Elementary Mathematics Professional Learning
Apprentissage professionnel en mathématiques à l'élémentaire

Additive Thinking

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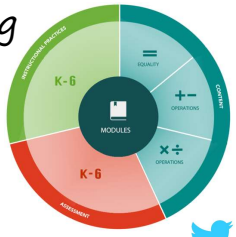

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Introduction

- What is it?
- Why is it important?
- What are the Big Ideas?

What is Additive Thinking?

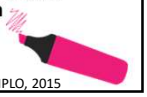
Introduction

What is Additive Thinking?

Additive Thinking

Students are able to manipulate numbers by joining, separating, and comparing while engaging in flexible mathematical reasoning. It is

- a capacity to work flexibly with the concepts, strategies and representations of addition and subtraction as they occur in a wide range of contexts. (mathematical reasoning)
- going beyond memorization of basic arithmetic skills
- the means to communicate additive understanding effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms).



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Introduction

Why is it important?

Additive Thinking

Research says

- Both mathematical reasoning and arithmetic skills are predictors of mathematical achievement, however, students ability to reason mathematically was the stronger predictor of success.

Nunes, Bryant, Barros, & Sylva, 2011

Introduction

What are the Big Ideas?

Additive Thinking

Once students trust "the count", they can flexibly manipulative numbers in order to make solving problems easier by

- Using Parts and Wholes
- Decomposing / Recomposing
- Partitioning
- Compensating
- Using Constant Difference

Introduction

What are the Big Ideas?

Additive Thinking

Students use mathematical reasoning to build connections between inverse problems.

- Addition is not just adding. It's subtraction as well.

Introduction

What are the Big Ideas?

Additive Thinking

Additive Thinking deals with questions where the start, change or result is unknown. It is joining, separating and comparing.

Introduction

Activity: Relating to POS

Additive Thinking

What Strategies Do Students Need?

MATHEMATICS KINDERGARTEN TO GRADE 9

Activity: Suggested Strategies

Additive Thinking

Standard / Traditional Algorithm	Doubles / Near Doubles	Facts of 10
Making Ten	Reordering	Partitioning by place value
Compensating	Think Addition for Subtraction	Keeping a Constant Difference

Just a Reminder

Additive Thinking

Strategies Must Be

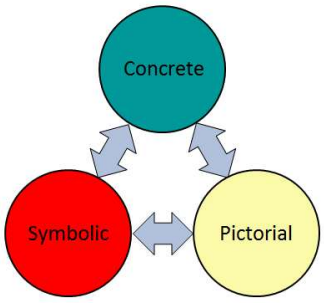
Efficient

Effective

Explainable

Using Manipulatives

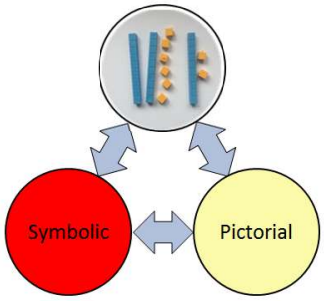
Additive Thinking



Introduction

Using Manipulatives

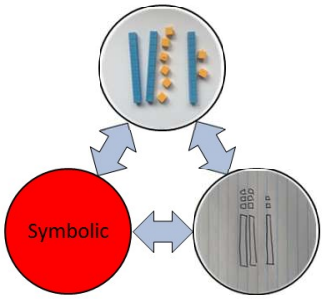
Additive Thinking



Introduction

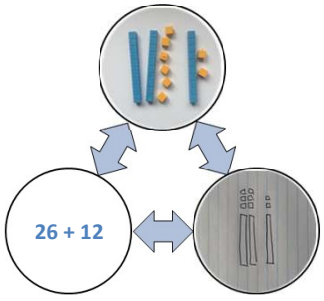
Using Manipulatives

Additive Thinking



Introduction

Using Manipulatives

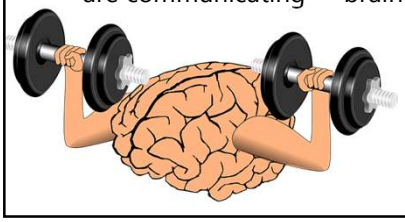


Using Manipulatives

Additive Thinking

Math learning and performance are optimized when the two sides of the brain are communicating – "brain crossing"

~Park & Brannon, 2013



Big Idea 1

Flexible Thinking

Additive Thinking

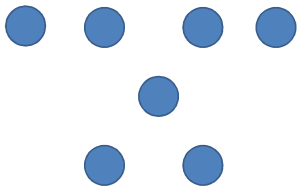
Once students trust "the count", they can flexibly manipulative numbers in order to make solving problems easier by

- Using Parts and Wholes
- Decomposing / Recomposing
- Partitioning
- Compensating
- Using Constant Difference

Big Idea 1

A Simple Test…

Additive Thinking



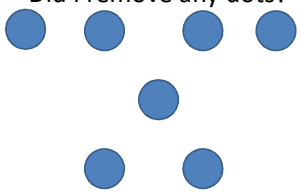
How many are there?
How do you know?

