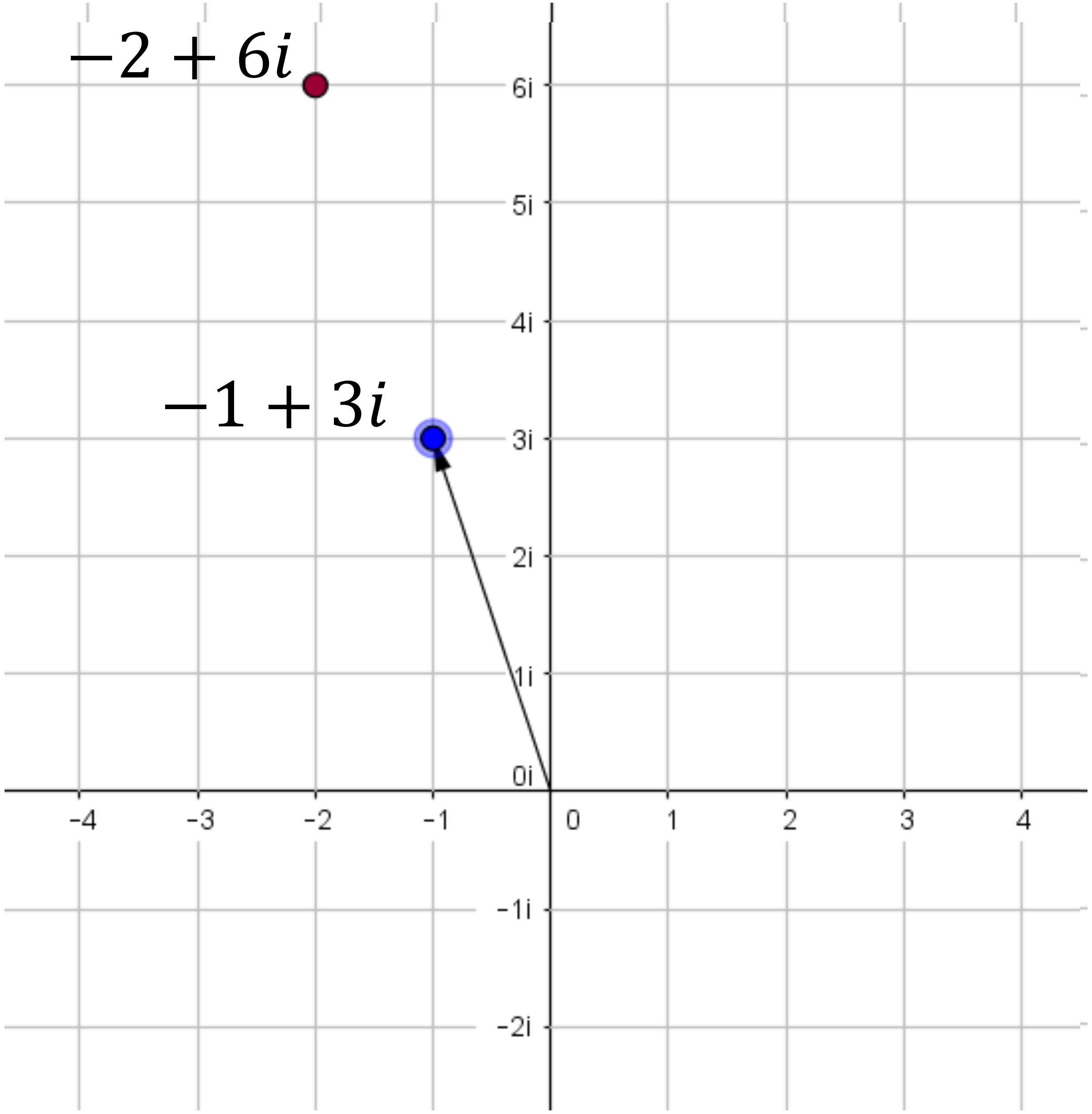
Plot $-1 + 3i$.

Multiply by 2.



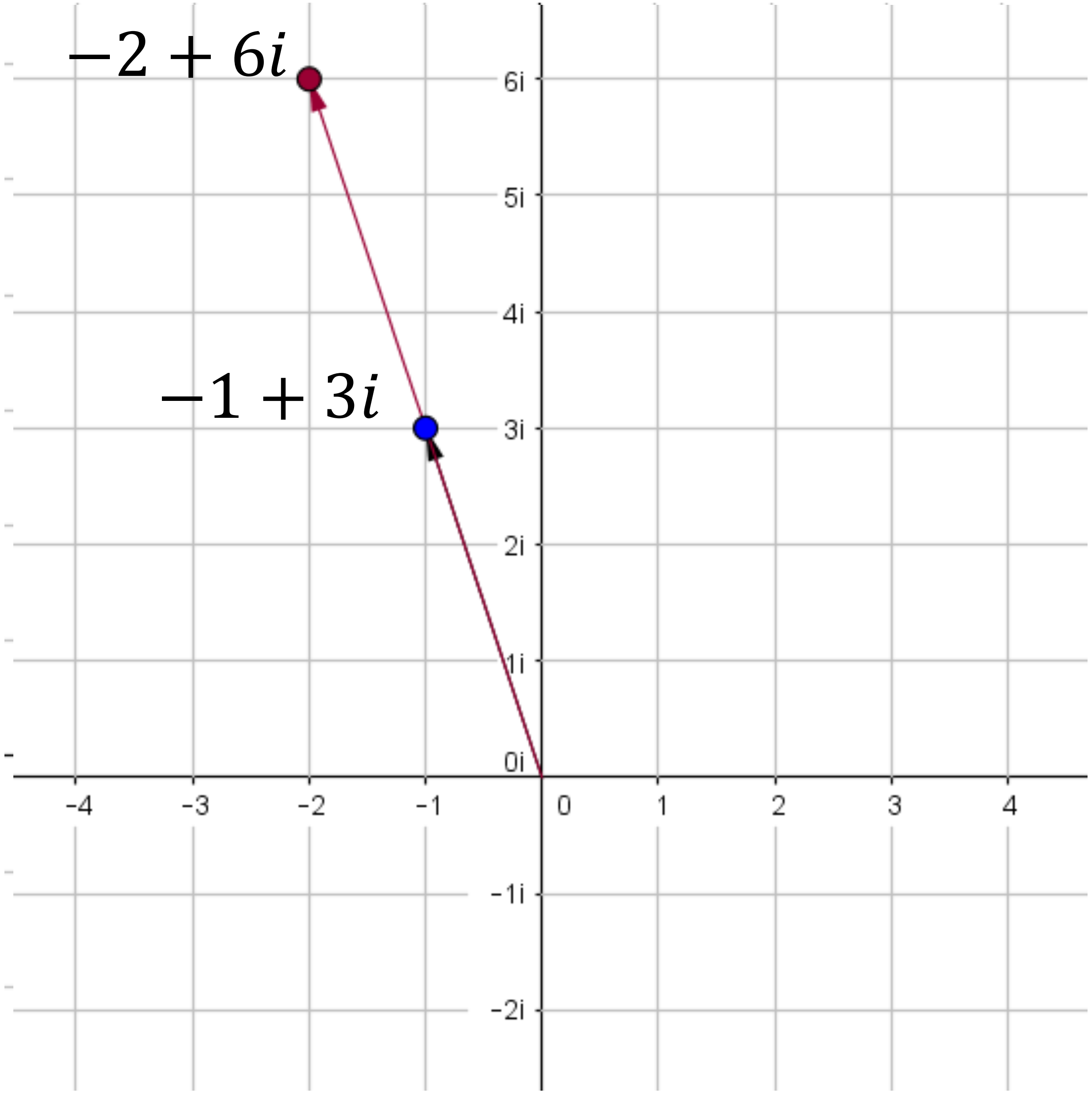
Plot $-1 + 3i$.

Multiply by 2.



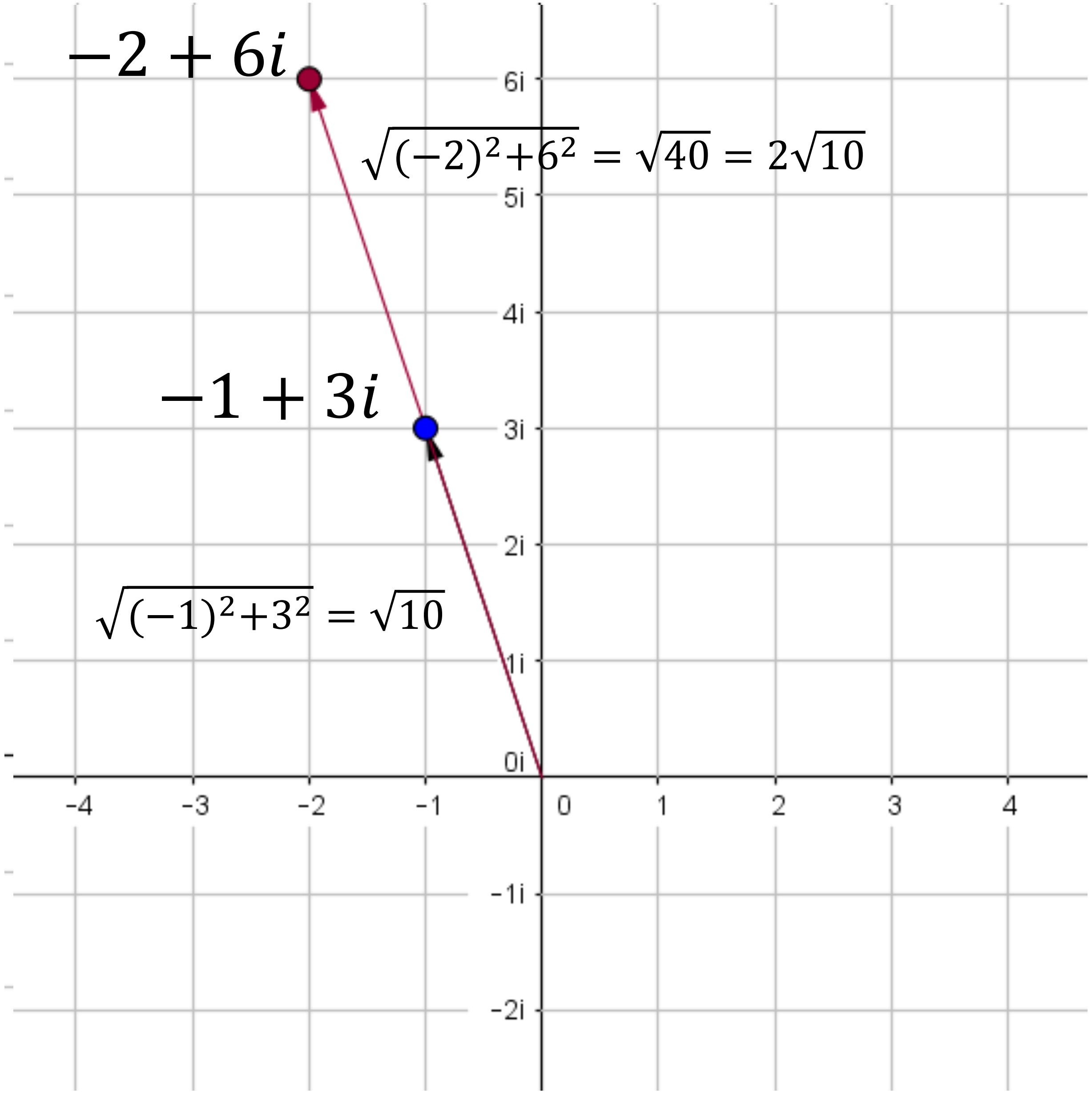
Plot $-1 + 3i$.

Multiply by 2.



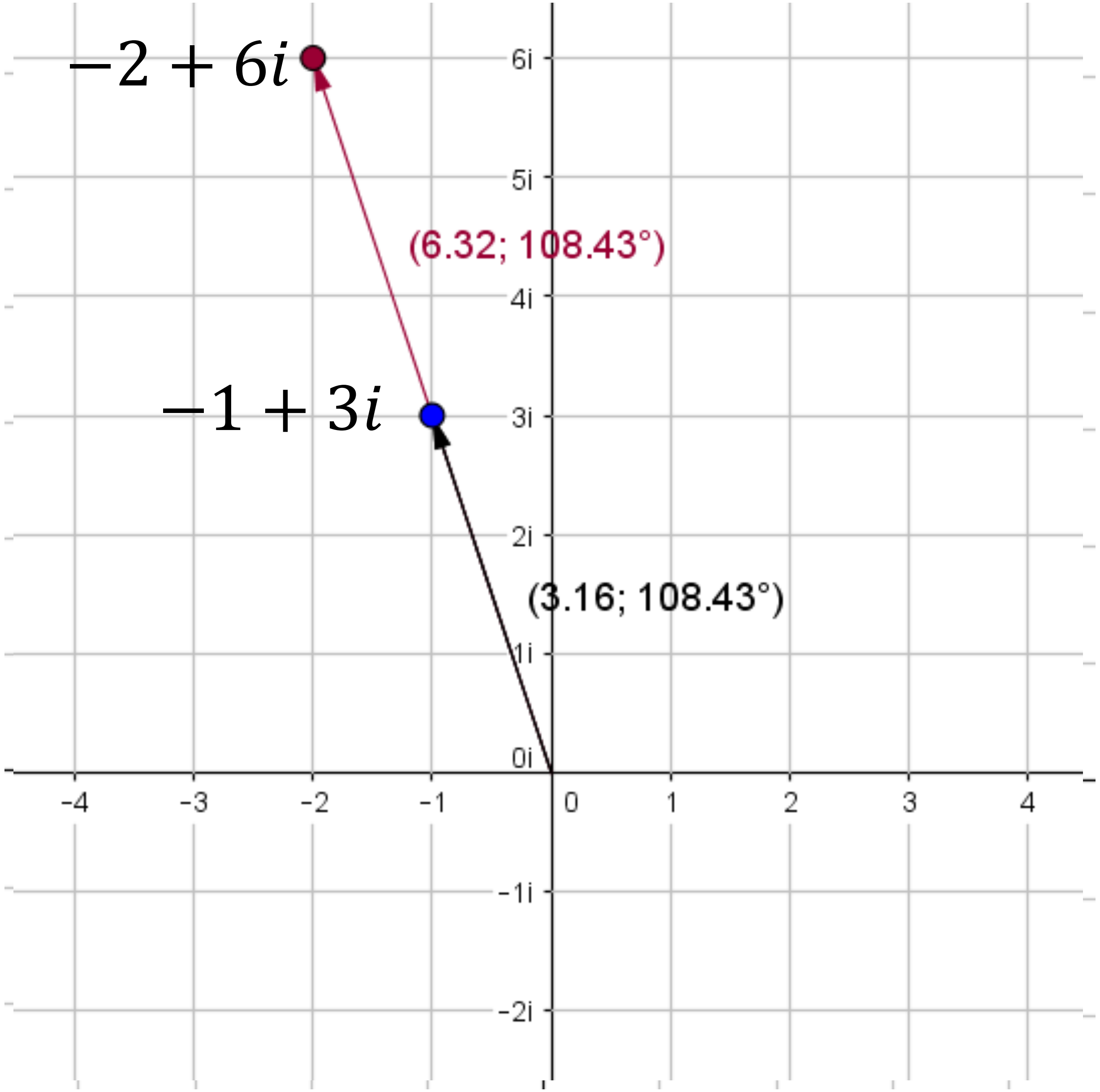
Plot $-1 + 3i$.

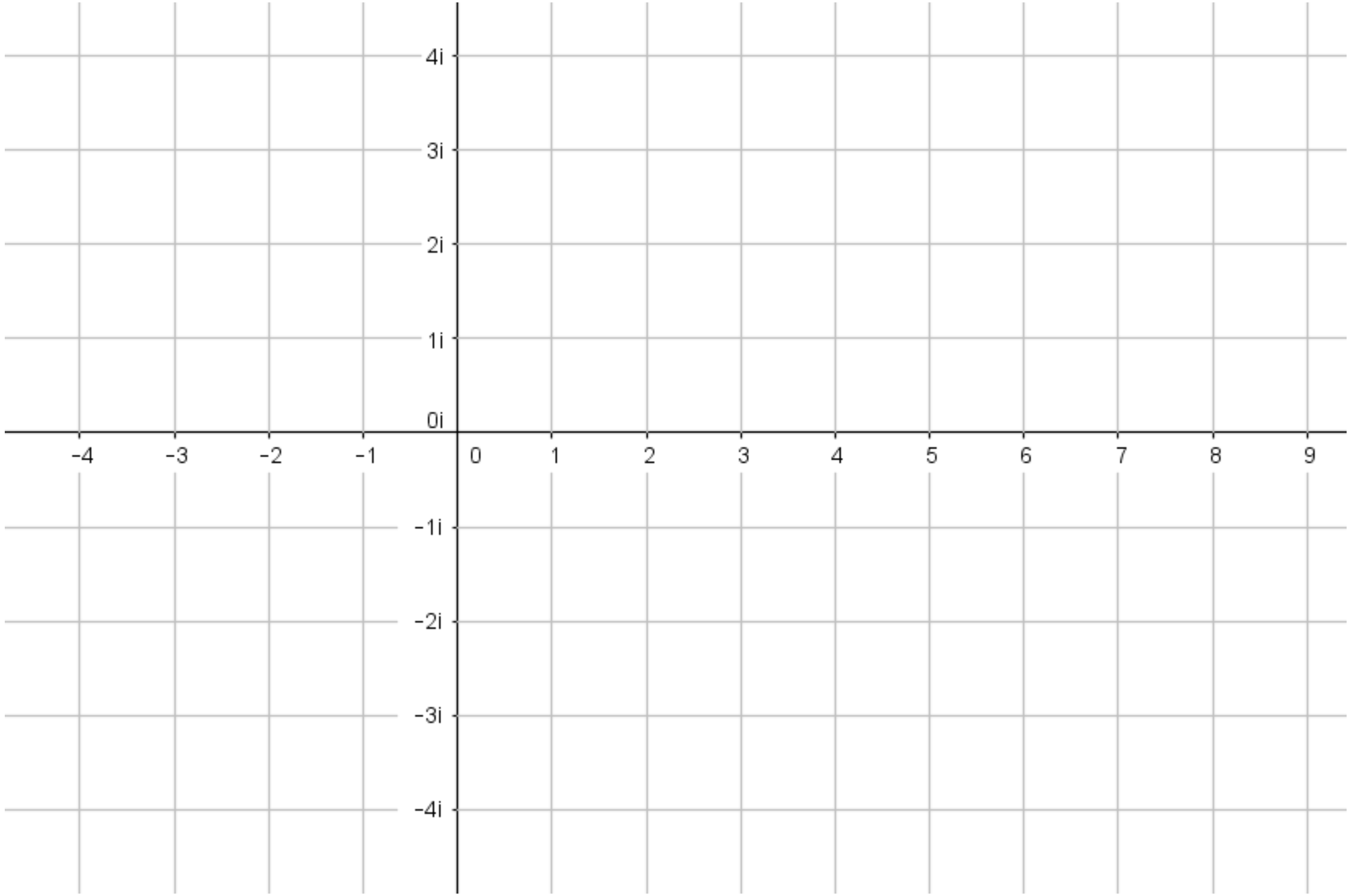
Multiply by 2.



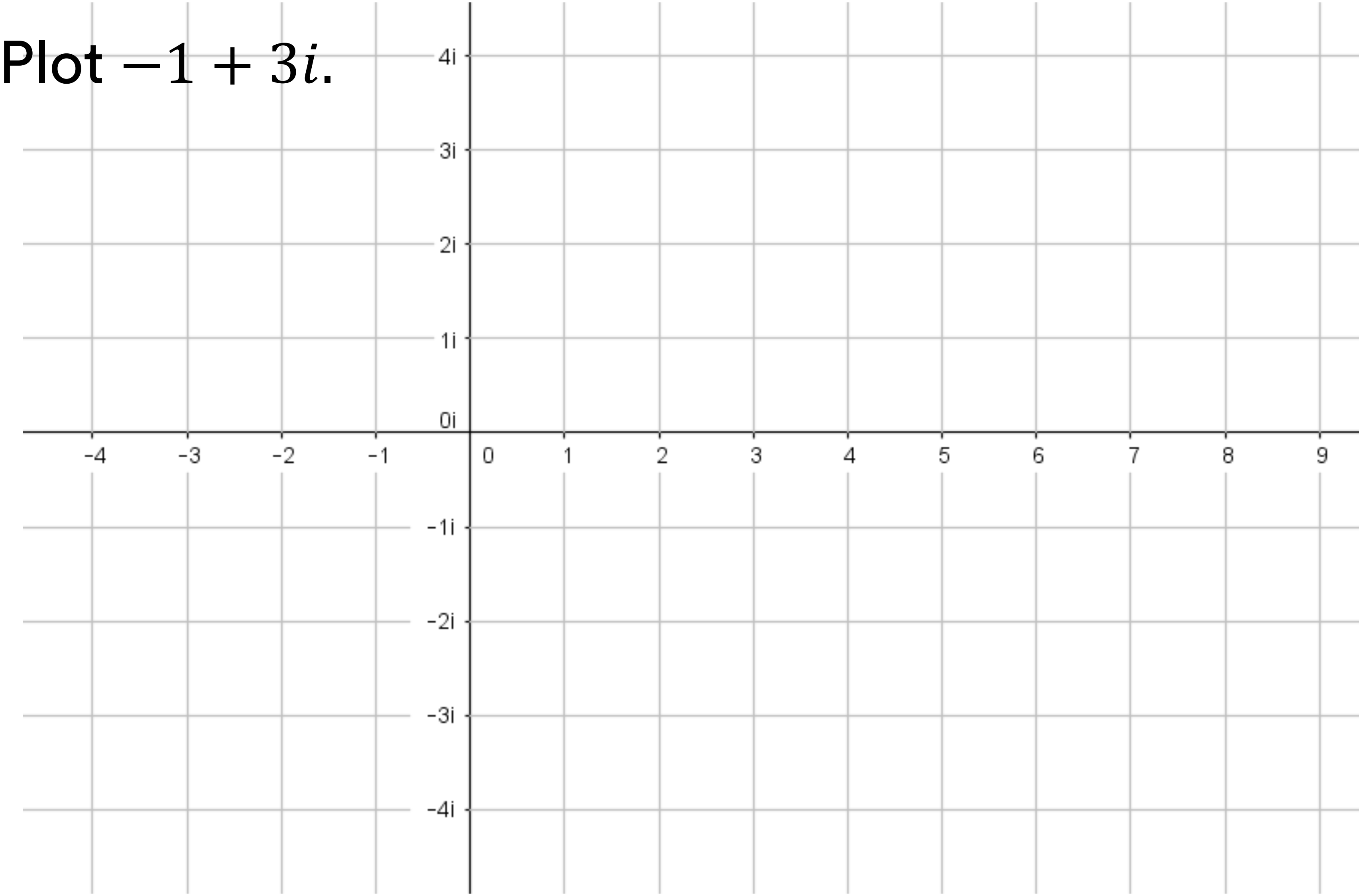
Plot $-1 + 3i$.

Multiply by 2.

