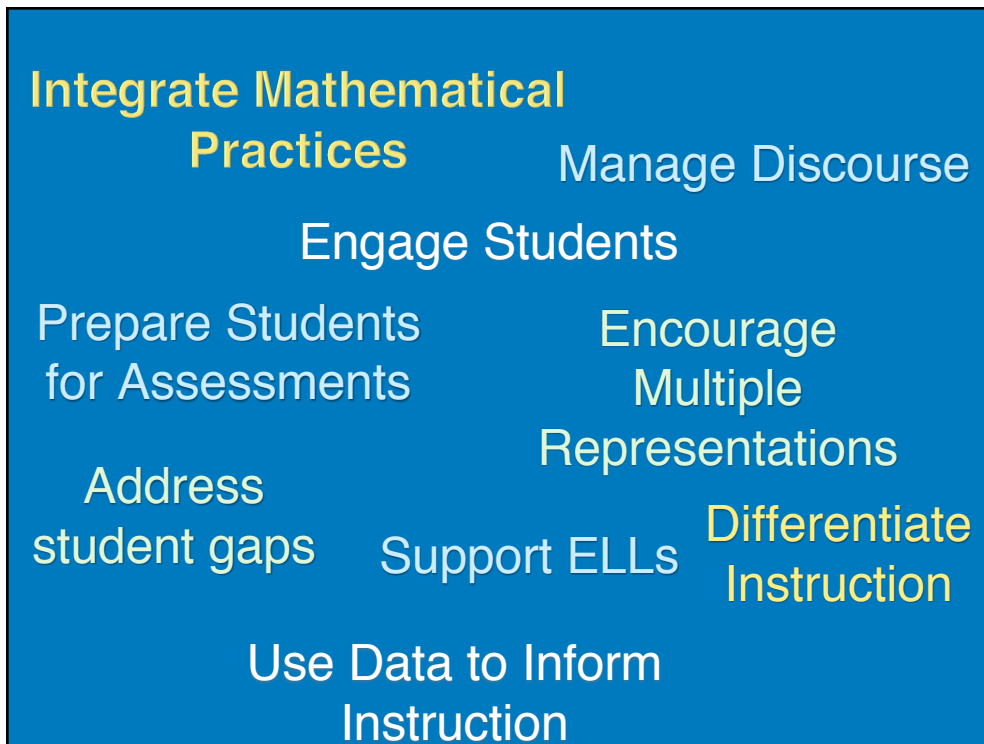


Helping Students Connect Mathematical Models
and Representations:

A Runway to Integrating the SMPs

Danielle Curran

daniellepcurran@gmail.com
Twitter: @danigirl1216



Integrate Mathematical Practices

Manage Discourse

Engage Students

Prepare Students
for Assessments

Encourage
Multiple
Representations

Address
student gaps

Support ELLs

Differentiate
Instruction

Use Data to Inform
Instruction

Some
representations
and strategies
may be new to
you...



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"Now, with the *new* math. . . ."

Learning & Teaching Mathematics

Standards for Mathematical Practice

Mathematically proficient students...

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Choose appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

(CCSS, 2010)

NCTM Teaching Practices

Effective mathematics educators...

1. Establish mathematical goals that focus learning
2. Implement tasks that promote reasoning and problem solving
3. Use and connect mathematical representations
4. Facilitate meaningful mathematical discourse
5. Pose purposeful questions
6. Build procedural fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

(NCTM, 2014)

Make instructional best practices manageable

Use an instructional routine

- Free up “brain power” to focus on the math
- Students and teachers know what to expect
- No script to follow
- Support teachers in facilitating discourse and making discourse easier to implement
- Instills good habits and breaks the bad
- Helps all teachers integrate the practice standards for students and teachers

Think-Share-Compare

THINK

- Make sense of the problem
- Solve and support your thinking

SHARE

- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE

- Make connections between strategies
- Apply strategies to new problems



Grade 4

4.NBT.B.5

Multiply a whole number of up to four digits by a one-digit whole number, and **multiply two two-digit numbers, using strategies based on place value** and the properties of operations. **Illustrate and explain the calculation by using** equations, rectangular arrays, and/or **area models**.

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Step 1 **Make Sense of the Problem**

- What is the problem about?
- What are you trying to find out?
- What information is important?

Problem Think about ways to solve the problem.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are set up?

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Step 2 **Solve and Support Your Thinking**

- Solve the problem.
- Record your process.
- Solve another way.