Big Idea 1

A Simple Test…

Additive Thinking

Did I add any dots?
Did I remove any dots?



How many dots are there now?
How do you know?

Big Idea 1

What do you do…

Additive Thinking

If they don't trust the count?

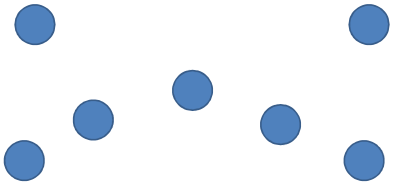
- Subitize familiar arrangements of objects or dots
- Demonstrate an understanding of conting

Big Idea 1

What do you do…

Additive Thinking

Did I add any dots?
Did I remove any dots?



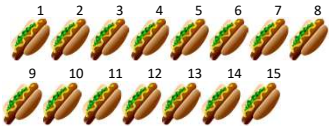
How many dots are there now?
Can you make a guess without counting?

Big Idea 1

How might a student solve this question?

Additive Thinking

On Monday, 8 hotdogs were ordered for Friday's fundraiser. On Tuesday, 7 hotdogs were ordered for Friday's fundraiser. How many hotdogs were ordered on Monday and Tuesday? Using pictures, words, numbers and/or symbols, show how you figured it out.

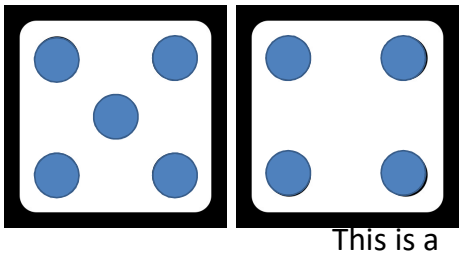


This is a
Counting Strategy

Big Idea 1

How might a student solve this question?

Additive Thinking



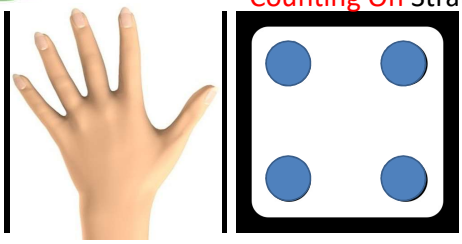
This is a
Counting Strategy

Big Idea 1

How might a student solve this question?

Additive Thinking

This is a Counting On Strategy



How do we help students move past just counting?

Big Idea 1

How might a student solve this question?

Additive Thinking

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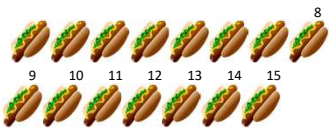


This is a Counting On Strategy

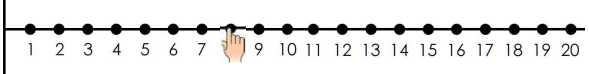
Big Idea 1

How might a student solve this question?

Additive Thinking

Grade 1
The Case of the Hotdogs!

On Monday, 8 hotdogs were ordered for Friday's fundraiser. On Tuesday, 7 hotdogs were ordered for Friday's fundraiser. How many hotdogs were ordered on Monday and Tuesday? Using pictures, words, numbers and/or symbols, show how you figured it out.



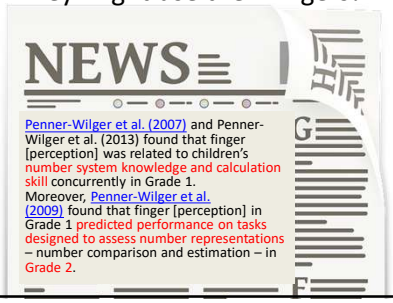
This is a Counting On Strategy

Big Idea 1

How might a student solve this question?

Additive Thinking

They might use their fingers!



Penner-Wilger et al. (2007) and Penner-Wilger et al. (2013) found that finger [perception] was related to children's number system knowledge and calculation skill concurrently in Grade 1. Moreover, Penner-Wilger et al. (2009) found that finger [perception] in Grade 1 predicted performance on tasks designed to assess number representations – number comparison and estimation – in Grade 2.