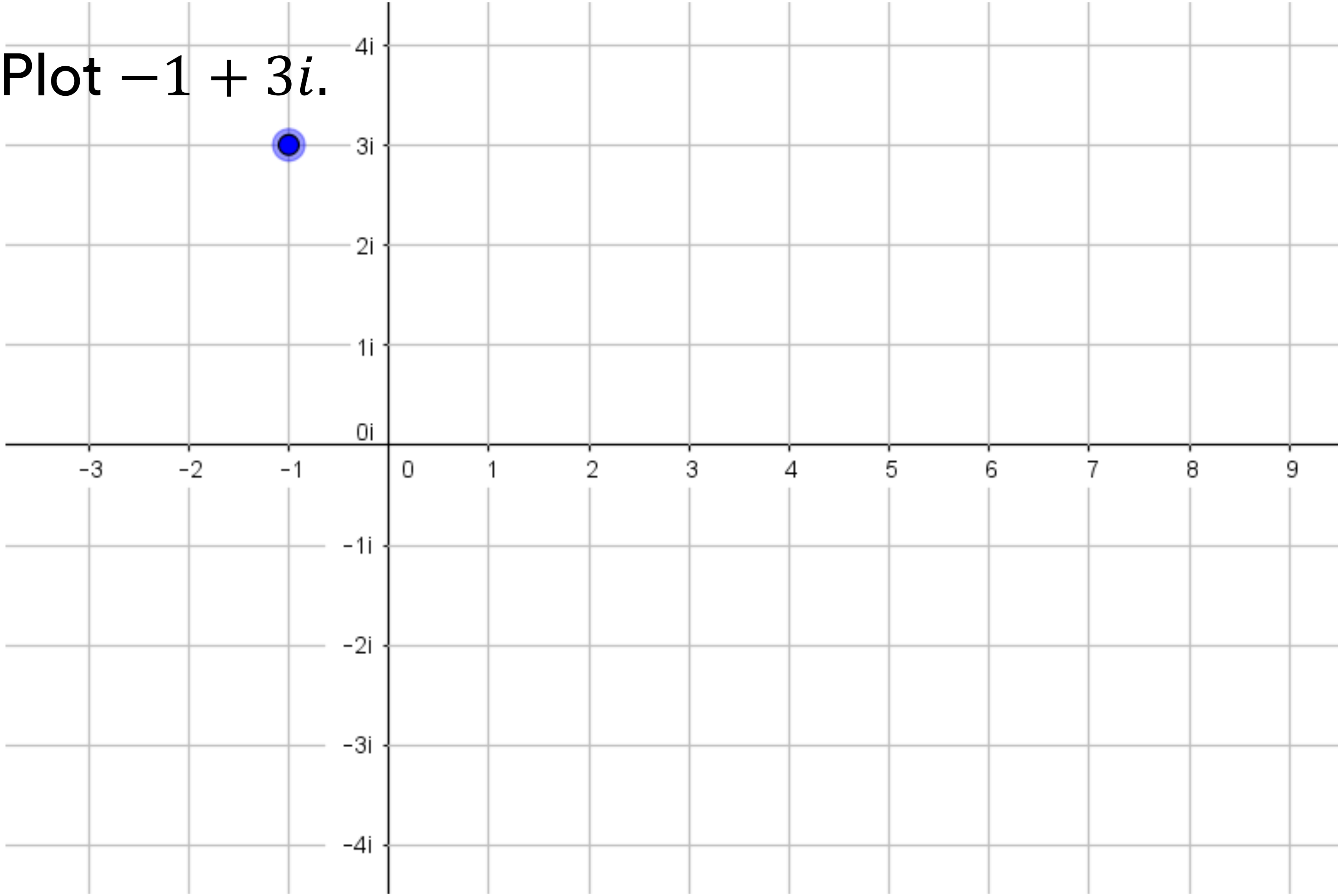



4i
3i
2i
1i
0i
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1i
-2i
-3i
-4i

Plot $-1 + 3i$.

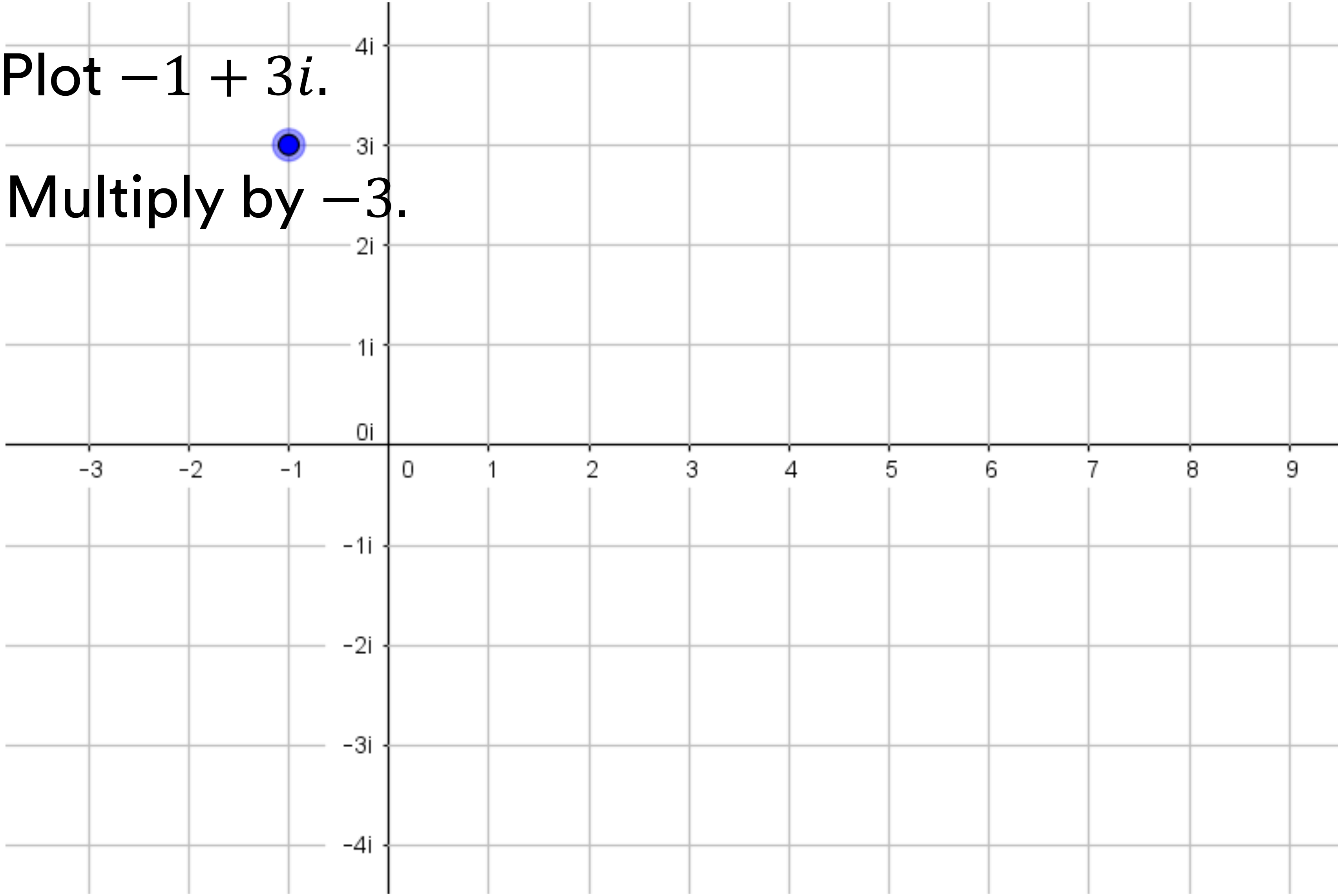


Plot $-1 + 3i$.



Plot $-1 + 3i$.

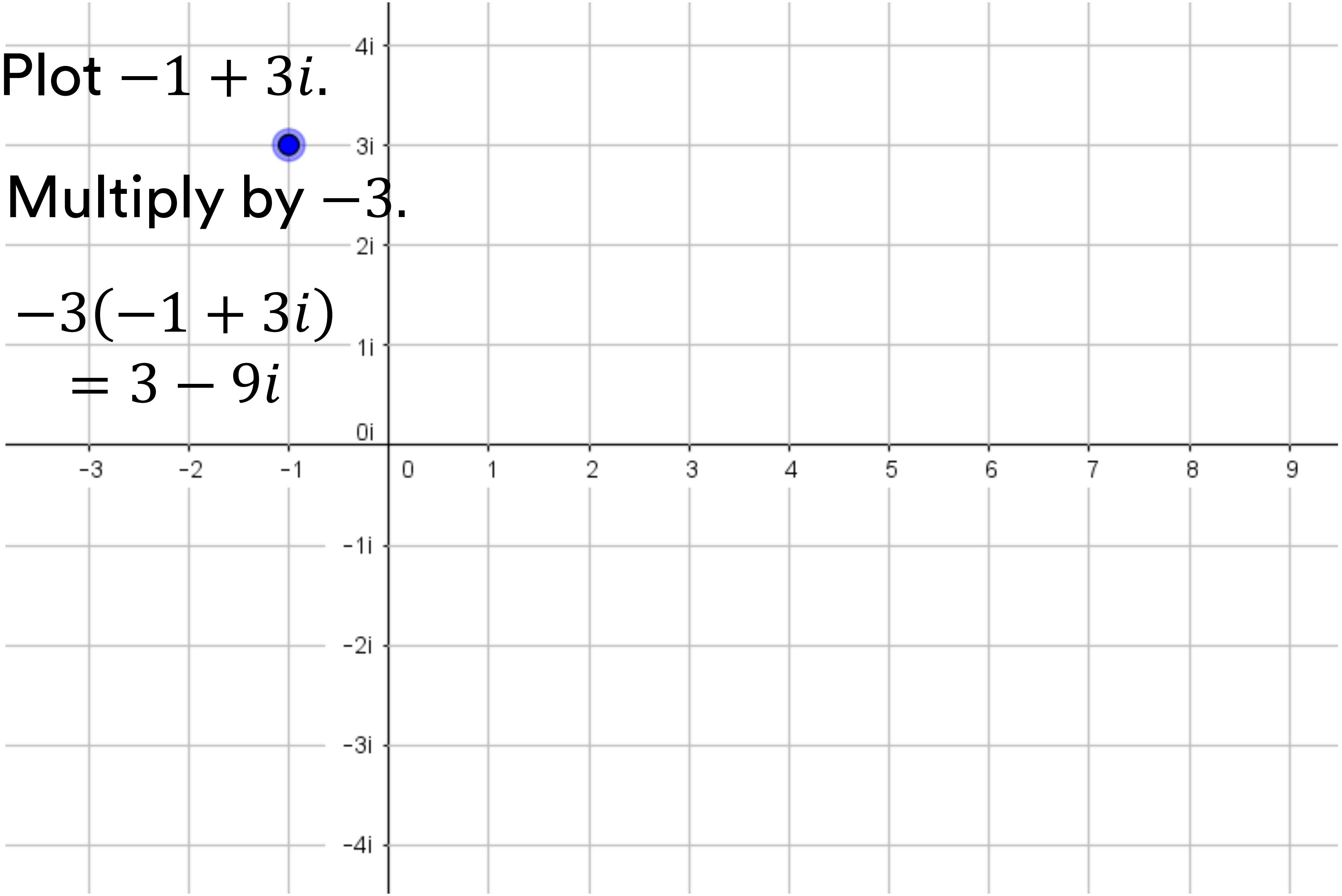
Multiply by -3.



Plot $-1 + 3i$.

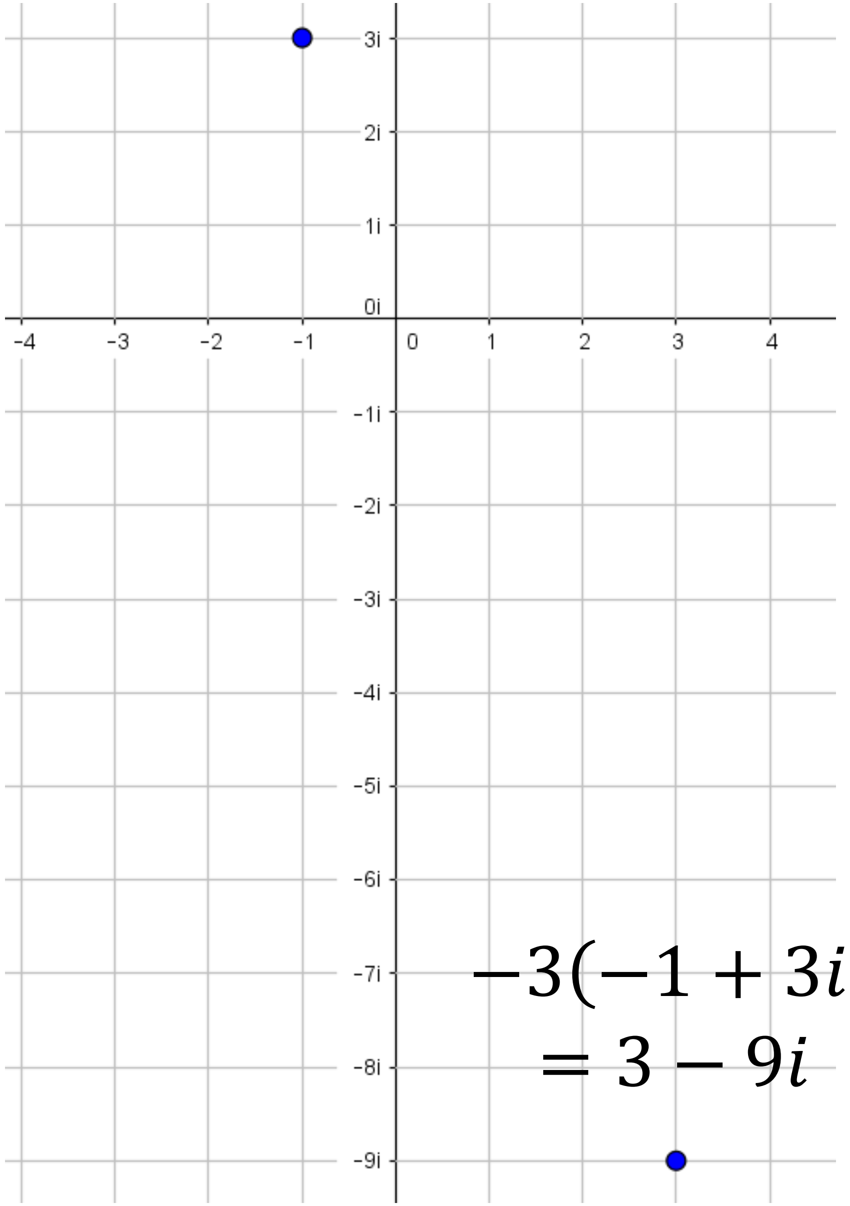
Multiply by -3.

$$-3(-1 + 3i)$$
$$= 3 - 9i$$



Plot $-1 + 3i$.

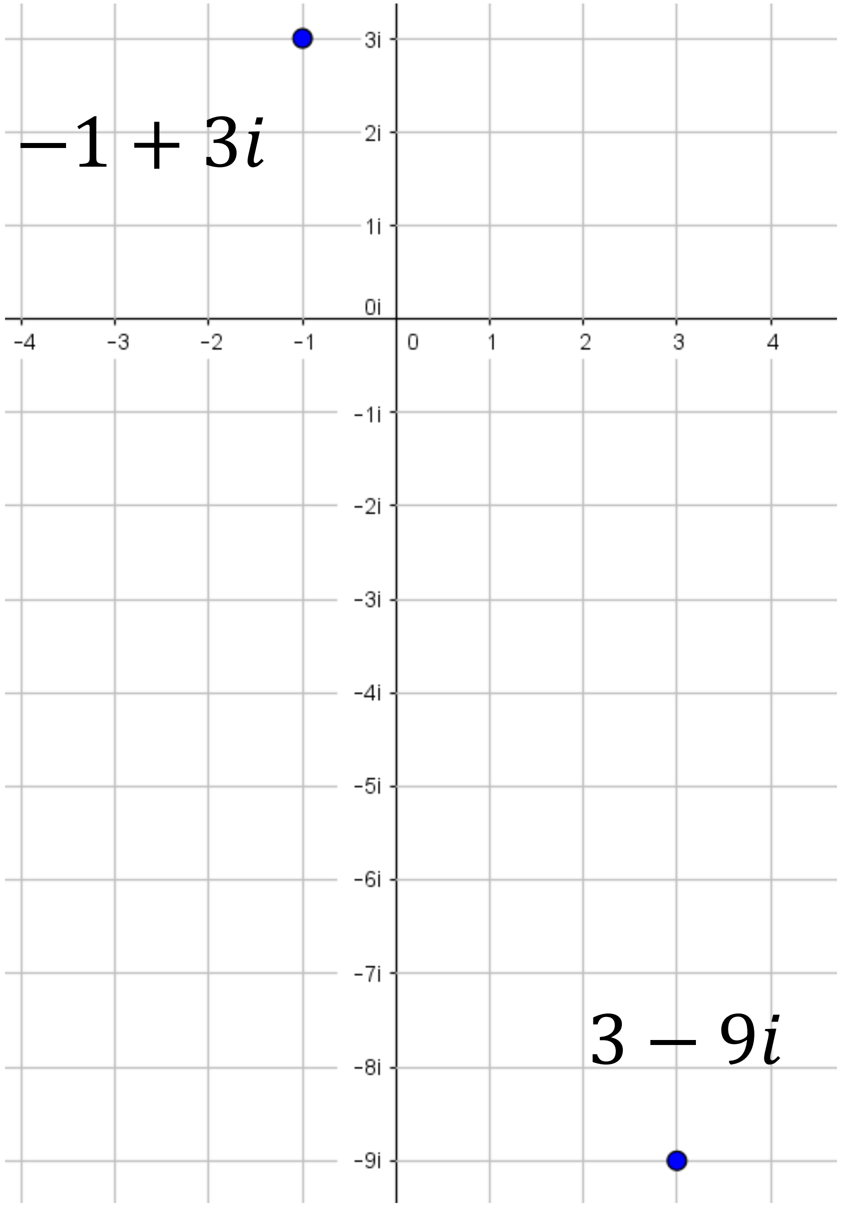
Multiply by -3.



$$-3(-1 + 3i)$$
$$= 3 - 9i$$

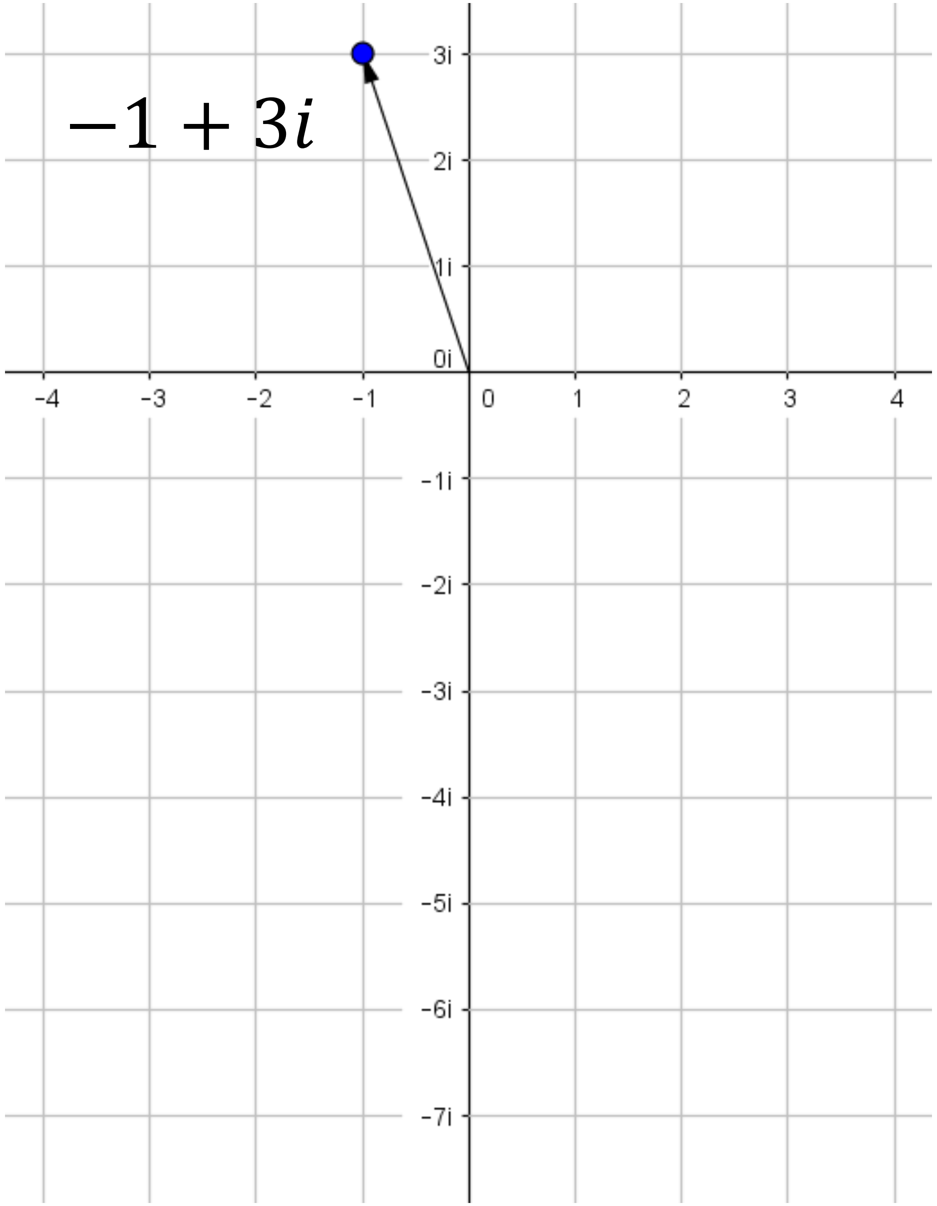
Plot $-1+3i$.

Multiply by -3.



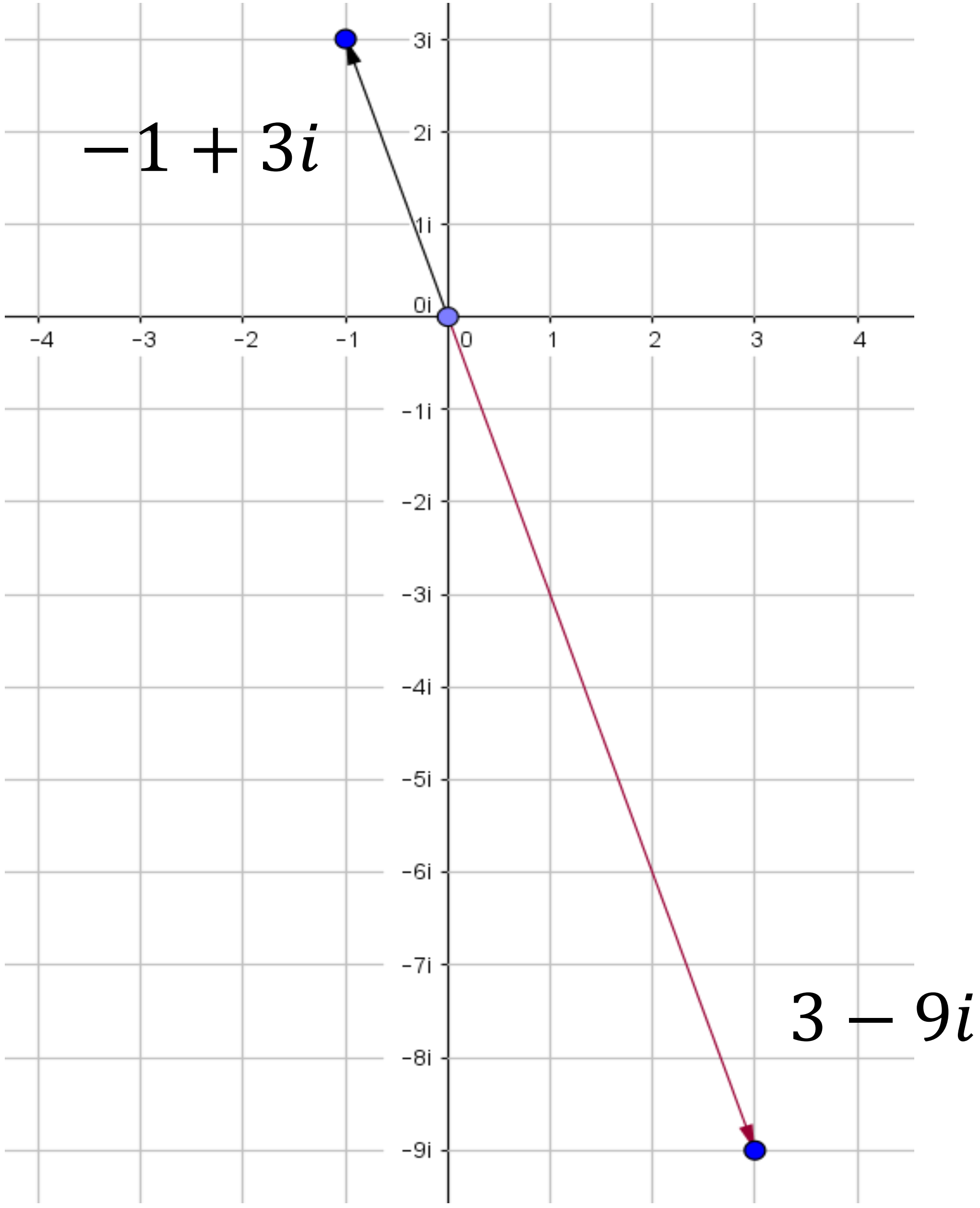
Plot $-1 + 3i$.

