

Using

# **Social Networks**

to Teach Graph Theory

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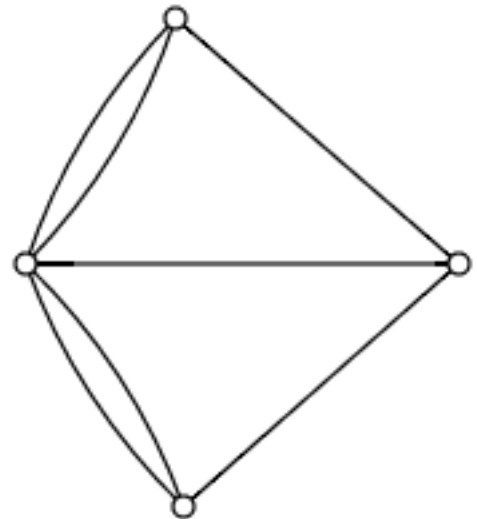
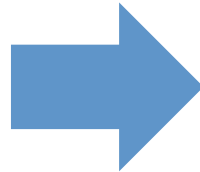
# Why Graph Theory?

- Mathematical Thinking (Habits of Mind, Mathematical Practices)
- Accessible to students at a variety of levels
- Mathematical Modeling

# One Possible Introduction

Bridges of Königsberg:

Can you cross all 7 bridges exactly once?



# One Possible Introduction

- Define terms

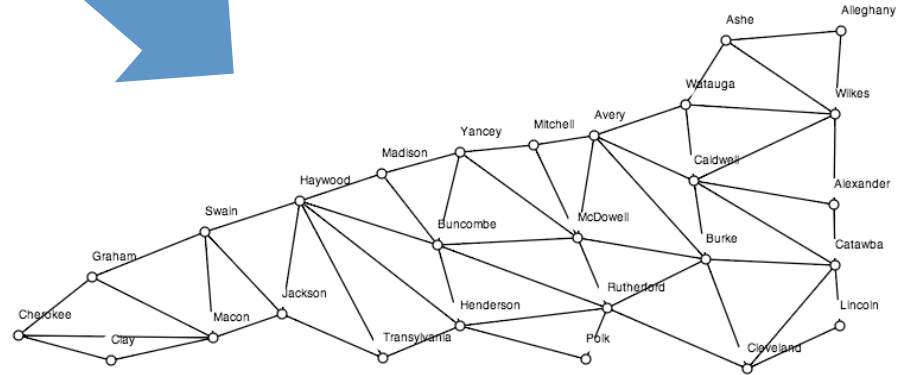
*vertex, edge, graph, adjacent, incident, connected, degree, subgraph, paths and circuits, adjacency matrix*

- What information can we add?

# One Possible Introduction

## Graph Coloring

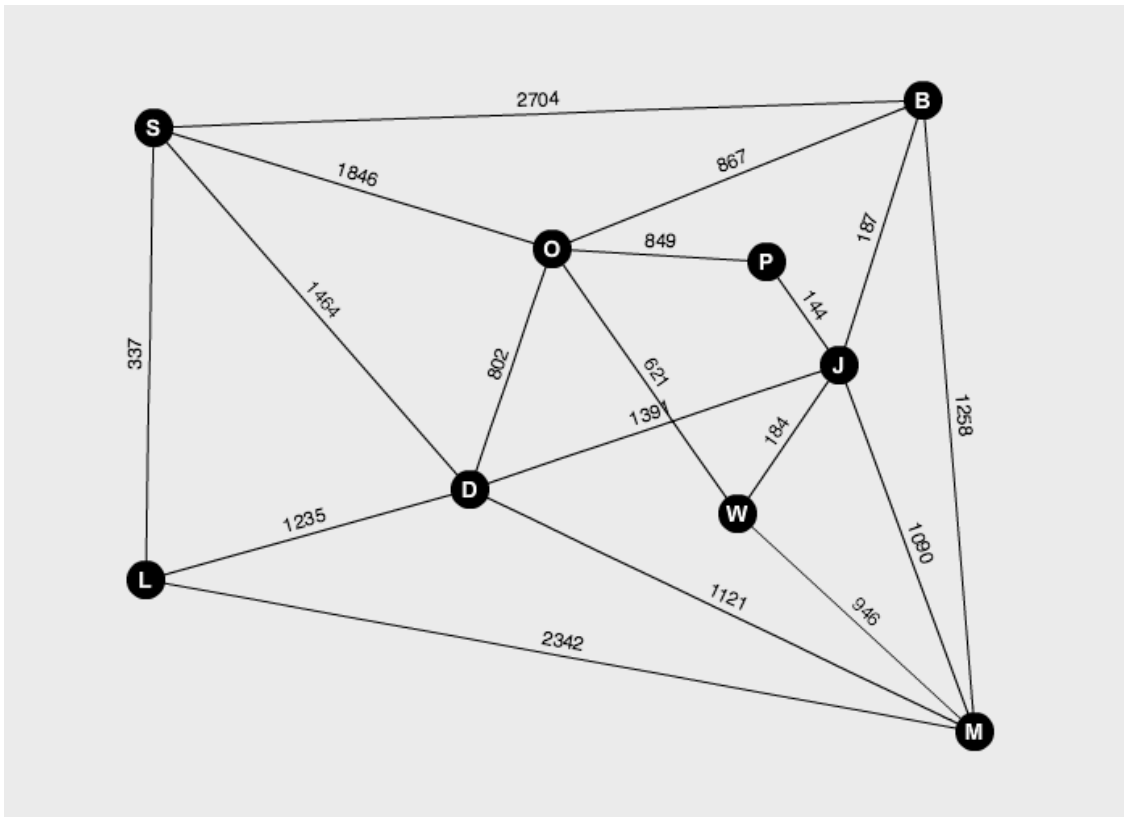
How many colors needed to color the map if no two regions sharing a border share a color?



# One Possible Introduction

Edge weights

What's the shortest path from B to L?

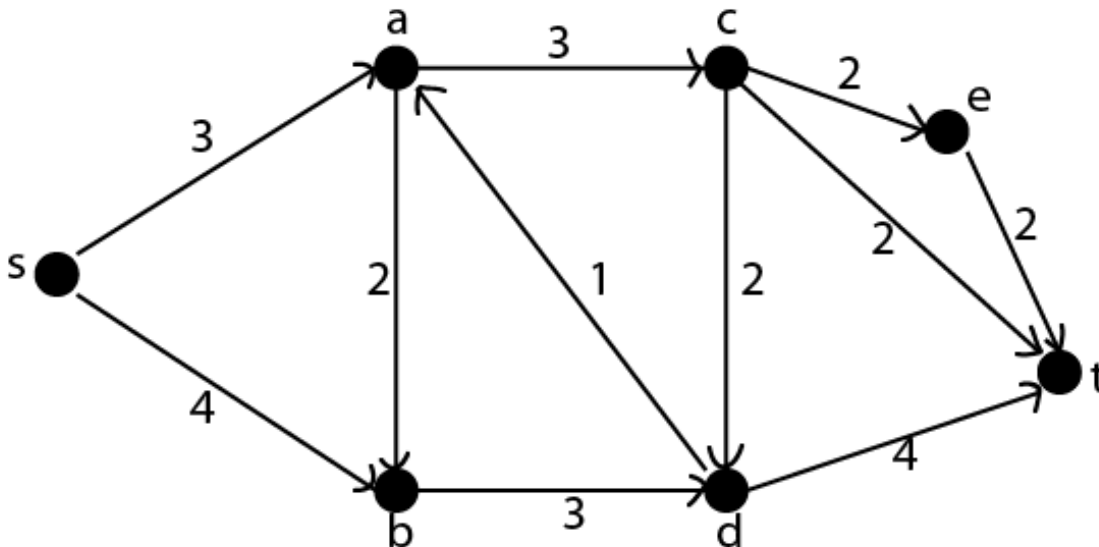


We can also motivate minimum spanning trees and traveling salesman problems, get into discussions about algorithms, efficiency, etc.