Problem

Think about ways to solve the problem.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are set up?

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Think-Share-Compare

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- Make sense of the problem
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- Share strategies as a class (whole class)

COMPARE

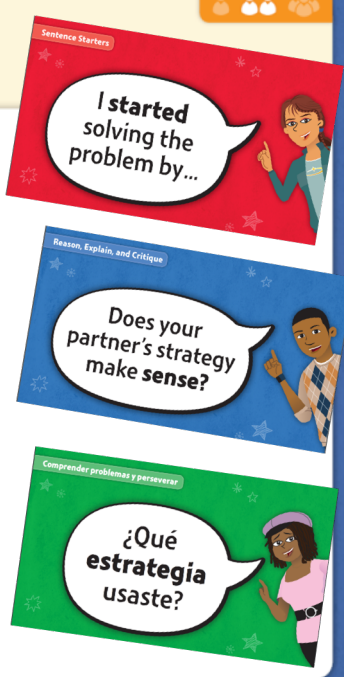
- Make connections between strategies
- Apply strategies to new problems

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Step 3 Discuss
• Share your thinking with a partner.

Discussion Starters

- The strategy I used was...
- Do you agree with me?
Why or why not?



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Step 4 Discuss
• Discuss strategies as a class

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are set up?

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Think-Share-Compare

THINK


- Make sense of the problem
- Solve and support your thinking


SHARE

- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE

- Make connections between strategies
- Apply strategies to new problems



 Ready
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Step 5 Compare

- Compare class strategies to these strategies.

$$\begin{array}{r}
 16 \\
 \times 28 \\
 \hline
 48 \\
 80 \\
 120 \\
 + 200 \\
 \hline
 448
 \end{array}$$

	10	+	6
20			
+			
8			

Reason, Explain, and Critique

How is **your** strategy different from or the same as another strategy?

Reflexionar y conectar

¿Cuál es la **relación** entre ____ y ____?

Reason, Explain, and Critique

What part of another person's solution do you want them to **explain** more specifically?

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Step 6 Apply

- Apply thinking to a new problem.
- Support your thinking.

Try It Use what you just learned to solve these problems. Show your work on a separate sheet of paper.

14 $27 \times 21 =$ _____

15 $37 \times 23 =$ _____

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Some
representations
and strategies
may be new to
you...



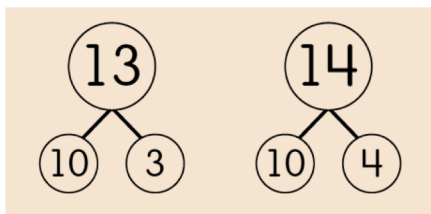
"Now, with the *new* math. . . ."

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**What are you doing the same
as you did 25 years ago?**



**The importance of
decomposing numbers in Grade 1...**



$$\begin{array}{r}
 10 + 3 \\
 10 + 4 \\
 \hline
 20 + 7
 \end{array}$$

...is essential when students multiply numbers in Grade 4...

	10	+	6	
20	200	120		
+				
8	80	48		

16	
× 28	
—	
48	
80	
120	
+ 200	
—	
448	

Ready Mathematics

...which lays the foundation for success in algebra...

What is “FOIL”?
When and why do you
“FOIL”?

Ready Mathematics

$$(x + 8)(x + 6)$$

First: $x \cdot x = x^2$

Outer: $x \cdot 6 = 6x$

Inner: $x \cdot 8 = 8x$

Last: $8 \cdot 6 = 48$

...which builds upon what
students do in algebra!

	x	$+$	6	
x	x^2		$6x$	
$+$				
8	$8x$		48	

$x^2 + 14x + 48$

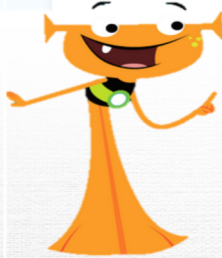
Do it again! Do it again!

Grade 1 & 2 (1.NBT.C.4; 2.NBT.B.5)

Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and (understand that) sometimes it is necessary to compose a ten.

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What do you notice?



- What are the **students** doing?
- What is the **teacher** doing?