Multiply by -3.



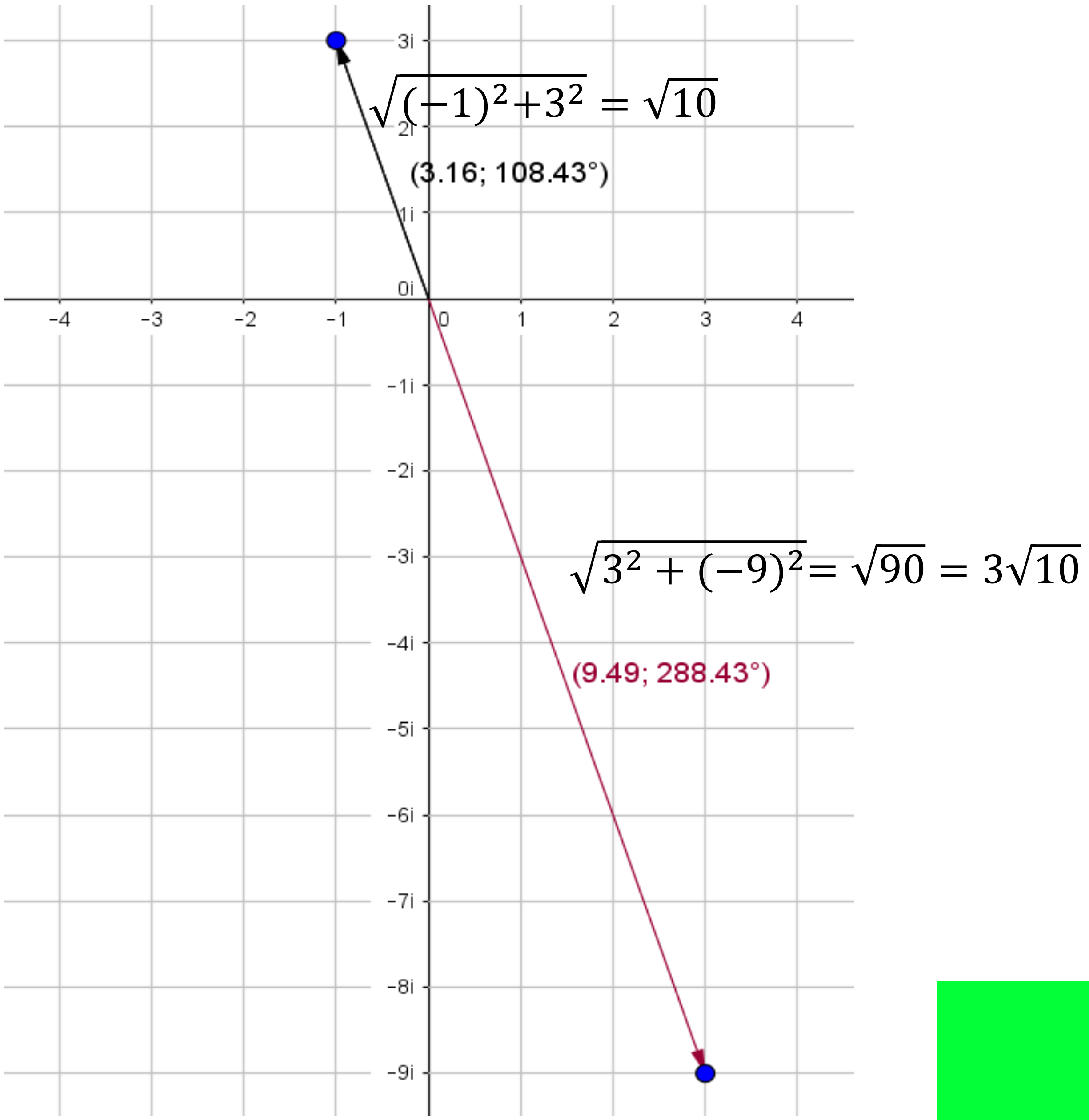
Plot $-1 + 3i$.

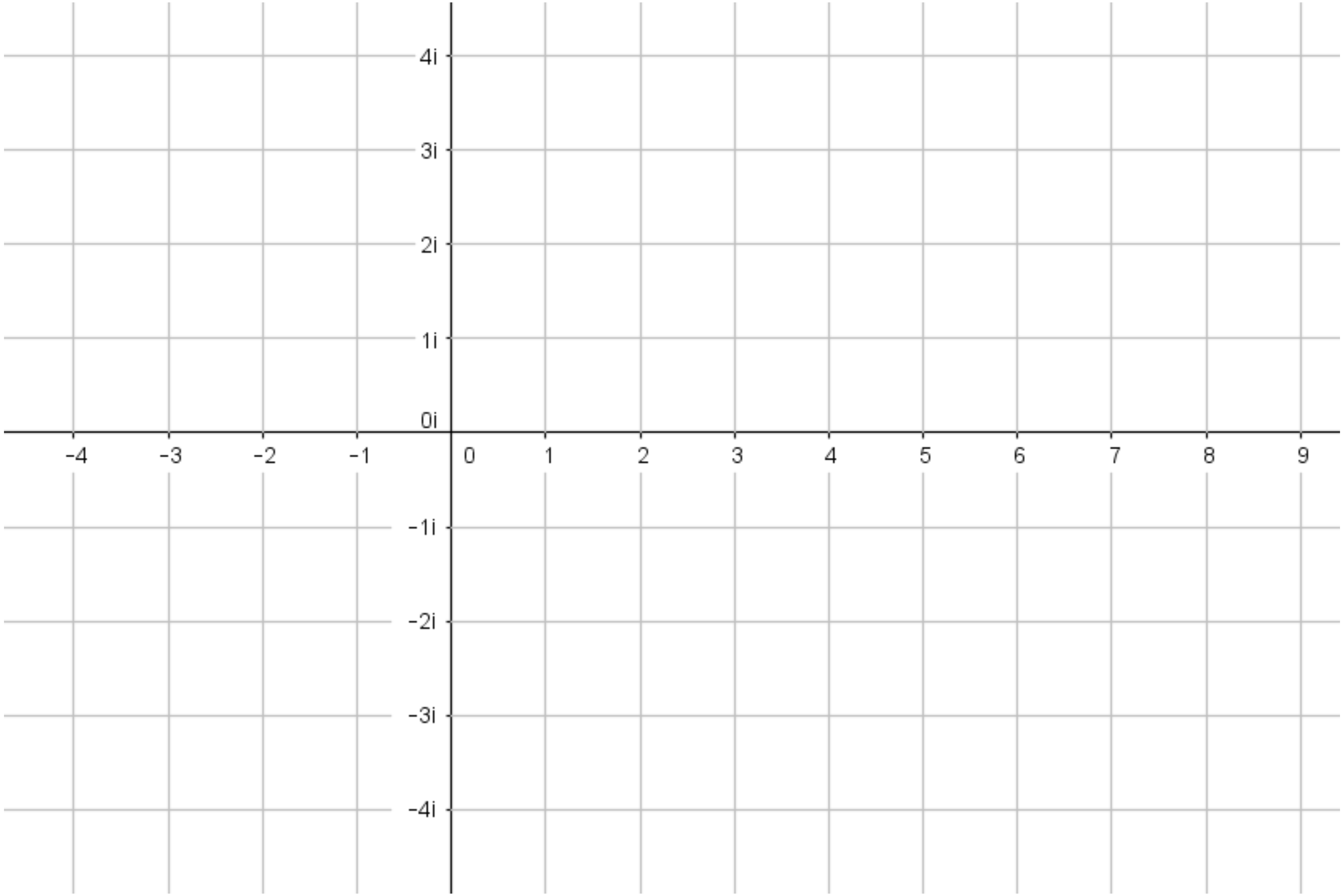
Multiply by -3.



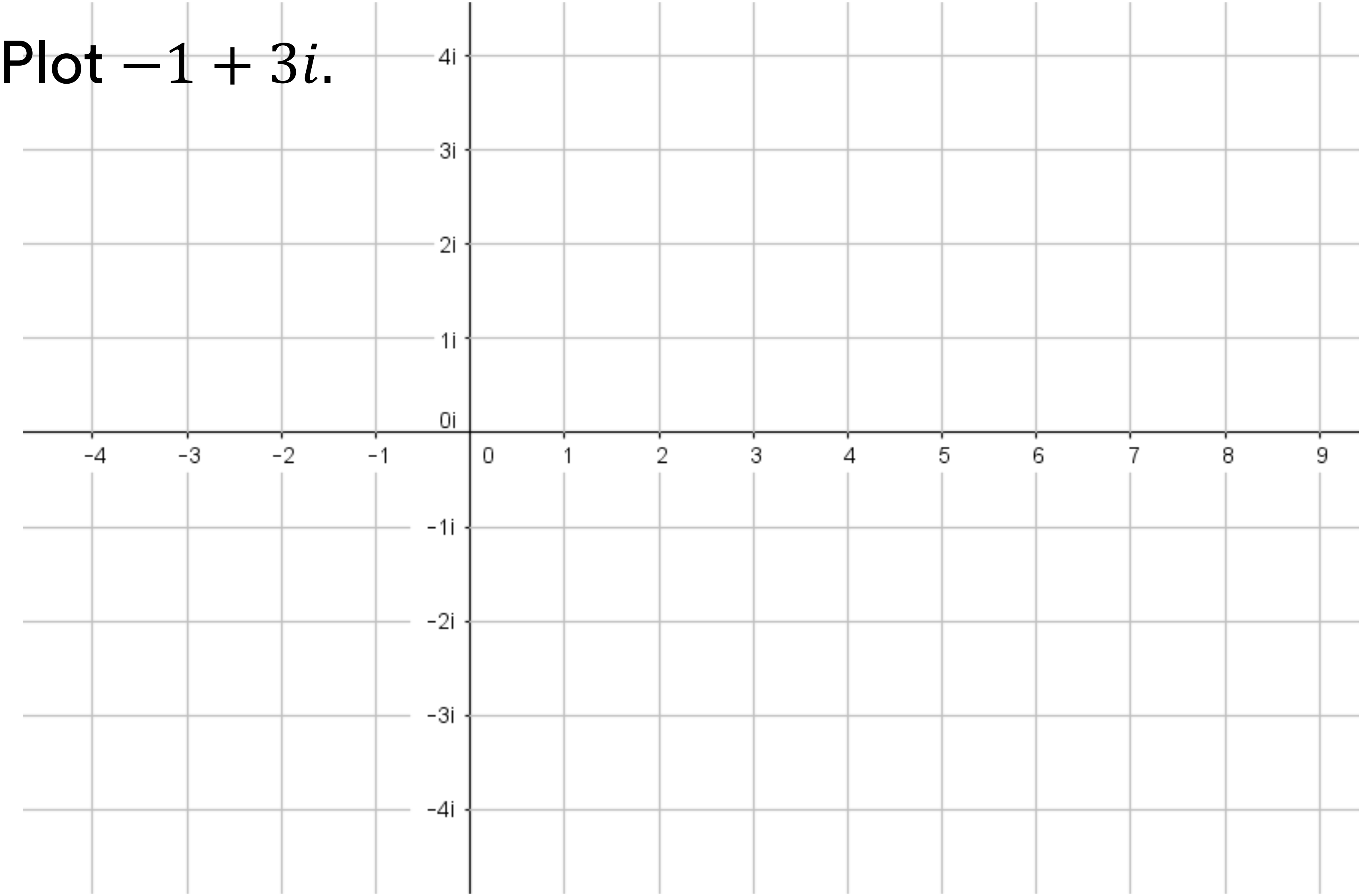
Plot $-1 + 3i$.

Multiply by -3.

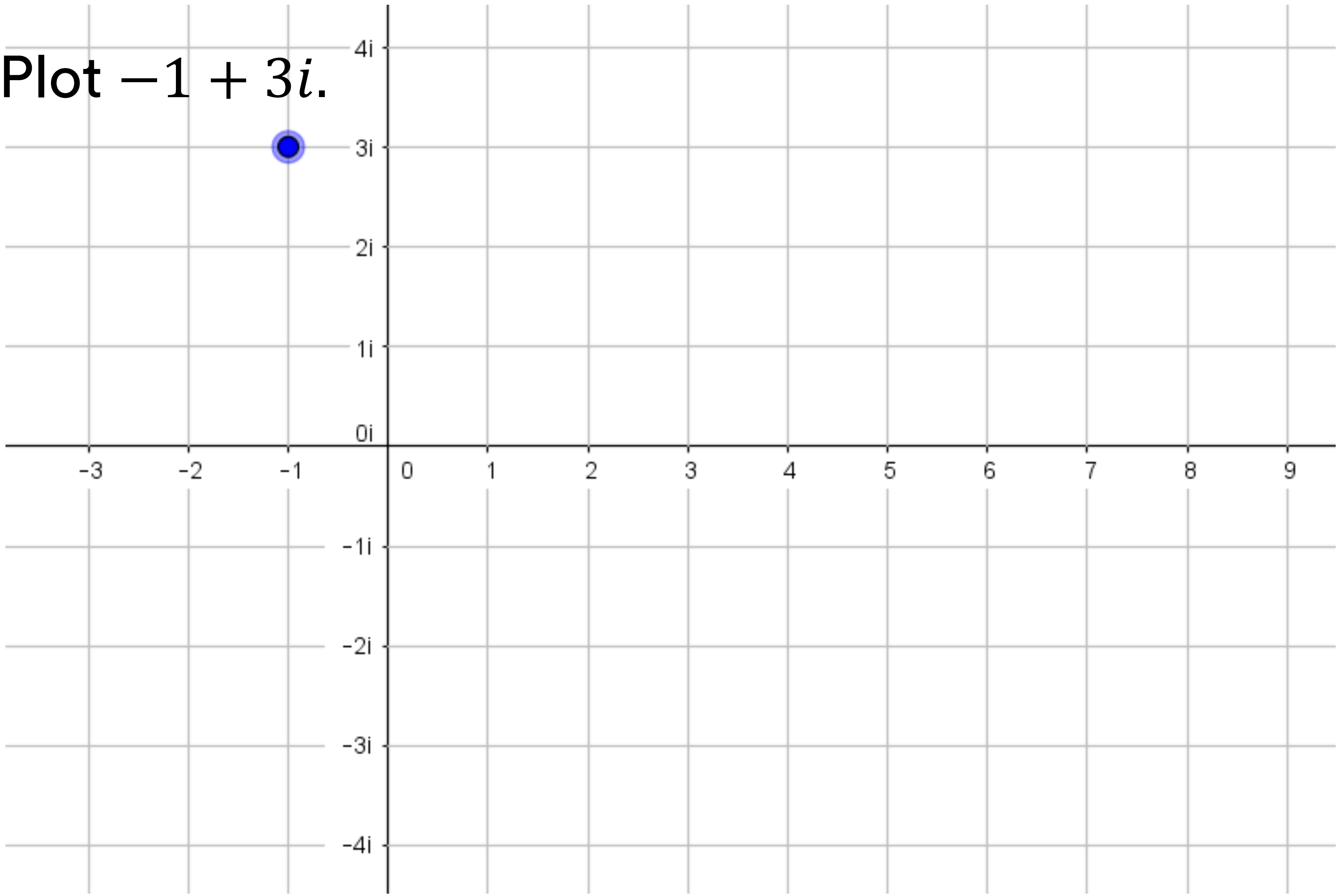



4i
3i
2i
1i
0i
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1i
-2i
-3i
-4i

Plot $-1+3i$.

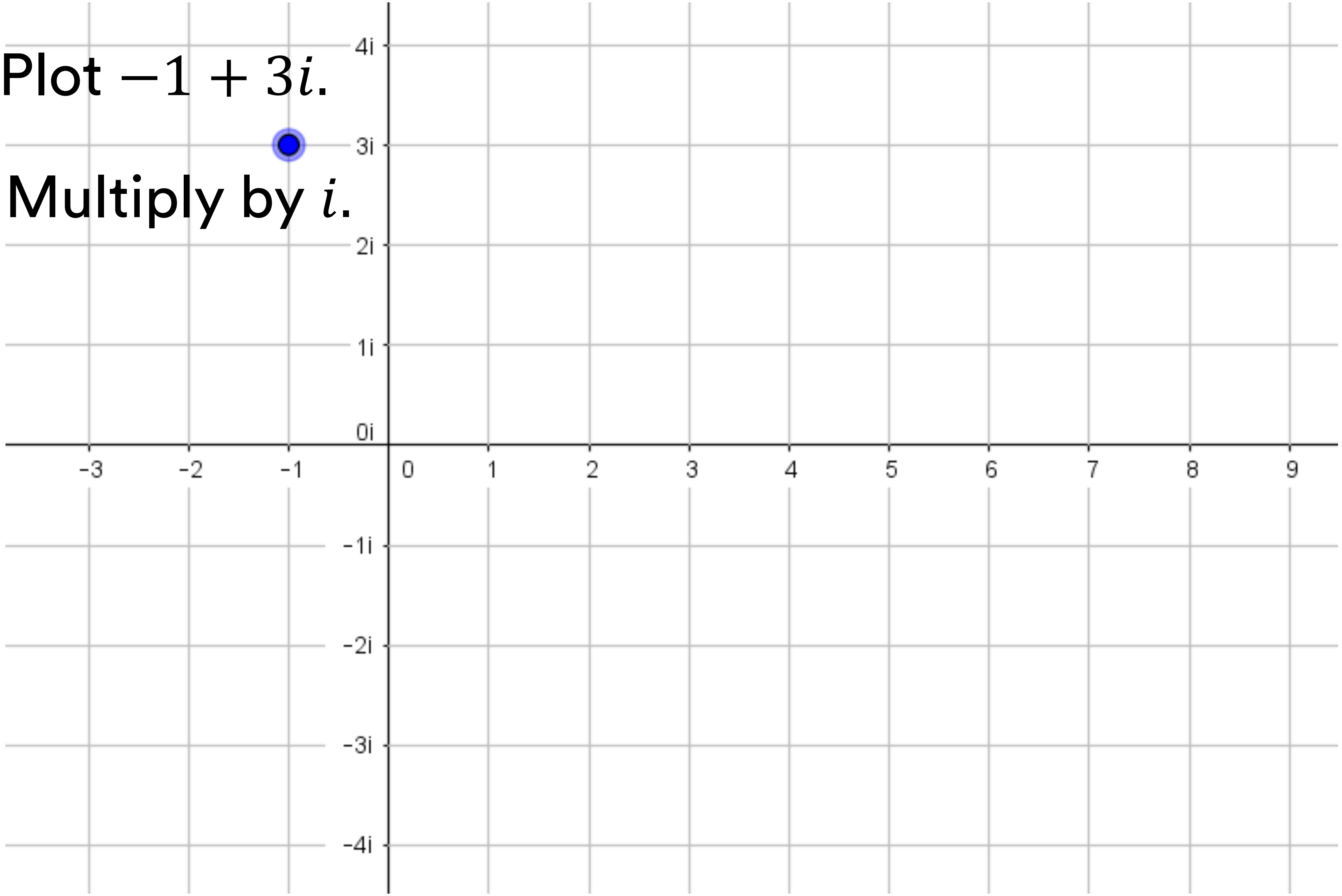


Plot $-1 + 3i$.



Plot $-1 + 3i$.

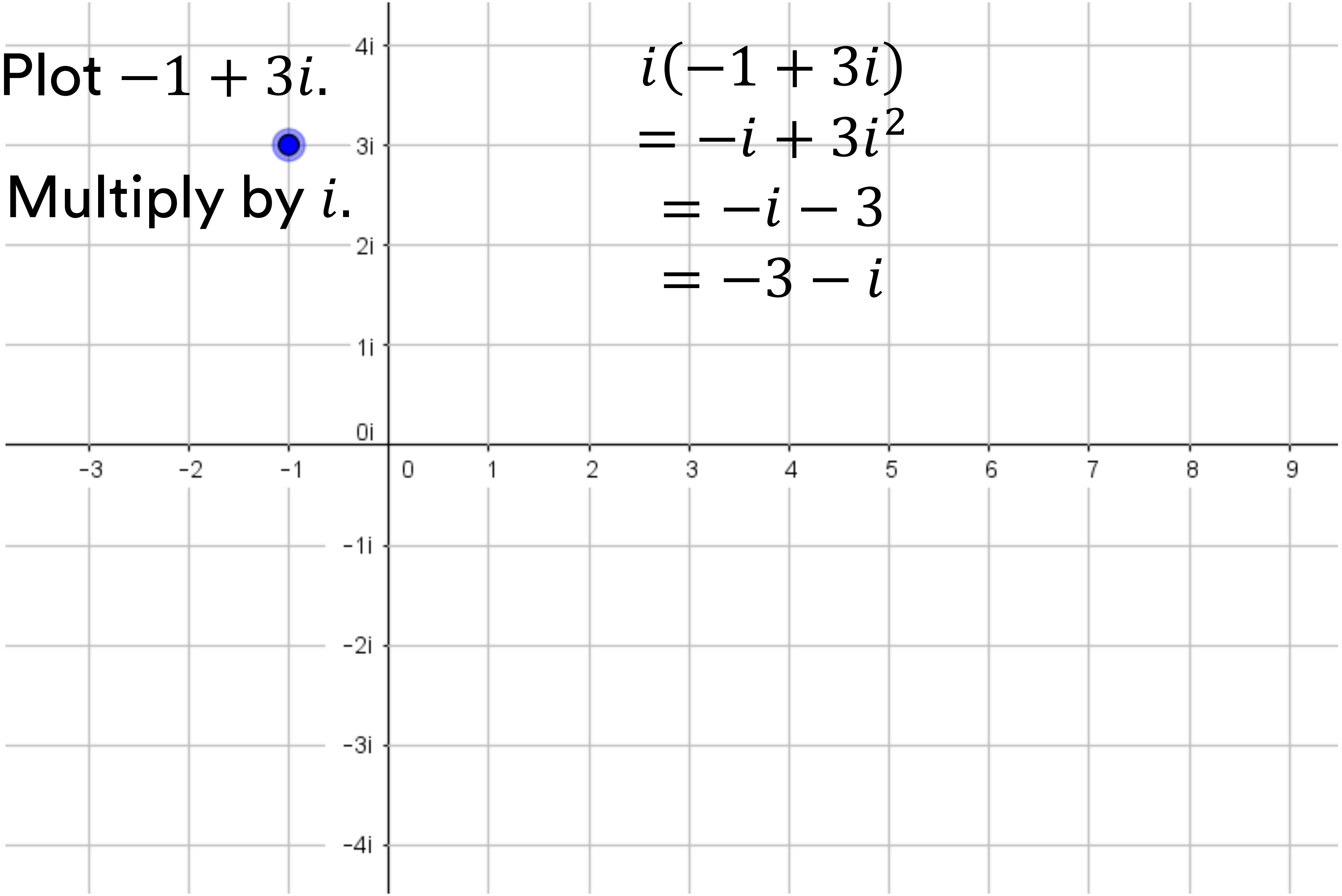
Multiply by i.



Plot $-1 + 3i$.

