Project LEAP:

Designing Tasks that Build Students' Algebraic Thinking

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TERC

The research reported here was supported in part by the US Department of Education under IES Award #R305A140092 . Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the US Department of Education.

- What is "algebra"?
- What "algebra" do we want children in elementary grades to learn?
- How does your curriculum and instruction address algebraic thinking?

Identify/Design a Task…

With a partner or group, identify an "algebra task" (or develop an idea for one) for the grade level you teach.

- *What is algebraic about this task?*
- *What are design features of this task? (What does it "do" that's important?)*

First…what is “algebra?”

Generalizing mathematical structure and relationships

- What is going on “in general”?
- What is the underlying structure?

Where does this happen in your curriculum?

First…what is “algebra?”

Representing generalizations

- How can students represent the relationships they notice?
- How do they move between representations flexibly?

Where does this happen in your curriculum?

First…what is “algebra?”

Justifying generalizations

- What types of arguments will children make to justify their generalizations?
- How do we help them develop more sophisticated arguments?

Where does this happen in your curriculum?

First…what is "algebra?"

Reasoning with generalizations

- How can students use generalizations that have already been accepted by the class to make sense of new claims and ideas?

Where does this happen in your curriculum?

Compute

$$432 + 0 \times (394.5 - 127.3) - 432 + 25 \times 12$$

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Compute

$$432 + 0 \times (394.5 - 127.3) - 432 + 25 \times 12$$

$$4y^2 + 3yz - 4y^2 + (3z - 3z)(4yz + 7 - y)$$

Compute

$$432 + 0 \times (394.5 - 127.3) - 432 + 25 \times 12$$

$$4y^2 + 3yz - 4y^2 + (3z - 3z)(4yz + 7 - y)$$

Compute

$3yz$

What are characteristics of “good” algebra tasks/lessons?

Tasks that build children’s algebraic thinking should…

- engage students in algebraic thinking practices
- be cognitively demanding (Smith & Stein, 1998)
  - non-algorithmic thinking, multiple representations, cognitive effort, use of relevant knowledge and experiences to solve problems
- require students to produce written and oral explanations of their thinking
- be aligned with Common Core Mathematical Practices and Standards

For the given LEAP task…

With your group…

- Solve the task.
- Identify how the task addresses the criteria for a "good algebra task."

Lesson 4:

Exploring Fundamental Properties:
Commutative Property of Addition

Lesson 4:
Exploring Fundamental Properties
(Commutative Property of Addition)

- How does this task address core algebraic thinking practices?
- In what ways is it cognitively demanding for Grade 3 students?
- How does it address *Common Core* Mathematical Practices?

