

CARNEGIE  
LEARNING



LONG + LIVE + MATH

## Long Live Fractions, Bar Models, & Equations

Courtney Lewis, Sr. Manager of School Partnerships, Raleigh, NC

Kelly W. Edenfield, Clinical Assistant Professor, University of Georgia, Athens, GA

## Which One Doesn't Belong?

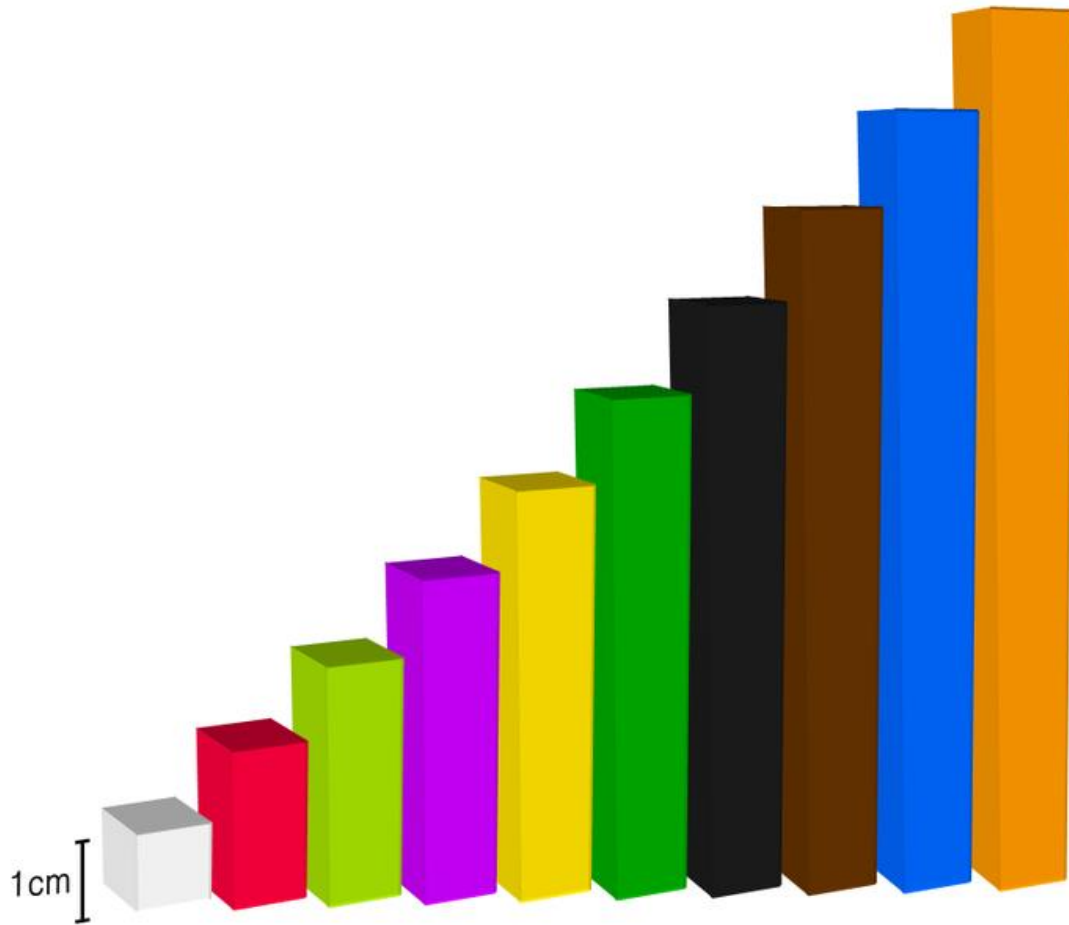
$y = 4x$	$y = x + 7$
$y = -2x + 4$	$y = 3x - 1$



# Learning Intentions

In this session, participants will...

- review using bar models (Cuisenaire Rods) to understand fraction equivalence and determining the whole when given a fractional piece
- transition from concrete representations (Cuisenaire Rods) of fractions to concrete representations of one-step equations
- use bar models and strategic problems to develop the algorithms for solving one-step addition and multiplication problems, including those involving fractions.
- discuss how the models empower students and promote classroom discourse.



