Linear Functions Roadmap

Making Connections Across Grades

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Illustrative Mathematics

6.RP Riding at a Constant Speed, Assessment Variation

Task

Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed,

a. How far did she ride in 15 minutes?

b. How long did it take her to ride 6 miles?

c. How fast did she ride in miles per hour?

d. What was her pace in minutes per mile?

6.RP Riding at a Constant Speed, Assessment Variation

Solutions

Solution: 1

	A	B	C	D	E	F
Number of Minutes	150	15	7.5	30	45	60
Number of Miles	20	2	1	4	6	8

The values in column B were found by dividing both values in column A by 10. The values in column C were found by dividing both values in column B by 2. The other columns contain multiples of the values in column B.

a. If we look in column B, we can see that she could ride 2 miles in 15 minutes.

b. If we look in column E, we can see that it would take her 45 minutes to ride 6 miles.

c. If we look in column F, we can see that she is riding 8 miles every 60 minutes (which is 1 hour), so she is riding her bike at a rate of 8 miles per hour.

d. If we look in column C, we can see that her pace is 7.5 minutes per mile.

Standards

6.RP.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with $b \neq 0$, and use rate language in the context of a ratio relationship.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Illustrative Mathematics

7.RP Molly's Run

Task

Molly runs $\frac{1}{3}$ of a mile in 4 minutes.

a. If Molly continues at the same speed, how long will it take her to run one mile?

b. Draw and label a picture showing why your answer to part (a) makes sense.

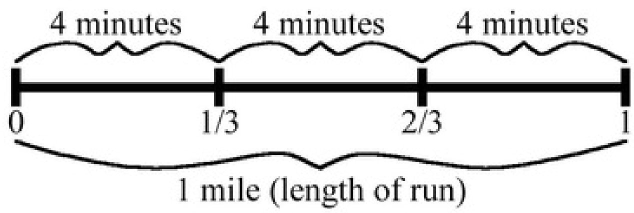
7.RP Molly's Run

Solutions

Solution: 1

a. If Molly runs $\frac{1}{3}$ of a mile every four minutes, then 2×4 minutes is is $\frac{2}{3}$ of this time. Similarly 3×4 minutes is $\frac{3}{3}$ of Molly's time to run a mile. Since $\frac{3}{3} = 1$, this means that it takes $3 \times 4 = 12$ minutes to run one mile.

b. The following picture illustrates this reasoning:



since there are 3 groups of 4 minutes, the picture shows why

$$3 \times 4$$

represents the solution to the problem. It is important to note that the constant speed Molly is running is shown in the picture by the equally spaced intervals of both time and distance: traveling at a constant speed means that equal distances are traveled in equal times.

Standards

7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units

.

Illustrative Mathematics

7.RP Gym Membership Plans

Task

In January, Georgia signed up for a membership at Anytime Fitness. The plan she chose cost $95 in start-up fees and then $20 per month starting in February. Edwin also signed up at Anytime Fitness in January. His plan cost $35 per month starting in February, and his start-up fees were waived.

a. Create tables for both Georgia and Edwin that compare the number of months since January to the total cost of their gym memberships. Continue this table for one year.

b. Plot the points from the two tables in part (a) on a coordinate plane.

c. Decide if either or both gym memberships are described by a proportional relationship, and write an equation representing any such relationship. Explain how parts (a) and (b) could be used to support your answer.

7.RP Gym Membership Plans

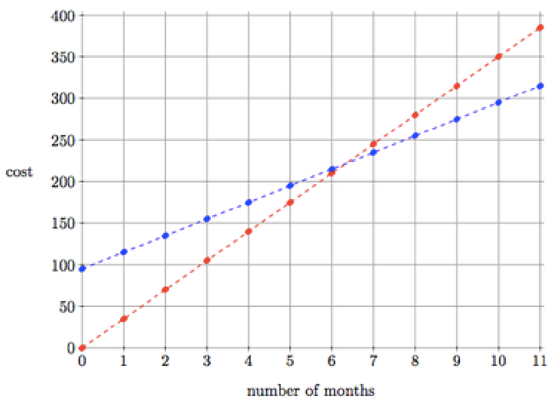
Solution

a. The table for Georgia's gym membership cost for 12 months is below:

number of months since January	0	1	2	3	4	5	6	7	8	9	10	11
total cost of Georgia's gym membership	95	115	135	155	175	195	215	235	255	275	295	315

The table for Edwin's gym membership cost for 12 months is below:

number of months since January	0	1	2	3	4	5	6	7	8	9	10	11
total cost of Edwin's gym membership	0	35	70	105	140	175	210	245	280	315	350	385



c. Georgia's plan does not represent a proportional relationship, and Edwin's plan does represent a proportional relationship. That Edwins' plan is proportional can be seen from the table by observing that whenever we multiply the number of months by a constant, the total cost multiplies by that same constant -- for example, doubling the number of months from 3 to 6 has the effect of doubling the cost from \$105 to \$210. This does not hold true for Georgia's plan, as can be seen by similarly doubling.

We could also see this from our response to part (b). Proportional relationships can be visualized graphically as being described by lines that go through the origin. Since Edwin's line (in red above) does go through the origin, it describes a proportional relationship, and likewise, Georgia's does not.

Finally, we find an equation to describe Edwin's plan. Since his relationship is proportional, every one month that passes will cost him \$35. So after n months, he will have paid \$35 dollars n times, for a total cost of \$35n dollars. Thus the total cost c of Edwin's plan is related to the number of months passed by the equation $c = 35n$.

Standards

7.RP.A.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.A.2c Represent proportional relationships by equations.

8.F Battery Charging

Task

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?

b. How much time would Sam need to charge the battery 100% on both devices?

8.F Battery Charging

Solution: Using tables

a. Since the video game player's battery charge is given in a table, we can extend the table and see what value it will give after 60 minutes. Note that the rate of change of the data in the table is constant: For every 10 minutes the charge increases by 12 percentage points. Assuming that this pattern continues, we have:

time charging (minutes)	0	10	20	30	40	50	60
video game player battery charge (%)	20	32	44	56	68	80	92

We can make a similar table for the MP3 player:

time charging (minutes)	0	15	30	45	60
MP3 player battery charge (%)	40	52	64	76	88

So after 60 minutes, the MP3 player's battery would be 88% charged and the video game player will be 92% charged.

b. We can see from the table above that the MP3 player would be fully charged in another 15 minutes, we just have to add one more column to the table to find that answer.