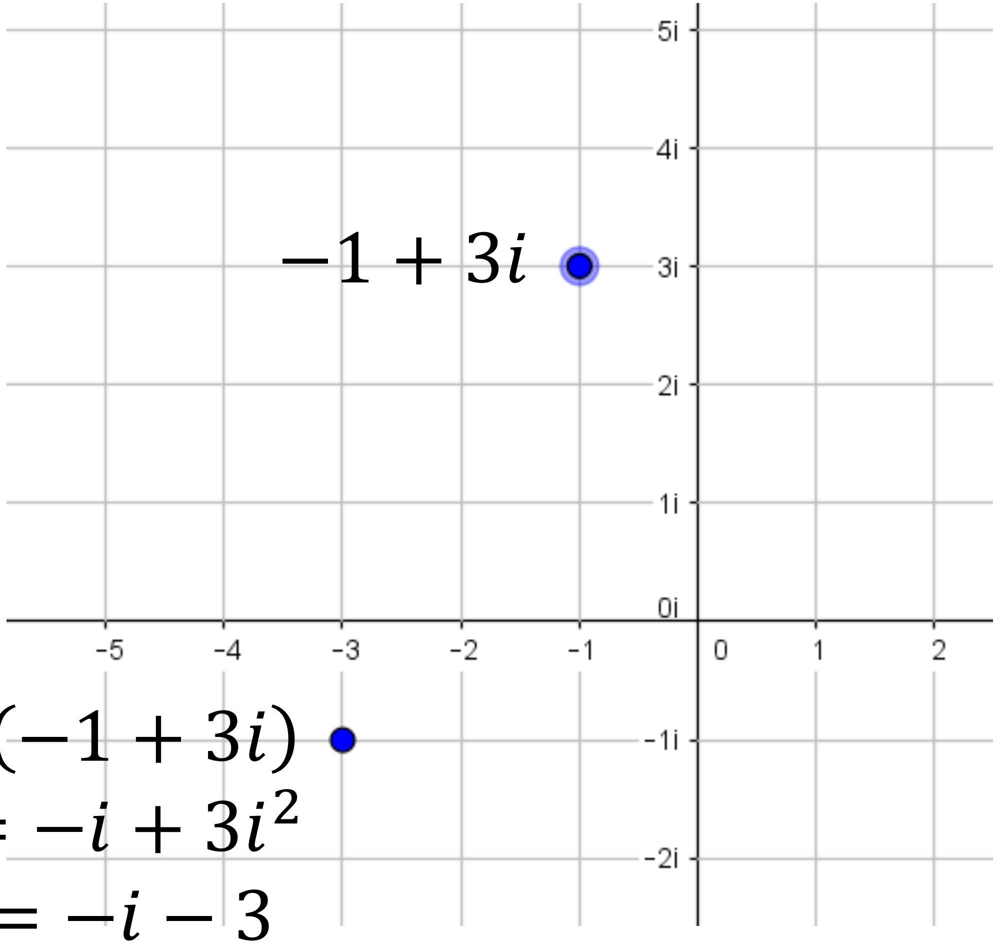
Multiply by i.

$$i(-1 + 3i)$$
$$= -i + 3i^2$$
$$= -i - 3$$
$$= -3 - i$$



Plot $-1 + 3i$.

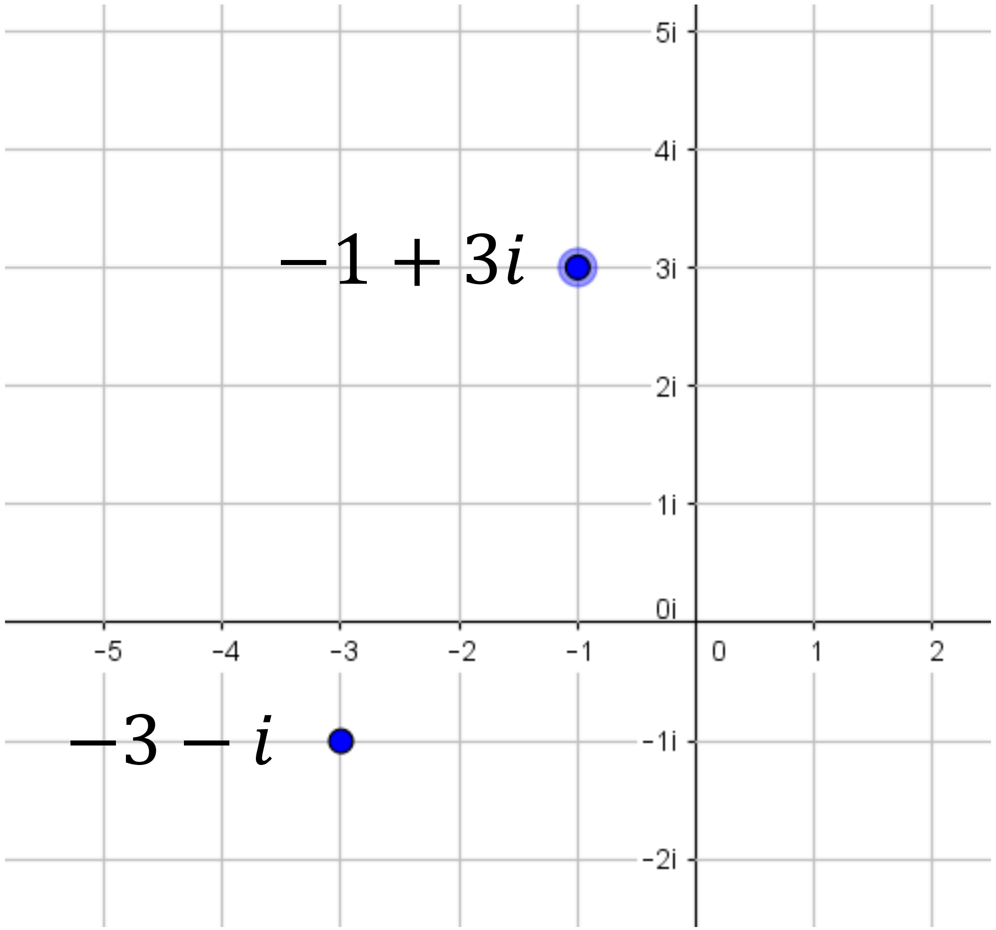
Multiply by i.



$$i(-1 + 3i)$$
$$= -i + 3i^2$$
$$= -i - 3$$
$$= -3 - i$$

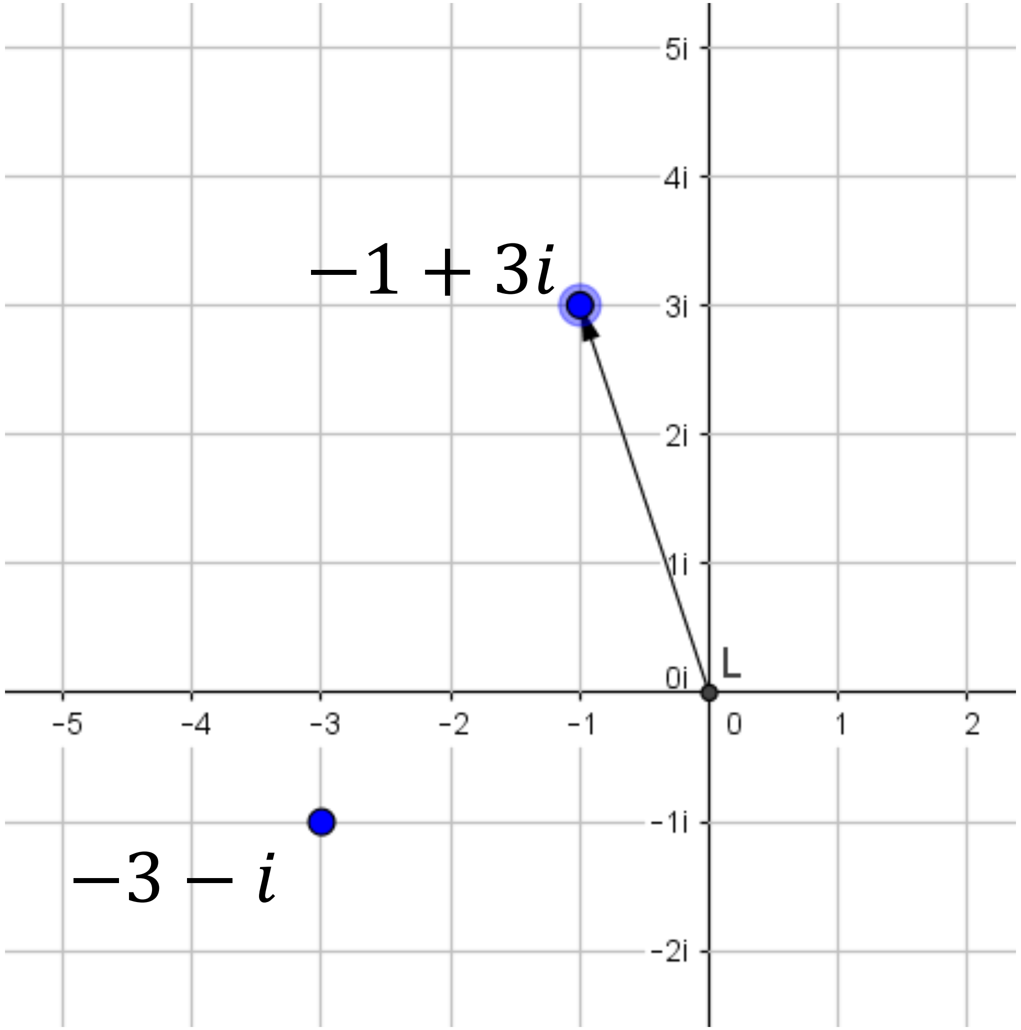
Plot $-1 + 3i$.

Multiply by i.



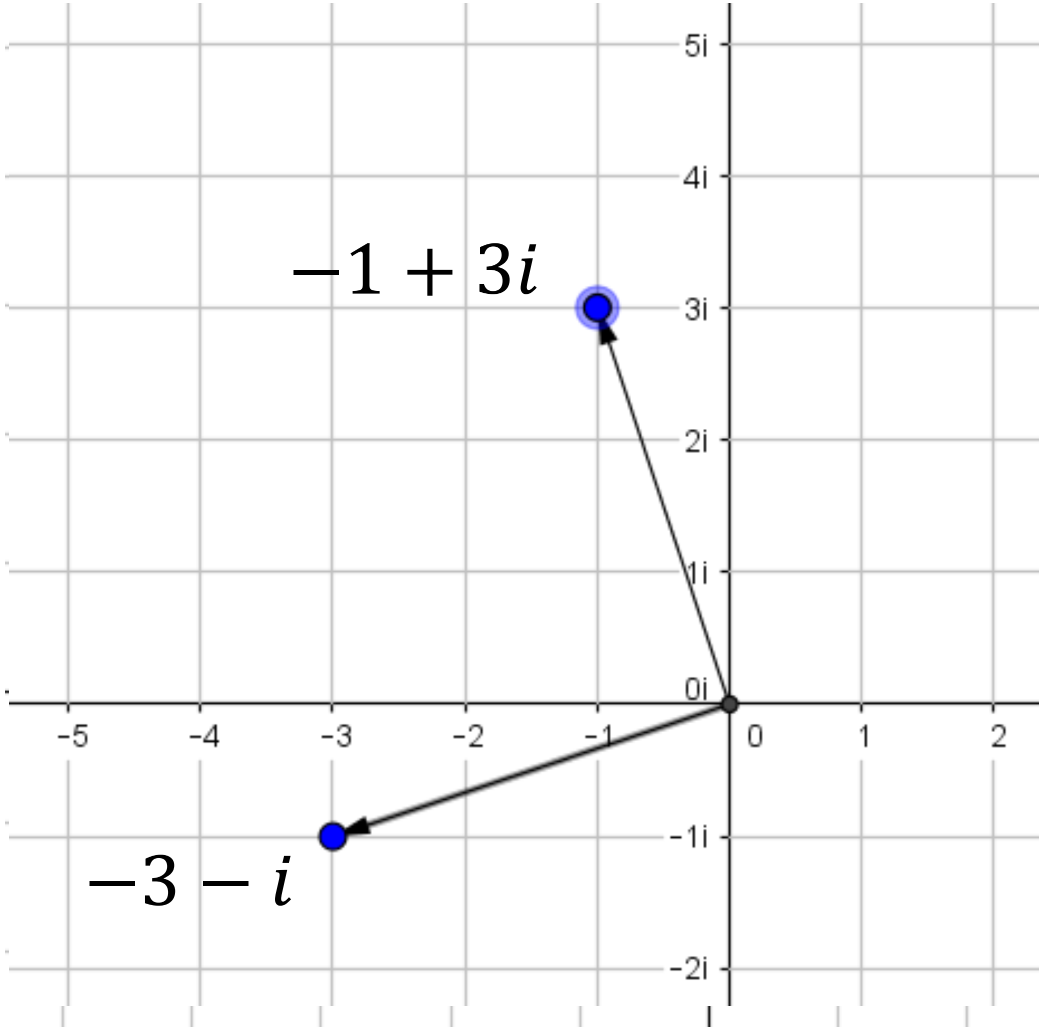
Plot $-1+3i$.

Multiply by i.



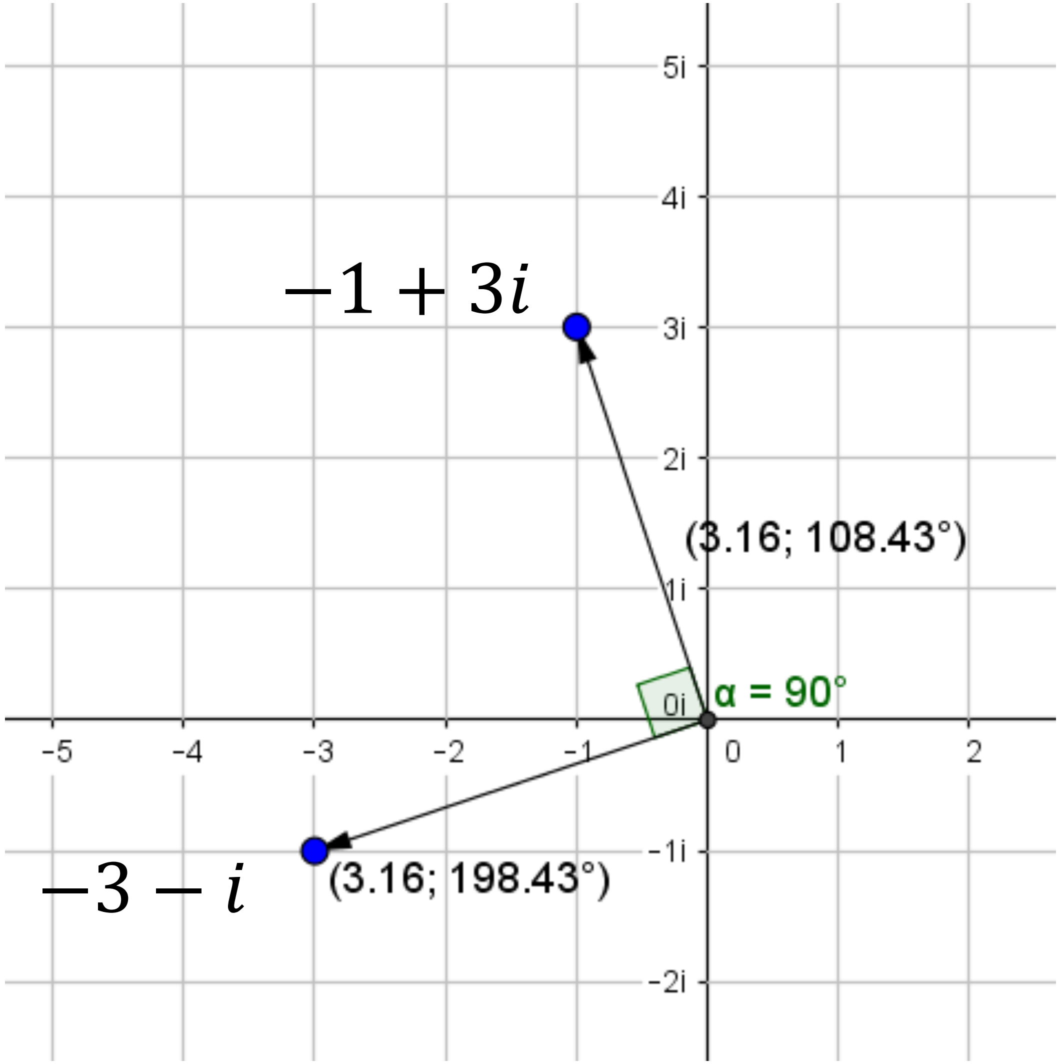
Plot $-1+3i$.

Multiply by i.



Plot $-1 + 3i$.

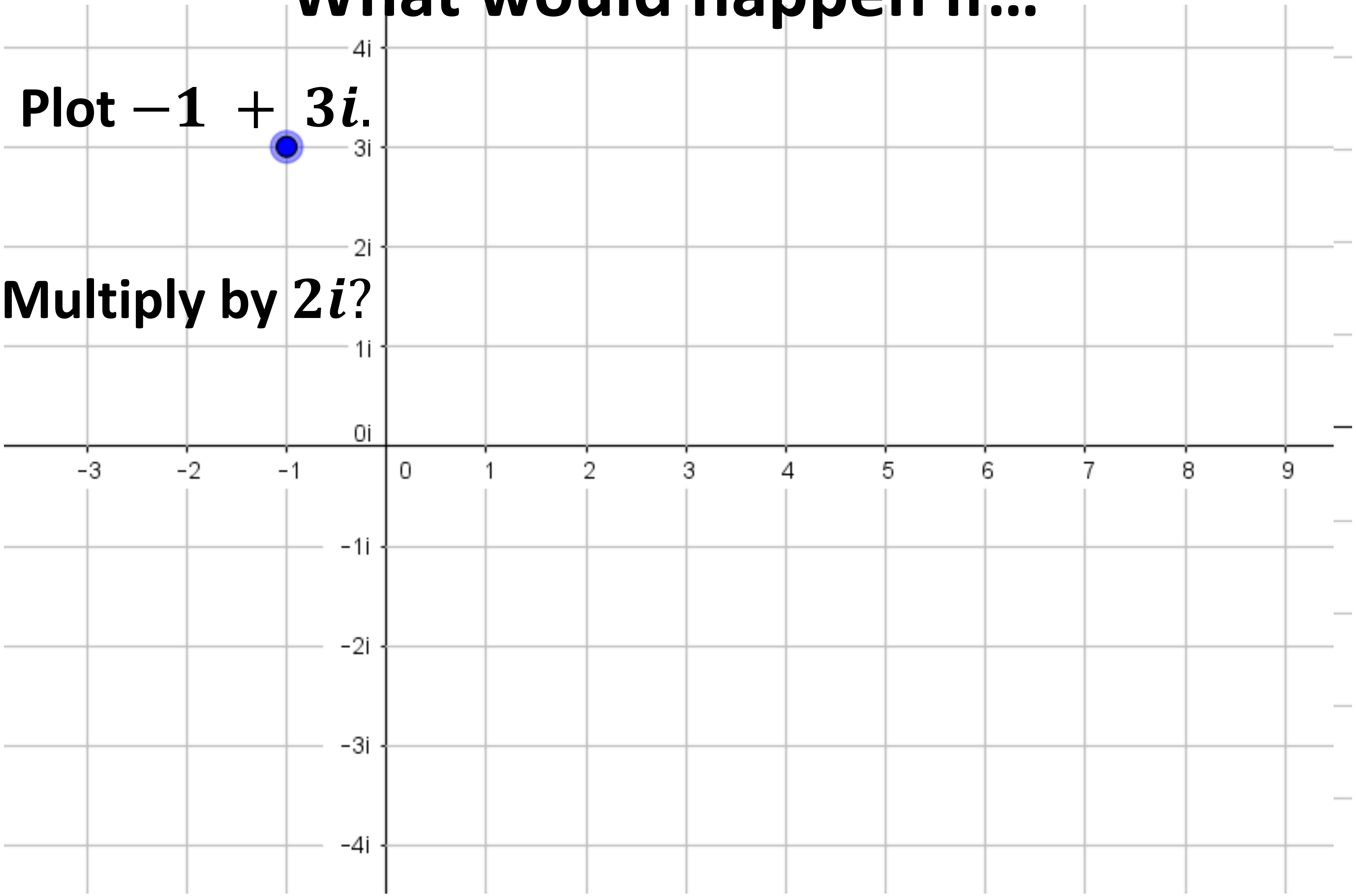
Multiply by i.



What would happen if…

Plot $-1 + 3i$.

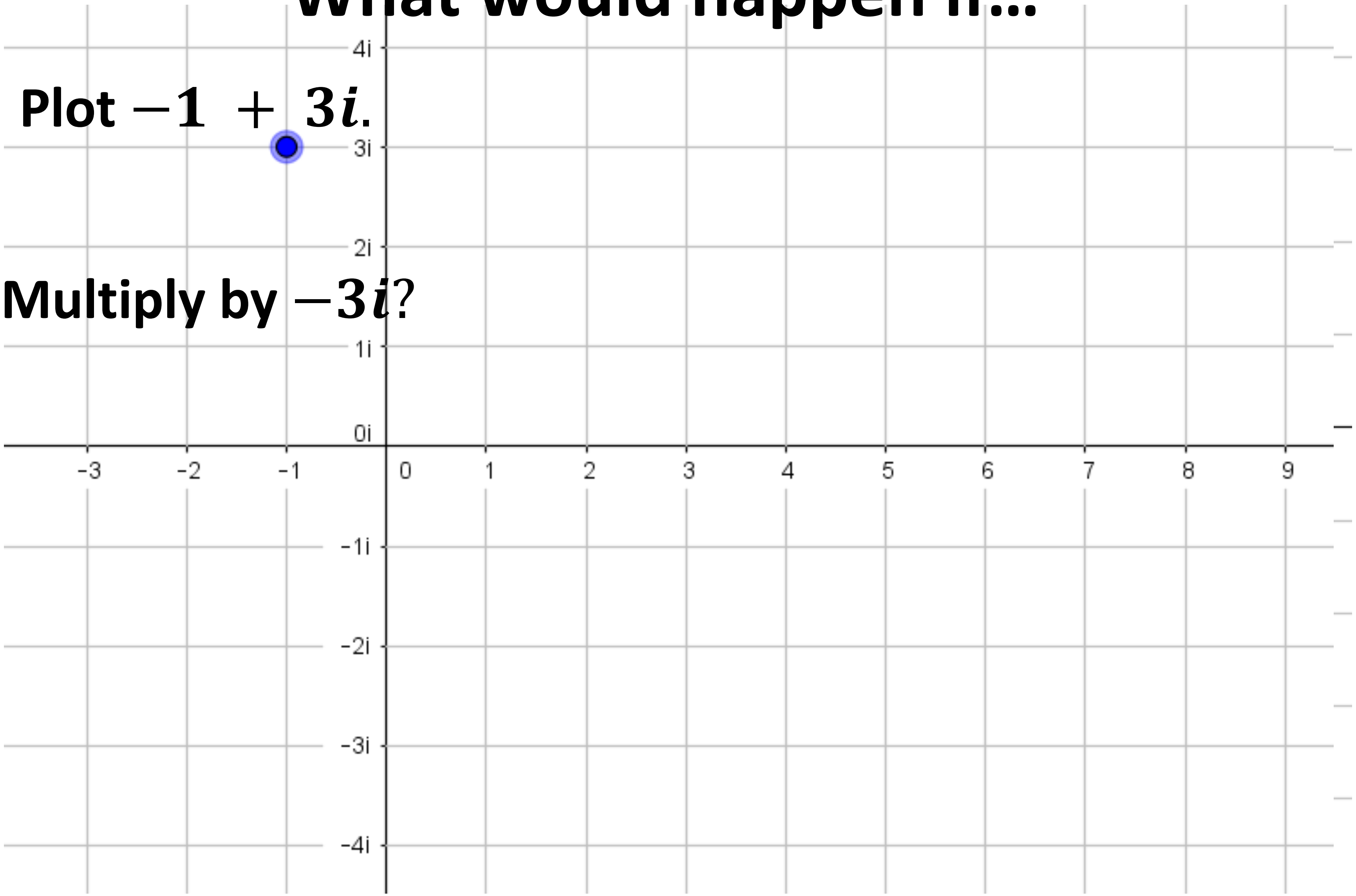
Multiply by $2i$?



What would happen if…

Plot $-1 + 3i$.

