NCTM ANNUAL MEETING & EXPOSITION 2018
April 25–28 | Washington, DC
PREMIER MATH EDUCATION EVENT
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NCTM

Talk NUMBER 2 Me:

The Ratio Table

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@virtuouscm
#TalkNumber2Me

CALIFORNIA MATHEMATICS COUNCIL Southern Section

NSBE NATIONAL SOCIETY OF BLACK ENGINEERS

Benjamin Banneker Association

A Word from my Sponsors…

- Board Member of the California Mathematics Council South (CMC-S). Equity and Empowerment Chair
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- National Society of Black Engineers Jr (NSBE), Southern California Chapter Advisor

CALIFORNIA MATHEMATICS COUNCIL
Southern Section

Benjamin Banneker Association

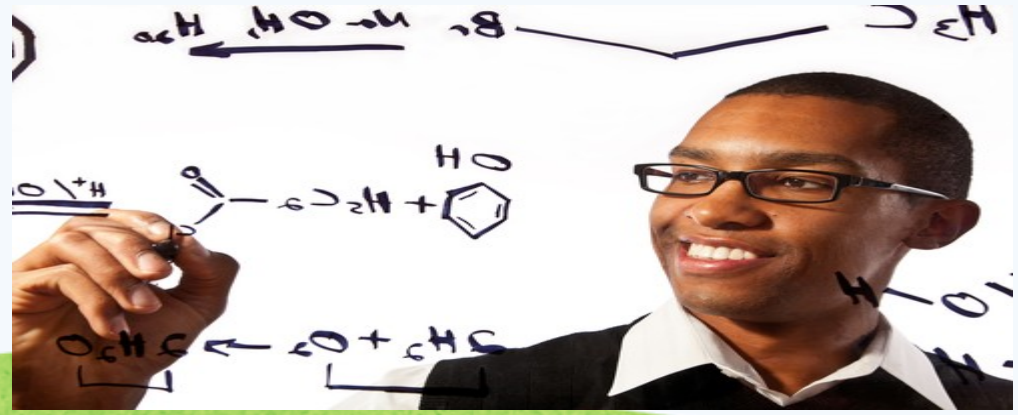
NSBE
NATIONAL SOCIETY OF BLACK ENGINEERS

California Mathematics Standards (CCSS)

- 3.OA.2 This standard focuses on two distinct models of division: partition models and quotative models. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
- 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted...Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- 5.NF.B.3 Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Did you know?

- Only 19% of African-Americans were proficient on the 2017 California's State Mathematics testing, 16% in 2016 (grades 3rd-11th)?
- By the eighth grade, less than one-third of African-American students nationwide are proficient in math and science. (Joint Center for Political and Economic Studies.)



Student Groups - Race/Ethnicity	Number of Students Tested with Scores 2017	Percent of Students Who Exceeded Standard in 2017	Percent of Students Who Met Standard in 2017	Percent of Students Who Nearly Met Standard in 2017	Percent of Students Who Did Not Meet Standard in 2017	Percent Point Change of Students Who Met or Exceeded Standard 2016–2017	Percent Point Change of Students Who Met or Exceeded Standard 2015–2017
Asian	300,380	49.42	23.27	16.11	11.20	0.69	3.69
Black or African American	175,068	5.92	13.10	25.48	55.50	1.02	3.02
Latin@x	1,752,405	8.31	16.89	29.10	45.70	1.20	4.20
White	752,059	26.81	26.04	25.59	21.56	-0.15	3.85

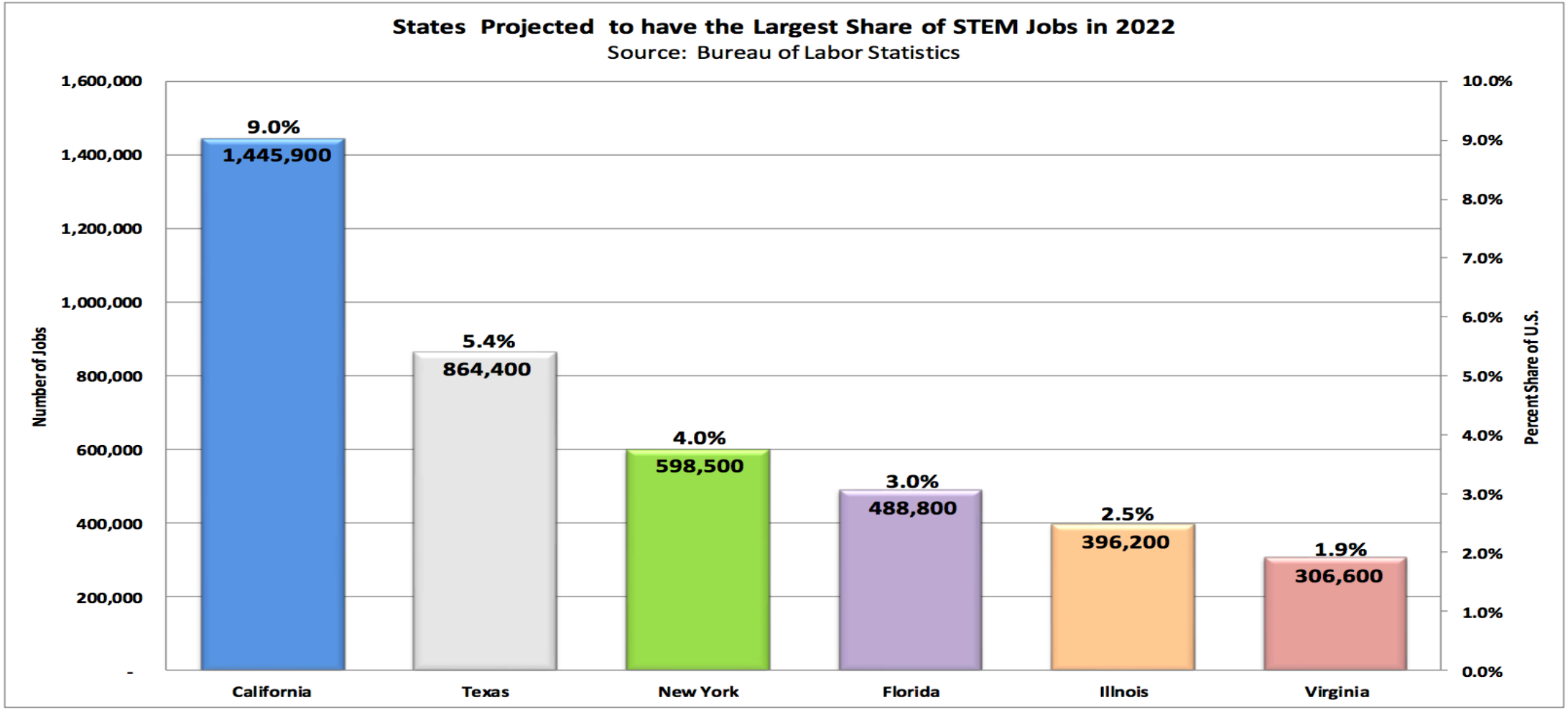
https://www.cde.ca.gov/nr/ne/yr17/yr17rel67a.asp

- African-Americans received just 7.6% of all STEM bachelor's degrees and 4.5% of doctorates in STEM. (https://nces.ed.gov/programs/digest/d14/tables/dt14_318.45.asp) 2013

- Even in the most diverse Silicon Valley tech firms, only 2% of the technology jobs are held by African-Americans