The video game player will need less than 10 minutes to fully charge, since we are only missing 8 percentage points after 60 minutes. To be exact, using the rate of increase, we will need 2/3 of 10 minutes, which is just under 7 minutes.

Standards

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

.

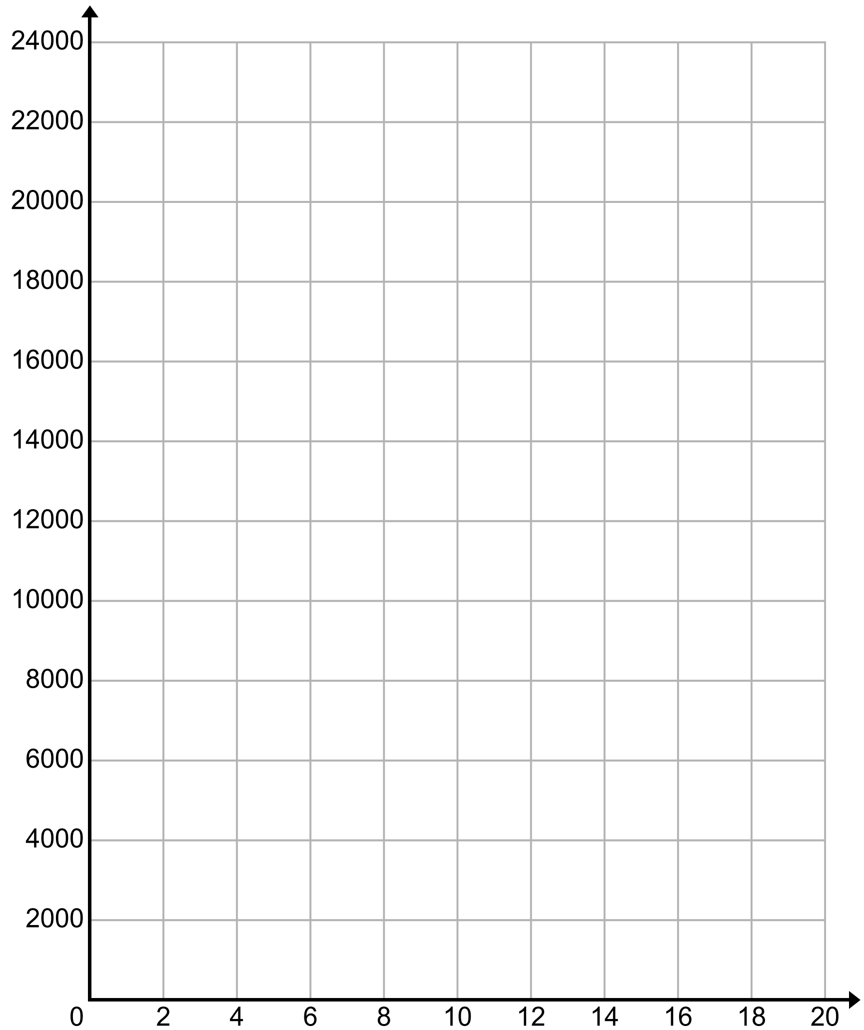
Would you rather…

Option (1) … have $1,000 per year for twenty years?
Option (2) … start with $1, double it the first year, double it again the second year, etc.?

Support your choice with mathematics:

1. Use the table below to find out how much money you would have at the end of twenty years for option (1) and option (2). Then graph each option.

Year	Option (1)	Option (2)
1	$ 1,000	$ 2
2	$ 2,000	$ 4
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		



2. Which model would you choose and why? Use the table and graph to support your answer.

3. What is the y-intercept for each option in your graph? What does the y-intercept mean in this situation?

4. Describe how each option grows. Explain what the growth means for each option in this situation.

5. (a) Is option (1) linear, exponential, or neither? Explain how you know.

   (b) Write a function rule for option (1).

6. (a) Is option (2) linear, exponential, or neither? Explain how you know.

   (b) Write a function rule for option (2).

7. How many years need to pass in order for you to have more money from option (2) than from option (1)? Support your answer using information from your table and your graph.

8. Use your function rules to figure out how much money you will have in 40 years for each option.

9. (a) What if, instead of $1.00, you started with one penny in option (2)? Then which option would you choose? Why? Support your answer with mathematics.

(b) How long would it be before you made more money from option (2) than from option (1)? Show your mathematical work and explain your thinking clearly in the space below.