Common Core Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

A. Which of the following number sentences are true? Use numbers, pictures, or words to explain your reasoning.

17 + 5 = 5 + 17   20 + 15 = 15 + 20   148 + 93 = 93 + 148

B. What numbers make the following number sentences true?

25 + 10 = _____ + 25   ____ + 237 = 237 + 395   38 + ____ = _____ + 38

C. What do you notice about these problems? Describe your conjecture in words.

D. Represent your conjecture using variables.

E. Can you express your conjecture in a different way using the same variables?

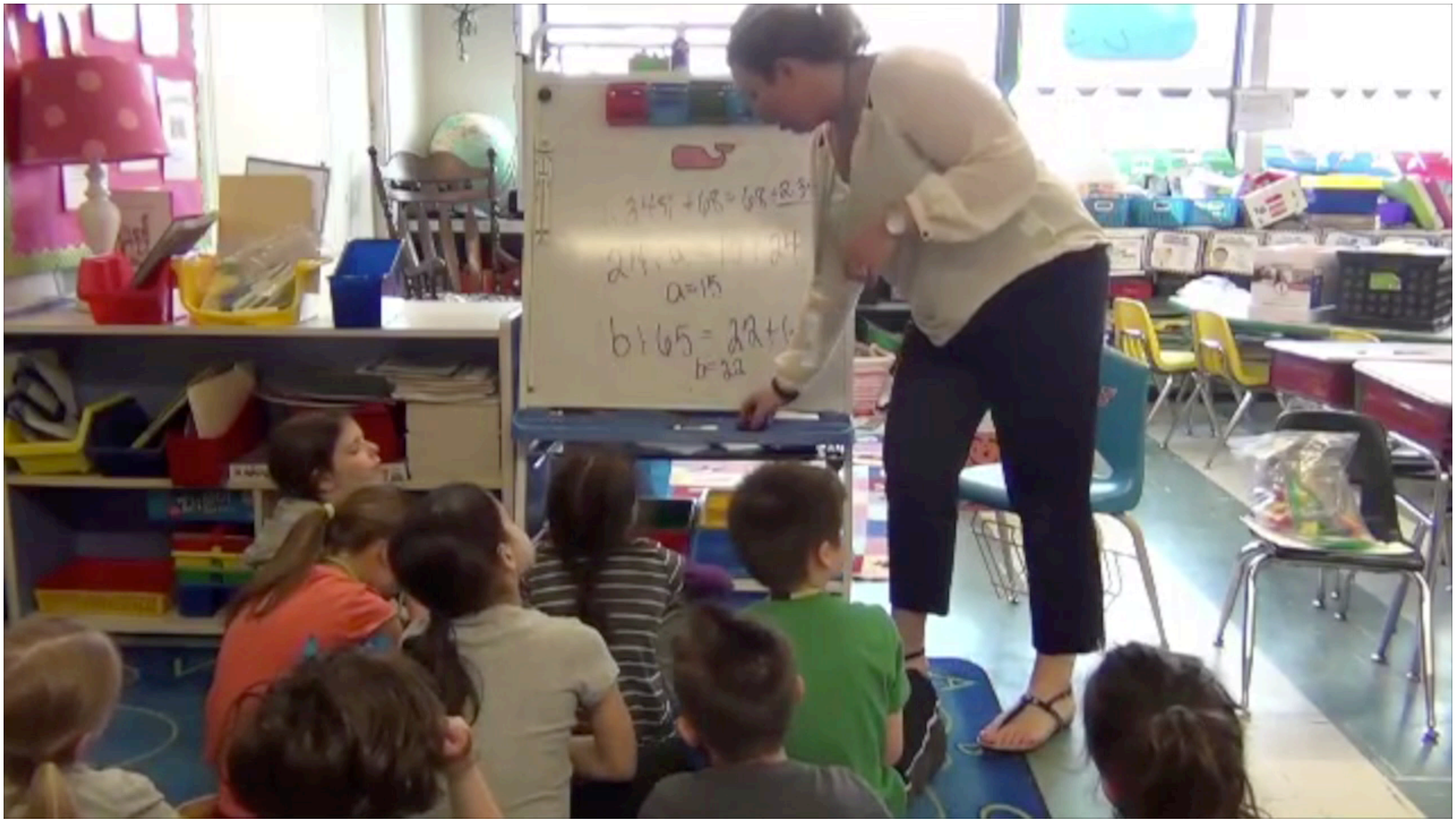
F. For what numbers is your conjecture true? Is it true for all numbers? Use numbers, pictures, or words to explain your thinking.

G. Find the following. Think about how you might use the properties you have learned.

95 + 39 – 39 + 12   68 + 27 + 32 – 27

First Grader

Representing a generalization



Lesson 5

Generalizations about Sums of Evens and Odds

Lesson 5:
Generalizations about Sums of Evens and Odds

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How Many Pairs?

How many pairs of cubes are in the number 6? How many cubes are left over after you've made your pairs? Use your cubes to complete the following table for the given numbers.

Number	Number of pairs created	Number of cubes left over
3		
4		
5		
6		
7		

- What do you notice? What (kinds of) numbers have no cubes left over after all pairs are made?
- What (kinds of) numbers have a cube left over?
- Write a sentence to describe each of your observations.