STEM in California

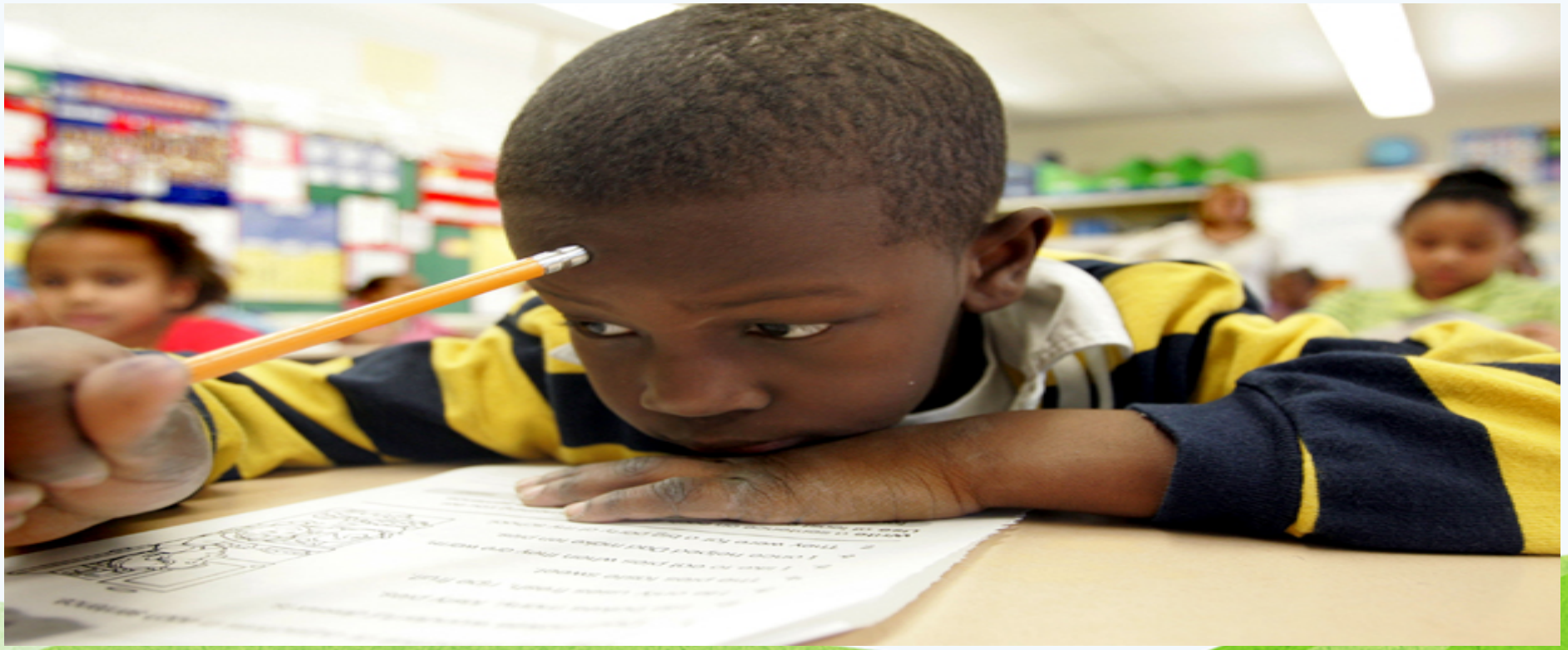
August 2015



Attributes of Science, Technology, Engineering, and Mathematics (STEM) Jobs[4]

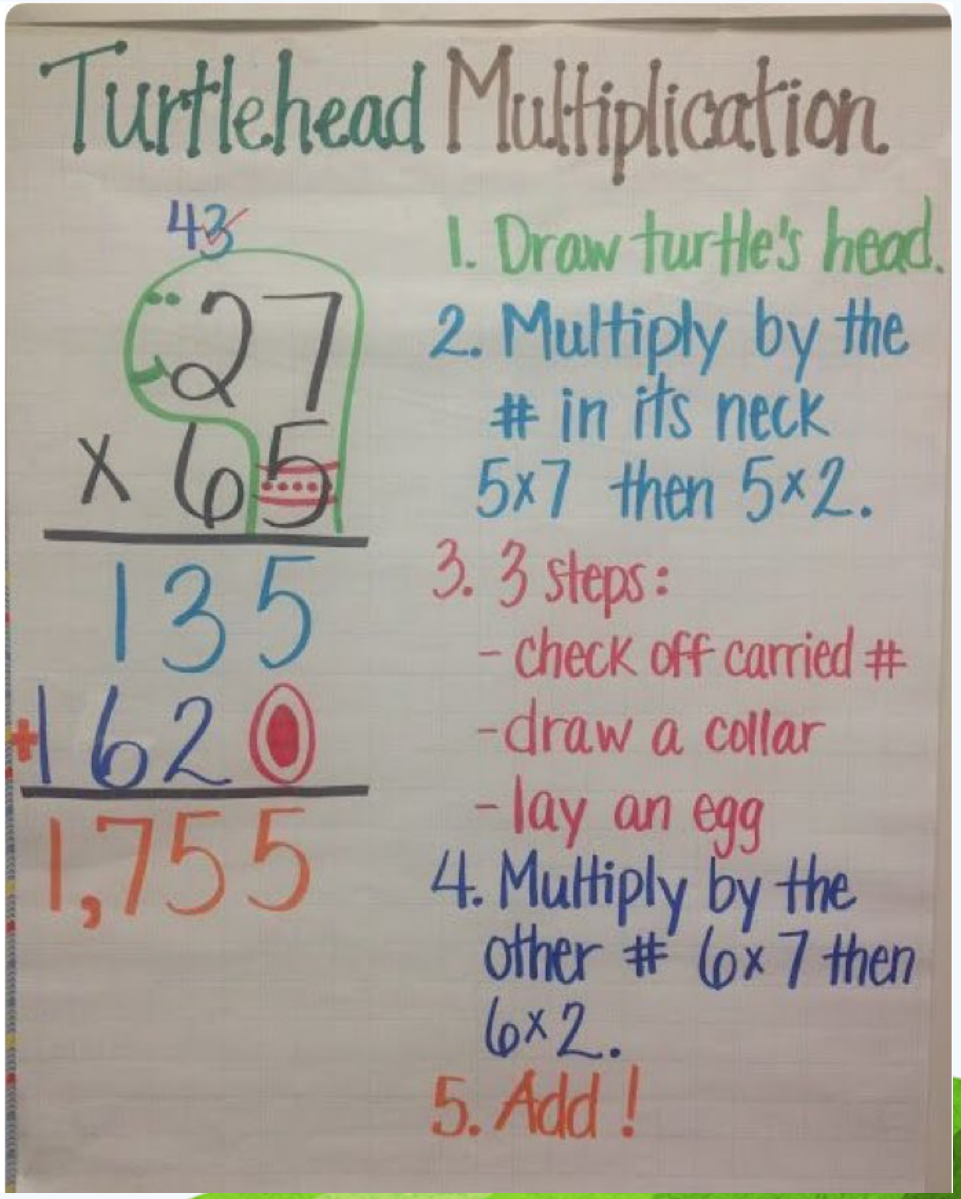
Why do students struggle with mathematics?

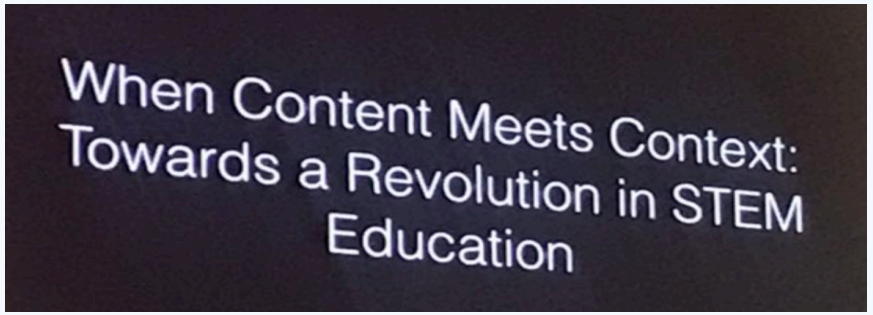
Why is mathematics a gatekeeper instead of a gateway?



Other Possible Misconceptions

- Primary experiences revolved around saying and writing numbers rather than counting and cardinality, understanding magnitudes and number relationships.
- NO CONTEXT; just computation, algorithms were introduced to them before they were developmentally ready.
- They lack strategies.
- They lack thinking tools to organize their thinking
- Seen as the sum of their deficits, not their strengths





NCTM, April 25, 2018
Opening Keynote

Agnosia-The ability to not make sense of what is right in front of you.

When natural intuitive knowledge is perceived as not being mathematical…You rob them of the opportunity to claim their place in math and science.

