Helping Students Connect Mathematical Models and Representations:

A Runway to Integrating the SMPs

Danielle Curran
daniellepcurran@gmail.com
Twitter: @danigirl1216

Integrate Mathematical Practices

Manage Discourse
Engage Students
Prepare Students for Assessments
Encourage Multiple Representations
Address student gaps
Support ELLs
Differentiate Instruction
Use Data to Inform Instruction
Some representations and strategies may be new to you...

Learning & Teaching Mathematics

**Standards for Mathematical Practice**
*Mathematically proficient students…*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Choose appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

*(CCSS, 2010)*

**NCTM Teaching Practices**
*Effective mathematics educators…*

1. Establish mathematical goals that focus learning
2. Implement tasks that promote reasoning and problem solving
3. Use and connect mathematical representations
4. Facilitate meaningful mathematical discourse
5. Pose purposeful questions
6. Build procedural fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

*(NCTM, 2014)*
Make instructional best practices manageable

Use an instructional routine
- Free up “brain power” to focus on the math
- Students and teachers know what to expect
- No script to follow
- Support teachers in facilitating discourse and making discourse easier to implement
- Instills good habits and breaks the bad
- Helps all teachers integrate the practice standards for students and teachers

Think-Share-Compare

THINK
- Make sense of the problem
- Solve and support your thinking

SHARE
- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE
- Make connections between strategies
- Apply strategies to new problems
Grade 4

4.NBT.B.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are set up?
Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are set up?

THINK
- Make sense of the problem
- Solve and support your thinking

SHARE
- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE
- Make connections between strategies
- Apply strategies to new problems
Step 3 Discuss

- Share your thinking with a partner.

Discussion Starters

- The strategy I used was...
- Do you agree with me? Why or why not?

Step 4 Discuss

- Discuss strategies as a class

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are set up?
Think-Share-Compare

Think
- Make sense of the problem
- Solve and support your thinking

Share
- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

Compare
- Make connections between strategies
- Apply strategies to new problems

Step 5 Compare
- Compare class strategies to these strategies.
Some representations and strategies may be new to you...

Try It: Use what you just learned to solve these problems. Show your work on a separate sheet of paper.

14 \(27 \times 21 = \) 
15 \(37 \times 23 = \)
What are you doing the same as you did 25 years ago?

The importance of decomposing numbers in Grade 1...

- $13 = 10 + 3$
- $14 = 10 + 4$
- $10 + 3$
- $10 + 4$
- $20 + 7$
...is essential when students multiply numbers in Grade 4...

\[
\begin{array}{c|c}
10 & 6 \\
20 & \hline \\
+ & 200 \\
\hline
8 & 120 \\
\hline
\end{array}
\]

\[
16 \times 28
\]

\[
\begin{array}{c|c}
 & 16 \\
\hline \\
\times 28 & 48 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
 & 80 \\
\hline \\
\downarrow & 120 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c}
 & 200 \\
\hline \\
\downarrow & \hline
\end{array}
\]

\[
448
\]

...which lays the foundation for success in algebra...

What is “FOIL”? When and why do you “FOIL”?
\[(x + 8)(x + 6)\]

**First:** \(x \cdot x = x^2\)

**Outer:** \(x \cdot 6 = 6x\)

**Inner:** \(x \cdot 8 = 8x\)

**Last:** \(8 \cdot 6 = 48\)

...which builds upon what students do in algebra!

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>+</td>
</tr>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>6x</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8x</td>
</tr>
<tr>
<td></td>
<td>48</td>
</tr>
</tbody>
</table>

\[x^2 + 14x + 48\]
Do it again! Do it again!

Grade 1 & 2 (1.NBT.C.4; 2.NBT.B.5)
Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and (understand that) sometimes it is necessary to compose a ten.

What do you notice?

– What are the students doing?
– What is the teacher doing?
## Practices for Students and Teachers

### Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Choose appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*(CCSS, 2010)*

### NCTM Teaching Practices

1. Establish mathematical goals that focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

*(NCTM, 2014)*

---

## Think-Share-Compare

**THINK**
- Make sense of the problem
- Solve and support your thinking

**SHARE**
- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

**COMPARE**
- Make connections between strategies
- Apply strategies to new problems
Step 1 THINK Make sense of the problem
- What is the problem about?
- What are you trying to find out?
- What information is important?