Ready

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Practices for Students and Teachers

Standards for Mathematical Practice <i>Mathematically proficient students...</i>	NCTM Teaching Practices <i>Effective mathematics educators...</i>
<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Choose appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning <p style="text-align: right; font-style: italic;">(CCSS, 2010)</p>	<ol style="list-style-type: none"> 1. Establish mathematical goals that focus learning 2. Implement tasks that promote reasoning and problem solving 3. Use and connect mathematical representations 4. Facilitate meaningful mathematical discourse 5. Pose purposeful questions 6. Build procedural fluency from conceptual understanding 7. Support productive struggle in learning mathematics 8. Elicit and use evidence of student thinking <p style="text-align: right; font-style: italic;">(NCTM, 2014)</p>

Think-Share-Compare

THINK


- Make sense of the problem
- Solve and support your thinking



SHARE

- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE

- Make connections between strategies
- Apply strategies to new problems



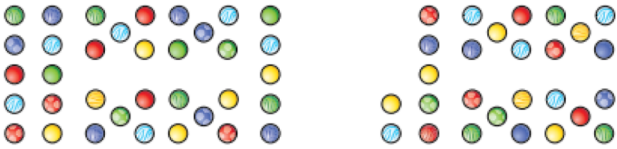
Step 1 THINK Make sense of the problem

- What is the problem about?
- What are you trying to find out?
- What information is important?

Problem Think about ways to solve the problem.

How many marbles?

35 marbles 27 marbles



Grade 5 Introduction

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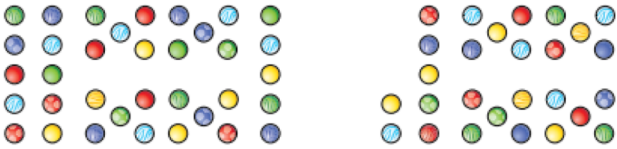
Step 2 THINK Solve and support your thinking

- Solve the problem.
- Record your process.
- Solve another way.

Problem Think about ways to solve the problem.

How many marbles?

35 marbles 27 marbles



Grade 5 Introduction

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Think-Share-Compare

THINK


- Make sense of the problem
- Solve and support your thinking

SHARE

- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE

- Make connections between strategies
- Apply strategies to new problems



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Step 3 SHARE: Discuss It

- Share your thinking with a partner.

Discussion Starters

- The strategy I used was...
- Do you agree with me? Why or why not?

Sentence Starters

I started solving the problem by...

Reason, Explain, and Critique

Does your partner's strategy make sense?

Comprender problemas y perseverar

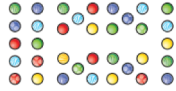
¿Qué estrategia usaste?

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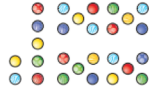
Step 4 SHARE:
Discuss It

How many marbles?

35 marbles



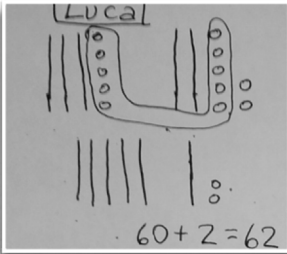
27 marbles



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Select and Sequence Solutions

Luca



$60 + 2 = 62$

Rob

$$\begin{array}{r} 35 \\ + 27 \\ \hline 12 \\ 50 \\ \hline 62 \end{array}$$

Maria

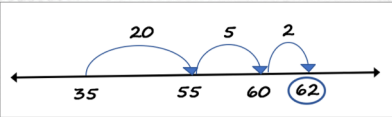
$$\begin{array}{r} 35 + 27 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 30 + 5 \quad 20 + 7 \\ 50 + 12 = 62 \end{array}$$

Olivia

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

James

$$\begin{array}{l} 35 + 20 = 55 \\ 55 + 5 = 60 \\ 60 + 2 = 62 \end{array}$$



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Think-Share-Compare

THINK


- Make sense of the problem
- Solve and support your thinking

SHARE

- Share strategies with a partner (pair/share)
- Share strategies as a class (whole class)

COMPARE

- Make connections between strategies
- Apply strategies to new problems



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Step 5**COMPARE: Connect It**

- Connect student strategies.
- Connect to book strategies.

**Model It Find $35 + 27$.**

Add the tens and ones.

$$\begin{array}{r}
 \begin{array}{|c|c|c|c|c|} \hline \text{○} \\ \hline \text{○} \\ \hline \text{○} \\ \hline \text{○} \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|} \hline \text{○} \\ \hline \text{○} \\ \hline \text{○} \\ \hline \text{○} \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|} \hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|} \hline \text{○} \\ \hline \text{○} \\ \hline \text{○} \\ \hline \text{○} \\ \hline \end{array} \\
 \text{5 tens} \quad \quad \quad \text{12 ones} \\
 50 \quad + \quad 12 \\
 50 + 10 + 2 = \underline{\quad} \\
 \mathbf{35 + 27 = \underline{\quad}}
 \end{array}$$

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Step 6**COMPARE: Apply It**

- Apply thinking to a new problem.
- Support your thinking.