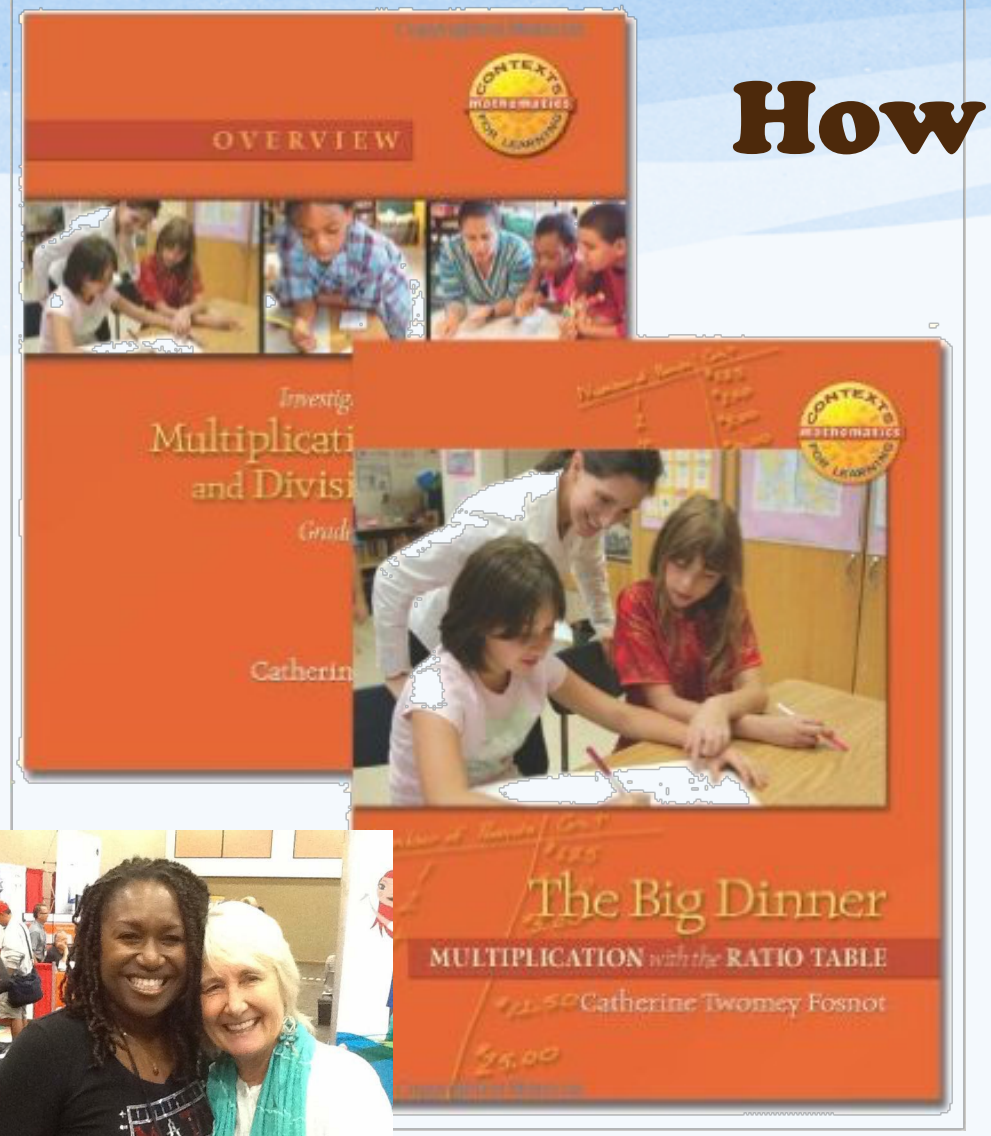
Make their knowledge privilege

Wounds imposed – inflicted by the pedagogy

If I'm always viewed as less than. If the math is not connected to my context…I have a phobia of the content.

Thesis:

- Algebra is the gateway to higher mathematics. Algebra is dependent on multiplicative reasoning and proportional relationships. Lack of multiplicative reasoning is a major reason students are failing algebra.
- The Ratio Table is a powerful tool for young students to shift from additive thinking to multiplicative, master the distributive property, partial products/quotients, and proportional reasoning; by exploring real world problems, crafting multiple solutions within contexts, and provide proofs of their own thinking.



How do we change the data?

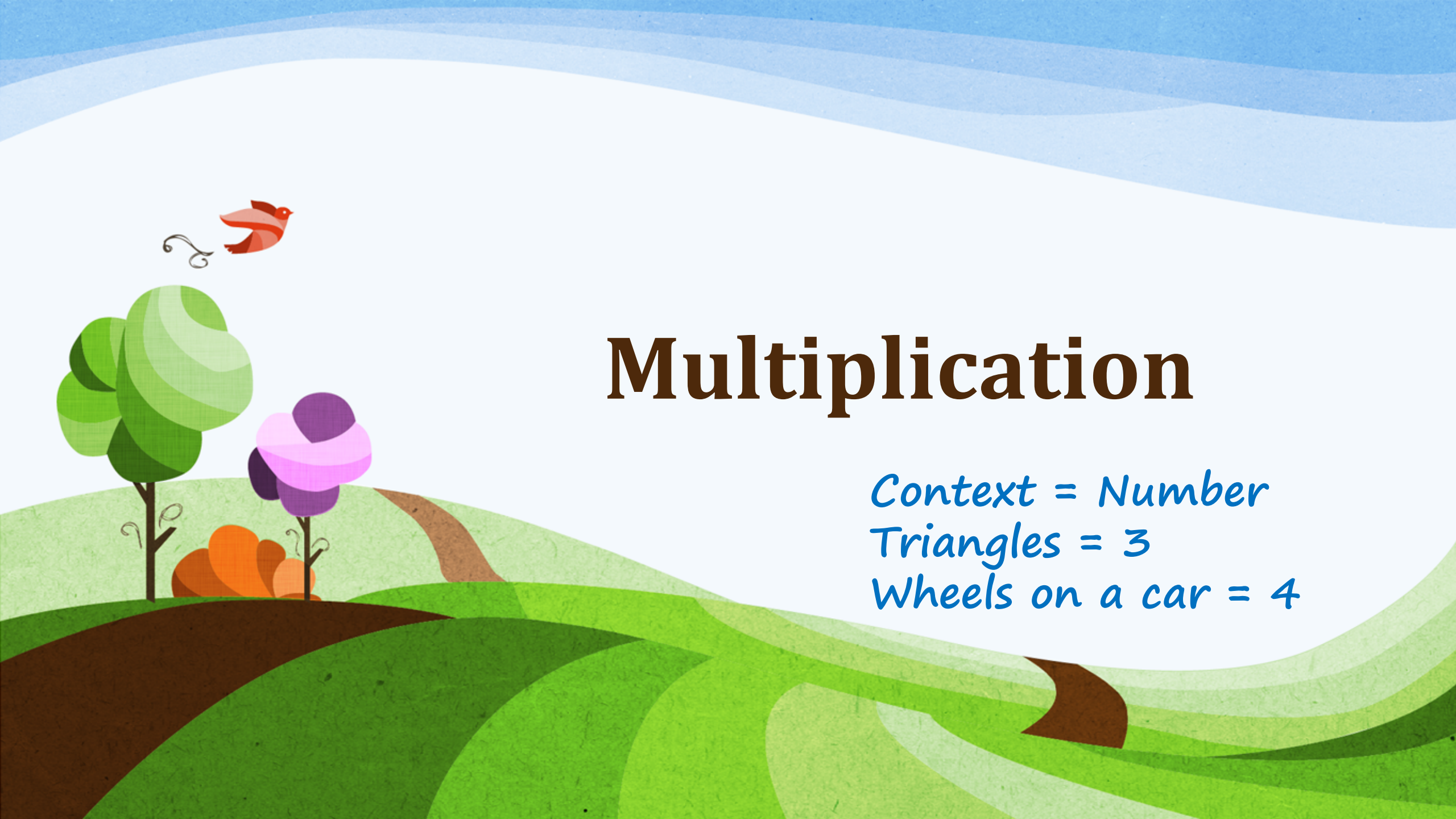
"Our work is driven by the desire to transform classrooms into communities of mathematicians: places where children explore interesting problems and like mathematicians, engage in crafting solutions, justifications, and proofs of their own making".