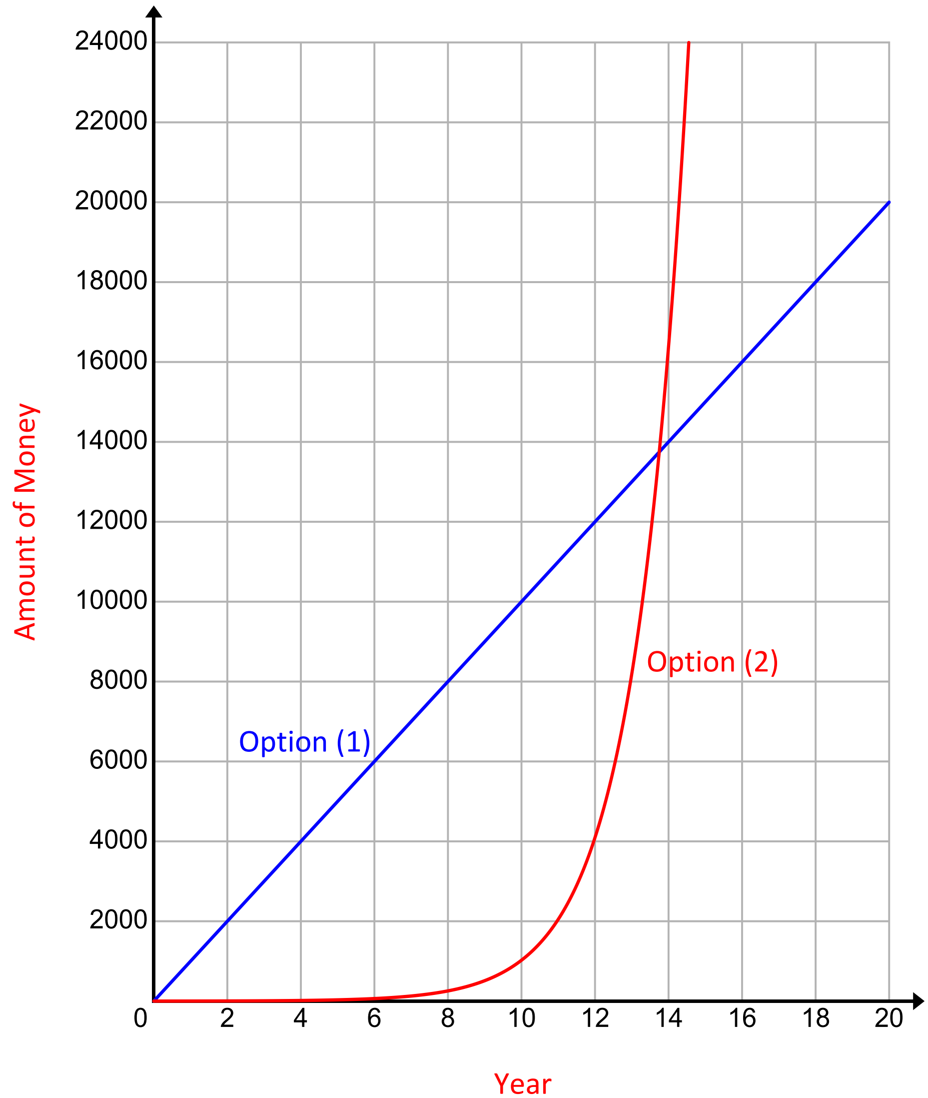
Would you rather…

Option (1) … have $1,000 per year for twenty years?
Option (2) … start with $1, double it the first year, double it again the second year, etc.?

Support your choice with mathematics:

1. Use the table below to find out how much money you would have at the end of twenty years for option (1) and option (2). Then graph each option.

Year	Option (1)	Option (2)
1	$ 1,000	$ 2
2	$ 2,000	$ 4
3	$ 3,000	$ 8
4	$ 4,000	$ 16
5	$ 5,000	$ 32
6	$ 6,000	$ 64
7	$ 7,000	$ 128
8	$ 8,000	$ 256
9	$ 9,000	$ 512
10	$ 10,000	$ 1,024
11	$ 11,000	$ 2,048
12	$ 12,000	$ 4,096
13	$ 13,000	$ 8,192
14	$ 14,000	$ 16,384
15	$ 15,000	$ 32,768
16	$ 16,000	$ 65,536
17	$ 17,000	$ 131,072
18	$ 18,000	$ 262,144
19	$ 19,000	$ 524,288
20	$ 20,000	$ 1,048,576



2. Which model would you choose and why? Use the table and graph to support your answer.

Answers vary. Sample answer given:

I would choose option (2). Based on my table, after 20 years I would have over a million dollars choosing option (2), but only $20,000 choosing option (1). In the graph, I can see that by the 14^{th} year, the amount of money in option (2) will be greater than the amount of money in option (1).

3. What is the y-intercept for each option in your graph? What does the y-intercept mean in this situation?

For option (1), the y-intercept is (0,0). This means that you start with no money in option (1).

For option (2), the y-intercept is (0,1). This means that you start with $1.00 in option (2).

4. Describe how each option grows. Explain what the growth means for each option in this situation.

In option (1), the amount of money grows at a constant rate of $1,000 per year. In option (2), the amount of money grows by a constant factor each year (the amount of money is multiplied by two each year).

5. (a) Is option (1) linear, exponential, or neither? Explain how you know.

Option (1) is linear because it has a constant rate of change: option (1) adds $1,000 to the total amount of money each year.

(b) Write a function rule for option (1).

$f(x) = 1000x$

6. (a) Is option (2) linear, exponential, or neither? Explain how you know.

Option (2) is exponential because the amount of money is being multiplied by a constant each year.

(b) Write a function rule for option (2).

$g(x) = 2^x$

7. How many years need to pass in order for you to have more money from option (2) than from option (1)? Support your answer using information from your table and your graph.

The amount of money in option (2) will surpass option (1) during the 14^{th} year. You can see this in the table when the amount of money in option (1) goes from $13,000 in year 13 to $14,000 in year 14, and the amount of money in option (2) doubles from $8,192 in year 13 to $16,384 in year 14. You can see this when the graph of option (2) intersects with and grows larger than option (1) between x=13 and x=14.

8. Use your function rules to figure out how much money you will have in 40 years for each option.

$f(40) = 1000(40) = 40{,}000$

$g(40) = 2^{40} = 1{,}099{,}511{,}627{,}776$

In 40 years, I will have $40,000 choosing option (1) and $1,0999,511,627,776 choosing option (2).

9. (a) What if, instead of $1.00, you started with one penny in option (2)? Then which option would you choose? Why? Support your answer with mathematics.

Option (1): $f(x) = 1000x$ Option (2): $h(x) = 0.01(2)^x$

$f(20) = 1000(2) = 20000$ $h(20) = 0.01(2)^{20} = 10485.76$

I would choose option (1) in this case. In twenty years, option (1) would yield $20,000 whereas option (2) would yield only $10,485.76.

(b) How long would it be before you made more money from option (2) than from option (1)? Show your mathematical work and explain your thinking clearly in the space below.

Students are not expected to solve the equation $f(x) = h(x)$ or the inequality $f(x) < h(x)$ algebraically. They should be able to show their work either in a graph or in a table. Both methods are shown below.