## Explore Cuisenaire Rods

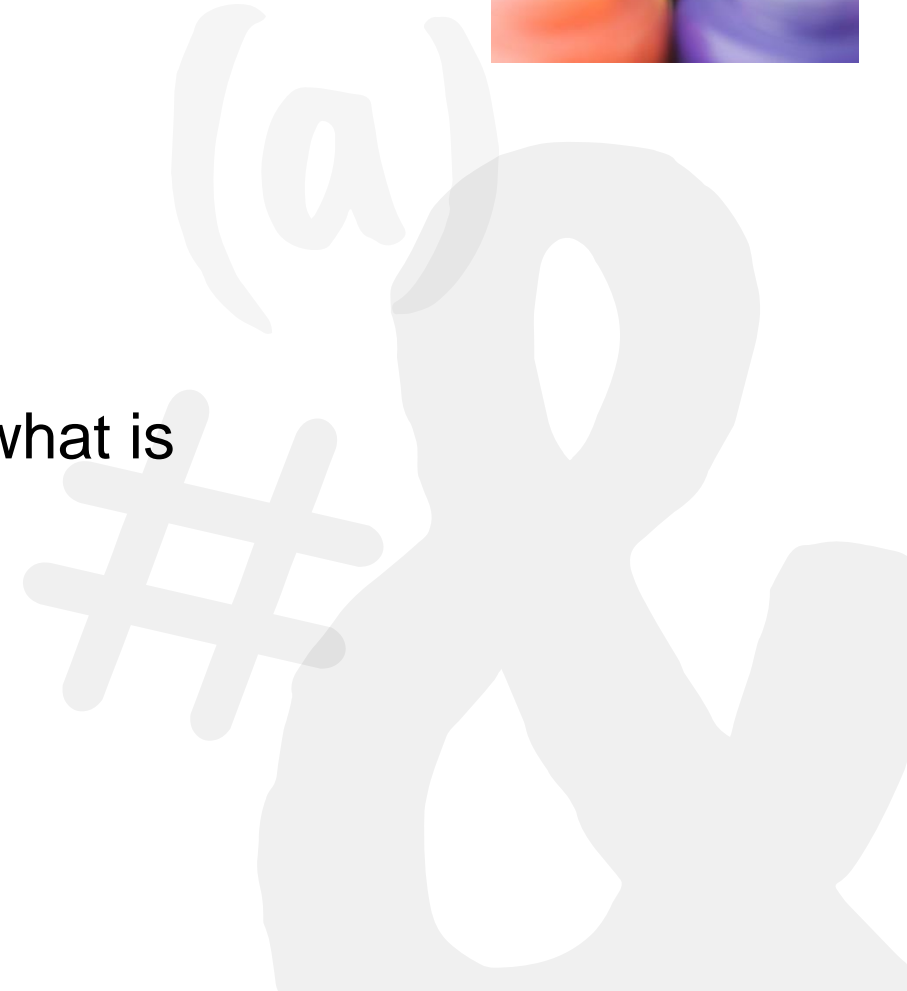
- Take 3 minutes to load and explore Cuisenaire Rods online!
- [http://www.learner.org/courses/learningmath/number/session8/part\\_b/try.html](http://www.learner.org/courses/learningmath/number/session8/part_b/try.html)



# Exploring Cuisenaire Rods



1. If **brown** is the whole, what is one-fourth?
2. If **white** is one-seventh, what is the whole?
3. If **orange** is one and one-fourth, what is the whole?