Big Idea 1

Developing finger perception

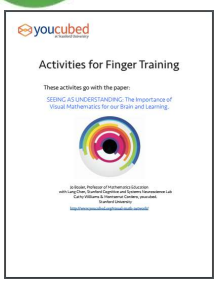
Additive Thinking

- Hold out one hand.
- Close your eyes.
- Partner touches one of your fingertips.
- Can you name the finger?

Big Idea 1

Developing finger perception

Additive Thinking


youcubed

Activities for Finger Training

These activites go with the paper:

SEEING AS UNDERSTANDING: The Importance of Visual Mathematics for our Brain and Learning.

https://www.youcubed.org/finger-activities/

Big Idea 1

Activity: Intro

Additive Thinking

The Case of the Hotdogs!

On Monday, __ hotdogs were ordered for Friday's fundraiser. On Tuesday, __ hotdogs were ordered for Friday's fundraiser. How many hotdogs were ordered on Monday and Tuesday? Using pictures, words, numbers and/or symbols, show how you figured it out.

A: 18 + 7
B: 18 + 37
C: 218 + 497
D: _____

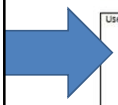
Big Idea 1

Activity: Intro

Additive Thinking

The Case of the Hotdogs!

On Monday, 18 hotdogs were ordered for Friday's fundraiser. On Tuesday, 37 hotdogs were ordered for Friday's fundraiser. How many hotdogs were ordered on Monday and Tuesday? Using pictures, words, numbers and/or symbols, show how you figured it out.



Use this space to solve it your way.

Strategy 1:

Big Idea 1

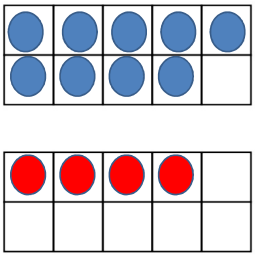
How many dots?

Additive Thinking

Big Idea 1

How many dots?

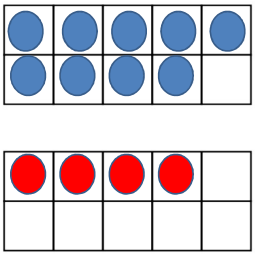
Additive Thinking



Big Idea 1

Intro: Making Ten

Additive Thinking

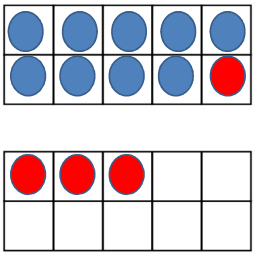


9 + 4

Big Idea 1

Intro: Making Ten

Additive Thinking

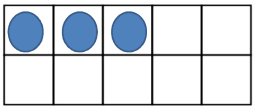


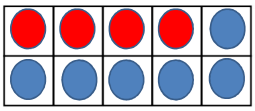
9 + 4

Big Idea 1

Intro: Making Ten

Additive Thinking



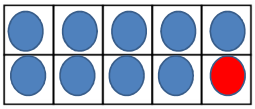


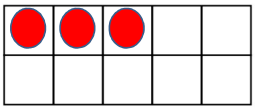
9 + 4

Big Idea 1

Intro: Making Ten

Additive Thinking



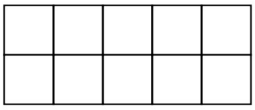


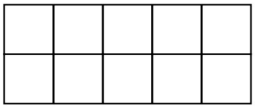
9 + 4 = 10 + 3

Big Idea 1

Intro: Try This

Additive Thinking



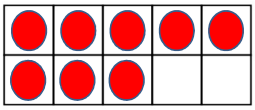


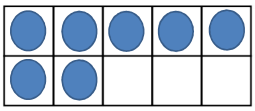
8 + 7

Big Idea 1

Intro: Try This

Additive Thinking





8 + 7

Big Idea 1

Extending: Making Ten

Additive Thinking

29 + 43

247 + 389

Big Idea 1

Practicing: Making Ten

Additive Thinking

Dragon Bump - Adding 9

11	15	14	13
14	10	15	12
13	12	10	11

Instruction:
Take turns with a partner. Roll the die. Add 9 to the number you roll. Put a cube on the number. If another player's cube is on that number, "bump" it off. If your cube is on that number, stack a second cube on top to "freeze" that spot. The player to use all of their cubes first wins!

Materials:
- One 6 sided die
- 10 cubes of one color
- 10 cubes of another color

Big Idea 1

Practicing: Making Ten

Additive Thinking

2 digit Addition Bump

Focusing on +8 +9

42	53	64	75
54	63	72	81
67	76	85	94

Instructions:
Take turns with a partner. Choose a card and add to find the sum. Put a cube on the sum. If another player's cube is on that number, "bump" it off. If your cube is on that number, stack a second cube on top to "freeze" that spot. The player to use all of their cubes first wins! If you run out of cards, shuffle.