**There are 36 white eggs
and 25 brown eggs.**

How many eggs?



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Which Practice Standards did you see in the routine?

Which Teaching Practices did you see in the routine?

Standards for Mathematical Practice <i>Mathematically proficient students...</i>	NCTM Teaching Practices <i>Effective mathematics educators...</i>
<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Choose appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning <p>(CCSS, 2010)</p>	<ol style="list-style-type: none"> 1. Establish mathematical goals that focus learning 2. Implement tasks that promote reasoning and problem solving 3. Use and connect mathematical representations 4. Facilitate meaningful mathematical discourse 5. Pose purposeful questions 6. Build procedural fluency from conceptual understanding 7. Support productive struggle in learning mathematics 8. Elicit and use evidence of student thinking <p>(NCTM, 2014)</p>

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6 Things to try immediately

- 1 Give students time to think before answering a question (use hand signals instead of hands up)
- 2 Encourage students to find more than one strategy to solve a problem
- 3 Have students turn and talk with a partner frequently
- 4 Have other students restate or explain other students' thinking/strategies/responses
- 5 Acknowledge and validate students' contributions, even if they don't advance the lesson
- 6 Use Think-Share-Compare to make it easier to integrate the SMPs and PtA Teacher Practices

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Use any contextual problem

- Think
- Share
- Compare

See the sample Lesson Guide for the Grade 1 problem

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Discourse-Based Lesson Guide

i-Ready | Ready

THINK

Make sense of the problem

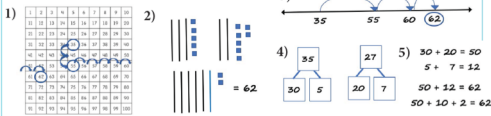
*What is a marble?
(size, shape, material)*

Solve and support your thinking

Possible error: Students may forget to regroup the tens.

SHARE

Anticipate, select, and sequence student solution strategies



Questions and sentence starters for whole class and partner discussions

I started the problem by...

What questions do you have about your partner's strategy?

COMPARE

Make connections between representations and strategies

Strategies to connect:

• *Strategies 1 and 3 connect:*

Start at 35; Jump 20, go to the next 10, add 2 "leftovers"

• *Strategies 2, 4 and 5 connect:*

Group tens and group ones together. Regroup 10 ones and 1 ten.

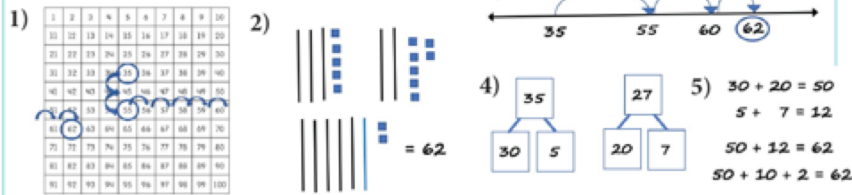
Apply It: Connect what students have learned to new problems

Try egg problem using two strategies, one that may be new to you.



SHARE

Anticipate, select, and sequence student solution strategies



COMPARE

Make connections between representations and strategies

Strategies to connect:

• *Strategies 1 and 3 connect:*

Start at 35; Jump 20, go to the next 10, add 2 "leftovers"

• *Strategies 2, 4 and 5 connect:*

Group tens and group ones together. Regroup 10 ones and 1 ten.

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ONE REASON PEOPLE
RESIST CHANGE IS
BECAUSE THEY FOCUS
ON WHAT THEY HAVE TO
GIVE UP, INSTEAD OF
WHAT THEY HAVE TO
GAIN.



Ready Tools to Support Practices

- 1 Make Sense of the Problem
- 2 Solve and Support Your Thinking
- 3 Discuss
- 4 Compare
- 5 Connect and Reflect
- 6 Apply







 Ready MathematicsCurriculum Associates

Helping Students Connect Mathematical Models and Representations: A Runway to Integrating the SMPs

Danielle Curran

daniellepcurran@gmail.com
dcurran@cainc.com
 Twitter: @danigirl1216