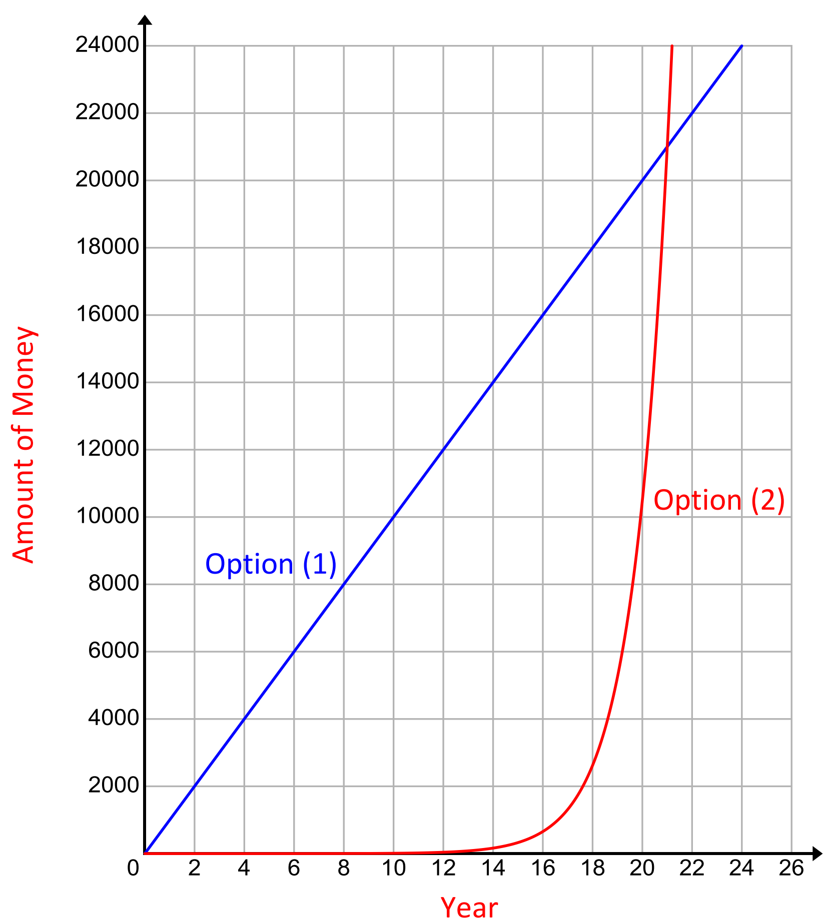
Year	Option (1)	Option (2)
20	$ 20,000	$ 10,485.76
21	$ 21,000	$ 20,971.52
22	$ 22,000	$ 41,943.04

I can see in the table that in year 21, the amounts of money in each option are very close to being equal, but option (1) is still greater. In year 22, the amount of money in option (2) surpasses option (1).

By looking at my graph, I can see that option (2) surpasses option (1) somewhere between year 20 and 22. I evaluated my two functions for $x = 21$ and $x = 22$, which showed me that it takes 22 years for option (2) to make more money than option (1).

$f(21) = 21000$ $h(21) = 20971.52$

$f(22) = 22000$ $h(22) = 41943.04$

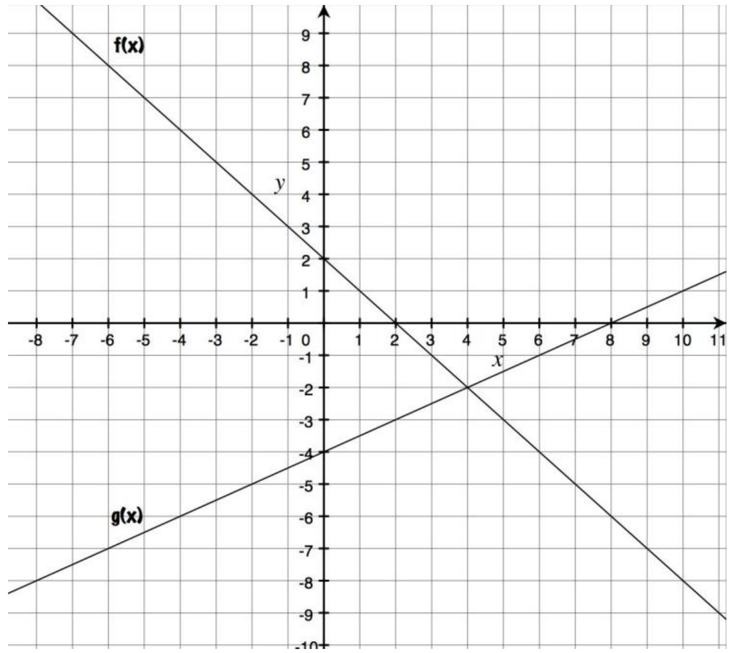


Generating Quadratics Graphically

READ THE DIRECTIONS CAREFULLY AND SHOW YOUR WORK WHEN NECESSARY.

1) Based on the graphs shown, fill in the table. Let $h(x) = f(x) \cdot g(x)$.

x	$f(x)$	$g(x)$	$h(x)$
-1			
0			
1			
2			
3			
4			
5			
8			
9			
10			



2) Plot all the points $(x, h(x))$.

3) What type of functions are $f(x)$ and $g(h)$? (linear or quadratic)

4) Write equations for the functions of $f(x)$ and $g(x)$.

5) Find the equation of $h(x) = f(x)g(x)$.

6) What type of function is this? Why do you get type of function?

7) Where are the x-intercepts of $h(x)$? How are they related to the graphs of $f(x)$ and $g(x)$?

8) Where is the y-intercept of $h(x)$? How is it related to the graphs of $f(x)$ and $g(x)$?

9) Identify the left-most *x*-intercept on the graph. With a straight-edge, cover everything to the right of that point. What connections do you see relating the signs of the *y*-values?

10) Identify the right-most intercept on the graph. With a straight-edge, cover up everything to the left of that point. What connections do you see relating to the signs of the *y*-values?

11) Cover up everything to the left of the left-most x-intercept and everything to the right of the right-most x-intercept so that the section between the x-intercepts is showing. What connections do you see relating to the signs of the *y*-values?

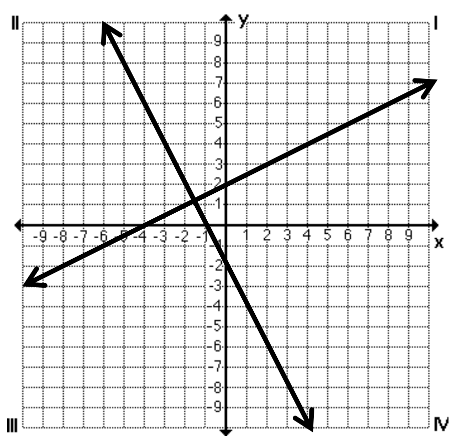
Complete the following sentences.

12) When both lines are **below** the *x*-axis, the *y*-values are ______________________ and the parabola __________________________.