# Exploring Cuisenaire Rods

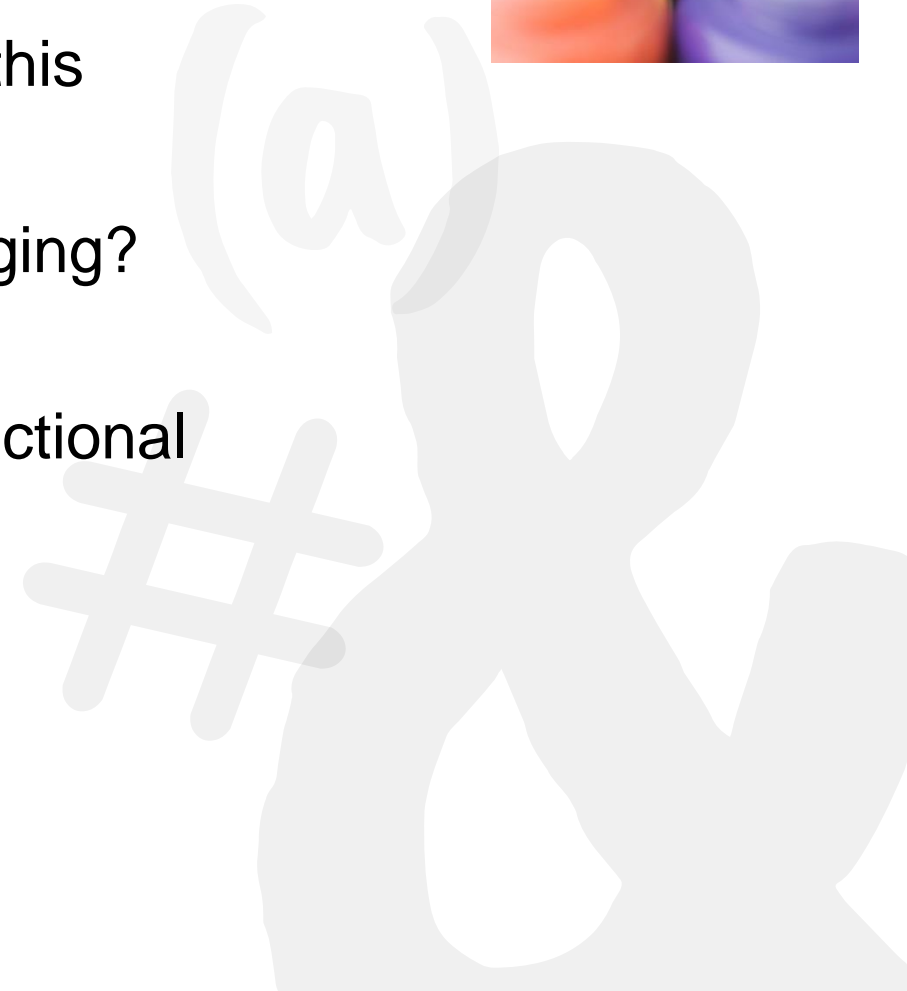


Line up the following Cuisenaire Rods: **white**, **red**, **light green**, **purple**, **dark green**, and **brown**.

1. If the **white** is one, what's the length of the other rods?
2. If the **dark green** is one, what's the length of the other rods?
3. If the **brown** is one, what's the length of the other rods?

# Reflection

- How is equivalency represented in this activity?
- Which problems were most challenging? Why?
- How can a color have a different fractional value in different problems?



# Concrete-Representational-Abstract

- **Concrete:** Students manipulate concrete objects to model the concept or skill.
- **Representational** (semi-concrete): Students draw pictures that represent the concrete objects.
- **Abstract:** The teacher and students use operation symbols and numbers to indicate the concept or skill.



# Using Rods to Solve Equations

A teal background with a collage of faint, light-blue mathematical symbols and geometric shapes. These include a star, a circle, a triangle, a square, a plus sign, a multiplication sign, a percent sign, a hash sign, a pi symbol, a checkmark, a large 'X', a large 'Y', a large '2', and a large 'a' in parentheses.

# A Strategic Addition Problem

Let's take a look at the addition equation

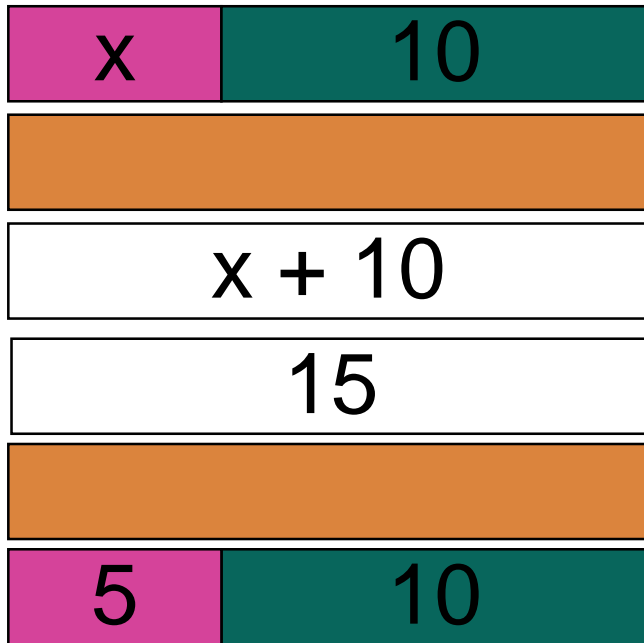
$$x + 10 = 15$$

- Model this equation using your Cuisenaire rods.
- How can you determine/prove the value of  $x$  using the model?
- Our focus here is on the *concrete* and then on the *representational*.

Keep in mind the values you already determined for white, red, light green, purple, dark green, and brown.