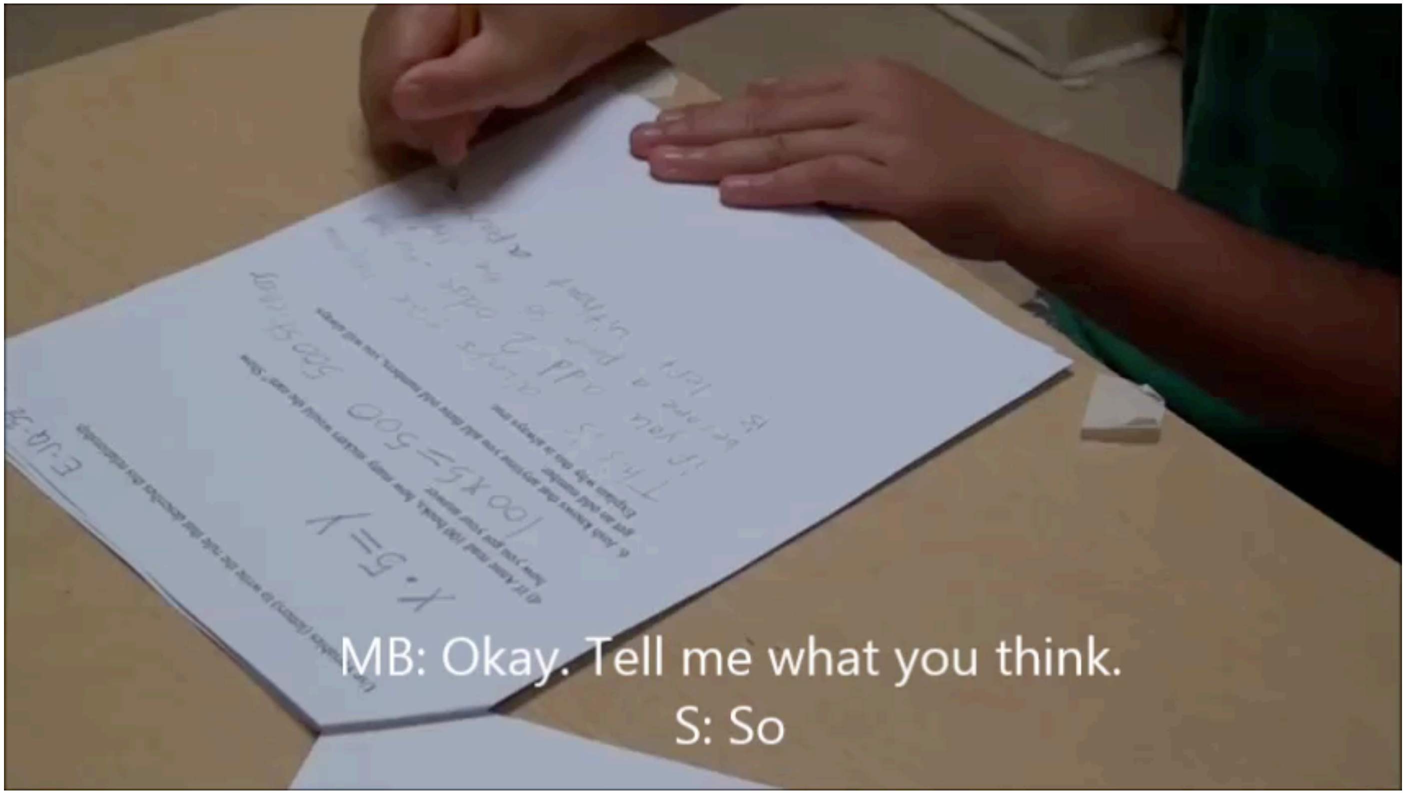
Exploring the sum of two even numbers

- Jesse is adding two even numbers. Do you think his answer will be an even number or an odd number?
- Develop a conjecture to describe what you found.
- Is your conjecture true for any two even numbers you add together? How do you know? Use numbers, pictures, cubes, or words to explain your thinking.

Third-grader

Justifying a generalization

Josh knows that anytime you add three odd numbers, you will always get an odd number. Explain why this is always true.



Lesson 7

Algebraic Expressions

(Candy Box*)

*Adapted from Carraher, Schliemann, & Schwarz, 2008)

Lesson 7:
Algebraic Expressions (Candy Box*)

- How does this task address core algebraic thinking practices?
- In what ways is it cognitively demanding for Grade 3 students?
- How does it address *Common Core* Mathematical Practices?

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Jack has a box of candies. He’s not sure how many pieces of candy are in his box. His mother gives him 4 more pieces. Can you draw a picture to illustrate this situation?

Write a mathematical expression to represent the number of pieces of candy Jack has.

What does the variable in your expression represent?

What are possible values of your variable?

Can you represent the number of pieces of candy Jack has in a different way using the same variable and number?

First grader

Generalizing and Representing a quantity

Lesson 12

Functional Thinking:
Trapezoid Desk Problem

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Functional Thinking: Trapezoid Desk Problem

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Suppose you could seat 3 people at a desk shaped like a trapezoid. If you joined 2 trapezoid desks end-to-end, you could seat 6 people (see the pictures below):

How many people could you seat if you joined 3 trapezoid desks end-to-end? What if you joined 4 desks?

What can you say about how the number of people that can be seated is changing as you add more desks?

Organize your information in a function table.

Find the number of people that could be seated at 7 desks.

In your own words, describe a relationship between the number of desks and the number of people who can be seated.

Use variables to write a rule (equation) that describes this relationship.