13) When one line is above the *x*-axis and the other is below the *x*-axis, the parabola __________________________.

y-value of *f*(*x*)	*y*-value of *g*(*x*)	parabola is above or below the *x*-axis?
+	-	
-	+	
-	-	
+	+	

14) Based on the patterns you saw above, draw a sketch of the quadratic function that would be obtained from the linear expressions of these lines.



15) Write the equation for each line.

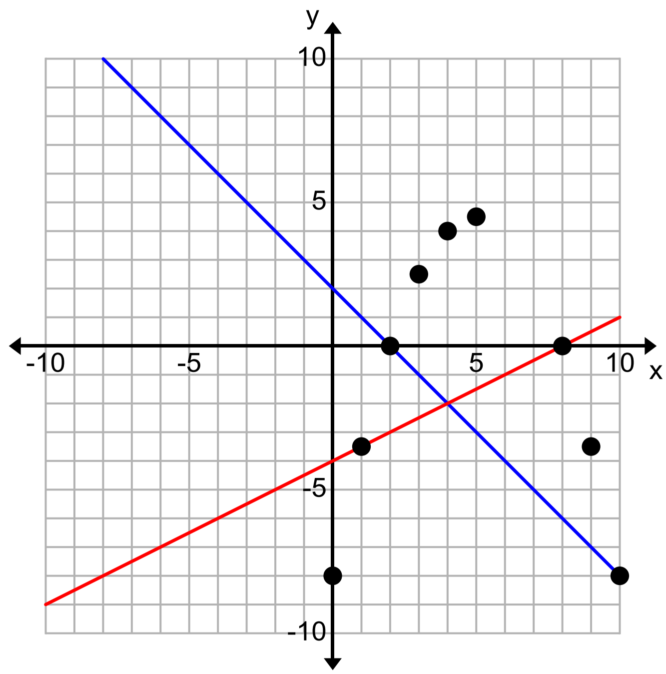
16) To check your sketch in Question 14, multiply the expressions together, and graph the resulting function on the grid above. How accurate was your sketch?

Generating Quadratics Graphically

READ THE DIRECTIONS CAREFULLY AND SHOW YOUR WORK WHEN NECESSARY.

1) Based on the graphs shown, fill in the table. Let $h(x) = f(x) \cdot g(x)$.

x	$f(x)$	$g(x)$	$h(x)$
-1	3	-4.5	-13.5
0	2	-4	-8
1	1	-3.5	-3.5
2	0	-3	0
3	-1	-2.5	2.5
4	-2	-2	4
5	-3	-1.5	4.5
8	-6	0	0
9	-7	0.5	-3.5
10	-8	1	-8



2) Plot all the points $(x, h(x))$.

3) What type of functions are $f(x)$ and $g(h)$? (linear or quadratic) They are both linear functions.

4) Write equations for the functions of $f(x)$ and $g(x)$.

$f(x) = -x + 2$

$g(x) = ½ x - 4$

5) Find the equation of $h(x) = f(x)g(x)$.

$h(x) = -½ x^2 + 5x - 8$

6) What type of function is this? Why do you get type of function?

The function $h(x)$ is a quadratic function. I know this because the degree of the trinomial is two and the graph of $h(x)$ is a parabola.

7) Where are the x-intercepts of $h(x)$? How are they related to the graphs of $f(x)$ and $g(x)$?
The x-intercepts are at (2, 0) and (8, 0). These coordinate points are the x-intercepts of $f(x)$ and $g(x)$, respectively.

8) Where is the y-intercept of $h(x)$? How is it related to the graphs of $f(x)$ and $g(x)$?
The y-intercept is at (0, -8). This coordinate point is related to the y-intercepts of the $f(x)$ and $g(x)$ because the y-coordinate is the product of the other 2 y-coordinates.

9) Identify the left-most *x*-intercept on the graph. With a straight-edge, cover everything to the right of that point. What connections do you see relating the signs of the *y*-values?
Since the values of *f(x)* are positive and the values of *g(x)* are negative when left of the point (2, 0), then the product of *f(x)* and *g(x)*, the quadratic function, *h*(*x*), has negative *y*-values in the defined region.

10) Identify the right-most intercept on the graph. With a straight-edge, cover up everything to the left of that point. What connections do you see relating to the signs of the *y*-values?
Since the values of *g(x)* are positive and the values of *f(x)* are negative when right of the point (8, 0), then the product of *f(x)* and *g(x)*, the quadratic function, *h*(*x*), again has negative *y*-values in that region.

11) Cover up everything to the left of the left-most x-intercept and everything to the right of the right-most x-intercept so that the section between the x-intercepts is showing. What connections do you see relating to the signs of the *y*-values?
In this region between the 2 *x*-intercepts, I notice that both linear functions have *y*-values that are negative and since we are multiplying them together, that causes the *y*-values of the quadratic function to be positive in this region.

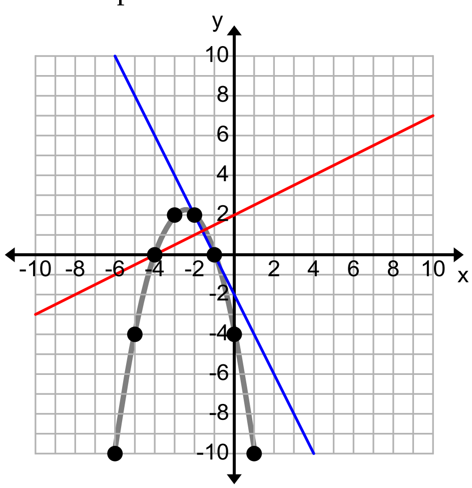
Complete the following sentences.

12) When both lines are **below** the *x*-axis, the *y*-values are negative and the parabola is above the *x*-axis.

13) When one line is above the *x*-axis and the other is below the *x*-axis, the parabola is below the *x*-axis.

y-value of *f*(*x*)	*y*-value of *g*(*x*)	parabola is above or below the *x*-axis?
+	-	below
-	+	below
-	-	above
+	+	above

14) Based on the patterns you saw above, draw a sketch of the quadratic function that would be obtained from the linear expressions of these lines.