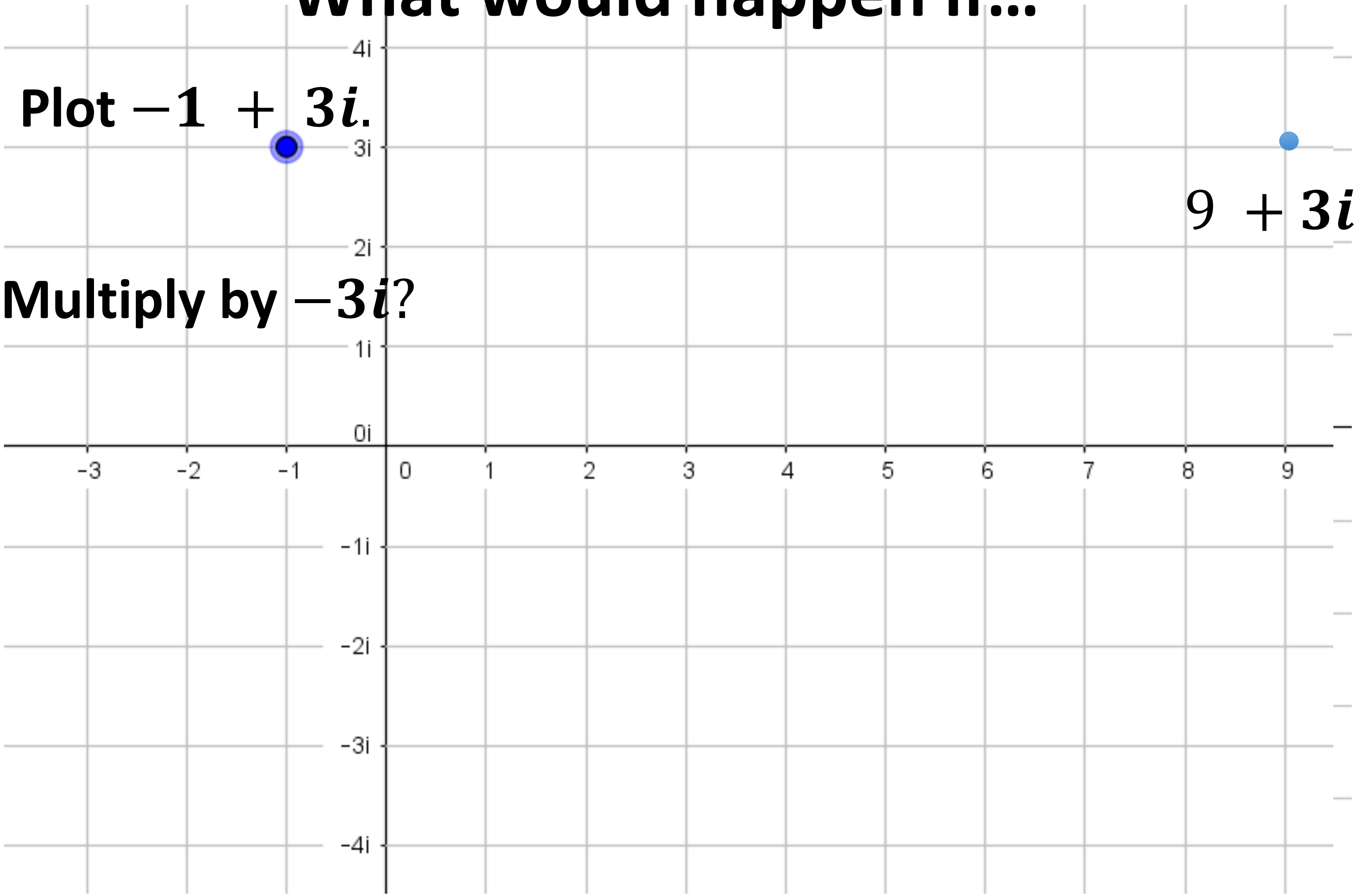
Multiply by $-3i$?



What would happen if…

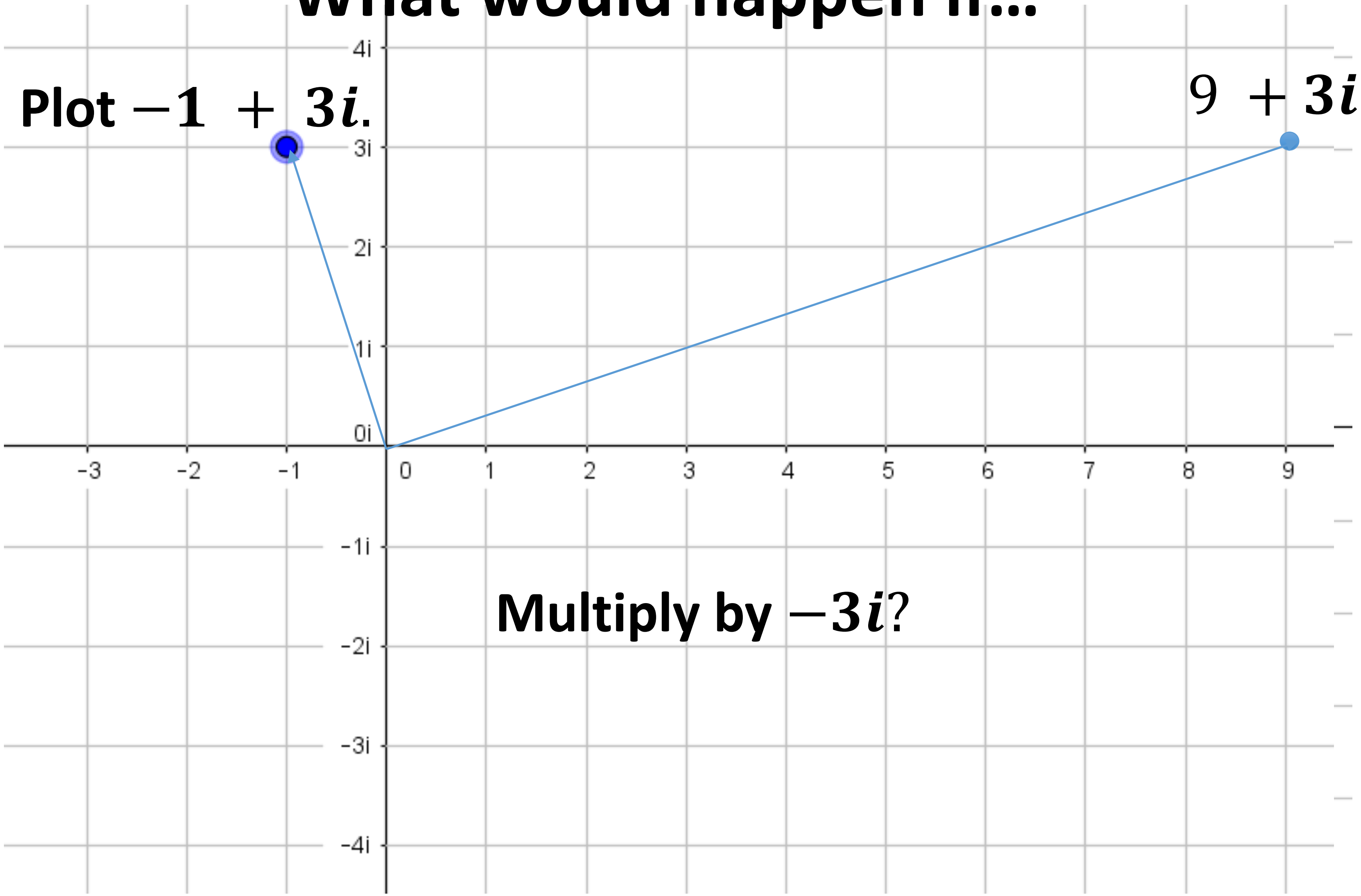
Plot $-1 + 3i$.

Multiply by $-3i$?

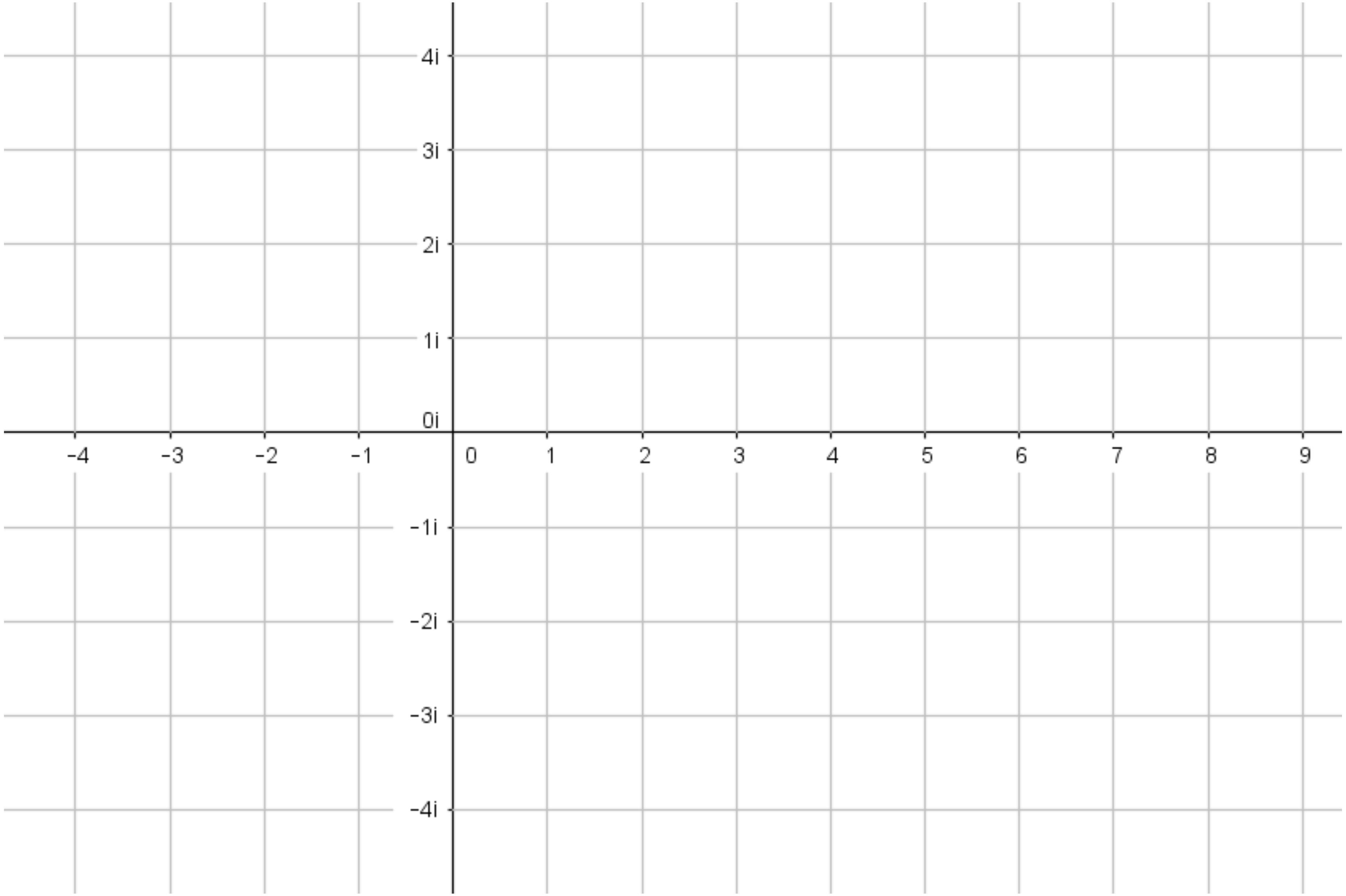


What would happen if…

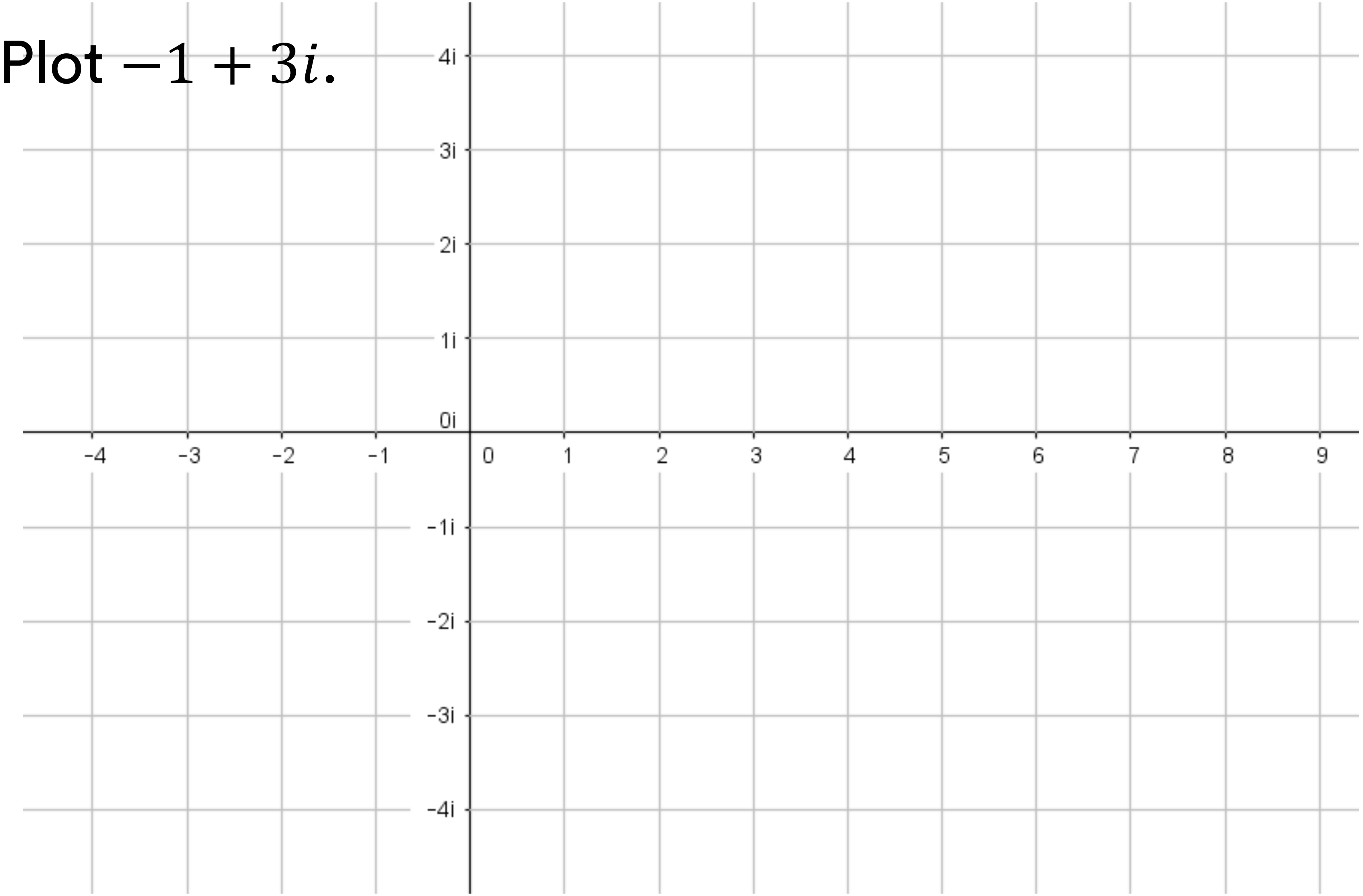
Plot $-1 + 3i$.



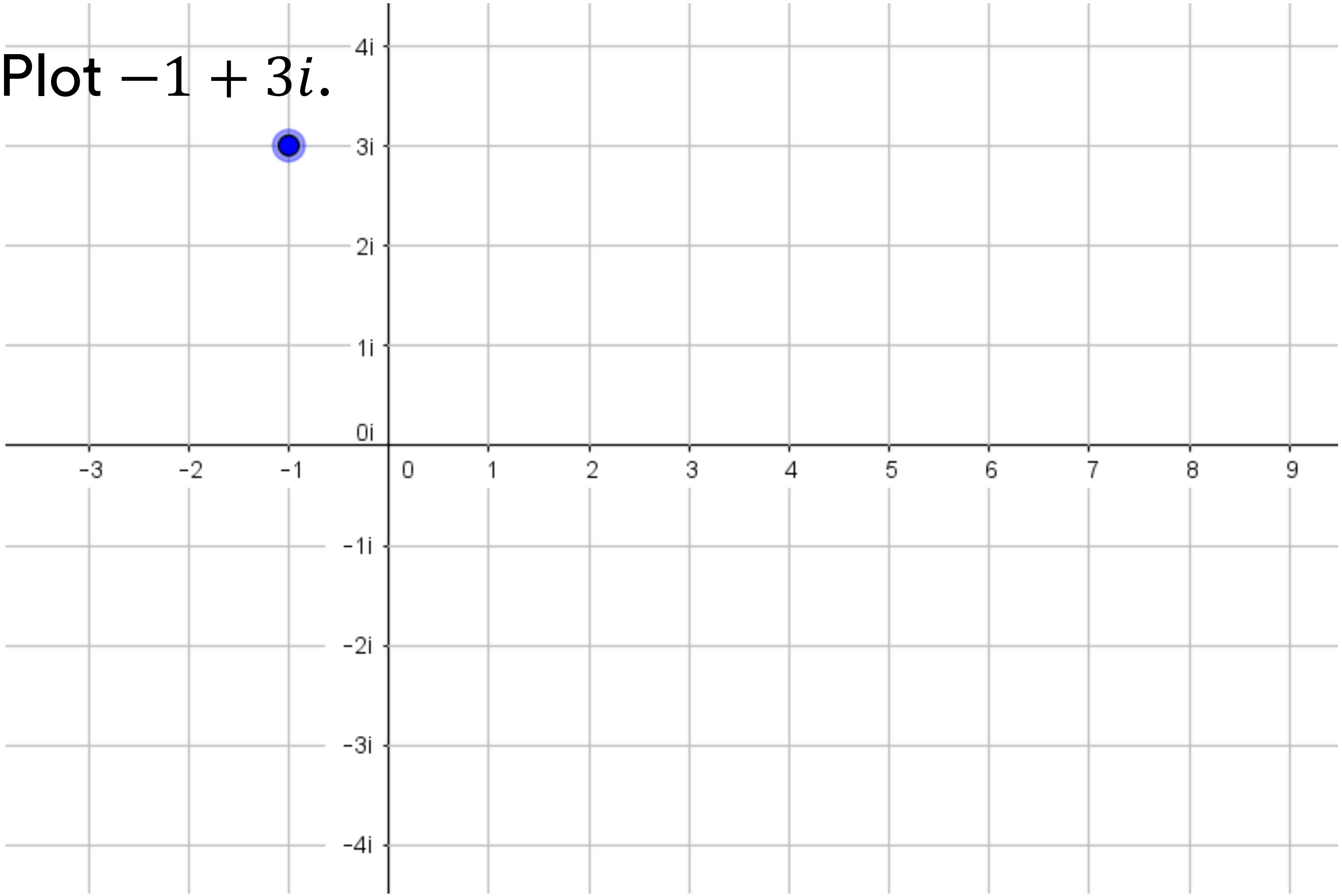
Multiply by $-3i$?


4i
3i
2i
1i
0i
-4
-3
-2
-1
0
1
2
3
4
5
6
7
8
9
-1i
-2i
-3i
-4i

Plot $-1 + 3i$.

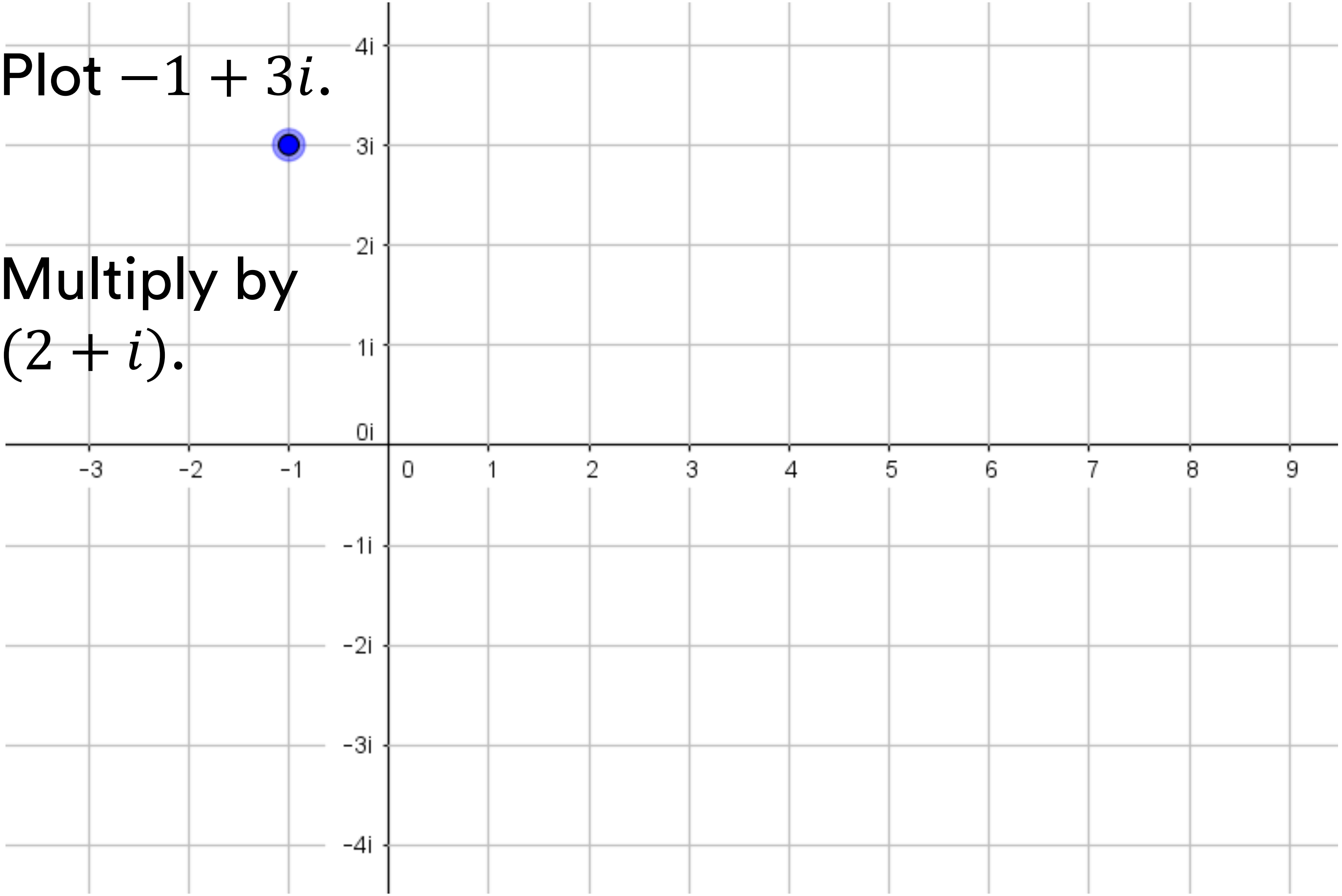


Plot $-1 + 3i$.



Plot $-1 + 3i$.

Multiply by $(2 + i)$.

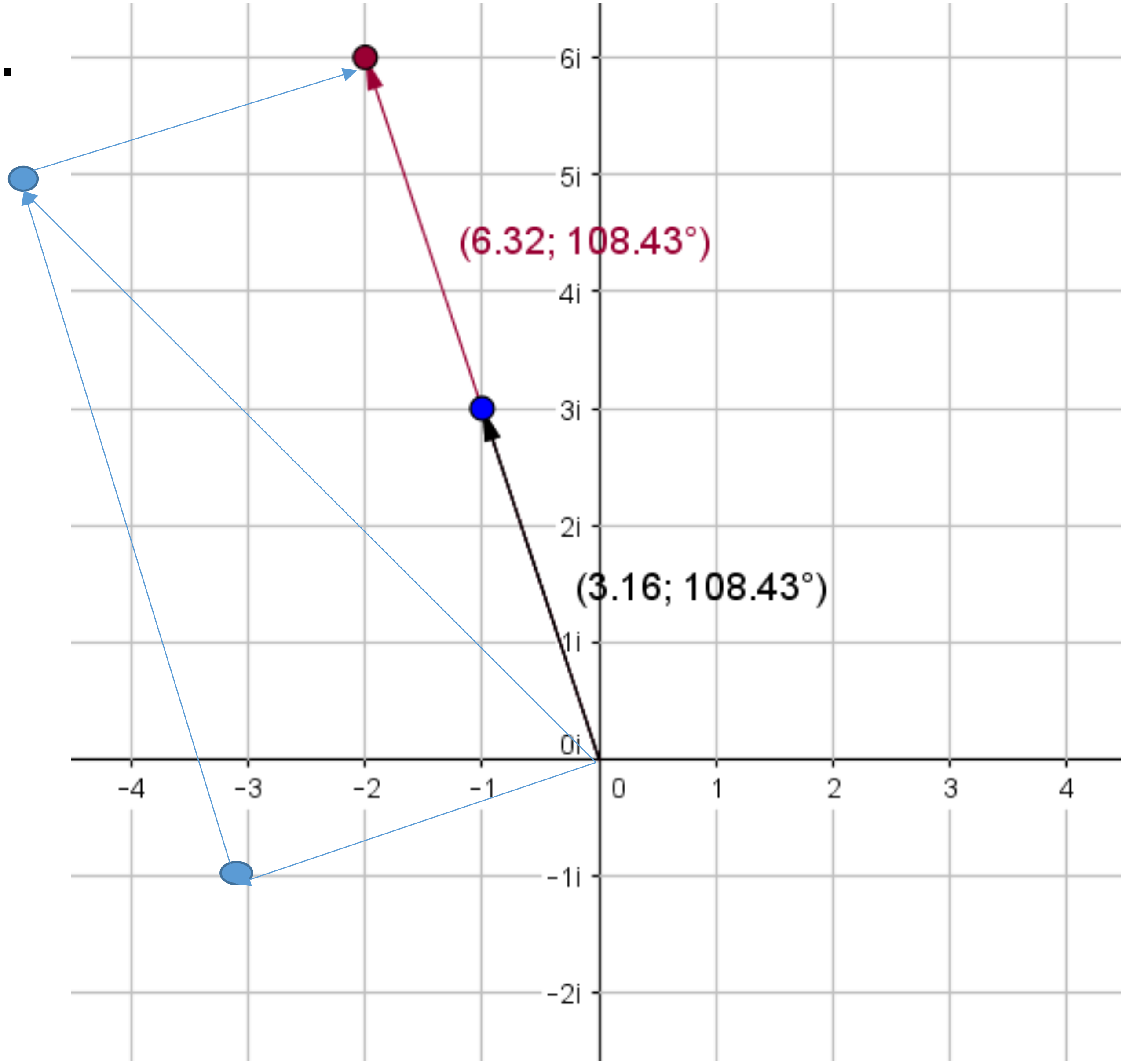


Plot $-1 + 3i$.