# One Possible Introduction

## Directed Edges

What if information only flows one direction?



We can ask some of the same questions with directed graphs, and also consider maximum flow, matching problems, and many others.

# How we often think about mathematical modeling:

Learn a  
new  
concept



Practice with  
it. Do a  
bunch of  
problems.



Now apply  
it!

**Student:** why  
should I care about  
this?

**Teacher:** You can  
do really cool stuff  
with it. Trust me.  
Just wait!

**Student:** why  
should I do this?

**Teacher:** You can  
do really cool stuff  
with it. Trust me.  
Just wait!

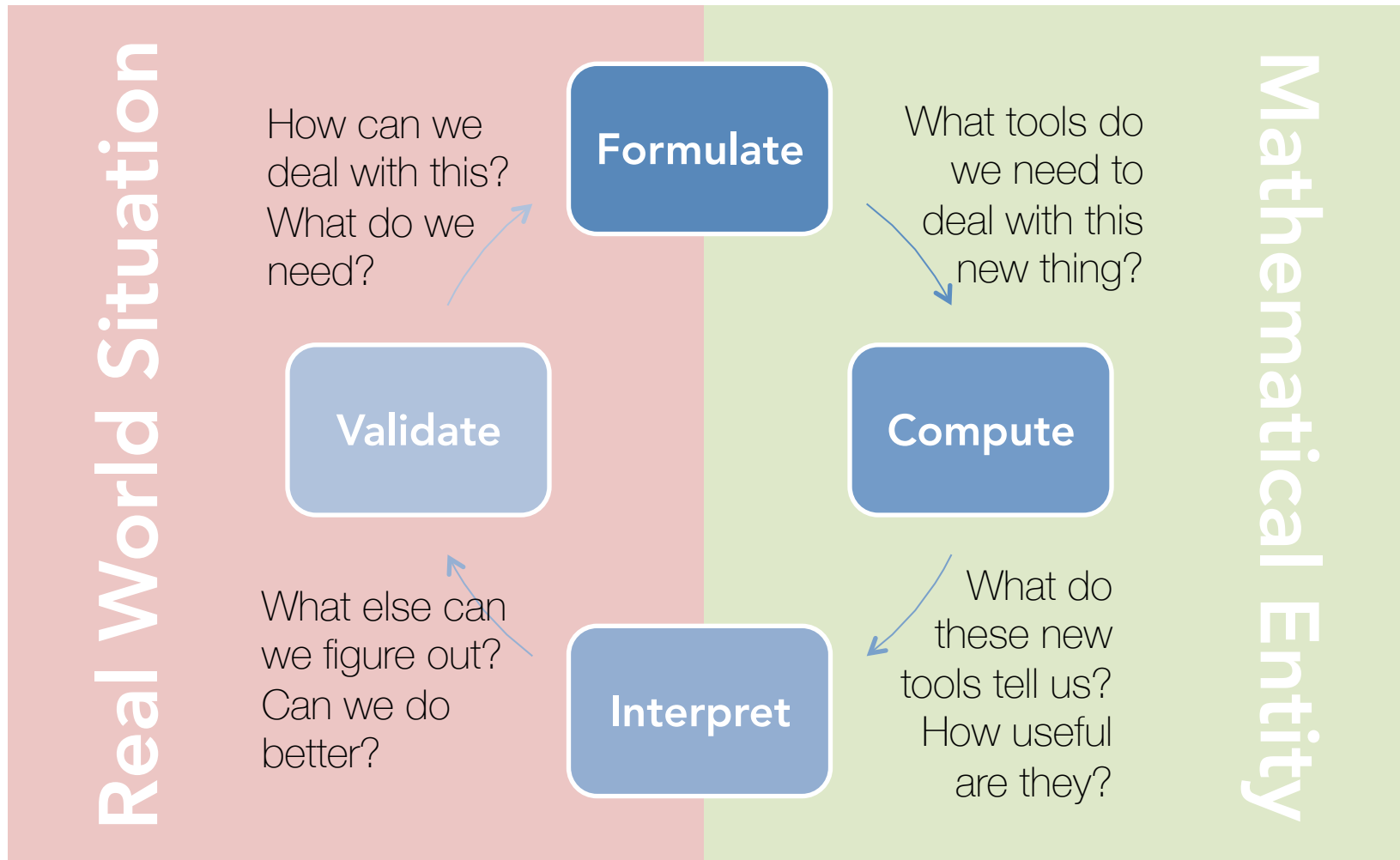
**Teacher:** Isn't this  
cool!

**Student 1:** why  
don't we always do  
this?

**Student 2:** . . .



# How we could (should) think about mathematical modeling:



The difficult part of this task is finding compelling situations that motivate useful, interesting, and accessible mathematics.

Most of my examples for teaching graph theory motivated interesting and accessible mathematics. They failed to be particularly compelling or particularly useful.

Enter .... Social Networks

# Motivating Graph Theory with Social Networks



Katheryn Deprill was abandoned as an infant in a Burger King in 1986. She posted this picture to Facebook on March 7, 2014.

## About

I am Katheryn (Hollis) Deprill, also known as The Burger King Baby. I have started the search for my biological mother in hope of reuniting with her!

## Biography

My birthday is September 15th, 1986 and I was left by my birth mother in the bathroom of an Allentown Burger King. Today I have launched a campaign to find my biological mother. I would love to be able to meet her, and hug her. I want her to know that I am not mad at her for what she did, however I have so many questions to ask her and also to start a relationship with my biological mother. If she comes forward please tell her to not be afraid and contact me as soon as possible.

# Motivating Graph Theory with Social Networks



It was shared over 30,000 times

She met her birth mother on March 24

## About

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## Biography

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# The most popular Facebook post of 2015:

**King Israel** ✓ with Jácý Àhũjā and 38 others.  
March 26, 2015 · 🌐

James (5:14-15) Is anyone among you sick? Let him call for the elders of the church, and let them pray over him, anointing him with oil in the name of the Lord. And the prayer of faith will save the one who is sick, and the Lord will raise him up.

This KID is blessed in [#Jesus](#) Name. CAN WE GET EVERYONE TO LIKE AND SHARE?





Àrèmúorin Composer, Yetmir and 10M others

Share

and 10M others

Top Comments ▾

622,823 shares

622,823 shares

84K comments

How does information move through a network?

How does something “go viral”?  
(besides involving cute pets)

There are some useful questions to ask ...