-Cathy Twomey Fosnot

Contexts for Learning Mathematics

Ratio Table: A thinking tool learners can use to organize their thinking which enables them to strategically solve problems. MP5

- Multiplication – whole number, find partial products
- Division – find partial quotients & develop proportional reasoning
- Fractions, decimals, percents



Multiplication

Context = Number
Triangles = 3
Wheels on a car = 4

- How many tires on a car?

Cars	Tires

Cars	Tires
1	4

Cars	Tires
1	4
2	8

Cars	Tires
1	4
2	8
4	16

Cars	Tires
1	4
2	8
4	16
8	32

Cars	Tires
1	4
2	8
4	16
8	32
10	40

Cars	Tires
1	4
2	8
4	16
8	32
10	40
9	36

Cars	Tires
1	4
2	8
4	16
8	32
10	40
9	36
12	48

Number Strings

What mathematical ideas or strategies are developed with this string?

2 x 4

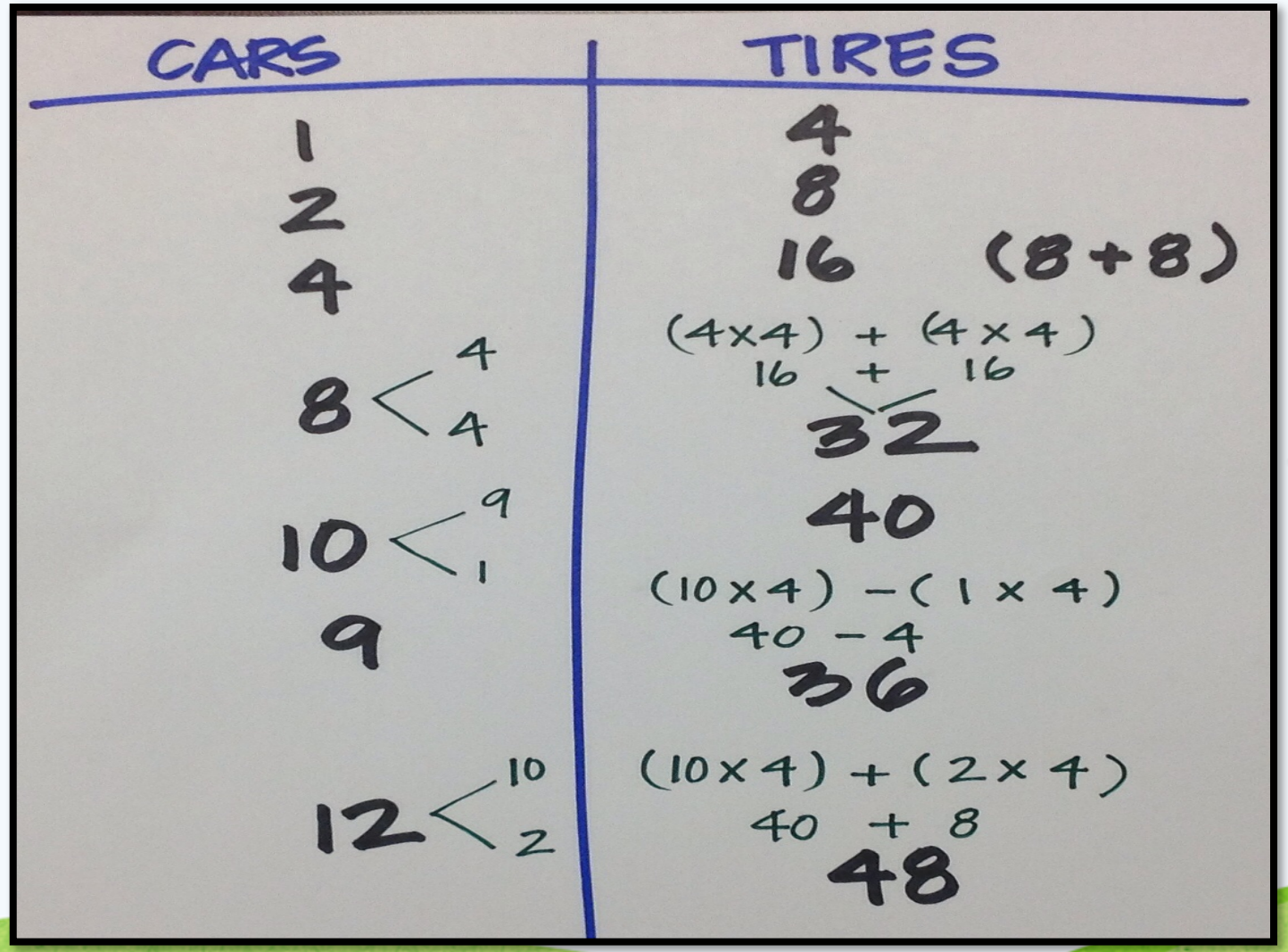
4 x 4

8 x 4

10 x 4

9 x 4

12 x 4



Number Strings

What mathematical ideas or strategies are developed with this string?

2 x 4

4 x 4

8 x 4

10 x 4

9 x 4

12 x 4

- Doubling
- Using Partial Products (place value understanding)
- Distributive Property over addition
- Distributive Property over subtraction

Regina is building a treehouse for her children. She bought 28 6ft blocks of wood. She needs 165ft of lumber. Did she buy enough?

Wood	Feet

Wood	Feet
1	6

Wood	Feet
1	6
2	12

Wood	Feet
1	6
2	12
3	18

Wood	Feet
1	6
2	12
3	18
4	24

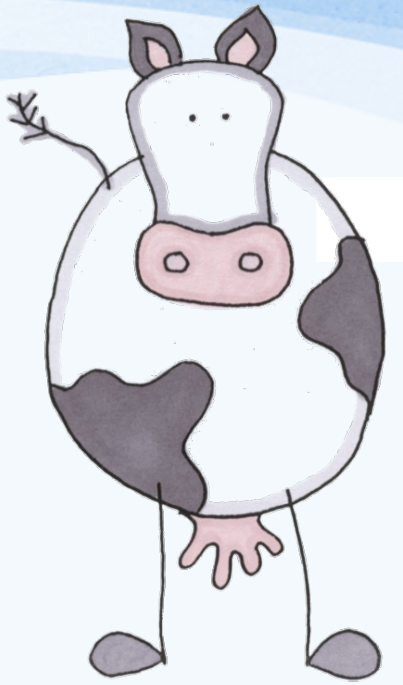
Wood	Feet
1	6
2	12
3	18
4	24
5	30

Wood	Feet
1	6
2	12
3	18
4	24
5	30
10	60

Wood	Feet
1	6
2	12
3	18
4	24
5	30
10	60
20	120

Wood	Feet
1	6
2	12
3	18
4	24
5	30
10	60
20	120
25	150

Wood	Feet
1	6
2	12
3	18
4	24
5	30
10	60
20	120
25	150
28	168



Ducks and Cows

Farmer Lisa has ducks and cows. One day she noticed that the animals had a total of 12 heads and 32 feet. How many of the animals were ducks and how many were cows?

Duck Heads	Cows Heads	Feet

Duck Heads	Cows Heads	Feet
1	1	6

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36

Too much

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36
6	5	32

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36
6	5	32

Only 11

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36
6	5	32
7	5	34

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36
6	5	32
7	5	34

Too much

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36
6	5	32
7	5	34
8	4	32

Duck Heads	Cows Heads	Feet
1	1	6
2	2	12
4	4	24
6	6	36
6	5	32
7	5	34
8	4	32

16+16

3336

A factory makes 1,200 shirts every 6 hours. The factory makes shirts for 9 hours each workday. Enter the **fewest** number of workdays the factory will need to make 12,600 shirts.

1	2	3
4	5	6
7	8	9
0	.	-

Hours	Shirts

Hours	Shirts
6	1,200

Hours	Shirts
6	1,200
60	12,000

Hours	Shirts
6	1,200
60	12,000
3	600

Hours	Shirts
6	1,200
60	12,000
3	600
63	12,600

Hours	Shirts
6	1,200
60	12,000
3	600
63	12,600

63 ÷ 9= 7days

Division

Context matters!

100 marbles, 12 in a bag.
How many bag can you fill?