15) Write the equation for each line. $y_1 = -2x - 2$ and $y_2 = \frac{1}{2}x + 2$

16) To check your sketch in Question 14, multiply the expressions together, and graph the resulting function on the grid above. How accurate was your sketch?
New equation: $y_3 = -x^2 - 5x - 4$

Describing Polynomial Functions

SOLUTIONS:

Graph	Zeros	Degree / end behavior	Increasing / decreasing	Positive / negative
G1	Z3	E6 or E9	I4	P10
G2	Z8	E1 or E5	I10	**P5**
G3	Z9	**E4 or E7**	I5	P4
G4	Z4	E3 or E10	I2	P9
G5	Z2	**E4 or E7**	I1	P8
G6	**Z5**	E6 or E9	I7	P3
G7	Z10	E2	**I6**	P2
G8	Z6	E1 or E5	I3	P7
G9	Z7	E3 or E10	I8	P6
G10	Z1	E8	I9	P1

BLANK CARDS (solutions)

Z5 The function has zeros of -2, 0, and 2.	E7 Third degree polynomial function End behavior: as $x \to \infty, y \to \infty$; as $x \to -\infty, y \to -\infty$.
I6 The function is increasing over the intervals $(-\infty, -1.5]$ and $[1.5, \infty)$, and decreasing over the interval $[-1.5, 1.5]$.	P5 The function is positive on the intervals $(-\infty, -4)$, $(-2, 0)$, and $(2, \infty)$. The function is negative on the intervals $(-4, -2)$ and $(0, 2)$.

Z3

The function has zeros of -4, -2, and 2.

Z8

The function has zeros of -4, -2, 0, and 2.

Z9

The function has zeros of -2 and 2 (multiplicity 2).

Z4

The function has zeros of -2 (multiplicity 2) and 2 (multiplicity 2).

Z2

The function has zeros of -2 (multiplicity 2) and 2.

Z5

Z10

The function has zeros of -2, 0 (multiplicity 3), and 2.

Z6

The function has zeros of -2, 0 (multiplicity 2), and 2.

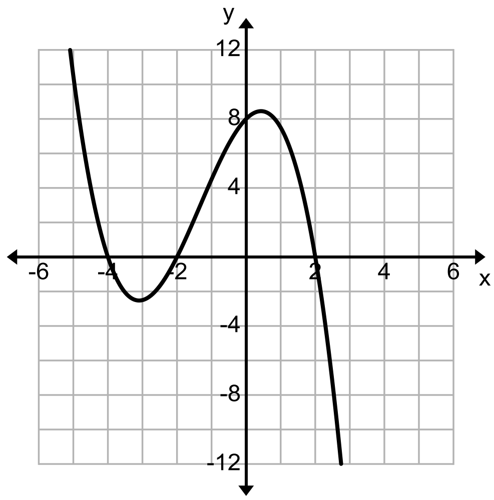
Z7

The function has zeros of -4, -2 (multiplicity 2), and 2.

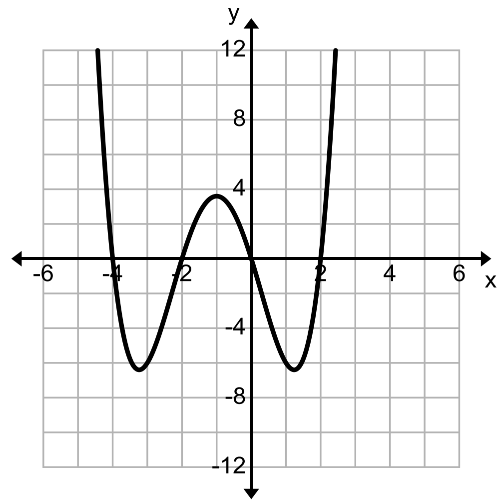
Z1

The function has zeros of -2 (multiplicity 2), 0, and 2 (multiplicity 3).