Multiply by $(2 + i)$.

Multiply by 2.

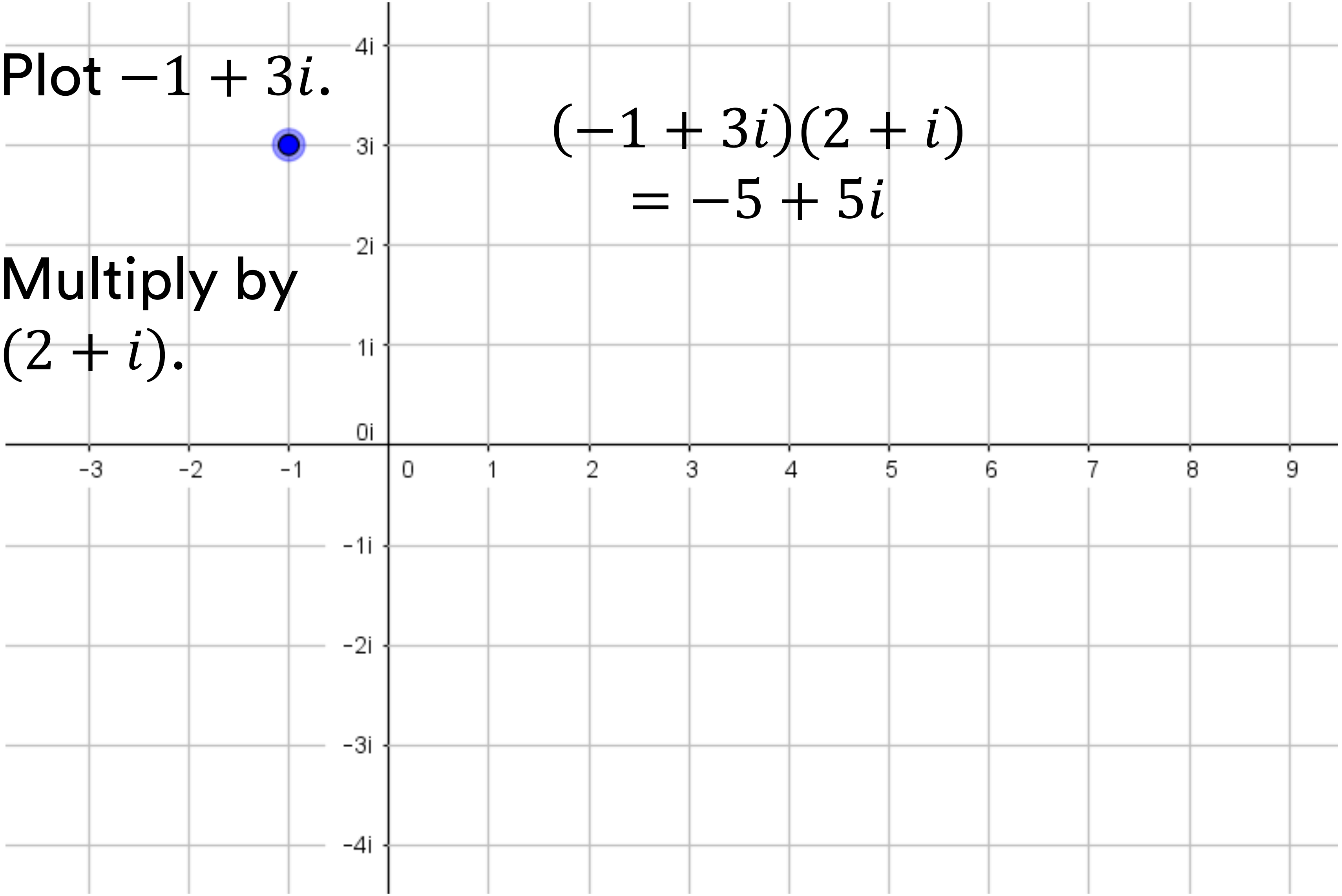
Rotate $90°$



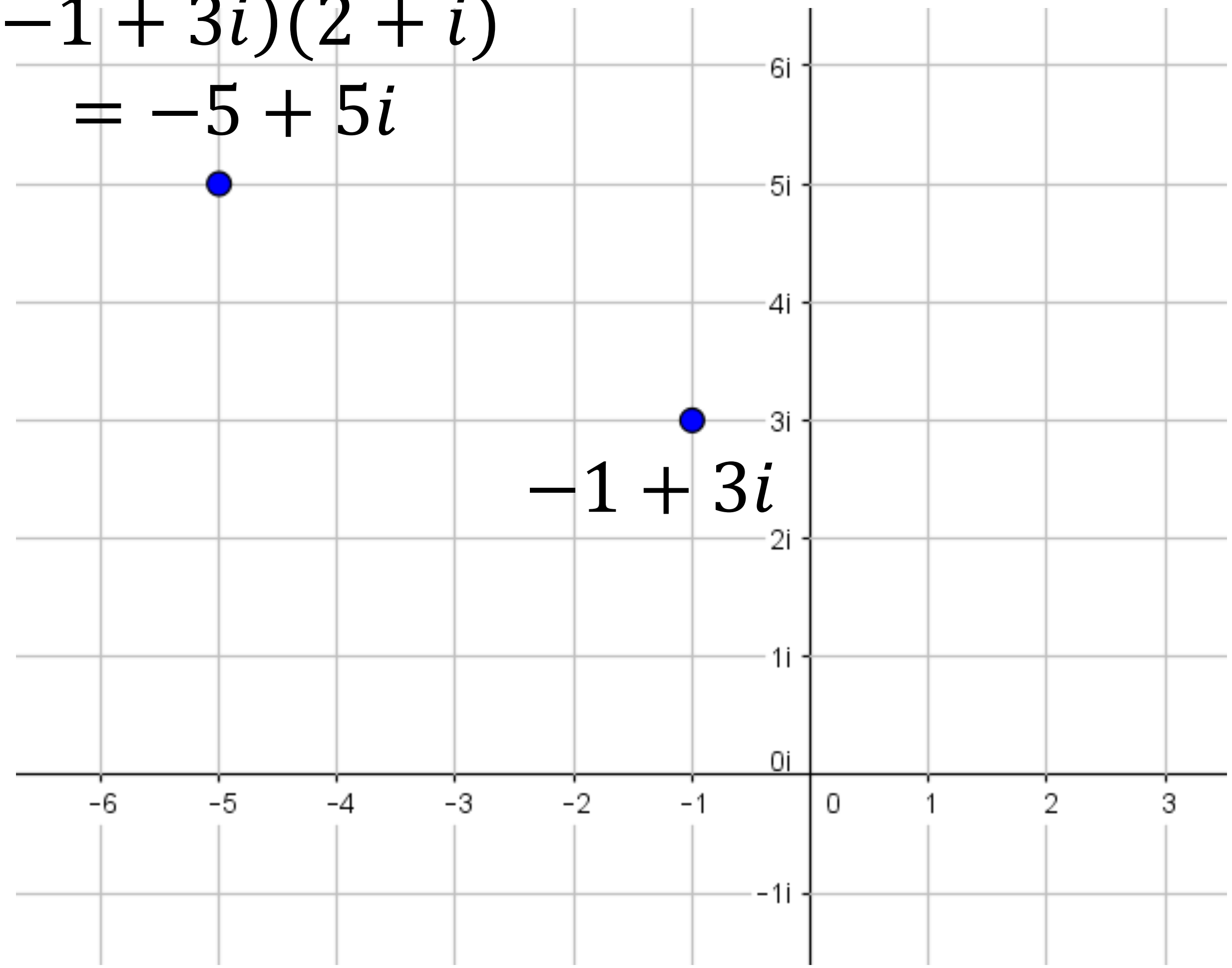
Plot $-1 + 3i$.

Multiply by $(2 + i)$.

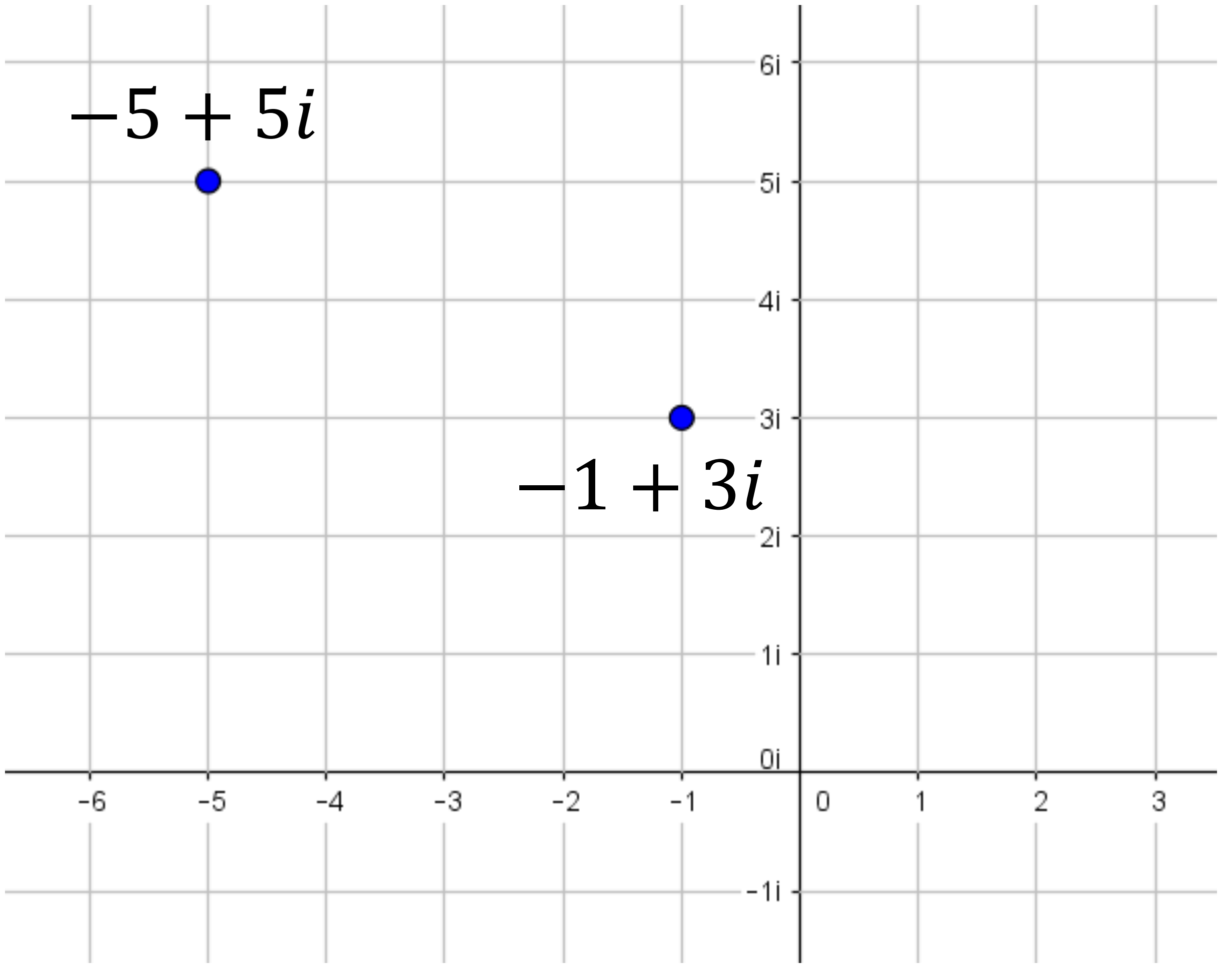
$$(-1 + 3i)(2 + i)$$
$$= -5 + 5i$$



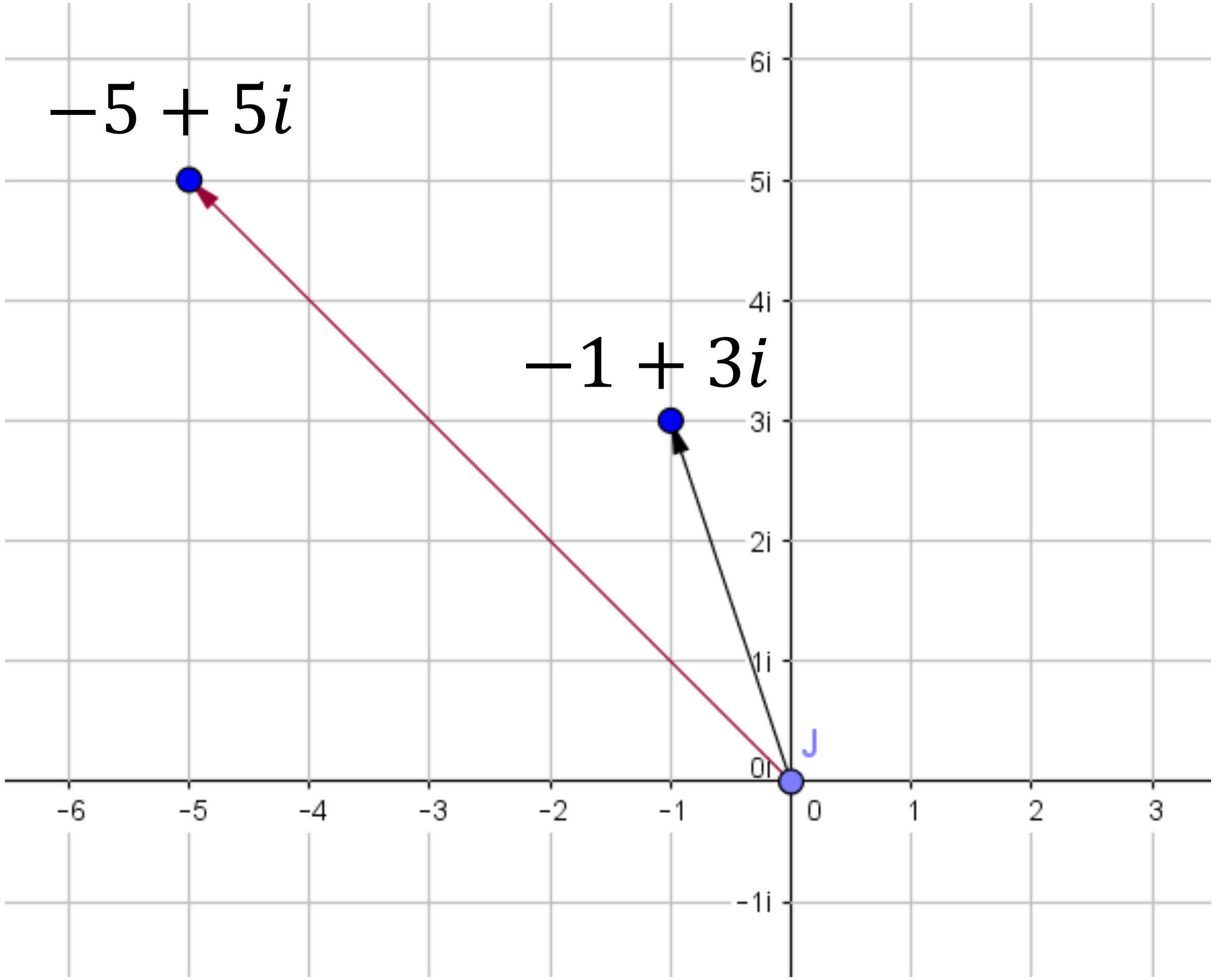
$$(-1 + 3i)(2 + i)$$
$$= -5 + 5i$$



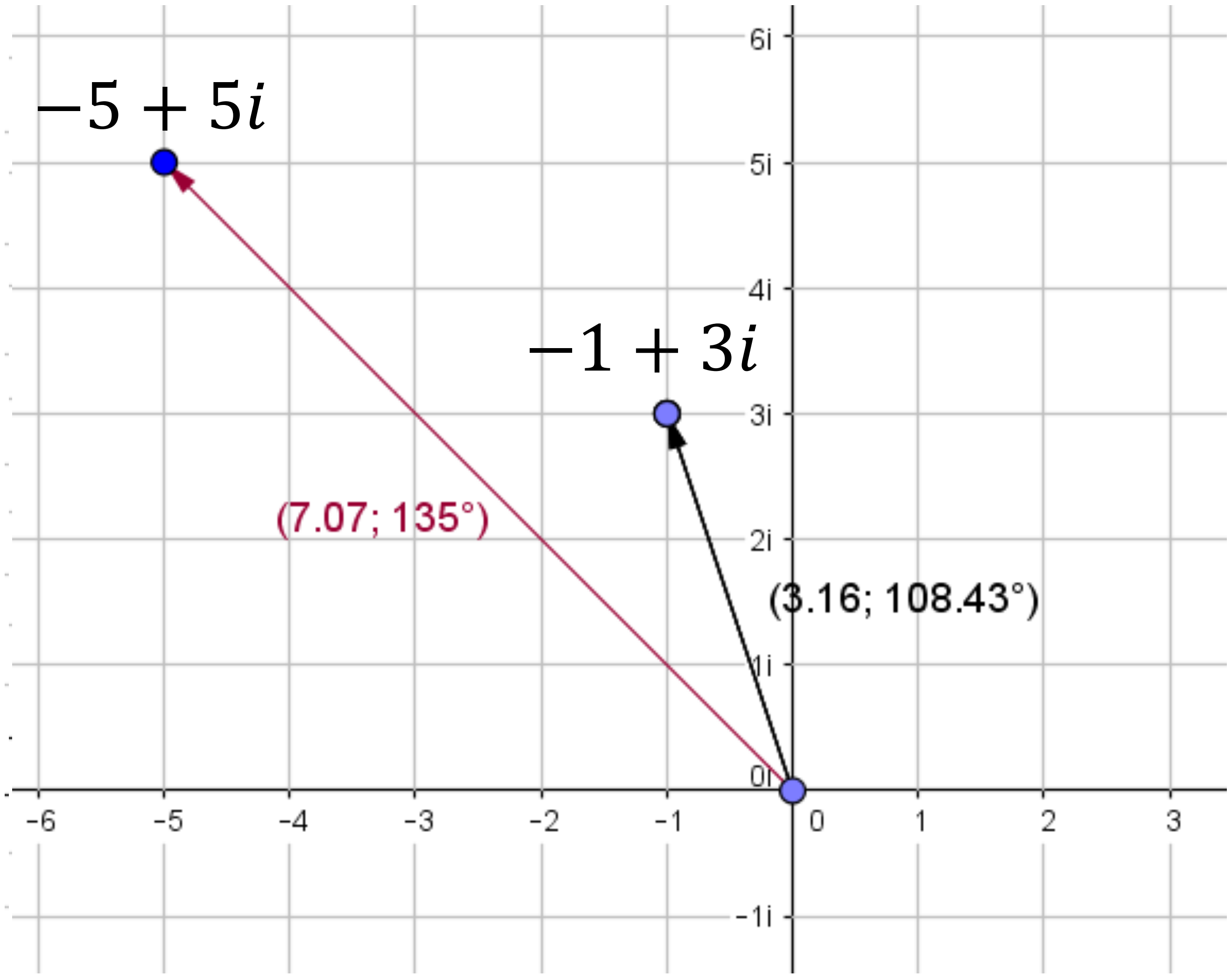
$$-1 + 3i$$



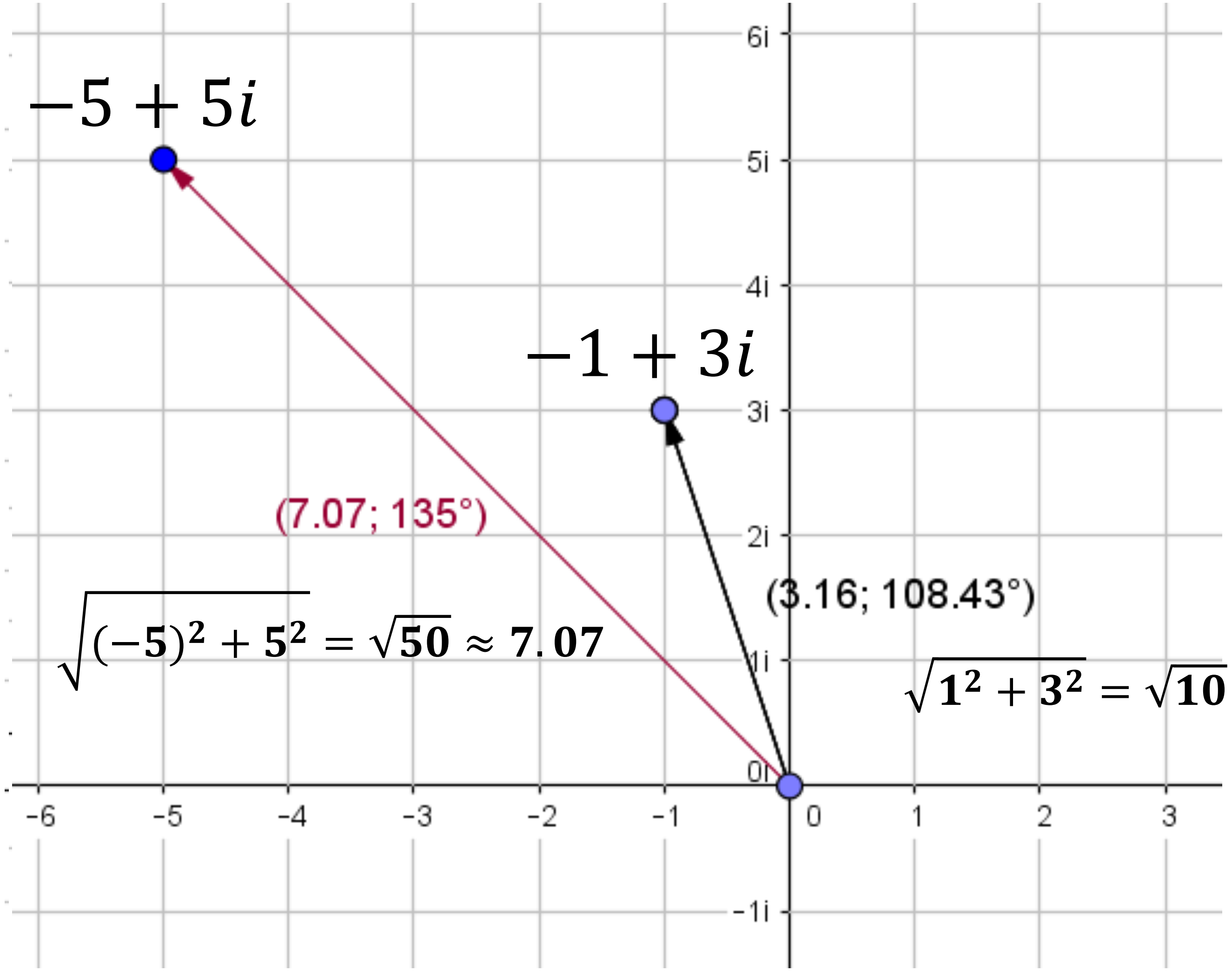
$-5+5i$
$-1+3i$
6i
5i
4i
3i
2i
1i
0i
-1i
-6
-5
-4
-3
-2
-1
0
1
2
3