Facebook over a billion users worldwide.  
To model this sort of behavior, perhaps we  
should start with something smaller:

Name	Is friends with ...
Riley	Jay, Joe, Henry, Stacey
Jay	Riley, Brett, Nathan
Joe	Riley, Henry, Stacey, Brett
Henry	Riley, Joe, Stacey
Stacey	Riley, Joe, Henry, Brett, Nathan
Brett	Joe, Jay, Stacey, Nathan
Nathan	Jay, Stacey, Brett, James, Chris
James	Nathan, Chris
Chris	Nathan, James, Ty, Alex, Mark
Ty	Chris, Alex
Alex	Chris, Ty, Mark
Mark	Chris, Alex

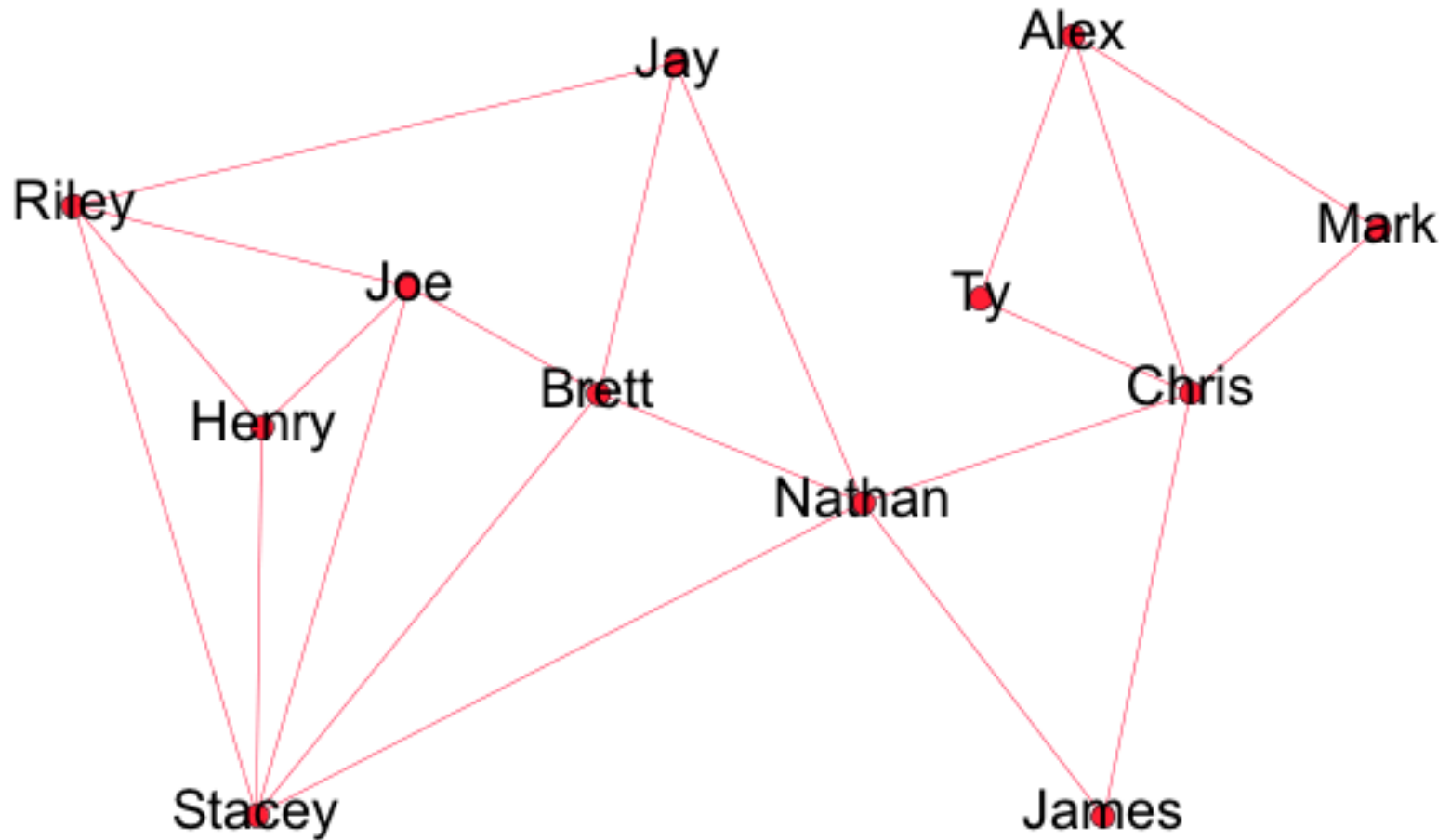
How can we  
represent these  
relationships?

We can use an adjacency matrix, or sociomatrix, to represent the network:

	Riley	Jay	Joe	Henry	Stacey	Brett	Nathan	James	Chris	Ty	Alex	Mark
Riley	-	1	1	1	1	0	0	0	0	0	0	0
Jay	1	-	0	0	0	1	1	0	0	0	0	0
Joe	1	0	-	1	1	1	0	0	0	0	0	0
Henry	1	0	1	-	1	0	0	0	0	0	0	0
Stacey	1	0	1	1	-	1	1	0	0	0	0	0
Brett	0	1	1	0	1	-	1	0	0	0	0	0
Nathan	0	1	0	0	1	1	-	1	0	0	0	0
James	0	0	0	0	0	0	1	-	1	0	0	0
Chris	0	0	0	0	0	0	1	1	-	1	1	1
Ty	0	0	0	0	0	0	0	0	1	-	1	0
Alex	0	0	0	0	0	0	0	0	1	1	-	1
Mark	0	0	0	0	0	0	0	0	1	0	1	-



Or we can visualize with a vertex-edge graph:



# Creating a Network

Create a contrived network from student input (**warning**: only with the right students)

- Example: Just ask a group of students to name their “friends” from a group. I make it clear that this is all in fun – and students do try and pick their friends to make the network more interesting.