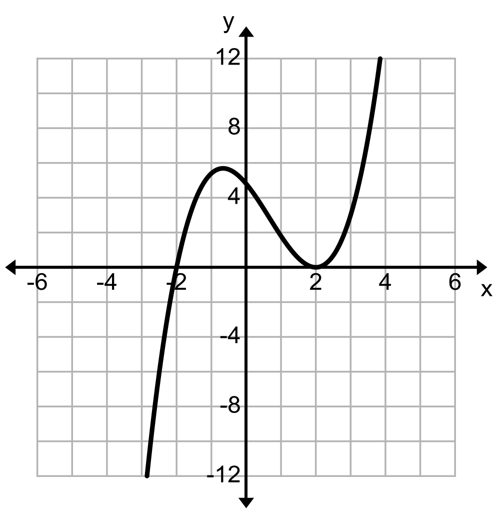
G1 $y = -0.5(x+4)(x+2)(x-2)$



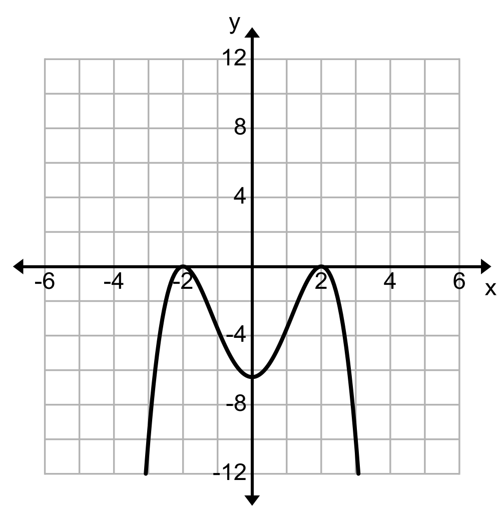
G2 $y = 0.4x(x+4)(x+2)(x-2)$



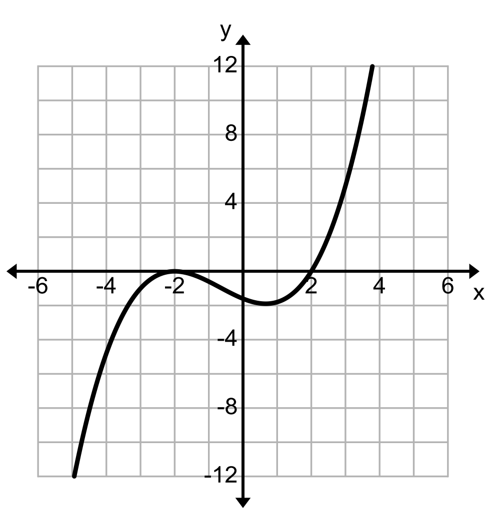
G3 $y = 0.6(x+2)(x-2)^2$



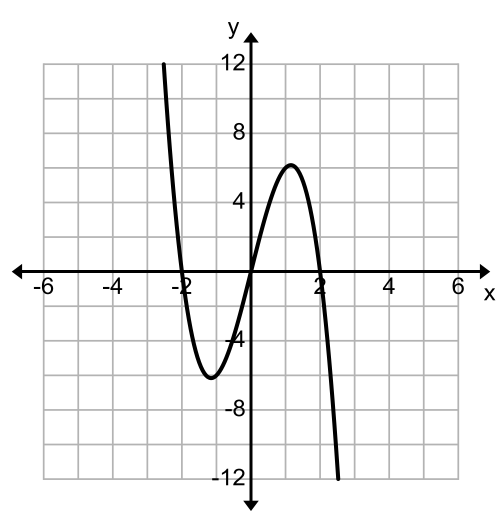
G4 $y = -0.4(x+2)^2(x-2)^2$



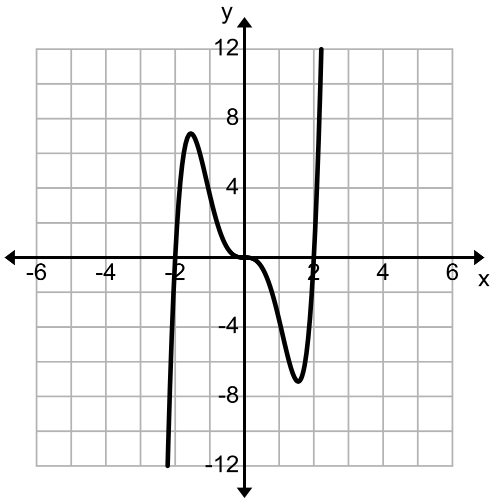
G5 $y = 0.2(x-2)(x+2)^2$



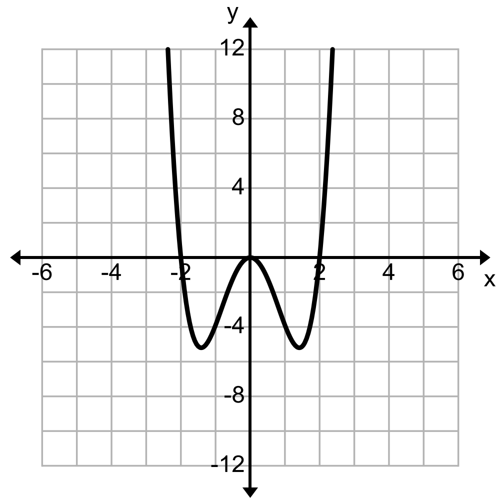
G6 $y = -2x(x+2)(x-2)$



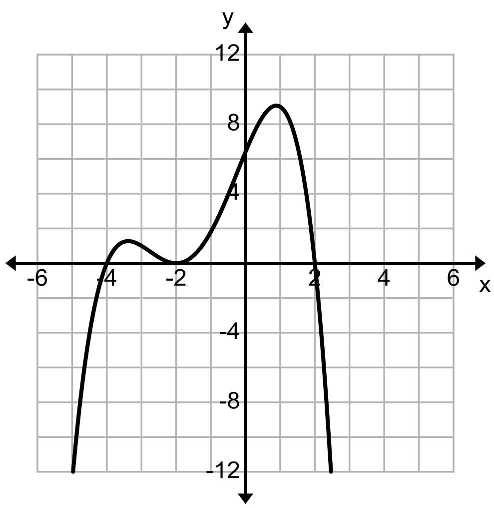
G7 $y = 1.2x^3(x+2)(x-2)$



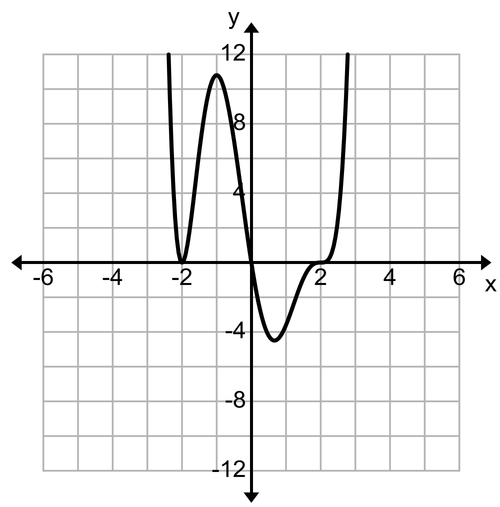
G8 $y = 1.3x^2(x+2)(x-2)$



G9 $y = -0.2(x+4)(x+2)^2(x-2)$



G10 $y = 0.4x(x+2)^2(x-2)^3$



E6

Third degree polynomial function
End behavior: as $x \to \infty, y \to -\infty$;
as $x \to -\infty, y \to \infty$.

E5

Fourth degree polynomial function
End behavior: as $x \to \infty, y \to \infty$;
as $x \to -\infty, y \to \infty$.

E7

E10

Fourth degree polynomial function
End behavior: as $x \to \infty, y \to -\infty$;
as $x \to -\infty, y \to -\infty$.

E4

Third degree polynomial function
End behavior: as $x \to \infty, y \to \infty$;
as $x \to -\infty, y \to -\infty$.

E9

Third degree polynomial function
End behavior: as $x \to \infty, y \to -\infty$;
as $x \to -\infty, y \to \infty$.

E2

Fifth degree polynomial function
End behavior: as $x \to \infty, y \to \infty$;
as $x \to -\infty, y \to -\infty$.

E1

Fourth degree polynomial function
End behavior: as $x \to \infty, y \to \infty$;
as $x \to -\infty, y \to \infty$.

E3

Fourth degree polynomial function
End behavior: as $x \to \infty, y \to -\infty$;
as $x \to -\infty, y \to -\infty$.

E8

Sixth degree polynomial function
End behavior: as $x \to \infty, y \to \infty$;
as $x \to -\infty, y \to \infty$.

I4

The function is decreasing over the intervals $(-\infty, -3.1]$ and $[0.4, \infty)$, and increasing over the interval $[-3.1, 0.4]$.