$-5+5i$
$-1+3i$
6i
5i
4i
3i
2i
1i
0i
-1i
J
-6
-5
-4
-3
-2
-1
0
1
2
3

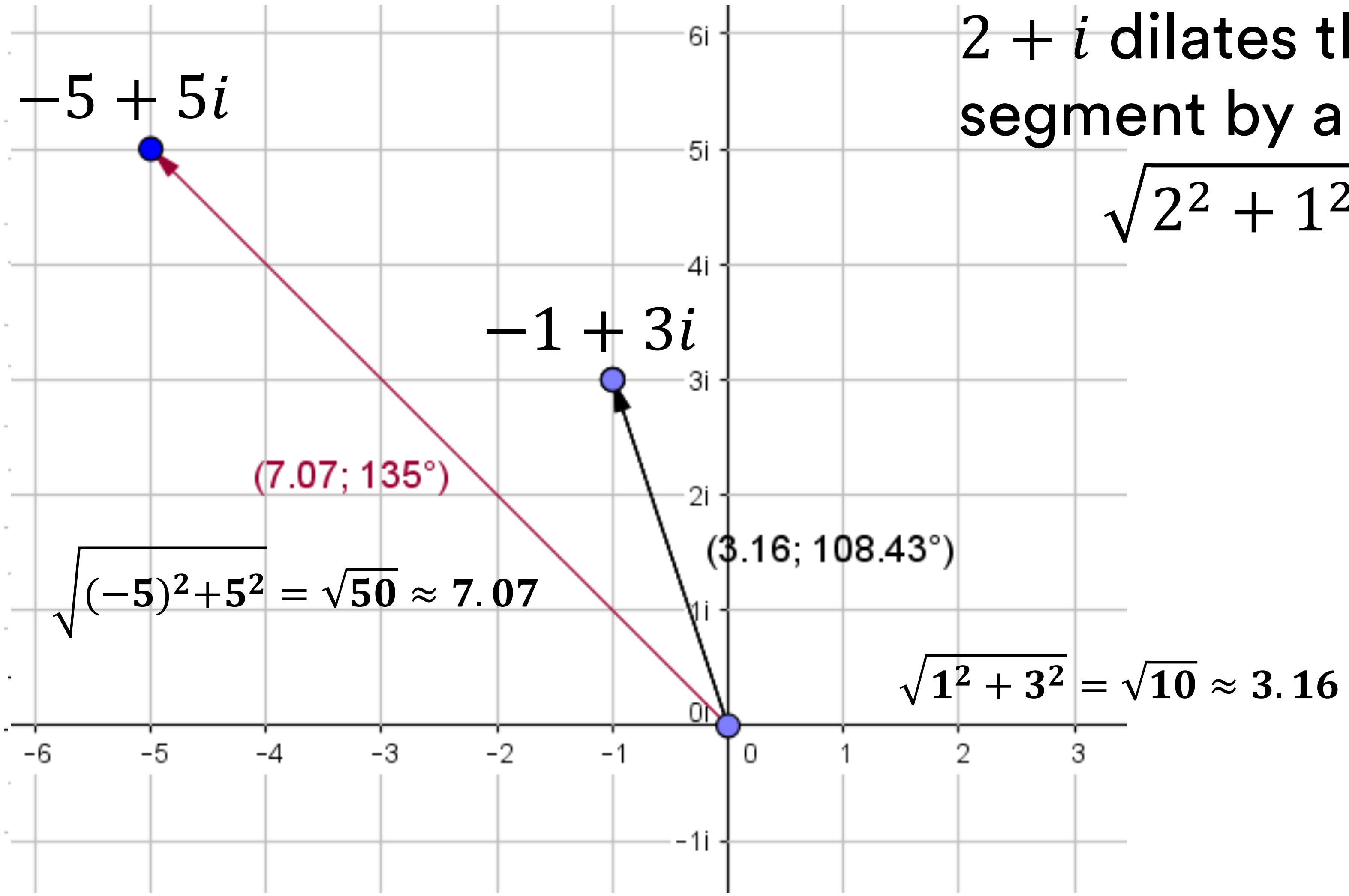


$-5+5i$
$-1+3i$
(7.07; 135°)
(3.16; 108.43°)
6i
5i
4i
3i
2i
1i
0i
-1i
-6
-5
-4
-3
-2
-1
0
1
2
3

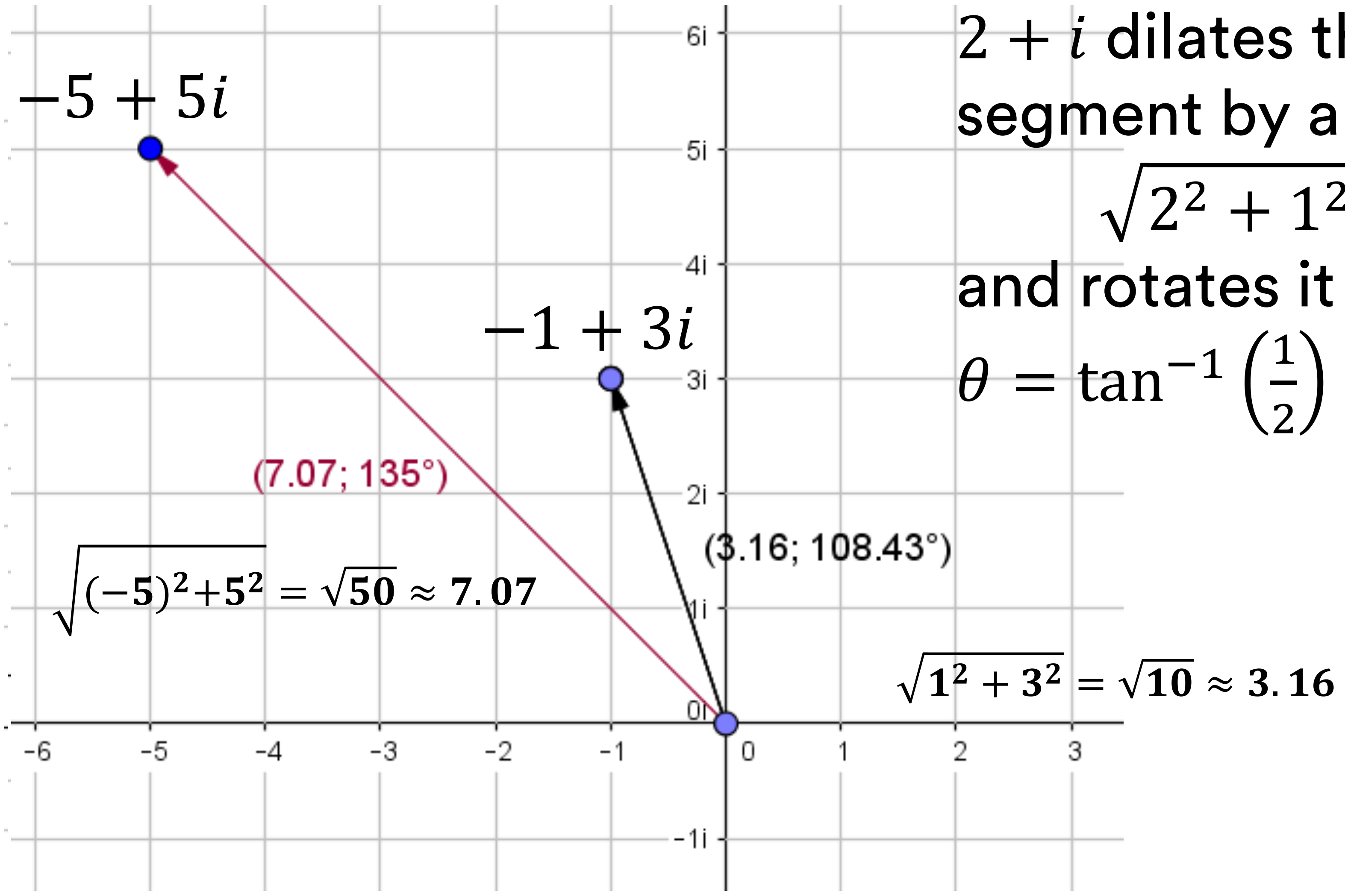


$\sqrt{(-5)^2+5^2}=\sqrt{50}\approx 7.07$

$\sqrt{1^2+3^2}=\sqrt{10}\approx 3.16$



$2+i$ dilates the original segment by a factor of $\sqrt{2^2+1^2} = \sqrt{5}$



$2 + i$ dilates the original segment by a factor of

$$\sqrt{2^2 + 1^2} = \sqrt{5}$$

and rotates it by

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$ degrees.

Using the complex number on the left as your starting point, predict what will happen when you multiply by the complex number on the right.

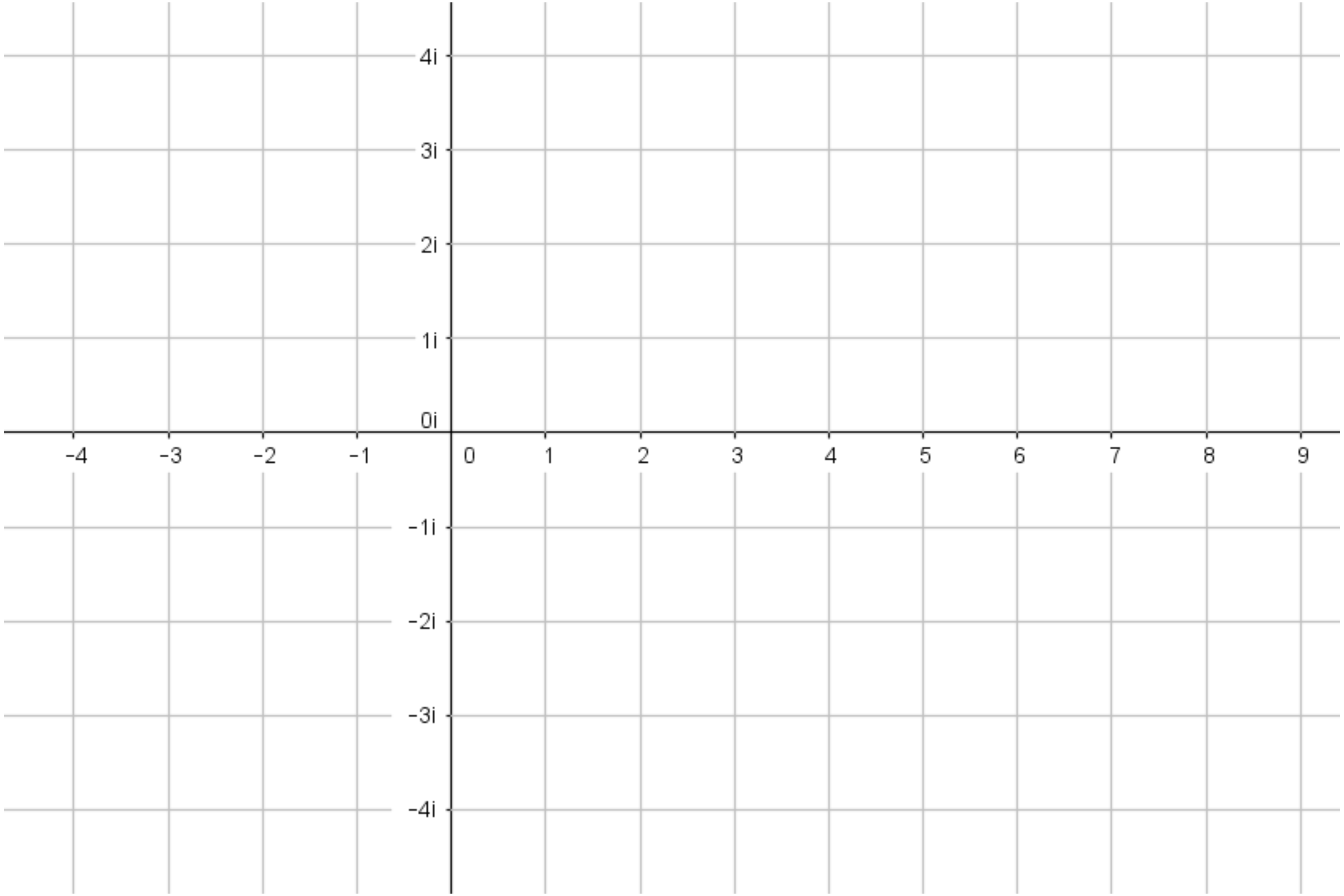
Plot your original point, the new point, and confirm your prediction by using algebra or by measuring.

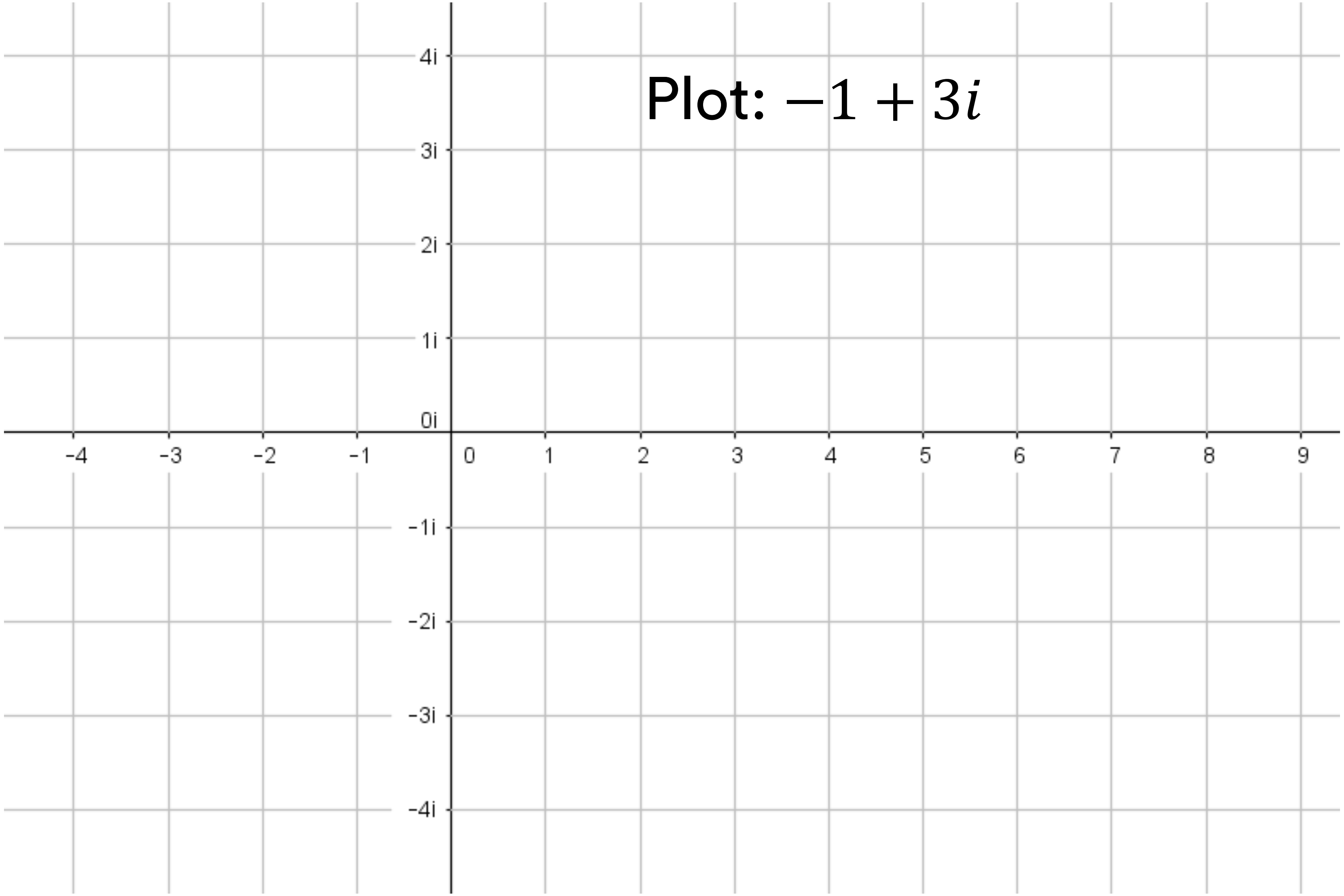
$$(-2 - i)(2 - i)$$
$$(3 + 2i)(2 - 3i)$$
$$(2 + i)(2 + i)$$
$$(4 - i)(4 + i)$$

Solutions

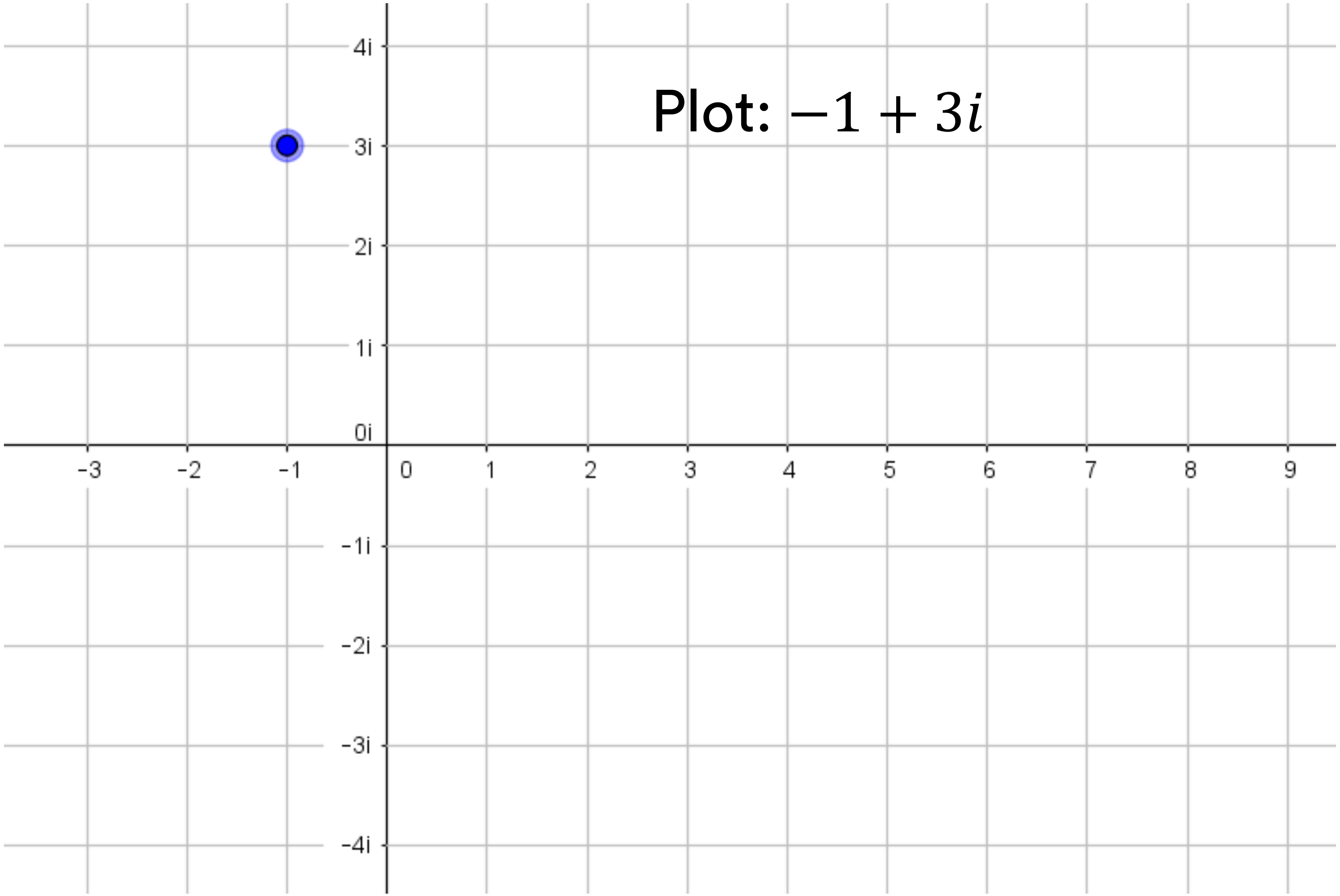
Initial Point	New Point	Dilation	Approximate Angle of Rotation
$-2-i$	-5	$\sqrt{5}$	$-26.5°$
$3+2i$	$12-5i$	$\sqrt{13}$	$-51.9°$
$2+i$	$3+4i$	$\sqrt{5}$	$26.5°$
$4-i$	17	$\sqrt{17}$	$14.0°$

What Does the Conjugate Represent Geometrically?


4i
3i
2i
1i
0i
-4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9
-1i
-2i
-3i
-4i