Bags	Marbles

Bags	Marbles
1	12

Bags	Marbles
1	12
2	24

Bags	Marbles
1	12
2	24
4	48

Bags	Marbles
1	12
2	24
4	48
8	96

100 marbles

How many bags can I fill?

Bags	Marbles
1	12
2	24
4	48
8	96

Context matters!

320 students are going on a field trip, 60 per bus. How many buses should be ordered?

Buses	Students

Buses	Students
1	60

Buses	Students
1	60
2	120

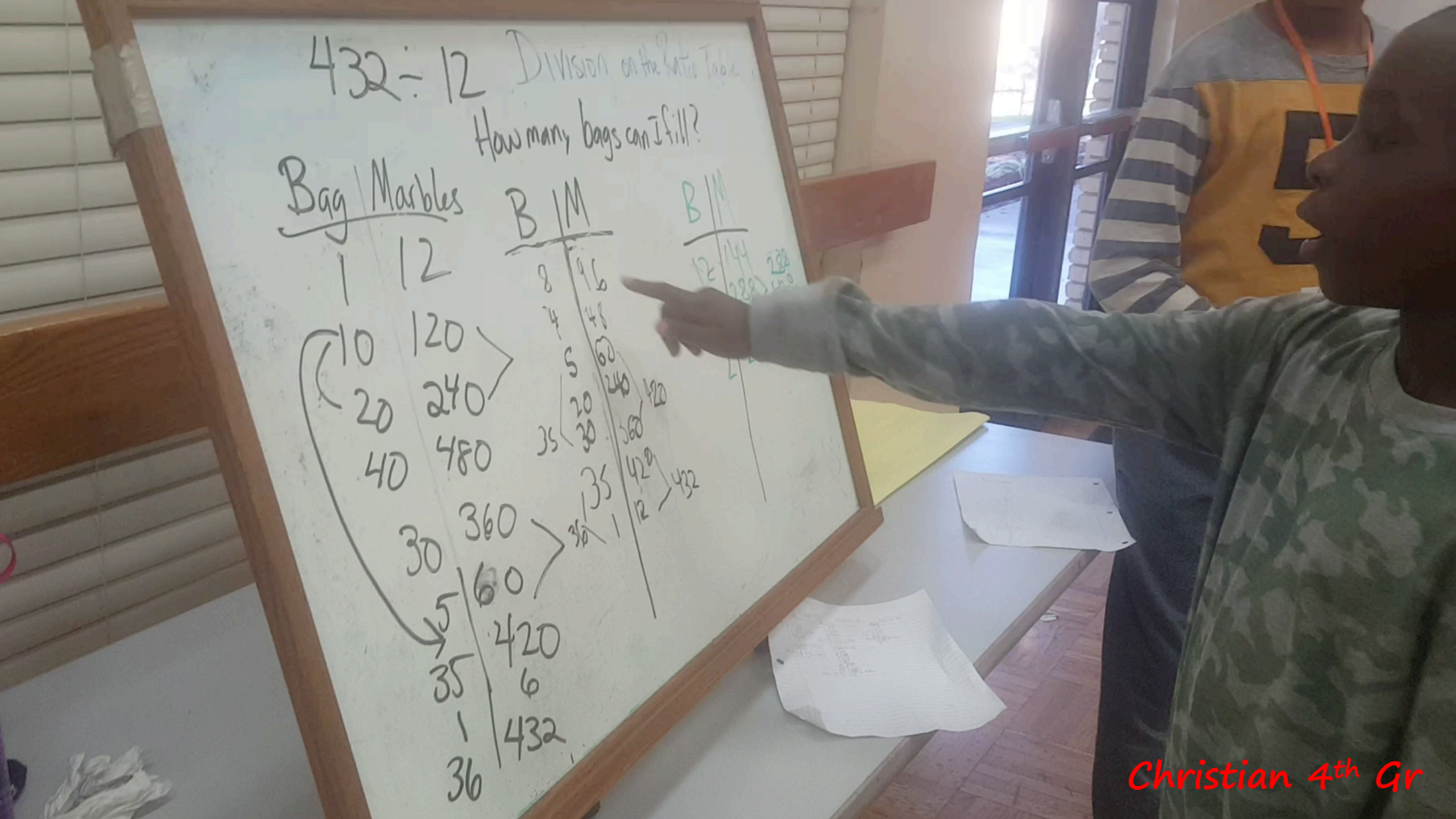
Buses	Students
1	60
2	120
4	240

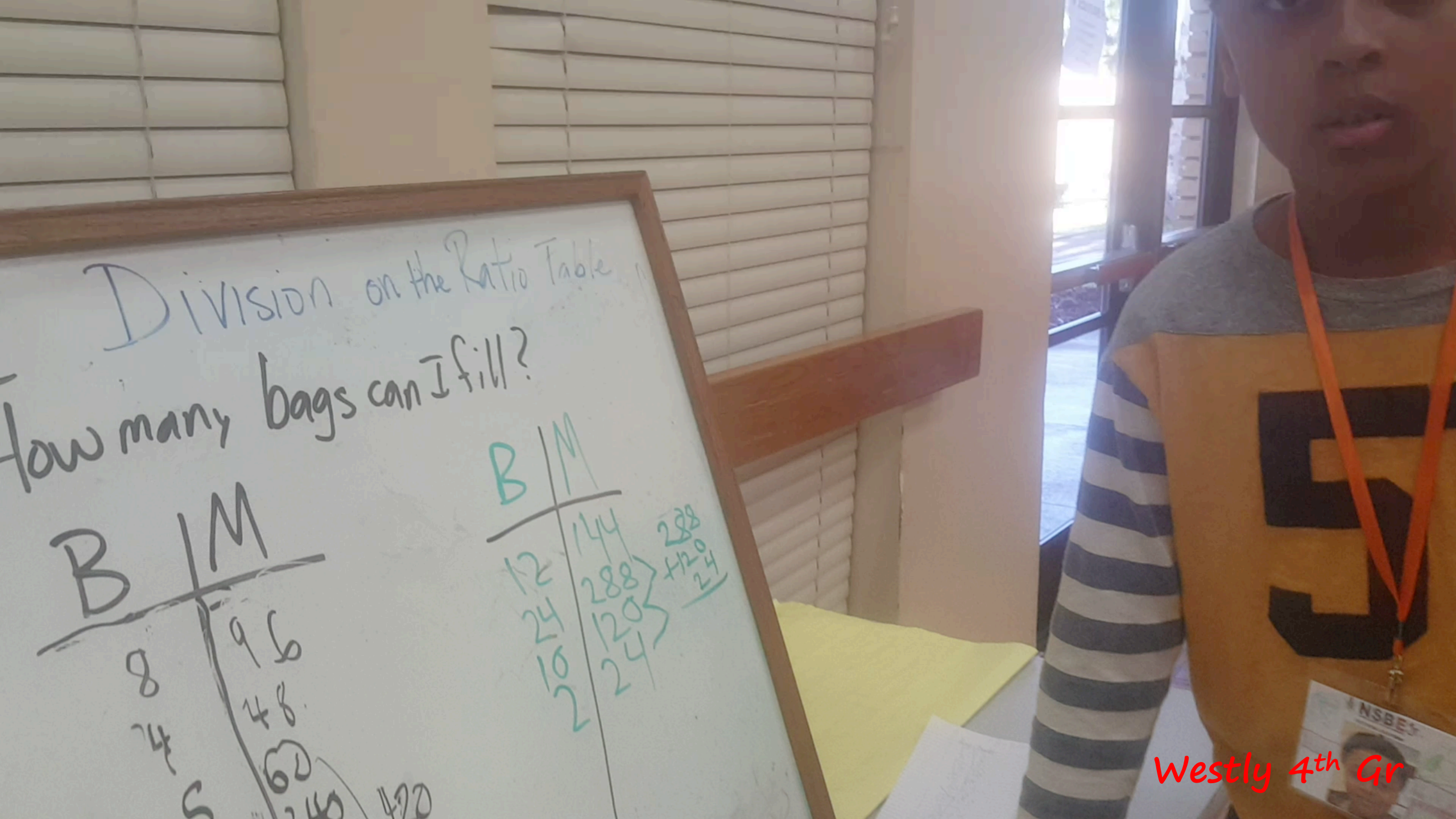
Buses	Students
1	60
2	120
4	240
6	360

Buses	Students
1	60
2	120
4	240
6	360
5	300

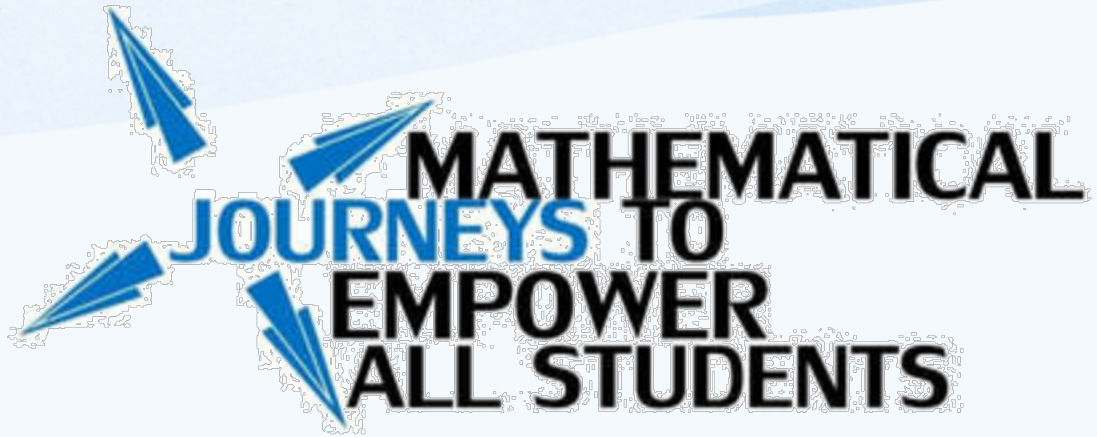
6	360
5	300

What about the other 20 kids?


432 ÷ 12
Division on the Satin Table
How many bags can I fill?
Bag | Marbles
1 | 12
10 | 120
20 | 240
40 | 480
30 | 360
5 | 60
35 | 420
1 | 6
36 | 432
B | M
8 | 96
4 | 48
5 | 60
20 | 240
30 | 360
35 | 420
1 | 12
B | M
12 | 144
Christian 4th Gr


Division on the Ratio Table
How many bags can I fill?
B | M
8 | 96
4 | 48
5 | 60
B | M
12 | 144
24 | 288
10 | 120
2 | 24
288
+120
24
NSBE
Westly 4th Gr

Do you remember these?



California Mathematics Council - South

59th Annual Mathematics Conference

November 2-3, 2018 in Palm Springs

Selesia left to go to the Palm Springs traveling 40 miles per hour. Malik left 30 minutes later traveling 60 miles per hour. When will they meet?

Hours	Selesia's Miles	Malik's Miles

Hours	Selesia's Miles	Malik's Miles
1	40	60

Hours	Selesia's Miles	Malik's Miles
1/2	20	30
1	40	60

Hours	Selesia's Miles	Malik's Miles
1/4	10	15
1/2	20	30
1	40	60

Hours	Selesia's Miles	Malik's Miles
1/4	10	15
1/2	20	30
3/4	30	45
1	40	60

Hours	Selesia's Miles	Malik's Miles
1/4	10	15
1/2	20	30
3/4	30	45
1	40	60
1 1/4	50	

Hours	Selesia's Miles	Malik's Miles
1/4	10	15
1/2	20	30
3/4	30	45
1	40	60
1 1/4	50	
1 1/2	60	

Hours	Selesia's Miles	Malik's Miles
1/4	10	15
1/2	20	30
3/4	30	45
1	40	60
1 1/4	50	
1 1/2	60	

Hours	Selesia's Miles	Malik's Miles
1/4	10	15
1/2	20	30
3/4	30	45
1	40	60
1 1/4	50	
1 1/2	60	