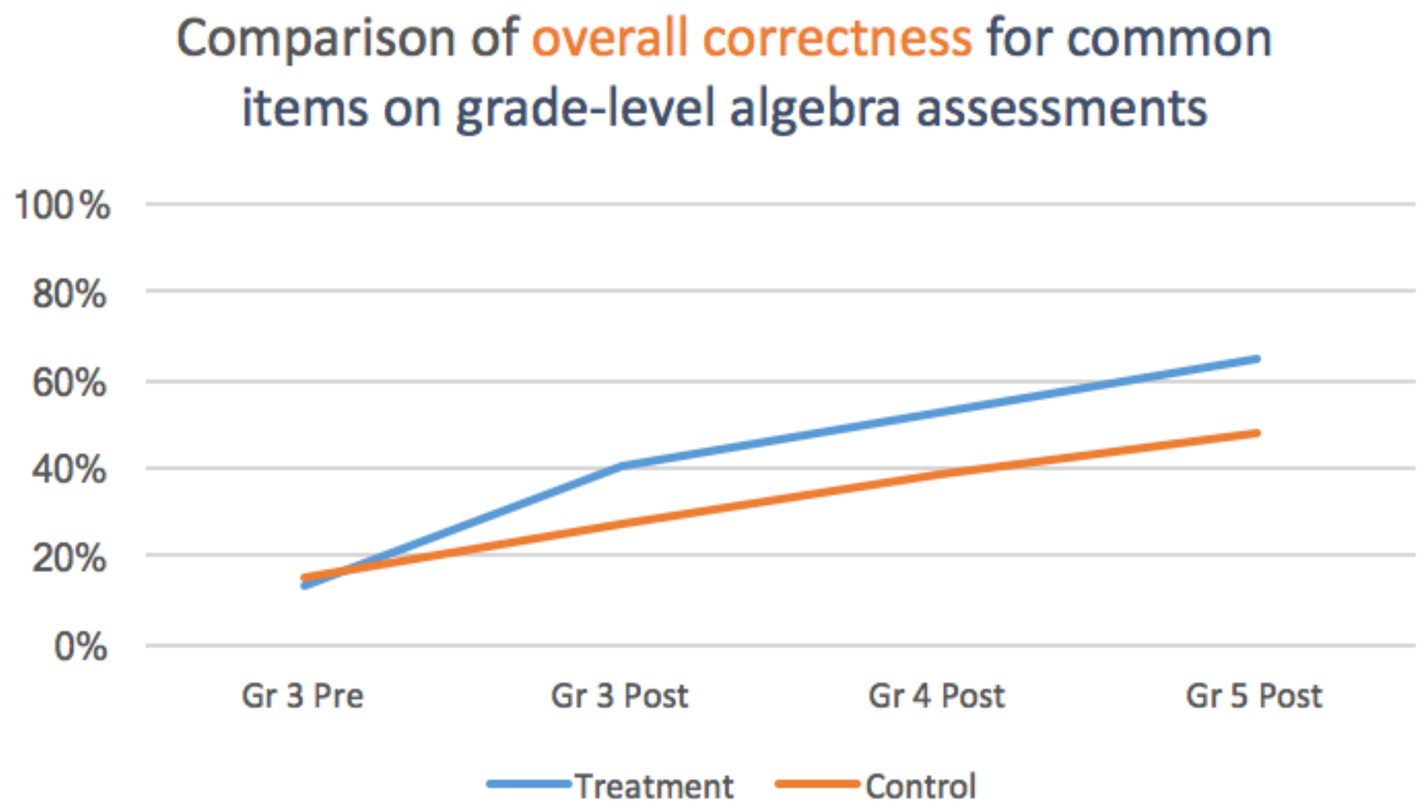
Write your rule in a different way.

First grader, *Growing Train*

Reasoning with a generalization

If we design tasks this way, how do we know they “work”?

LEAP 3 Grades 3-5 Results



Final thoughts…

- In our work, we follow certain design principles for creating tasks (e.g., tasks must engage students in algebraic thinking practices)
- But actually building an algebraic task is simple:
  - ✓ Lesson 4&5: Build on naturally occurring computational work to focus children's attention on fundamental properties such as the commutative property and other arithmetic generalizations such as those involving evens and odds
  - ✓ Lesson 7: Take simple arithmetic problems and make one of the knowns unknown
  - ✓ Lesson 12: Vary the parameter in an arithmetic task to create a functional thinking task

Caveat

- It is risky to “pick” algebra problems and drop them randomly into regular instruction.
- Beware of treatment of early algebra as simply solving equations with unknowns and use of procedural rules!
- Algebraic concepts should be thoughtfully developed in a comprehensive, cohesive way across K-12 (see our overview of Grade 3 instructional sequence)

Questions?

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for Grades 3 – 5 LEAP lessons