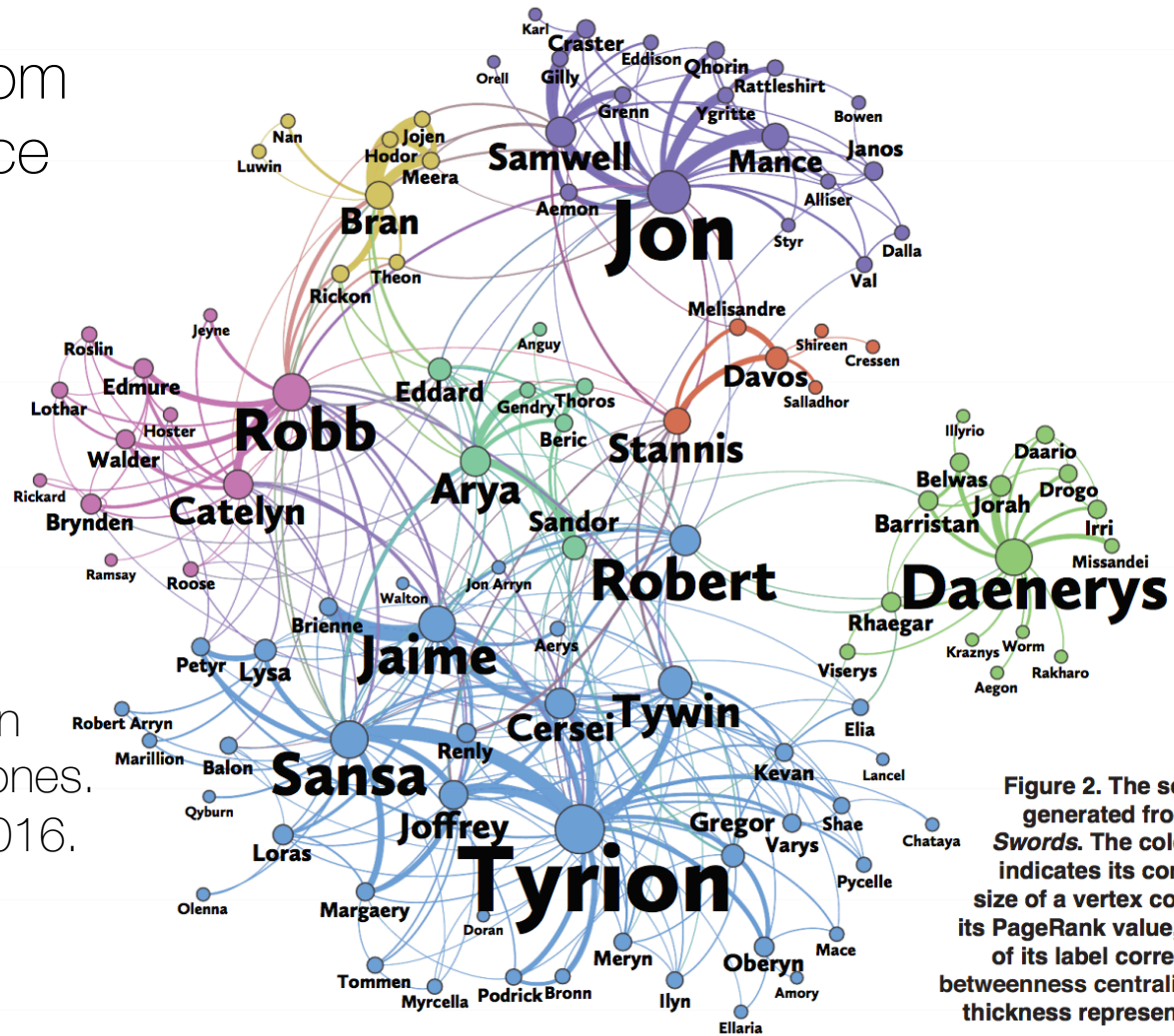
# Creating a Network

Use an activity to model interactions or “connections”

- Example: Give each student a die. They cycle through the whole class, rolling dice in pairs. If the sum is a certain number(s), they are “friends”. If not, then they’re not. You can rig the probability to create more or less connections.

# Creating a Network

Use a network from  
some other source

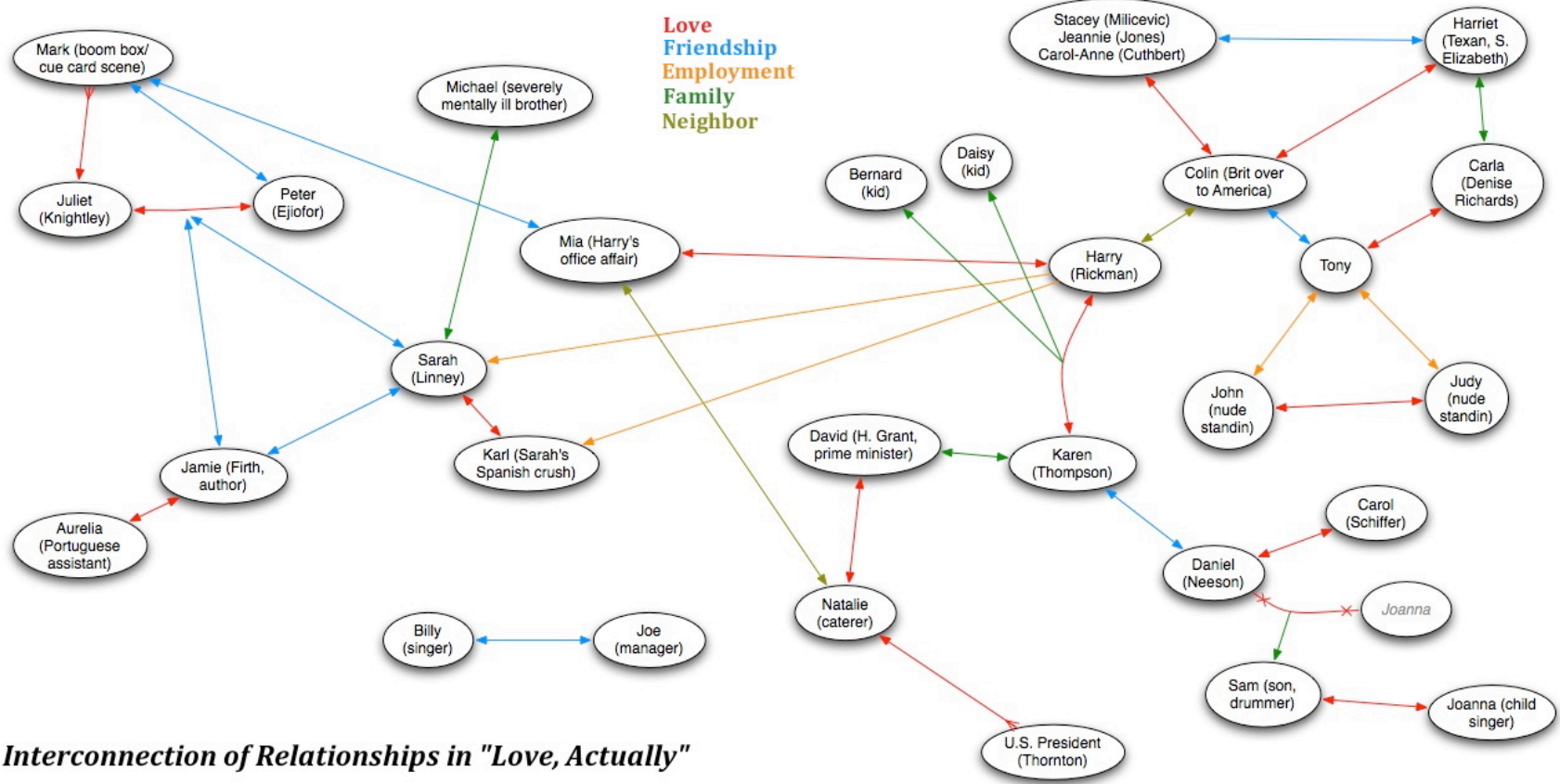


From Beveridge & Shan  
(2016). Network of thrones.  
*Math Horizons*, April 2016.  
(see [www.maa.org/mathhorizons](http://www.maa.org/mathhorizons))

Figure 2. The social network generated from *A Storm of Swords*. The color of a vertex indicates its community. The size of a vertex corresponds to its PageRank value, and the size of its label corresponds to its betweenness centrality. An edge's thickness represents its weight.