Problem Think about ways to solve the problem.

How many marbles?

35 marbles 27 marbles

Step 2 THINK Solve and support your thinking
- Solve the problem.
- Record your process.
- Solve another way.

Problem Think about ways to solve the problem.

How many marbles?

35 marbles 27 marbles
THINK
• Make sense of the problem
• Solve and support your thinking

SHARE
• Share strategies with a partner (pair/share)
• Share strategies as a class (whole class)

COMPARE
• Make connections between strategies
• Apply strategies to new problems

Step 3  SHARE: Discuss It
• Share your thinking with a partner.

Discussion Starters
• The strategy I used was...
• Do you agree with me? Why or why not?
• Discuss class strategies

**SHARE:** Discuss It

How many marbles?

- 35 marbles
- 27 marbles

---

**Select and Sequence Solutions**

- **Local**: 60 + 2 = 62
- **Rob**:
  - 35 + 27
  - \[ \frac{12}{50} \]
  - \[ \frac{62}{50} \]
- **Maria**:
  - 35 + 27
  - \[ \frac{12}{50} \]
  - \[ \frac{62}{50} \]
- **Olivia**:
  - 35 + 20 = 55
  - 55 + 5 = 60
  - 60 + 2 = 62
- **James**:
  - 35 + 20 = 55
  - 55 + 5 = 60
  - 60 + 2 = 62

---

*Curriculum Associates*
Think-Share-Compare

**THINK**
- Make sense of the problem
- Solve and support your thinking

**SHARE**
- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

**COMPARE**
- Make connections between strategies
- Apply strategies to new problems
Connect It

- Connect student strategies.
- Connect to book strategies.

Model It

Find $35 + 27$.
Add the tens and ones.

\[
\begin{array}{c}
5 \text{ tens} \\
\hline
50
\end{array}
+\begin{array}{c}
12 \text{ ones} \\
\hline
12
\end{array}
=\begin{array}{c}
10 \text{ tens} \\
\hline
60
\end{array}
+\begin{array}{c}
2 \text{ ones} \\
\hline
2
\end{array}
=\begin{array}{c}
50 + 10 + 2
\end{array}
=\begin{array}{c}
62
\end{array}
\]

Apply It

There are 36 white eggs and 25 brown eggs.
How many eggs?
Which Practice Standards did you see in the routine? Which Teaching Practices did you see in the routine?

### Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Choose appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

*(CCSS, 2010)*

### NCTM Teaching Practices

1. Establish mathematical goals that focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

*(NCTM, 2014)*

---

### 6 Things to try immediately

1. Give students time to think before answering a question (use hand signals instead of hands up).
2. Encourage students to find more than one strategy to solve a problem.
3. Have students turn and talk with a partner frequently.
4. Have other students restate or explain other students’ thinking/strategies/responses.
5. Acknowledge and validate students’ contributions, even if they don’t advance the lesson.
6. Use Think-Share-Compare to make it easier to integrate the SMPs and PtA Teacher Practices.

*Curriculum Associates*
Allow for Multiple Approaches

- Think
- Share
- Compare

See the sample Lesson Guide for the Grade 1 problem
ONE REASON PEOPLE RESIST CHANGE IS BECAUSE THEY FOCUS ON WHAT THEY HAVE TO GIVE UP, INSTEAD OF WHAT THEY HAVE TO GAIN.
Ready Tools to Support Practices

1. Make Sense of the Problem
2. Solve and Support Your Thinking
3. Discuss
4. Compare
5. Connect and Reflect
6. Apply

I started solving the problem by...
Does your partner's strategy make sense?
¿Qué estrategia usaste?

Helping Students Connect Mathematical Models and Representations: A Runway to Integrating the SMPs

Danielle Curran
daniellepcurran@gmail.com
dcurran@cainc.com
Twitter: @danigirl1216