(1hr + 30mins)
Meet same time

Task Model 4 **DOK Levels 2, 3** **Target D: Interpret results in the context of a situation.**	**Task Expectations:** Students need to link their answer(s) back to the problem's context. This might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem's context. **Example Item 1 (Grade 4):** Primary Target 4D (Content Domain OA), Secondary Target 1A (CCSS 4.OA.A), Tertiary Target 4A, Quaternary Target 1A (CCSS 4.OA.3) A group of 137 students and 15 adults go to a museum. The students and adults have to take the elevator up to the 6th floor. • The elevator can hold a maximum of 12 people. • At least one adult must ride with each group of students on the elevator. ***Part A:*** What is the **fewest** number of elevator trips it will take to get all of the students and adults to the 6th floor? Enter your response in the first response box. ***Part B:*** What is the **fewest** number of people on the final elevator trip? Enter your response in the second response box.

12 on an Elevator

Trips/Adults	Students
1	11
2	22
4	44
8	88
12	132
(4+8=12)	(44+88=132)

12 trips will have 12 adults 132 students

Adults 15-12=3 Students 137-132=5

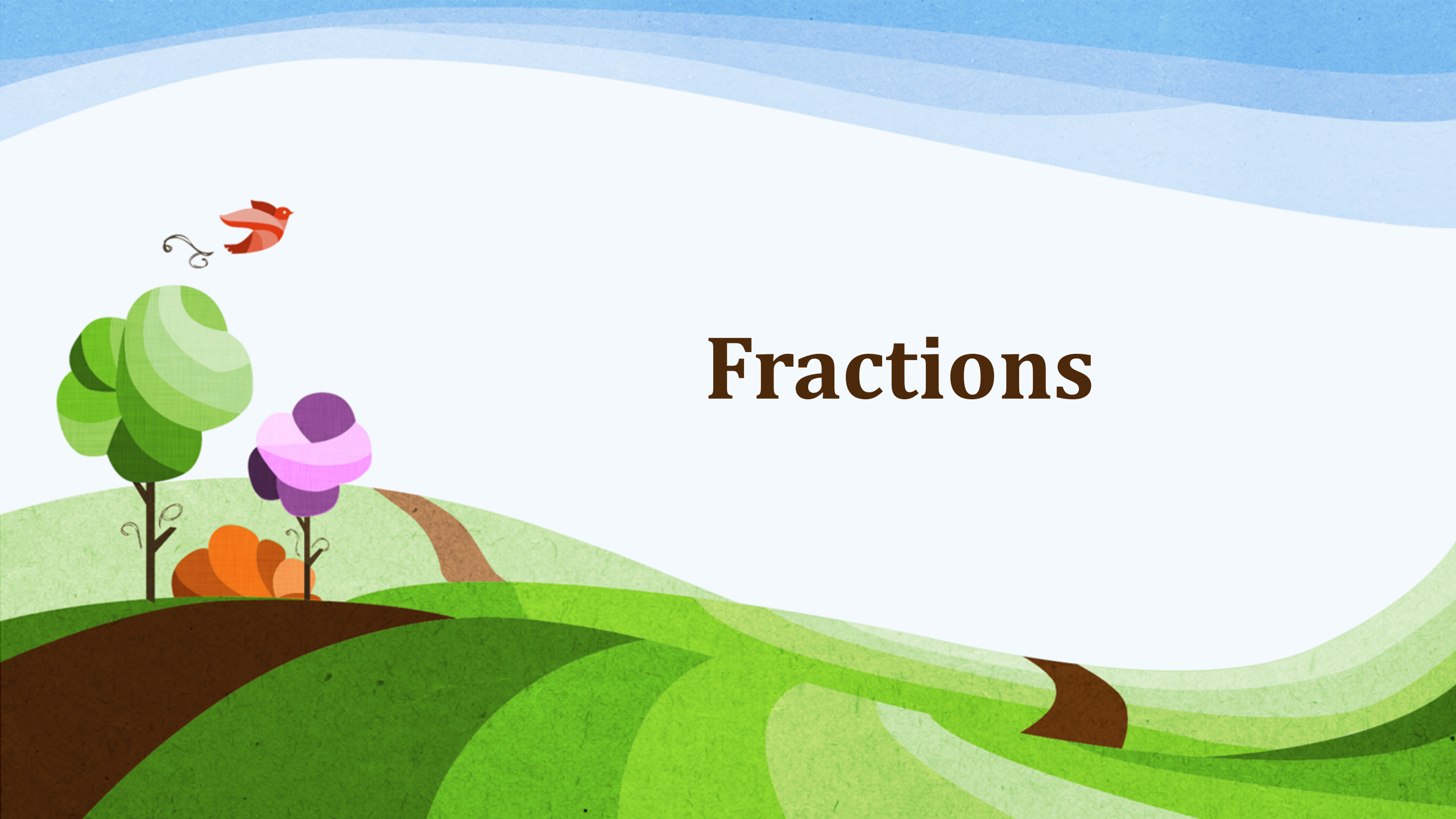
1 trip will have 3 adults 5 students

Part A: Fewest number of trips
13 trips

Part B: Fewest number of people on the final trip
8 on the last trip (3 adults/5 students)

Catherine Fosnot

The ratio table will show the partial products and will lead eventually to the algebra of, again, another example of the use of the distributive property, which is the big idea that partial products gets you to. The doubling and halving gets you to tripling and thirding, quadrupling and quartering, really what you're doing is just associating a factor differently, and that leads to the associative property. This is the, this landscape that I'm talking about, is the foundation of the development of algebra, and this is what we need to be focused on in the elementary school.