# Creating a Network

Use a network from  
some other source



*Interconnection of Relationships in "Love, Actually"*

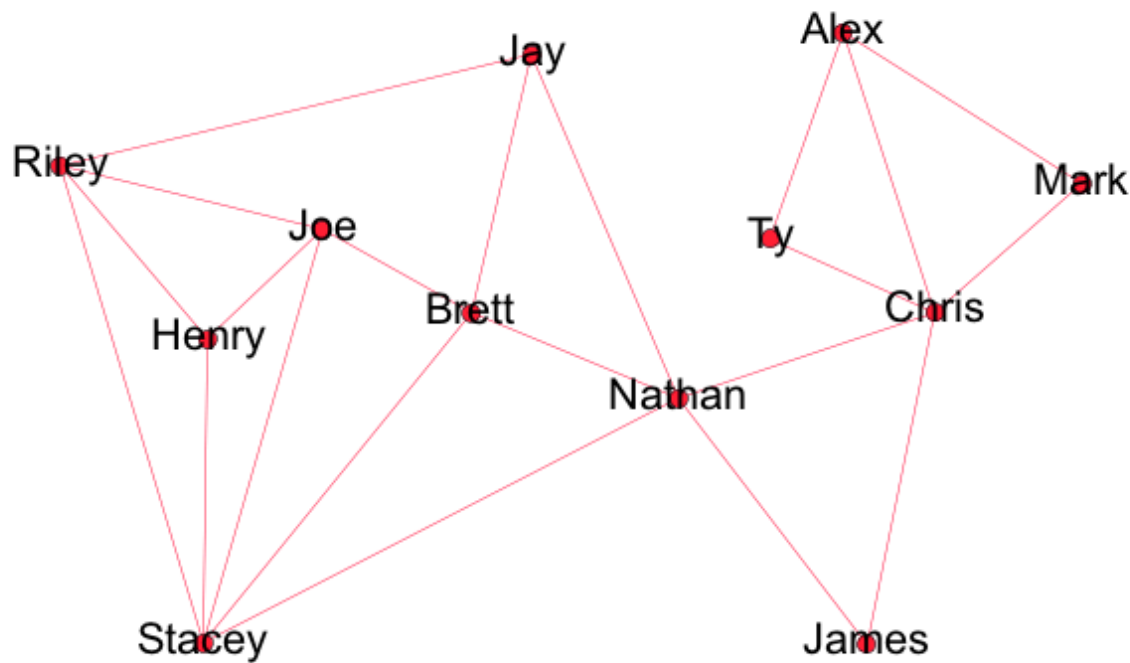
# Creating a Network

## Mine Facebook connections

- Students mine their own. Have them track their interactions using their timeline for some length of time, then look at their friends' pages to track their interactions with other friends.
  - [www.teachengineering.org](http://www.teachengineering.org) has decent instructions using this method, though they still need some tweaking to be implemented.
- WolframAlpha will create a Facebook network for any user
- NodeXL is a free Excel plug-in (though it only works with Windows), and will import information from a Facebook page or group.
- Gephi is a free network visualization tool
- Facebook has updated its settings in the last couple of years so most apps and plugins that will create graphs of your friends no longer work

# What Can We Do With Our Network?

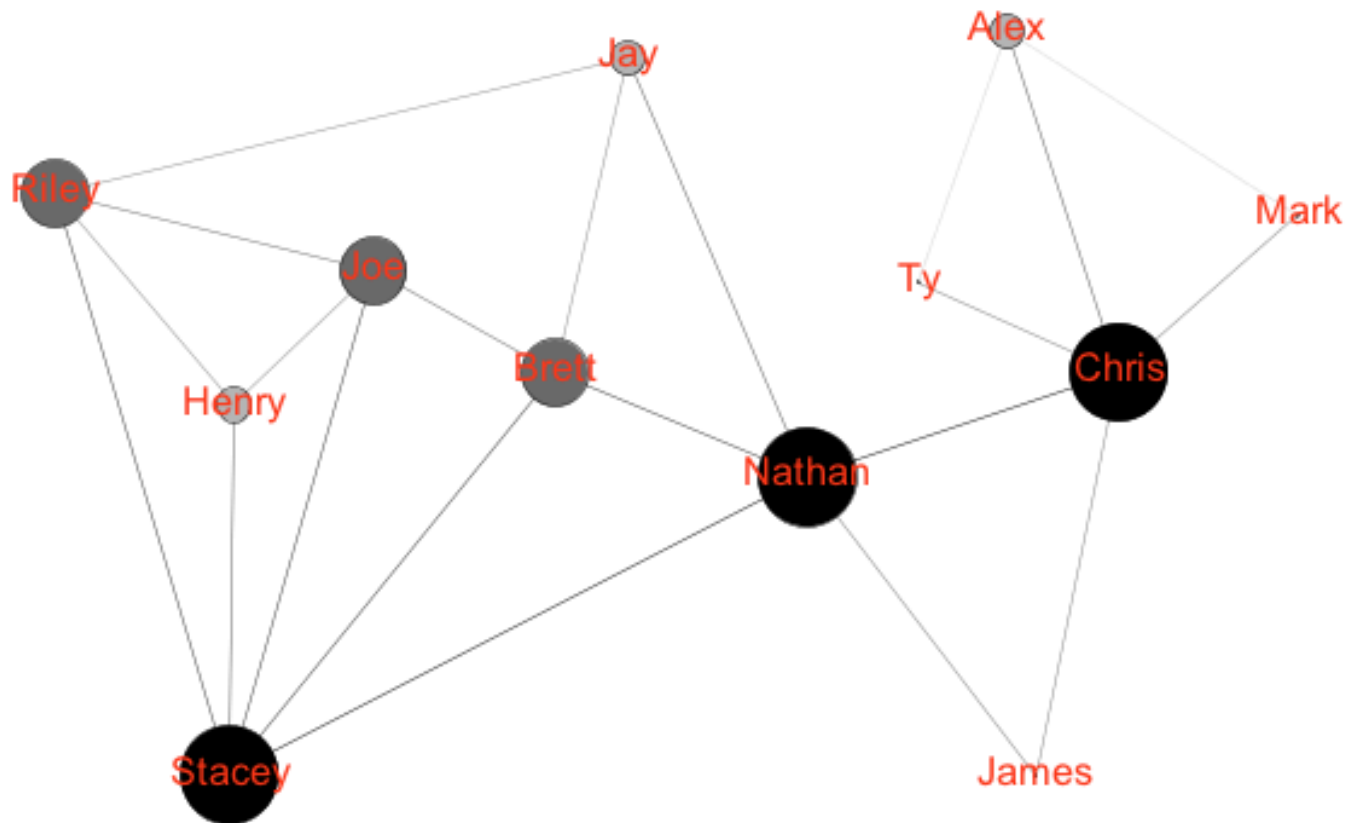
Let's start with initial impressions.



How might posts move through this network?

Who is the most important for making a post go viral? - Can you quantify this somehow?

The nodes have different *degrees*. Stacey, Nathan, and Chris have the largest degree (5)