# A Strategic Addition Problem

$$x + 10 = 15$$



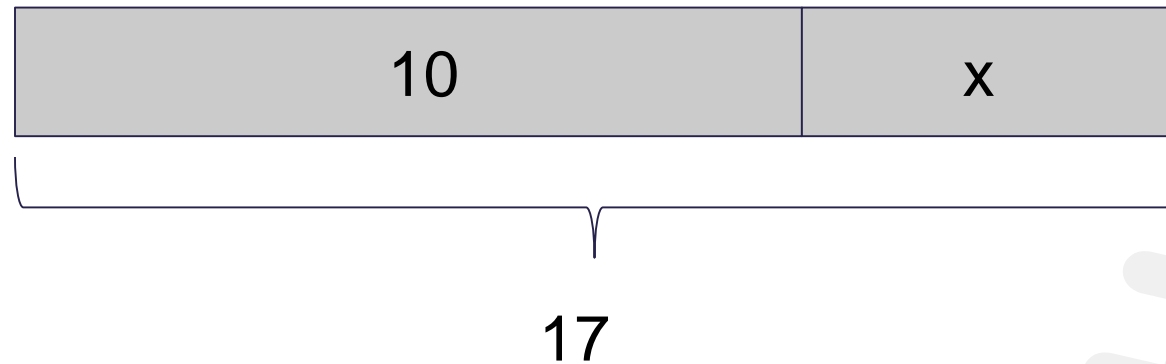
What must  $x$  equal?

What was “strategic” about this problem?

On what types of addition problems could you use the rods? When might you want to shift to bar models?

## Representational Stage: Bar Models

Let's create a bar model for  $x + 10 = 17$



Determine the value of  $x$  based on the model.

Be able to justify your reasoning.

# A Strategic Multiplication Problem

Let's consider the multiplication equation

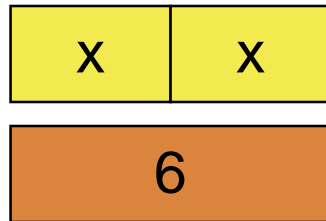
$$2x = 6$$

Build a model for the equation using the Cuisenaire rods. Determine the value of  $x$  based on the model. Be able to justify, using the rods, the value of  $x$ .

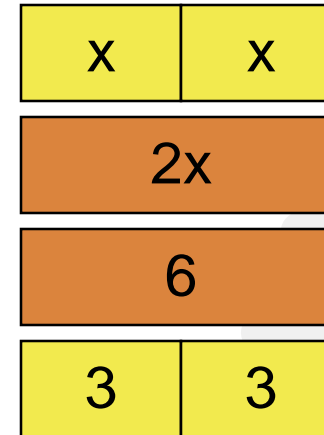
# A Strategic Multiplication Problem

Let's consider the multiplication equation

$$2x = 6$$



What must  $x$  equal?



“How do these bar models relate to the bar models you used to solve addition equations?”



# A Strategic Multiplication Problem

Let's consider the multiplication equation

$$2x = 6$$

x	x
2x	
6	
3	3

$$2x = 6$$

$$x + x = 6$$

$$x + x = 3 + 3$$

$$\text{so, } x = 3$$

## Two More Similar Problems

Explore with these multiplication equations.

Build a model for the equation using the Cuisenaire rods. Determine the value of  $x$  based on the model. Be able to justify your answer.

1.  $3x = 12$

2.  $7x = 63$

In each bar model, how did you determine how to decompose or compose the given expression?

How are these three problems alike?

How does our process relate to the algorithm?



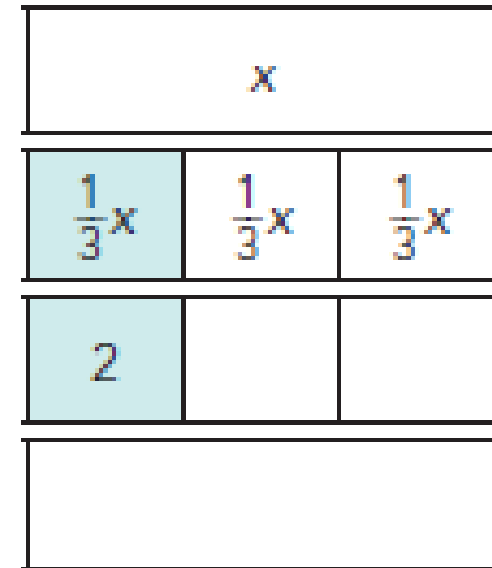
## Equations of the Form: $\frac{1}{a}x = b$

$$\frac{1}{3}x = 2$$

How would you model this with the Cuisenaire rods?