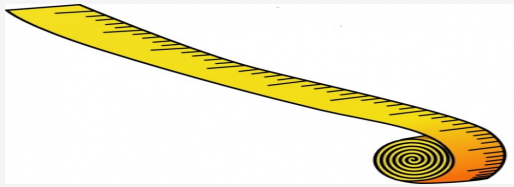


Fractions

Justine likes to make pillowcases. It takes 2/3 of a yard of material to make a pillowcase. How many yards of material would it take to make 15 pillowcases?

Ratio Table

- Doubling
- Using Partial Products (place value understanding)
- Distributive Property over addition
- Number greater than a whole



1	2/3
2	4/3
3	6/3 = 2
6	12/3 = 4
12	24/3 = 8
12+3 = 15 pillows	8+2 = 10 yards

Grade 4 Mathematics

Item	Claim	Domain	Target	DOK	CCSS-MC	CCSS-MP
#31	2	NF	C	2	4.NF.B.4	4

A class is making 7 flags. It takes $\frac{3}{4}$ of a yard of felt to make each flag.

The total number of yards of felt needed is between which two numbers?

Ⓐ 1 and 2

Ⓑ 3 and 4

Ⓒ 5 and 6

Ⓓ 7 and 8

Flags	Yards

Flags	Yards
1	3/4

Flags	Yards
1	3/4
2	6/4

Flags	Yards
1	3/4
2	6/4
4	12/4=3

Flags	Yards
1	3/4
2	6/4
4	12/4=3
8	24/4

Flags	Yards
1	3/4
2	6/4
4	12/4=3
8	24/4=6

Flags	Yards
1	3/4
2	6/4
4	12/4=3
8	24/4=6
7	5 1/4

8-1=7

6-3/4=5 1/4

Flags	Yards
1	3/4
2	6/4
4	12/4=3
8	24/4=6
7	5 1/4

- (A) 1 and 2
- (B) 3 and 4
- 5 and 6
- (D) 7 and 8

8-1=7

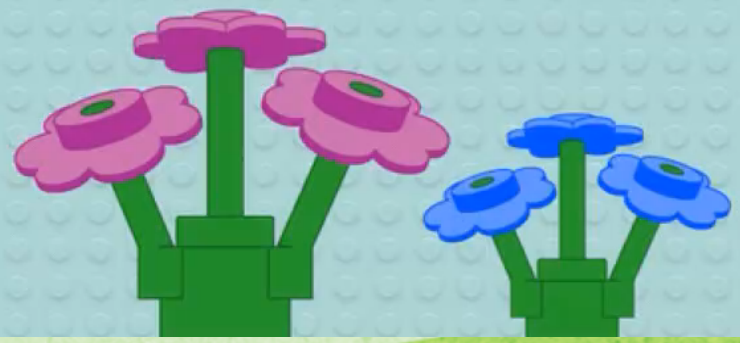
6-3/4=5 1/4

Context: Granny Frances is having her three grandchildren over afterschool. She wants to serve snacks.

How much chocolate will each child get if three share 1/2lb of chocolate equally?

3 PEOPLE & 1/2LB CANDY

DIRECTED BY CHRISTINA MOORE



https://www.mathlearningcenter.org/web-apps/pattern-shapes/

Persons	Lbs
3	1/2
3	3/6
2	2/6
1	1/6

each gets

Your Turn – Use The Ratio Table

Grades 3-5, Claim 2

Task Model 3 **DOK Level 2** **Target C: Interpret results in the context of a situation.**	**Example Item 3 (Grade 5):** Primary Target 2C (Content Domain NF), Secondary Target 1F (CCSS 5.NF.B) Carl feeds his dog $2\frac{1}{2}$ cups of dog food every day. Each bag contains 64 cups of dog food. What is the maximum number of days that Carl can feed his dog exactly $2\frac{1}{2}$ cups of dog food from one full bag? **Rubric:** (1 point) The student is able to determine the maximum number of servings from one bag of food and interpret the remainder as not being enough for another serving **Response Type:** Equation/Numeric

Days	Cups of Dog Food
1	2 1/2
2	5
20	50
4	10
24	60
25	62 1/2
26	65

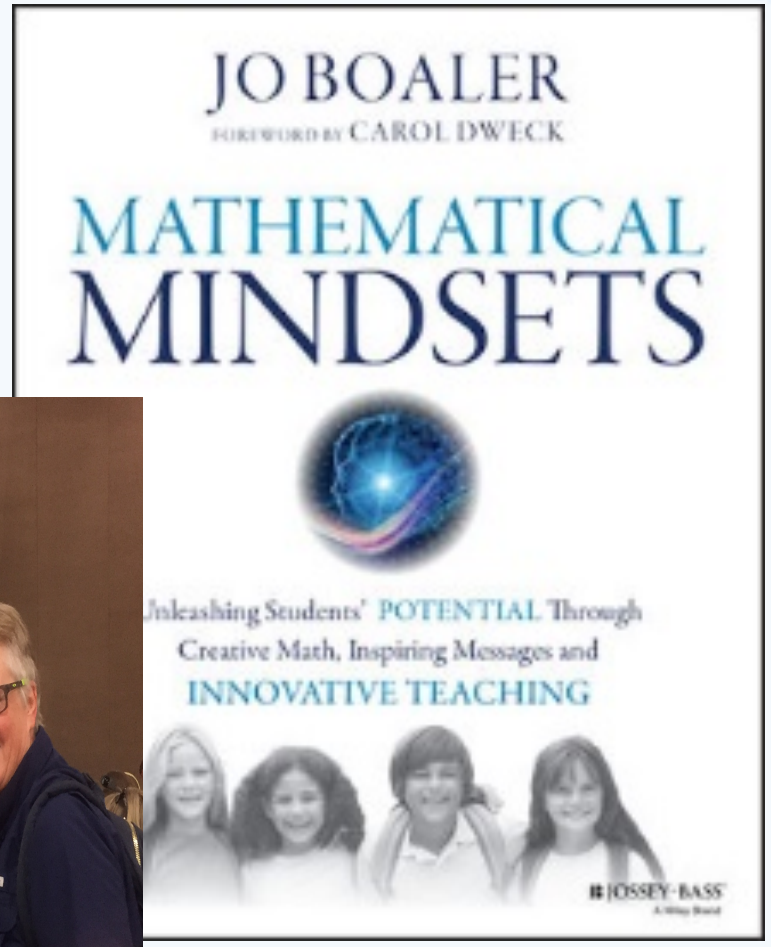
Days	Cups of Dog Food
1	2 1/2
2	5
20	50
4	10
24	60
25	62 1/2
26	65

Too much

We can change the data!

"Math problems should encourage and acknowledge the different ways in which people see mathematics and the different pathways they take to solve problems. When these changes happen, students engage with math more deeply and well."
pg.xii
– Jo Boaler

CMC-South 10/2017



The ratio table is a powerful tool…

- Additive Thinking: Distributive Property of Multiplication Over Addition and Partial Products
- Subtractive Thinking: Distributive Property od Multiplication over Subtraction and Using Friendly Numbers
- Multiplicative Thinking: Ratio and Proportional Reasoning
- Algebra

Strategies and thinking tools lay the foundation for algebraic thinking…

"Algebra is the more generalized mathematical application of strategies students should have been taught K-6."