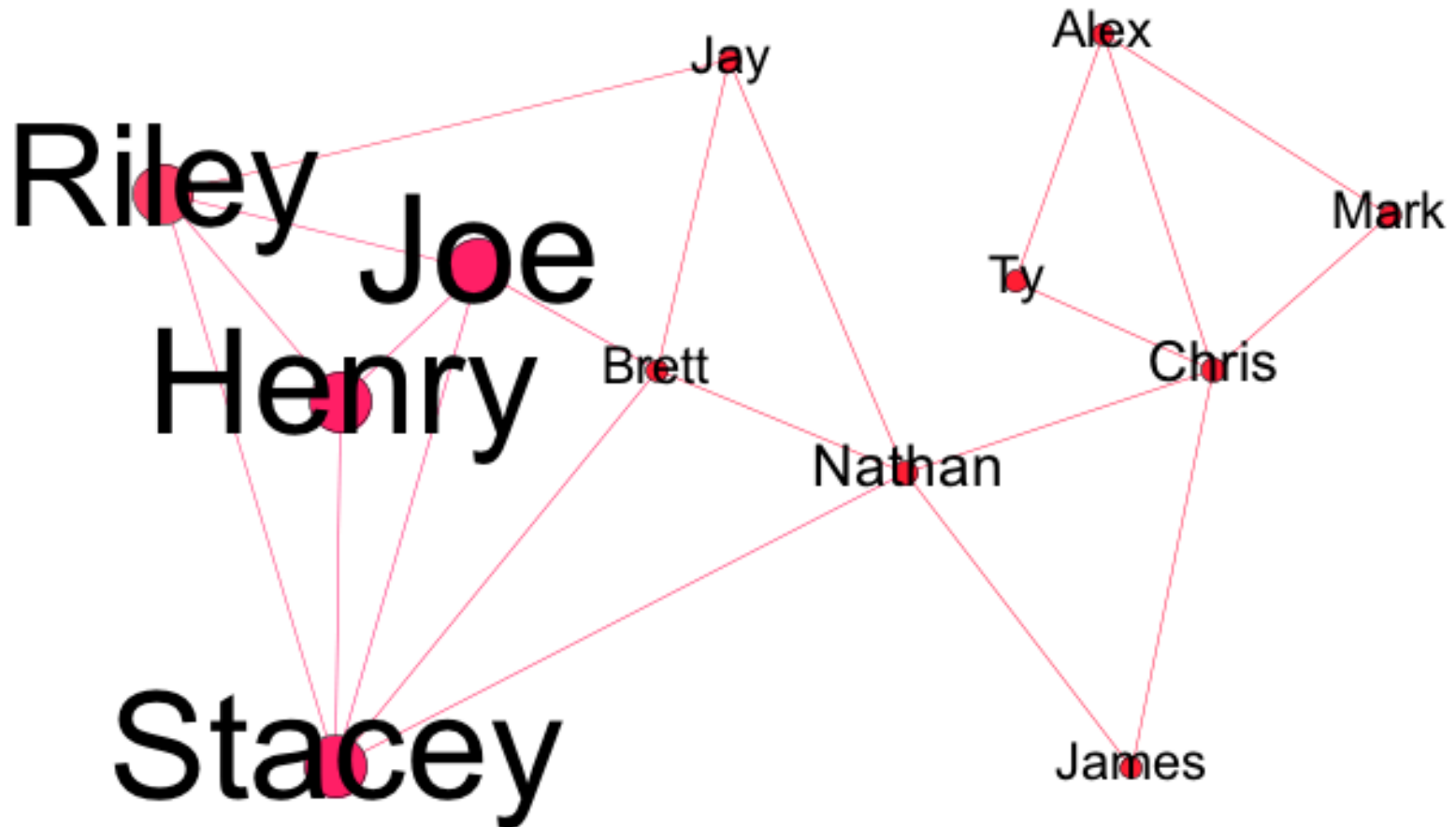


But just those three may not be sufficient to have a post go viral – they don't reach everyone (Joe).

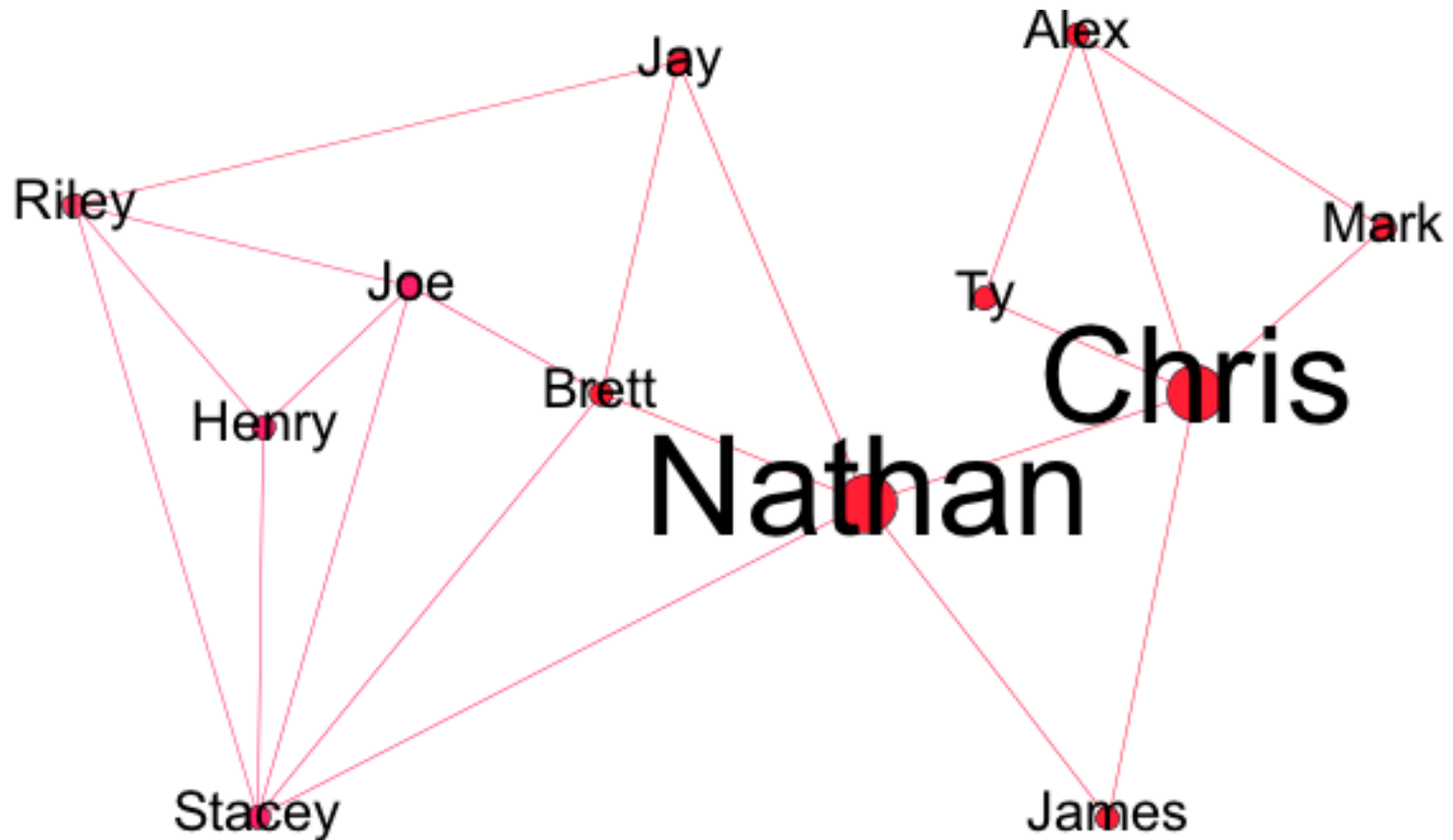


Riley, Joe, Henry, and Stacey form a *clique*. A completely connected subgraph.

- Allows us to introduce *subgraphs*, *complete graphs*, and the idea of *connectivity*.