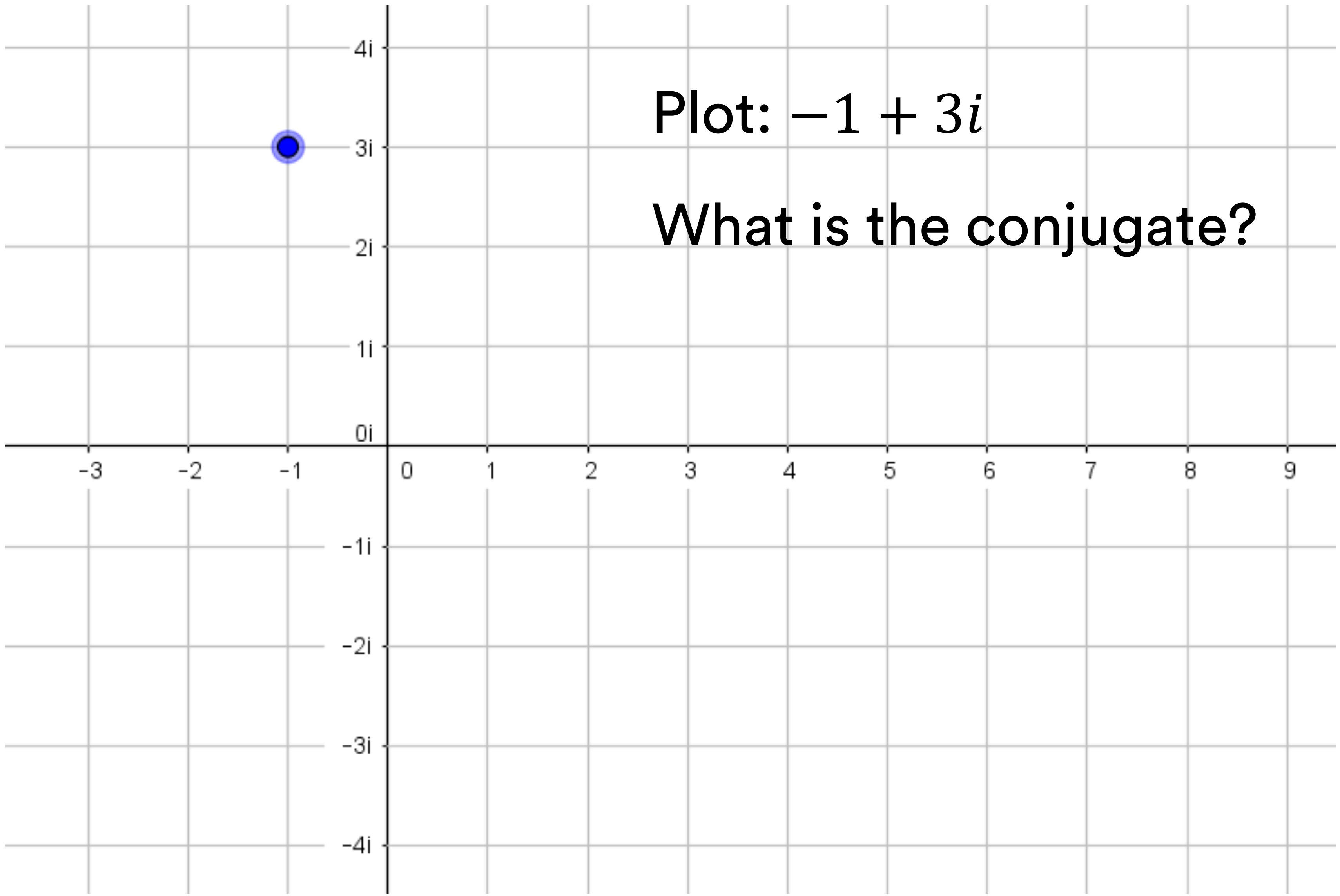
Plot: $-1 + 3i$

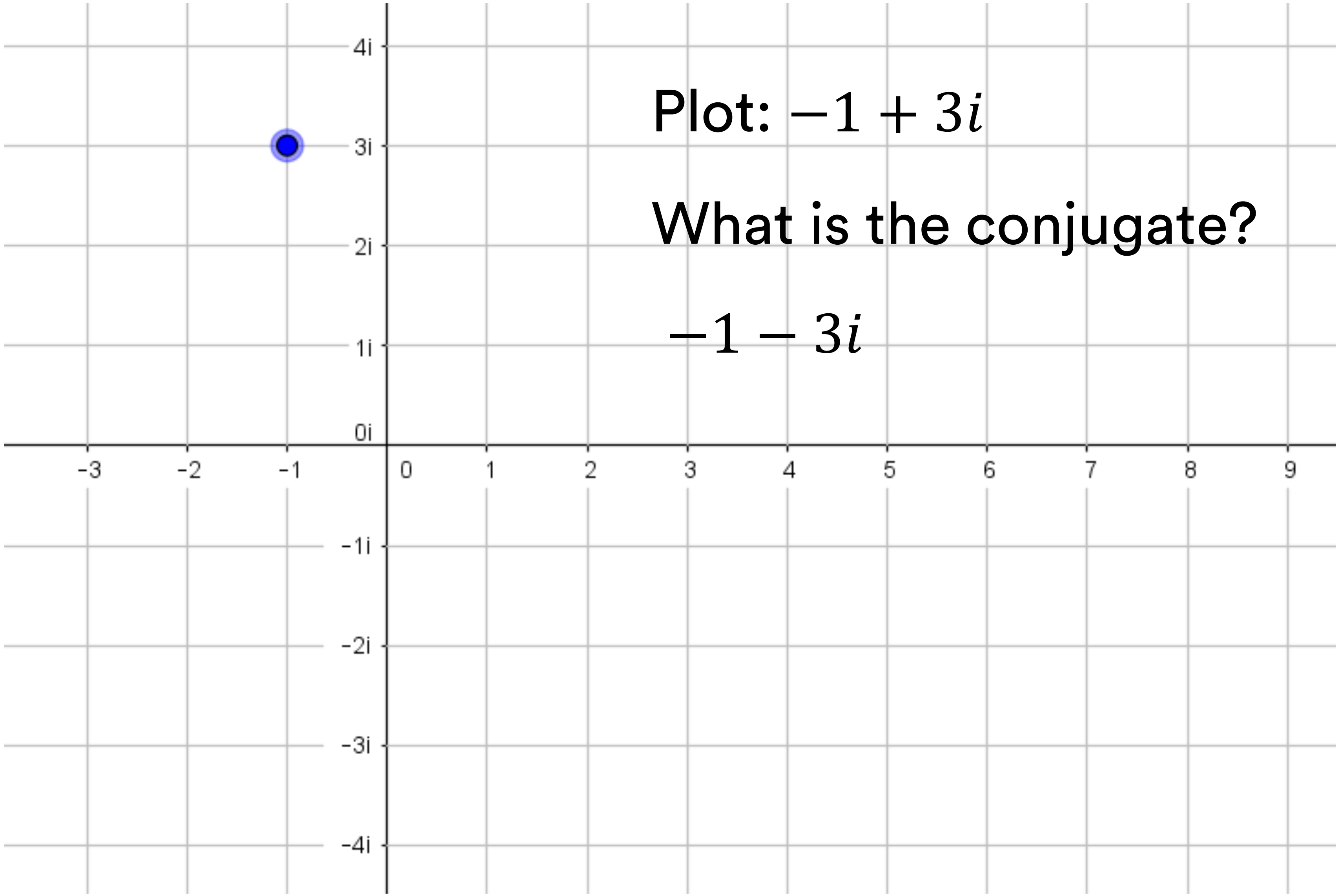


Plot: $-1 + 3i$

Plot: $-1 + 3i$

What is the conjugate?

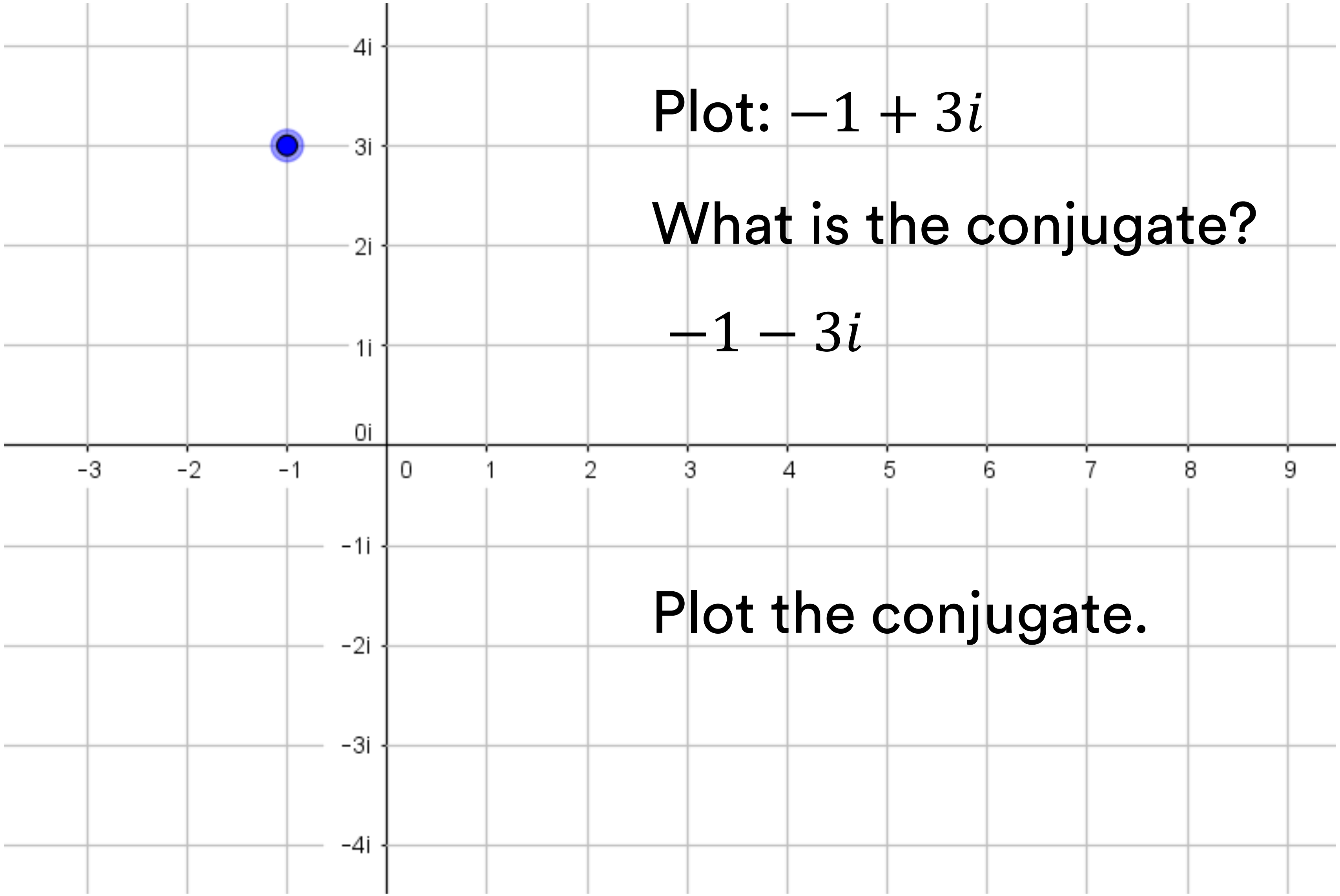




Plot: $-1 + 3i$

What is the conjugate?

$-1 - 3i$

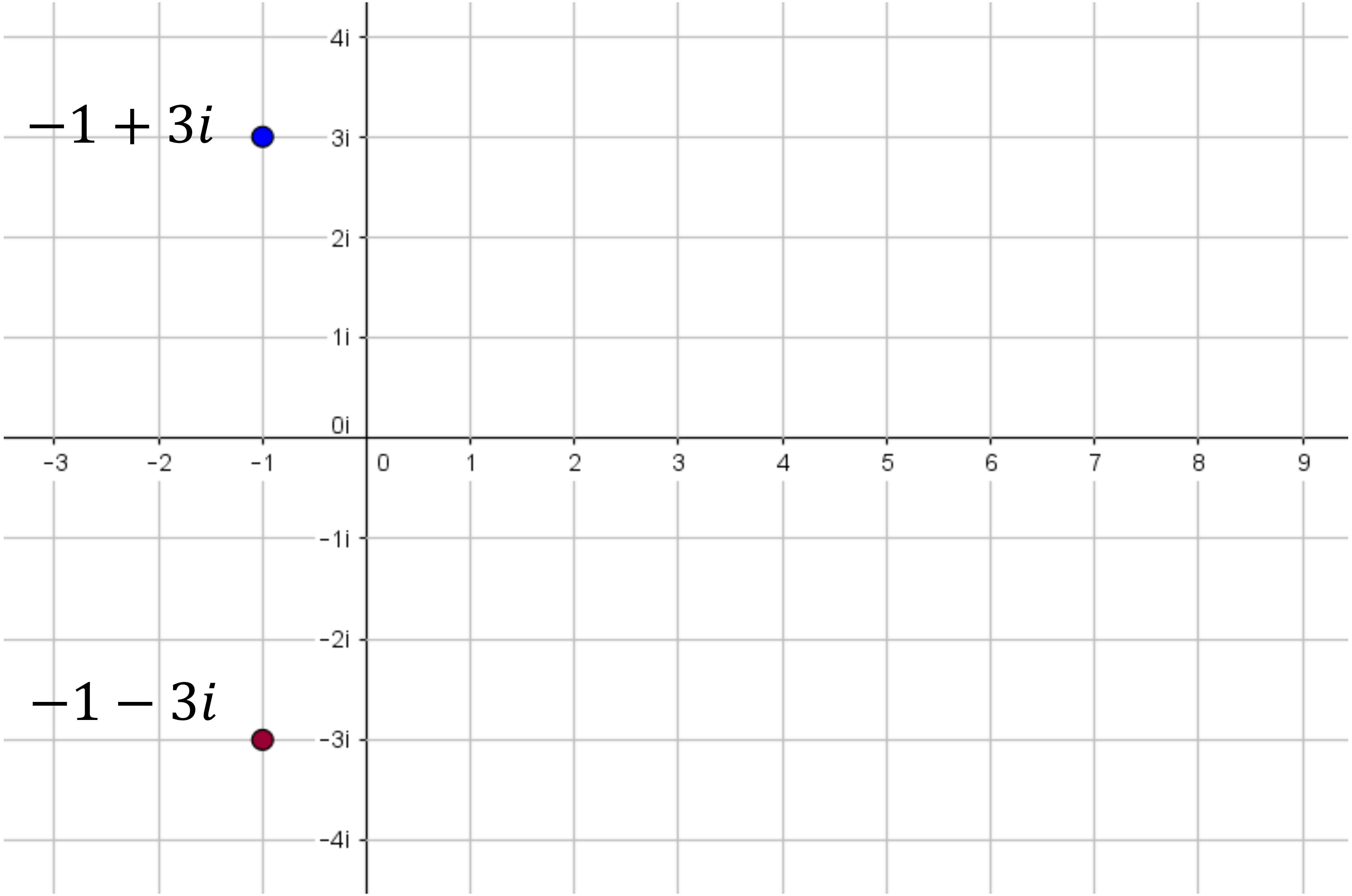


Plot: $-1 + 3i$

What is the conjugate?

$-1 - 3i$

Plot the conjugate.


$-1+3i$
$-1-3i$
4i
3i
2i
1i
0i
-1i
-2i
-3i
-4i
-3
-2
-1
0
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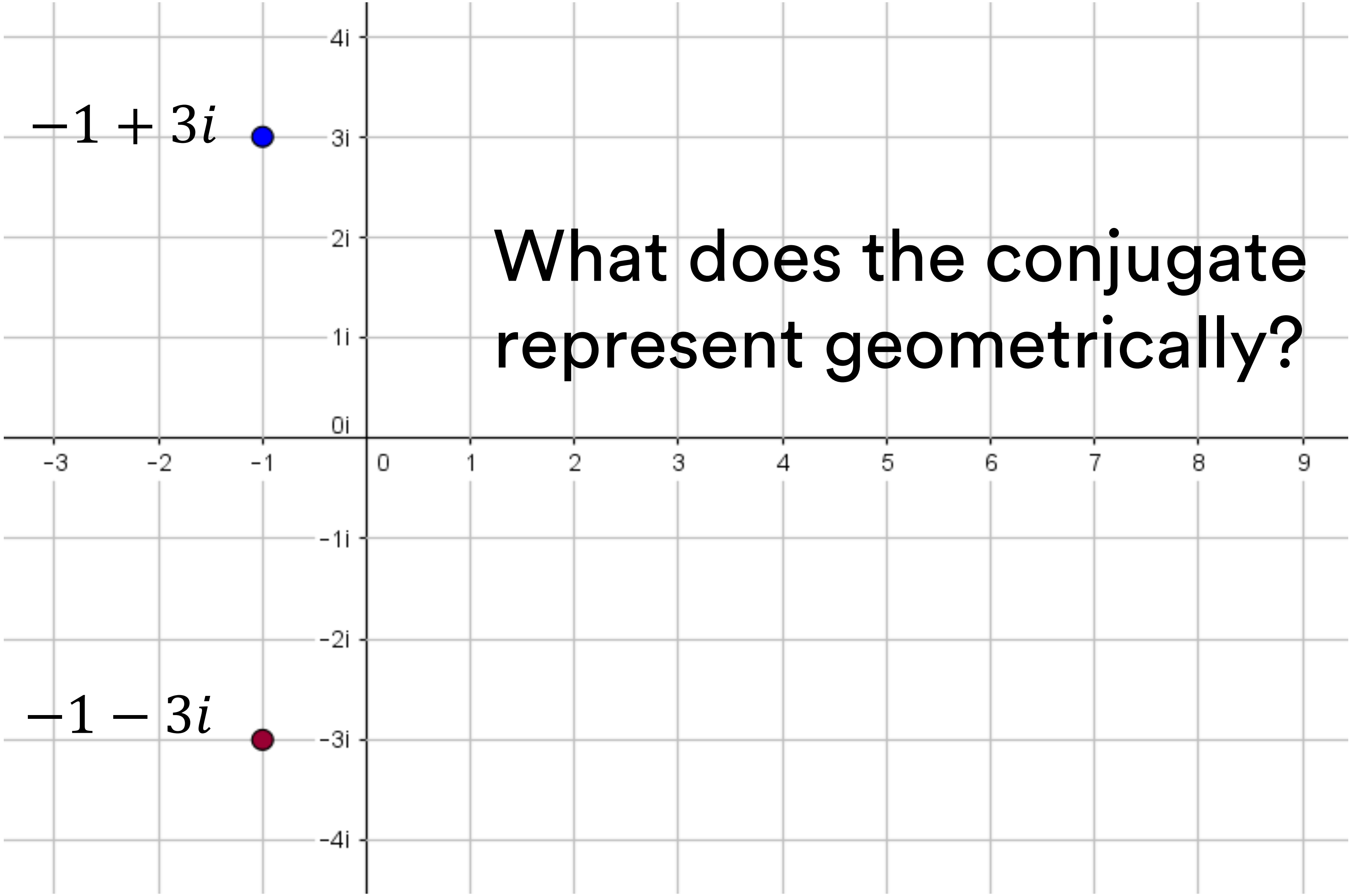
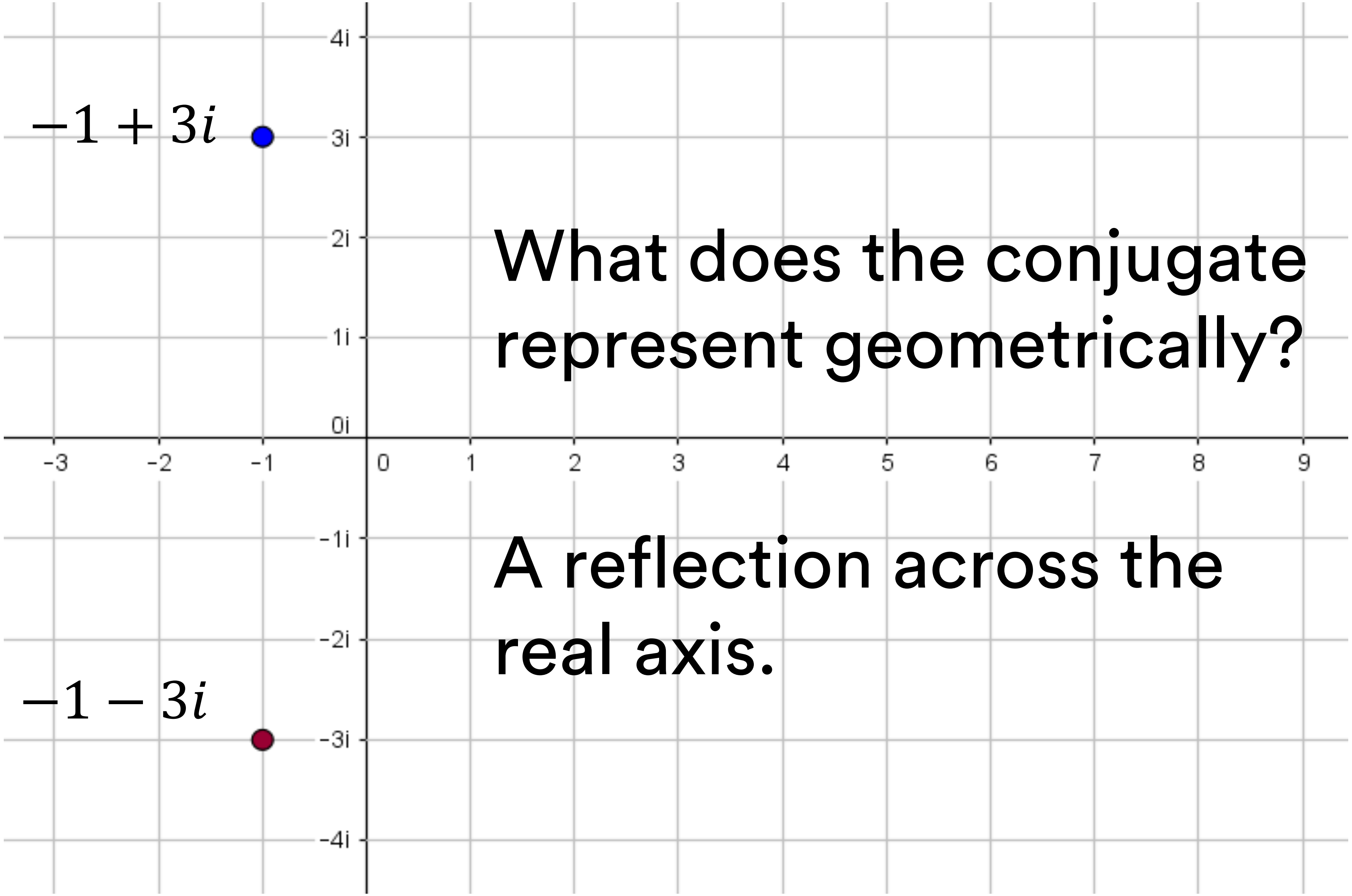
4
5
6
7
8
9



What does the conjugate represent geometrically?



What does the conjugate represent geometrically?

A reflection across the real axis.

Let's Summarize.

Operation	Geometric Result
Adding or subtracting a complex number $(a + bi)$	
Multiplying by a constant, k	
Multiplying by i	
Multiplying by a complex number, $(a + bi)$	
The conjugate of a complex number	

Let's Summarize.

Operation	Geometric Result
Adding or subtracting a complex number $(a + bi)$	Horizontal translation by a and a vertical translation by b
Multiplying by a constant, k	Dilation by k
Multiplying by i	Rotation of $90°$
Multiplying by a complex number, $(a + bi)$	Dilation by $\sqrt{a^2 + b^2}$ and rotation of $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
The conjugate of a complex number	Reflection across the real axis

Lagniappe: A Little Extra Work with Complex Numbers

What is a multiplicative inverse?

What is a multiplicative inverse?

A number that when multiplied by a given number yields the multiplicative identity of 1

Does $3 + 4i$ have a multiplicative inverse?

Does $3 + 4i$ have a multiplicative inverse?

Is there a number $p + qi$
such that
$(3 + 4i)(p + qi) = 1$?

$$p + qi = \frac{1}{3 + 4i}$$

$$p + qi = \frac{1}{3 + 4i}$$

$$\frac{1}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i}$$

$$= \frac{3 - 4i}{9 - 12i + 12i - 16i^2}$$

$$= \frac{3 - 4i}{9 + 16}$$

$$= \frac{3 - 4i}{25}$$

$$p + qi = \frac{1}{3 + 4i}$$

$$\frac{1}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i}$$

$$= \frac{3 - 4i}{9 - 12i + 12i - 16i^2}$$

$$= \frac{3 - 4i}{9 + 16}$$

$$= \frac{3 - 4i}{25}$$

$$p + qi = \frac{3}{25} - \frac{4}{25}i$$

Let's find a general formula!

Is there a number
$p + qi$ such that
$(a + bi)(p + qi) = 1$?

$$p + qi = \frac{1}{a + bi}$$

$$p + qi = \frac{1}{a + bi}$$

$$\frac{1}{a + bi} \cdot \frac{a - bi}{a - bi}$$

$$p + qi = \frac{1}{a + bi}$$

$$\frac{1}{a + bi} \cdot \frac{a - bi}{a - bi}$$

$$= \frac{a - bi}{a^2 - abi + abi - b^2 i^2}$$

$$= \frac{a - bi}{a^2 + b^2}$$

$$p + qi = \frac{1}{a + bi}$$

$$\frac{1}{a + bi} \cdot \frac{a - bi}{a - bi}$$

$$= \frac{a - bi}{a^2 - abi + abi - b^2 i^2}$$

$$= \frac{a - bi}{a^2 + b^2}$$

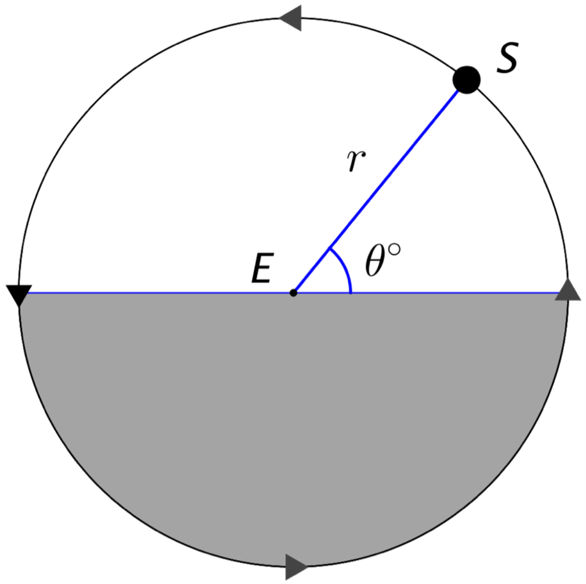
$$p + qi = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$$

Math Trivia

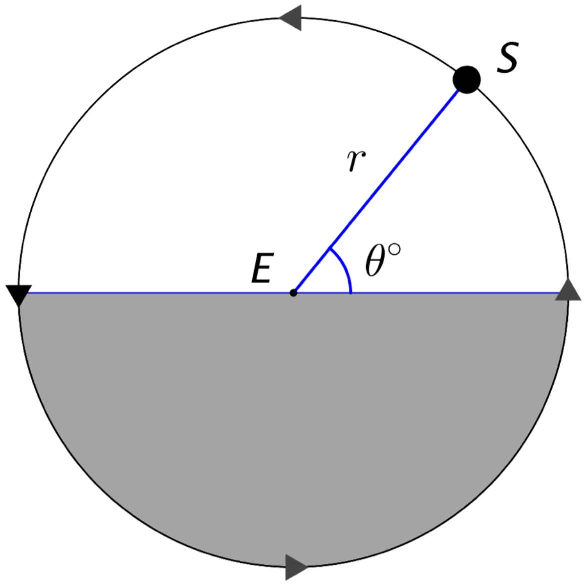
Math Trivia

Why does positive angle rotation move in a counterclockwise direction?

Why does positive angle rotation move in a counterclockwise direction?



Why does positive angle rotation move in a counterclockwise direction?



Questions?

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