-Fosnot

Strategy:

Doubles Doubles +1

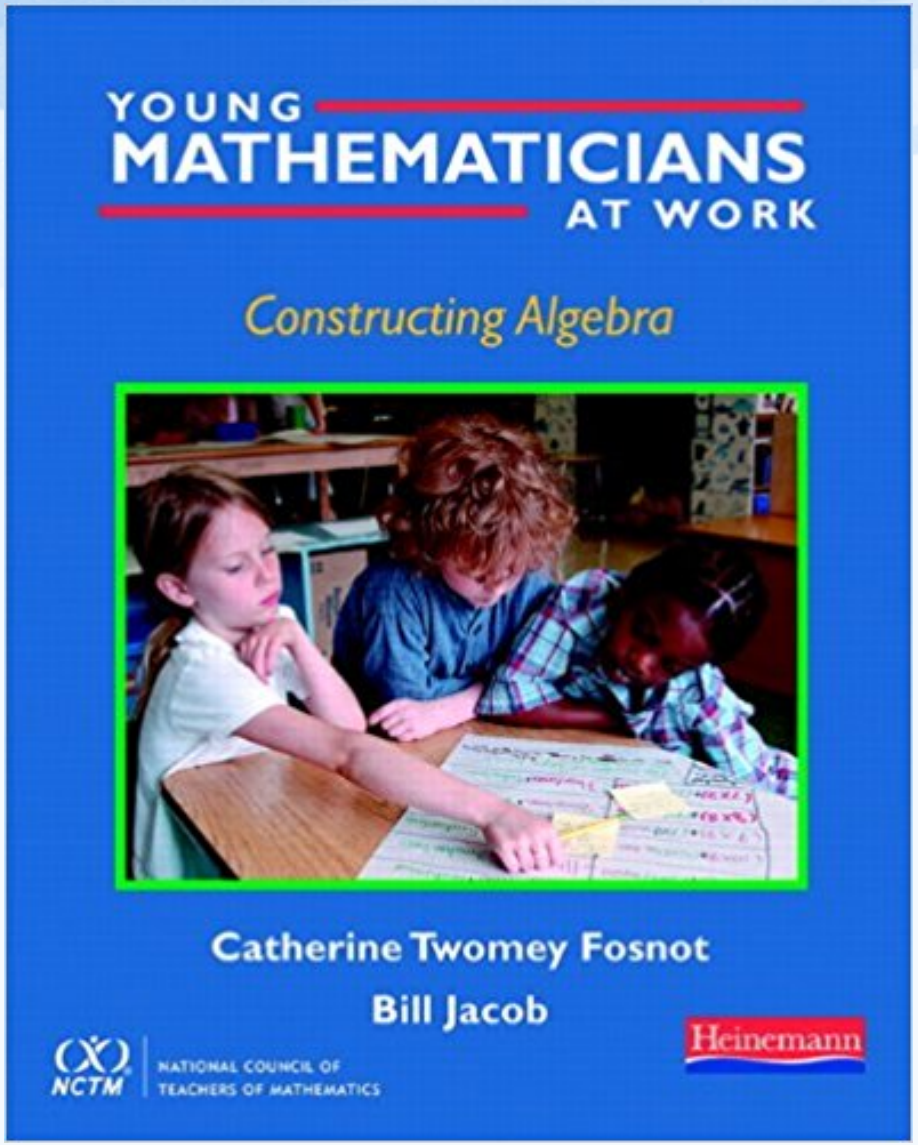
Additive Thinking

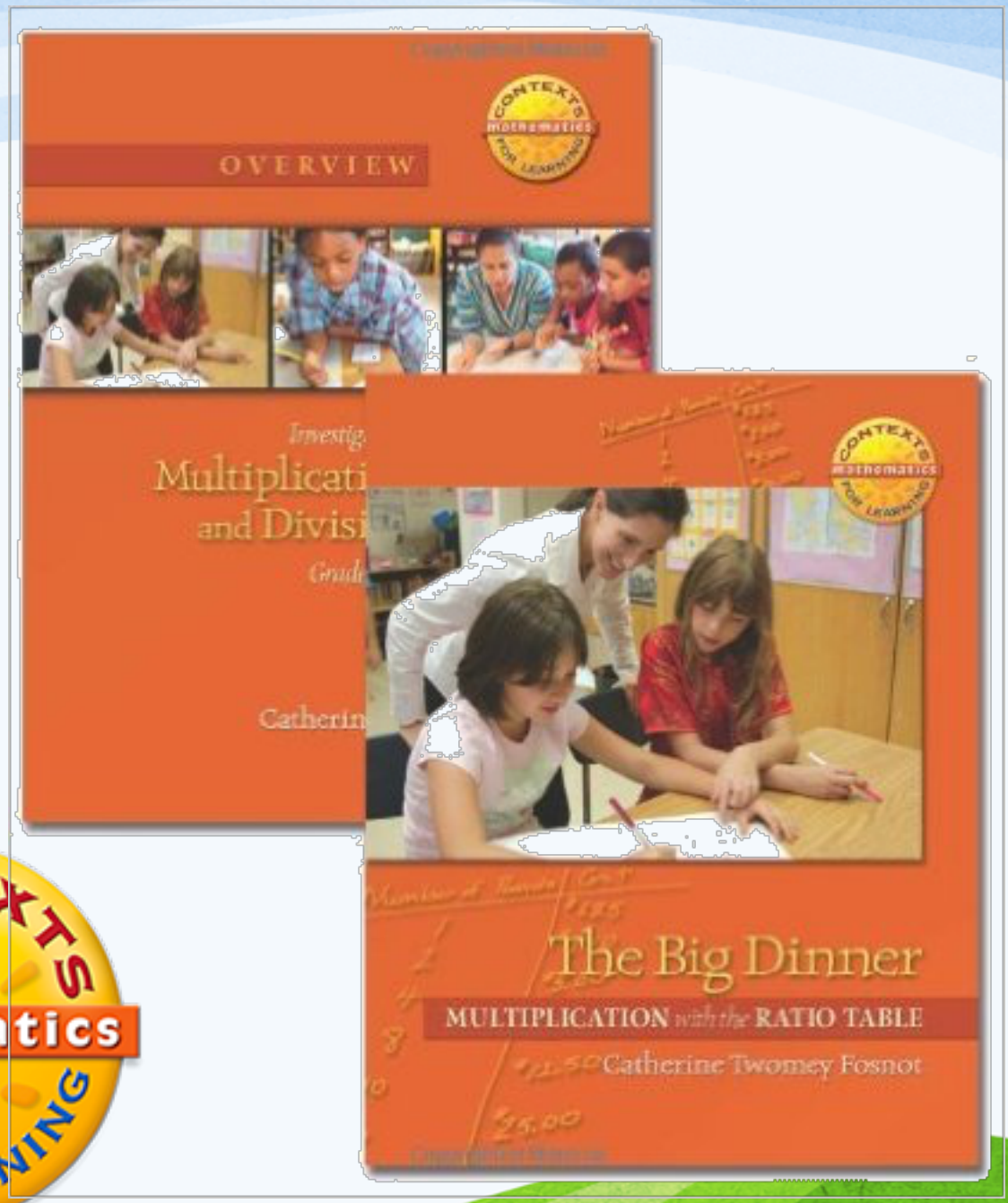
(n + n) (n + n) + 1

Multiplicative Thinking

2n 2n + 1

Leads to Multiplicative Number Sense





CONTEXTS
mathematics
FOR LEARNING

PREMIER MATH EDUCATION EVENT

NCTM ANNUAL MEETING & EXPOSITION 2018

April 25–28 | Washington, DC

Empowering the Mathematics Community

Learn more at **nctm.org/annual** #NCTMannual

NCTM

MATHEMATICAL JOURNEYS TO EMPOWER ALL STUDENTS

California Mathematics Council - South

59th Annual Mathematics Conference

November 2-3, 2018 in Palm Springs

Talk Number 2 Me: The Ratio Table Webinar

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