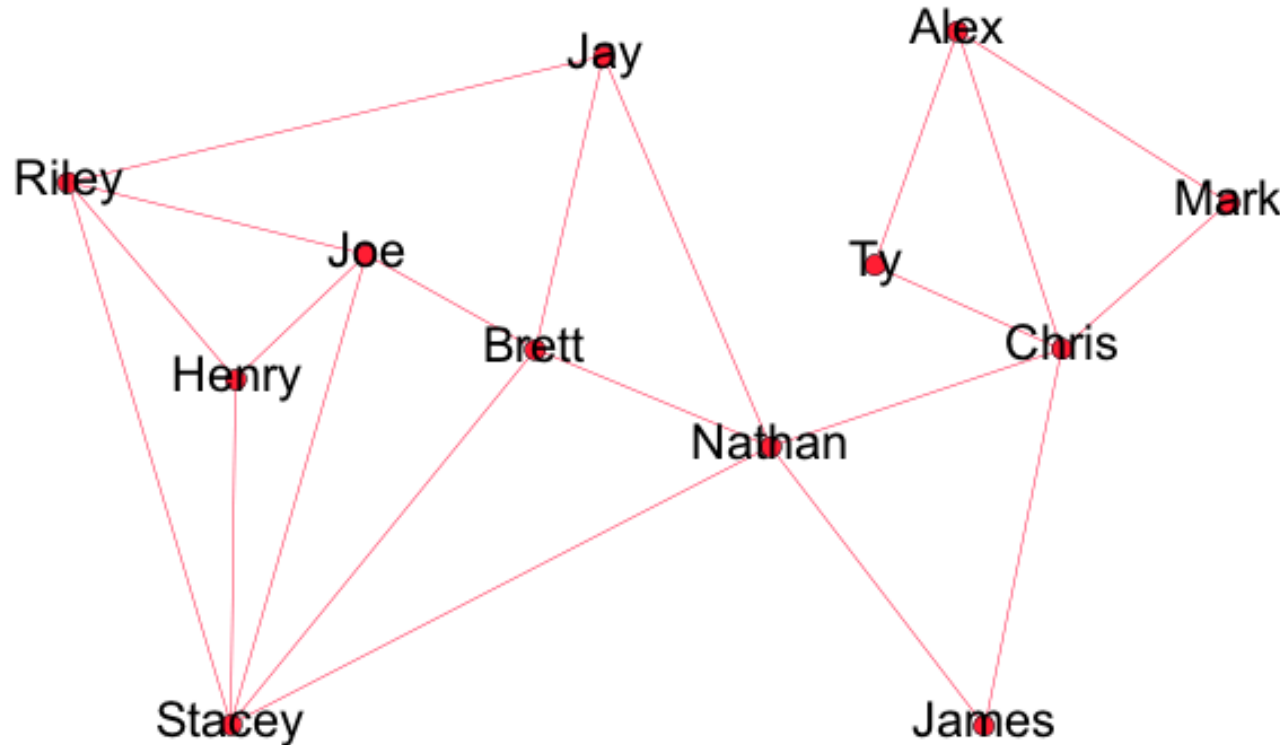


Nathan and Chris are *gateways*. There are pairs of nodes for which every path between the nodes must run through either Nathan or Chris.



The distances between vertices varies. The *eccentricity* of a node is its maximum distance from another node.

- The *diameter* of a graph is the maximum eccentricity.
- The *radius* is the minimum eccentricity.



Diameter = 4  
Radius = 2

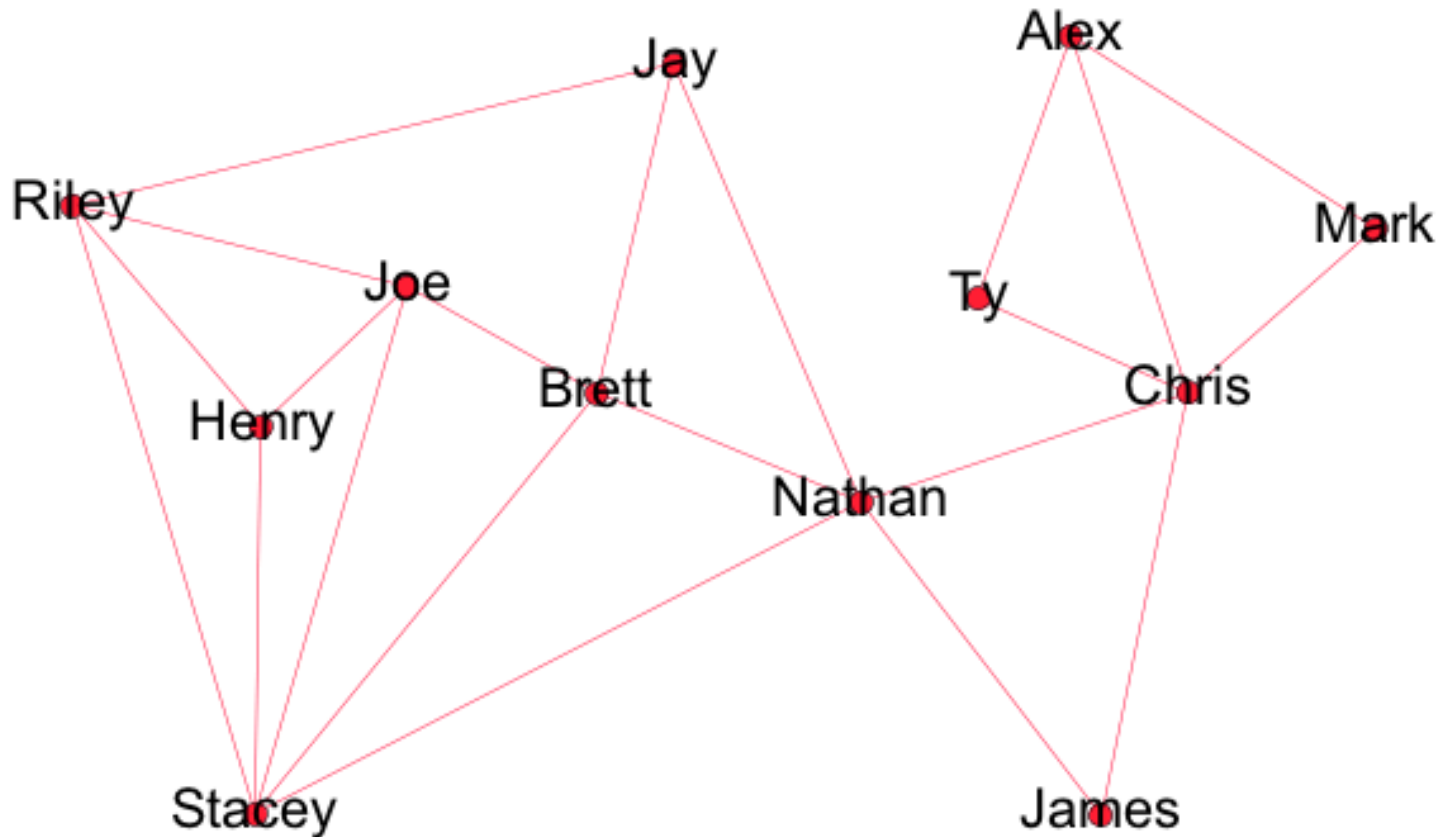
Alex, Mark, Ty,  
Riley, Henry,  
and Joe are  
*peripheral*