I10

The function is increasing over the intervals $[-3.2, -1]$ and $[1.2, \infty)$, and decreasing over the intervals $(-\infty, -3.2]$ and $[-1, 1.2]$.

I5

The function is increasing over the intervals $(-\infty, -0.7]$ and $[2, \infty)$, and decreasing over the interval $[-0.7, 2]$.

I2

The function is decreasing over the intervals $[-2, 0]$ and $[2, \infty)$, and increasing over the intervals $(-\infty, -2]$ and $[0, 2]$.

I1

The function is increasing over the intervals $(-\infty, -2]$ and $[0.7, \infty)$, and decreasing over the interval $[-2, 0.7]$.

I7

The function is decreasing over the intervals $(-\infty, -1.2]$ and $[1.2, \infty)$, and increasing over the interval $[-1.2, 1.2]$.

I6

I3

The function is increasing over the intervals $[-1.4, 0]$ and $[1.4, \infty)$, and decreasing over the intervals $(-\infty, -1.4]$ and $[0, 1.4]$.

I8

The function is increasing over the intervals $(-\infty, -3.4]$ and $[-2, 0.9]$, and decreasing over the intervals $[-3.4, -2]$ and $[0.9, \infty)$.

I9

The function is increasing over the intervals $[-2, -1]$ and $[0.7, \infty)$, and decreasing over the intervals $(-\infty, -2]$ and $[-1, 0.7]$.

P10

The function is negative on the intervals $(-4,-2)$ and $(2,\infty)$, and positive on the intervals $(-\infty,-4)$ and $(-2,2)$.

P5

P4

The function is positive on the intervals $(-2,2)$ and $(2,\infty)$, and negative on the interval $(-\infty,-2)$.

P9

The function is negative on the intervals $(-\infty,-2)$, $(-2,2)$, and $(2,\infty)$. The function is never positive.

P8

The function is positive on the interval $(2,\infty)$, and negative on the intervals $(-\infty,-2)$ and $(-2,2)$.

P3

The function is negative on the intervals $(-2,0)$ and $(2,\infty)$, and positive on the intervals $(-\infty,-2)$ and $(0,2)$.

P2

The function is positive on the intervals $(-2,0)$ and $(2,\infty)$, and negative on the intervals $(-\infty,-2)$ and $(0,2)$.

P7

The function is positive on the intervals $(-\infty,-2)$ and $(2,\infty)$, and negative on the intervals $(-2,0)$ and $(0,2)$.

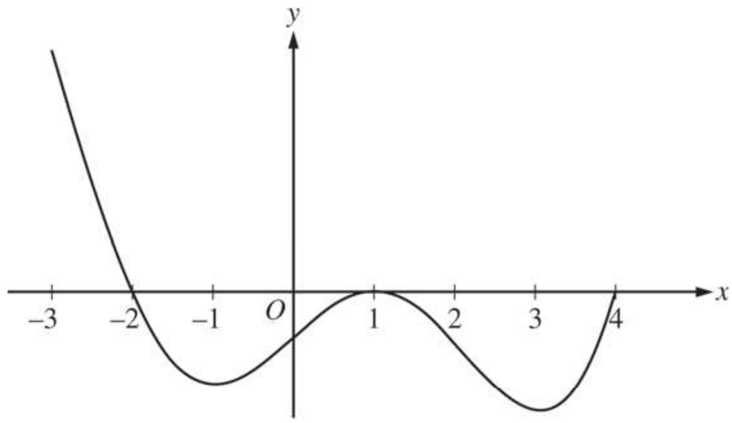
P6

The function is positive on the intervals $(-4,-2)$ and $(-2,2)$, and negative on the intervals $(-\infty,-4)$ and $(2,\infty)$.

P1

The function is positive on the intervals $(-\infty,-2)$, $(-2,0)$, and $(2,\infty)$. The function is negative on the interval $(0,2)$.

AP Calculus AB Exam (2015): Free-Response Question #5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

EU 2.2: A function's derivative, which is itself a function, can be used to understand the behavior of the function.

LO 2.2A: Use derivatives to analyze properties of a function.

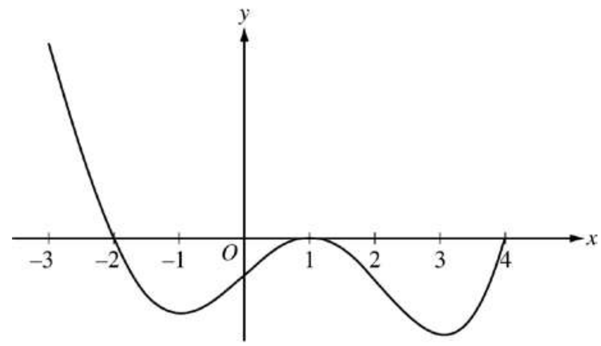
EK 2.2A1: First and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward or downward concavity, and points of inflection.

EK 2.2A2: Key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations.

EK 2.2A3: Key features of the graphs of f, f', and f'' are related to one another.

Question 5

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.



Graph of f'

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

(a) $f'(x) = 0$ at $x = -2$, $x = 1$, and $x = 4$.
$f'(x)$ changes from positive to negative at $x = -2$.
Therefore, f has a relative maximum at $x = -2$.

$2 : \begin{cases} 1 : \text{identifies } x = -2 \\ 1 : \text{answer with reason} \end{cases}$

(b) The graph of f is concave down and decreasing on the intervals $-2 < x < -1$ and $1 < x < 3$ because f' is decreasing and negative on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) The graph of f has a point of inflection at $x = -1$ and $x = 3$ because f' changes from decreasing to increasing at these points.

The graph of f has a point of inflection at $x = 1$ because f' changes from increasing to decreasing at this point.

$2 : \begin{cases} 1 : \text{identifies } x = -1, 1, \text{ and } 3 \\ 1 : \text{reason} \end{cases}$

(d) $f(x) = 3 + \int_1^x f'(t)\,dt$

$f(4) = 3 + \int_1^4 f'(t)\,dt = 3 + (-12) = -9$

$f(-2) = 3 + \int_1^{-2} f'(t)\,dt = 3 - \int_{-2}^{1} f'(t)\,dt$
$= 3 - (-9) = 12$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression for } f(x) \\ 1 : f(4) \text{ and } f(-2) \end{cases}$