Materials:
Addition Cards
Ten cubes of one color
Ten cubes of another color

Big Idea 1

Consolidating Your Learning

Additive Thinking

The Case of the Hotdogs!

On Monday, 18 hotdogs were ordered for Friday's fundraiser. On Tuesday, 37 hotdogs were ordered for Friday's fundraiser. How many hotdogs were ordered on Monday and Tuesday? Using pictures, words, numbers and/or symbols, show how you figured it out.

Use this space to solve it your way.

Strategy 1:

Compensating

Big Idea 1

Intro: Partition by Place Value

Additive Thinking

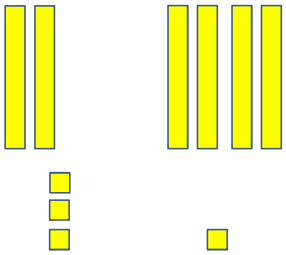
23 + 41

Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41



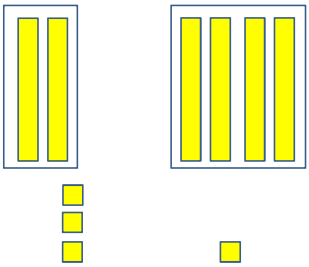
Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41

20
+ 40
60



Big Idea 1

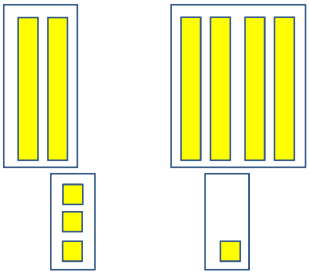
Intro: Partition by Place Value

Additive Thinking

23 + 41

20
+ 40
60

3
+ 1
4



Big Idea 1

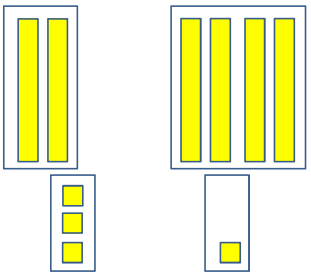
Intro: Partition by Place Value

Additive Thinking

23 + 41

20
+ 40
60

3
+ 1
4



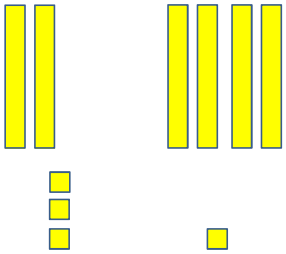
60
+ 4
64

Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41



Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41

3
+ 1
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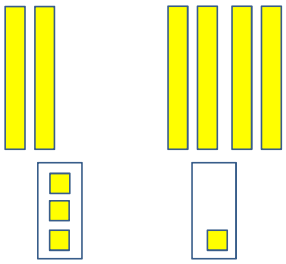


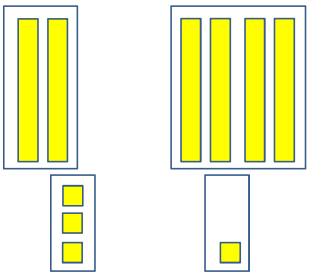
Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41

20
+ 40
60



3
+ 1
4

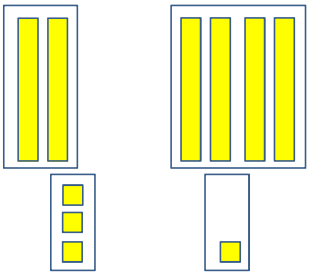
Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41

20
+ 40
60



3
+ 1
4

4
+ 60
64

Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41

23
+ 41
4
60
64

Big Idea 1

Intro: Partition by Place Value

Additive Thinking

23 + 41

23

+ 41

60

4

64

Big Idea 1

Intro: Partition by Place Value

Additive Thinking

Try with

29 + 43

247 + 389

Big Idea 1

Consolidating Your Learning

Additive Thinking

Strategy 2:

Partitioning by Place Value

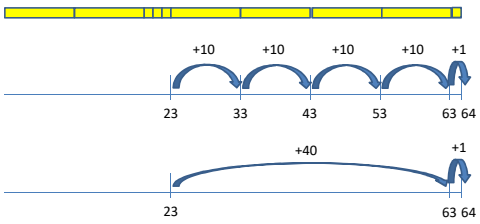
Strategy 3:

Big Idea 1

Intro: Open Number Lines

Additive Thinking

23 + 41

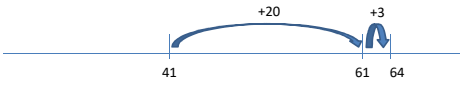


Big Idea 1

Intro: Open Number Lines

Additive Thinking

23 + 41



Big Idea 1

Intro: Try This

Additive Thinking

29 + 43

247 + 389

Big Idea 1

Consolidating Your Learning

Additive Thinking

Strategy 2:

Strategy 3:

Open Number Line

Big Idea 1

Quick Assessment

Additive Thinking

Quick Assessment

	Counting Strategy	Addition Strategy	Multiplication Strategy
Inventory of Strategies	☐ Counting ☐ Counting on/back	☐ Known Facts ☐ Using doubles (3+3) ☐ Skip Counting / Repeated Addition ☐ Making 10 ☐ Compensation ☐ Partitioning by Place Value ☐ Open Number Line ☐ Associative Property ☐ Commutative Property ☐ Traditional Algorithm	☐ Known Facts ☐ Using Doubles (3x2) ☐ Arrays ☐ Associative Property ☐ Commutative Property ☐ Distributive Property ☐ Traditional Algorithm
	☐ Other Strategies Used		

Answer	Communication	Mathematics
	☐ Obvious ☐ Inferred a little ☐ Inferred a lot	☐ Is correct ☐ Has a minor mistake ☐ Has a misunderstanding

Notes/Next Steps

Follow up Questions to Ask the Student

- ☐ Explain what you did
- ☐ Why did you choose this strategy?
- ☐ Will this always work?
- ☐
- ☐

The Relationships and Connections this student made:

Follow up Steps for Student

Big Idea 1

Quick Assessment

Additive Thinking

	Counting Strategy	**Addition** Strategy	**Multiplication** Strategy
Inventory of Strategies	☐ Counting ☐ Counting on/back	☐ Known Facts ☐ Using doubles (3+3) ☐ Skip Counting / Repeated Addition ☐ Making 10 ☐ Compensation ☐ Partitioning by Place Value ☐ Open Number Line ☐ Associative Property ☐ Commutative Property ☐ Traditional Algorithm	☐ Known Facts ☐ Using Doubles (3x2) ☐ Arrays ☐ Associative Property ☐ Commutative Property ☐ Distributive Property ☐ Traditional Algorithm
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Big Idea 1

Quick Assessment

Additive Thinking

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Big Idea 1

Quick Assessment

Additive Thinking

Notes/Next Steps

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- ☐

The Relationships and Connections this student made:

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Big Idea 1

Quick Assessment

Additive Thinking

This Quick Assessment Tool

Formative Assessment
This tool is not meant to be used as a summative assessment tool. Instead, it is merely one tool in your formative assessment toolbox that could be used to record students' thinking on a single assessment item. This will provide a snapshot in time.

"I can use a strategy" compared to "I understand a strategy"
Using a strategy accurately does not reflect students' understanding of the strategy. When interviewing students, using probing questions to determine if they are merely following a procedure or if they truly understand the strategy.

The answer
A correct answer does not necessarily indicate understanding. Students may be able to follow procedures without understanding the strategy or the final answer. Focus more on mathematical understanding.

An incorrect answer does not necessarily indicate misunderstanding of mathematical concepts. Don't judge students' work based on minor mathematical mistakes. Focus more on mathematical understanding.

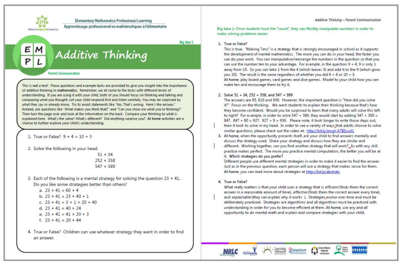
Follow-up Questions
Use follow-up questions to probe students' more deeply about their thinking. We often infer when interpreting students' work. Instead, ask them to explain what they've done and why they've chosen to use a particular strategy.

"The mathematics instruction we provide children should emphasize meaning, relationships, and connections, and we should be mindful of what our students understand, not merely what they can do."
~Marilyn Burns~

Big Idea 1

Communicating with Parents

Additive Thinking



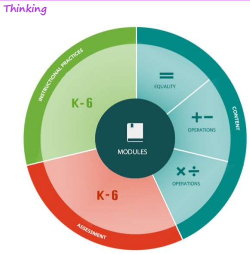
Conclusion

- The EMPL Website
- Survey

Conclusion

The Website

Additive Thinking



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Conclusion
Additive Thinking
Thank You !