- Determining the Whole:
- If red is  $\frac{1}{3}$  of a whole, what color is 1 whole?

- Solve  $\frac{1}{3}x = 2$ :
- If the red has a value of 2 units, what is the value of the whole?



## Equations of the Form: $\frac{1}{a}x = b$

$$\frac{1}{3}x = 2$$

$x$		
$\frac{1}{3}x$	$\frac{1}{3}x$	$\frac{1}{3}x$
2		

$$\frac{1}{3}x = 2$$

$$\frac{1}{3}x + \frac{1}{3}x + \frac{1}{3}x = 2 + 2 + 2$$

$$1x = 6$$

or

$$3\left(\frac{1}{3}x\right) = 3(2)$$

$$1x = 6$$

## Equations of the Form: $\frac{1}{a}x = b$

• Build a model for each equation using the Cuisenaire rods. Determine the value of  $x$  based on the model. Be able to justify your answer.

1.  $\frac{1}{4}x = 7$

2.  $\frac{1}{2}x = 5$

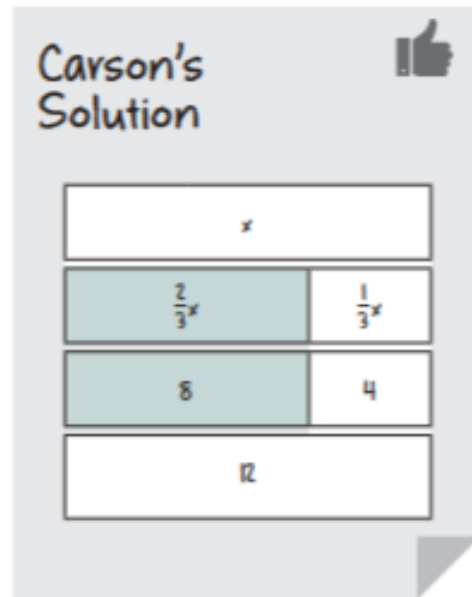
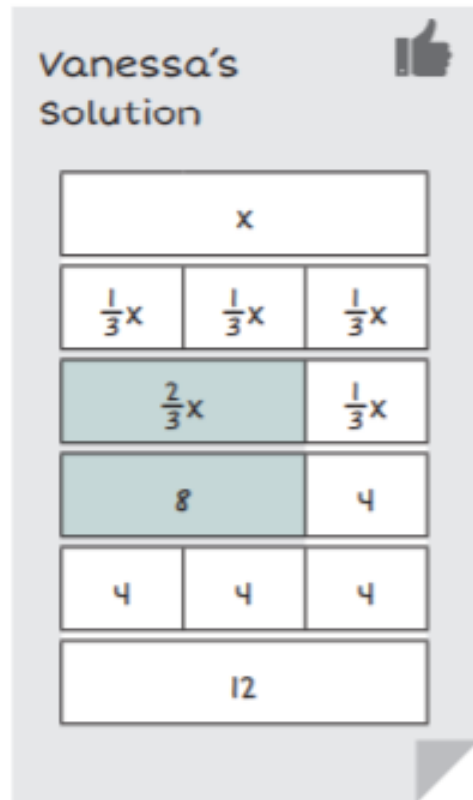
In each bar model, how did you determine how to decompose or compose the given expression?

- Determine the Whole
- Solve the equation

- How does our process relate to the algorithm?

# Equations of the form $\frac{a}{b}x = c$

Consider how to use Cuisenaire rods or bar models to solve  $\frac{2}{3}x = 8$ .



- Analyze each strategy.
- How are Vanessa's and Carson's strategies similar to your strategy?



## Tweet Reflection

Tweet your response to...

How can using Cuisenaire rods empower students in your classrooms and promote discourse?

Tweet us!

@Clewis\_carnegie

Courtney Lewis

[CLEWIS@carnegielearning.com](mailto:CLEWIS@carnegielearning.com)

@kwedenfield75

Kelly Edenfield

[kedenfield@uga.edu](mailto:kedenfield@uga.edu)

**Courtney Lewis**  
**Sr. Manager of School**  
**Partnerships**  
**Carnegie Learning**

**CLEWIS@carnegielearning.com**

**Kelly Edenfield**  
**Clinical Assistant Professor**  
**University of Georgia**

**kedenfield@uga.edu**

CARNEGIE  
LEARNING

LONG + LIVE + MATH

How can using Cuisenaire rods empower students in your classrooms and promote discourse?