Nathan is  
*central*

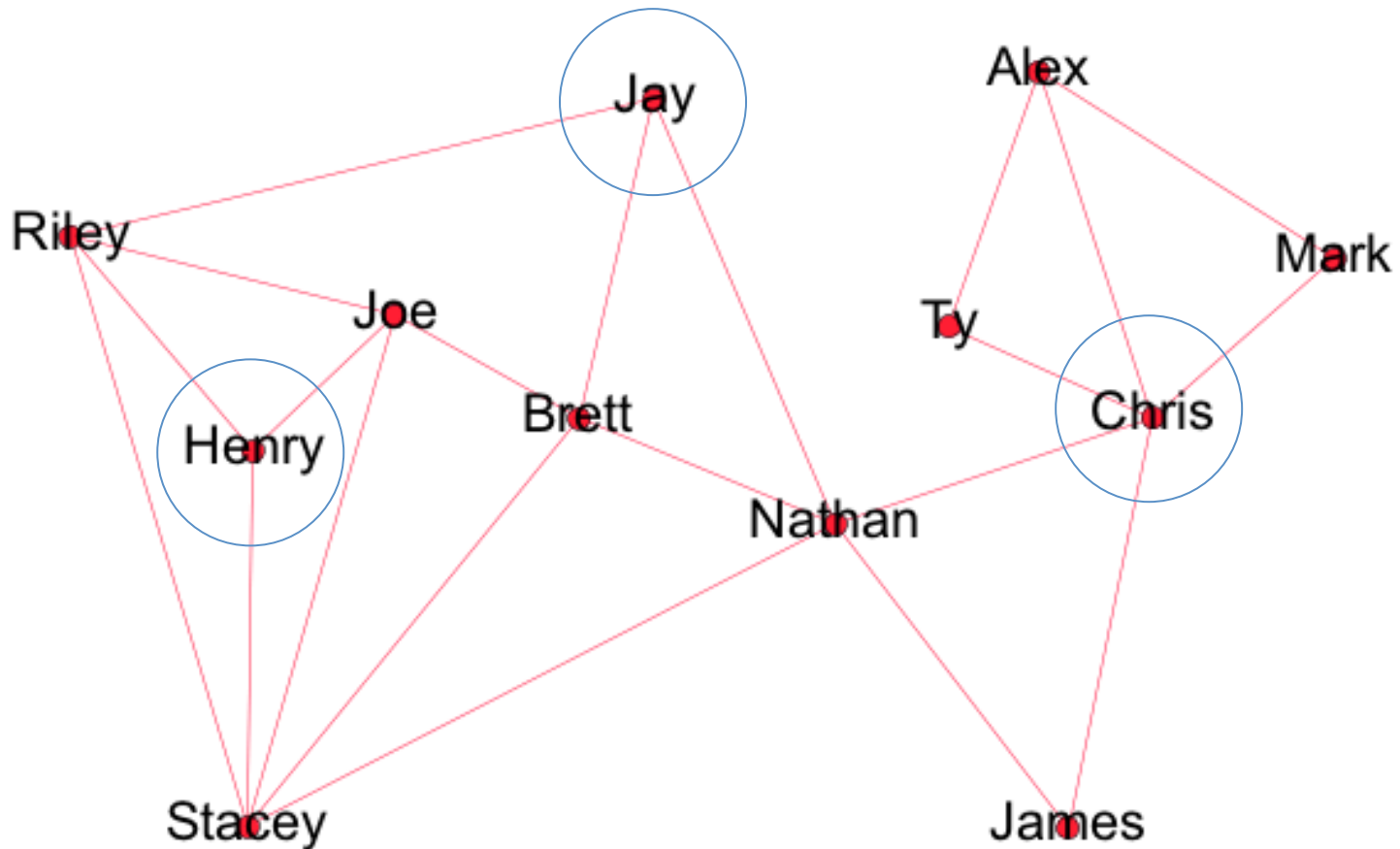
Interesting, but I want my  
cat photo to go viral. How  
do I make it go viral?

The high-degree nodes may not be enough – they don't reach everyone.

- What group of individuals could post something so that *everyone* saw it?

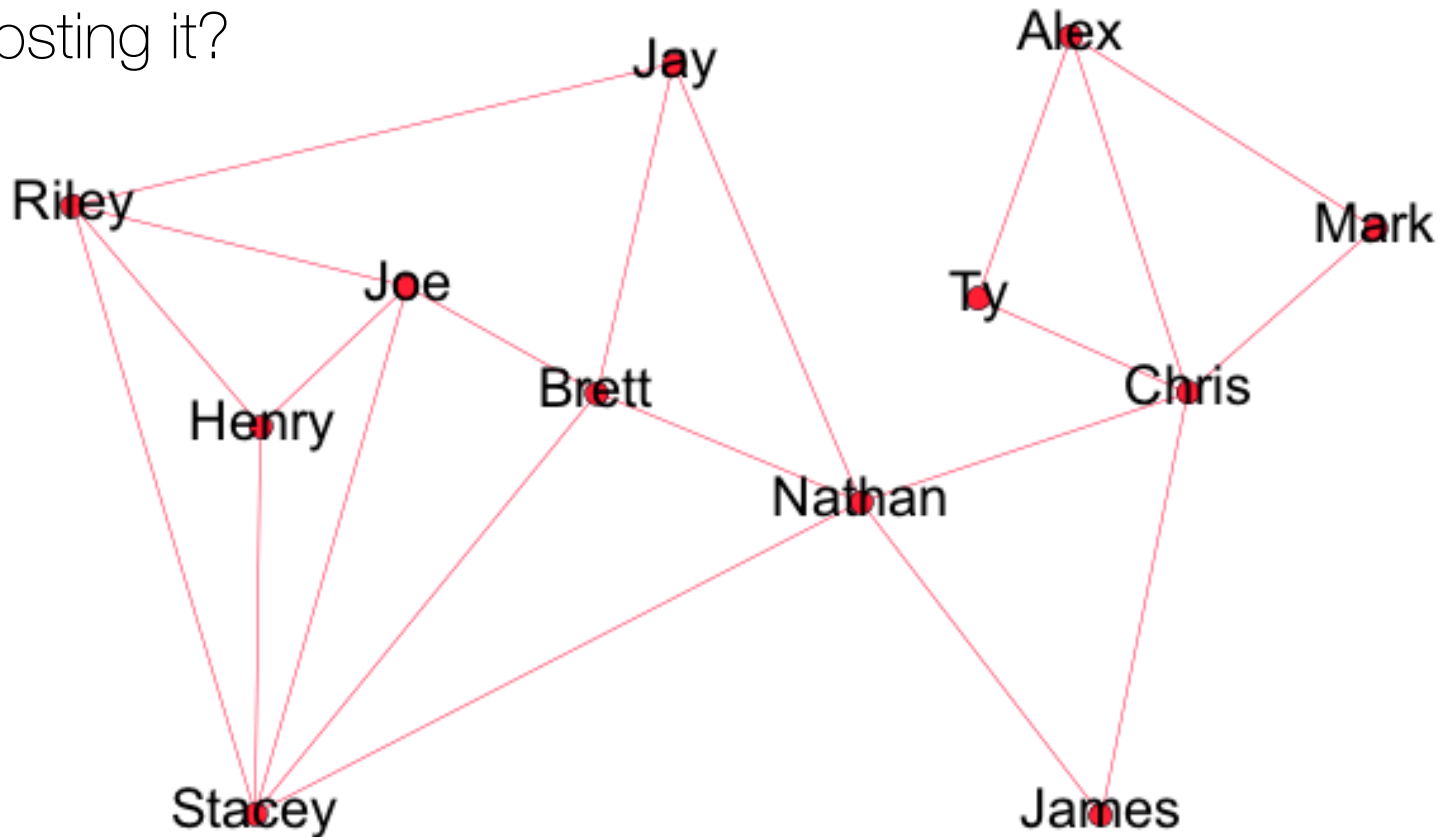


All nodes are included in the *neighborhoods* of these three nodes.

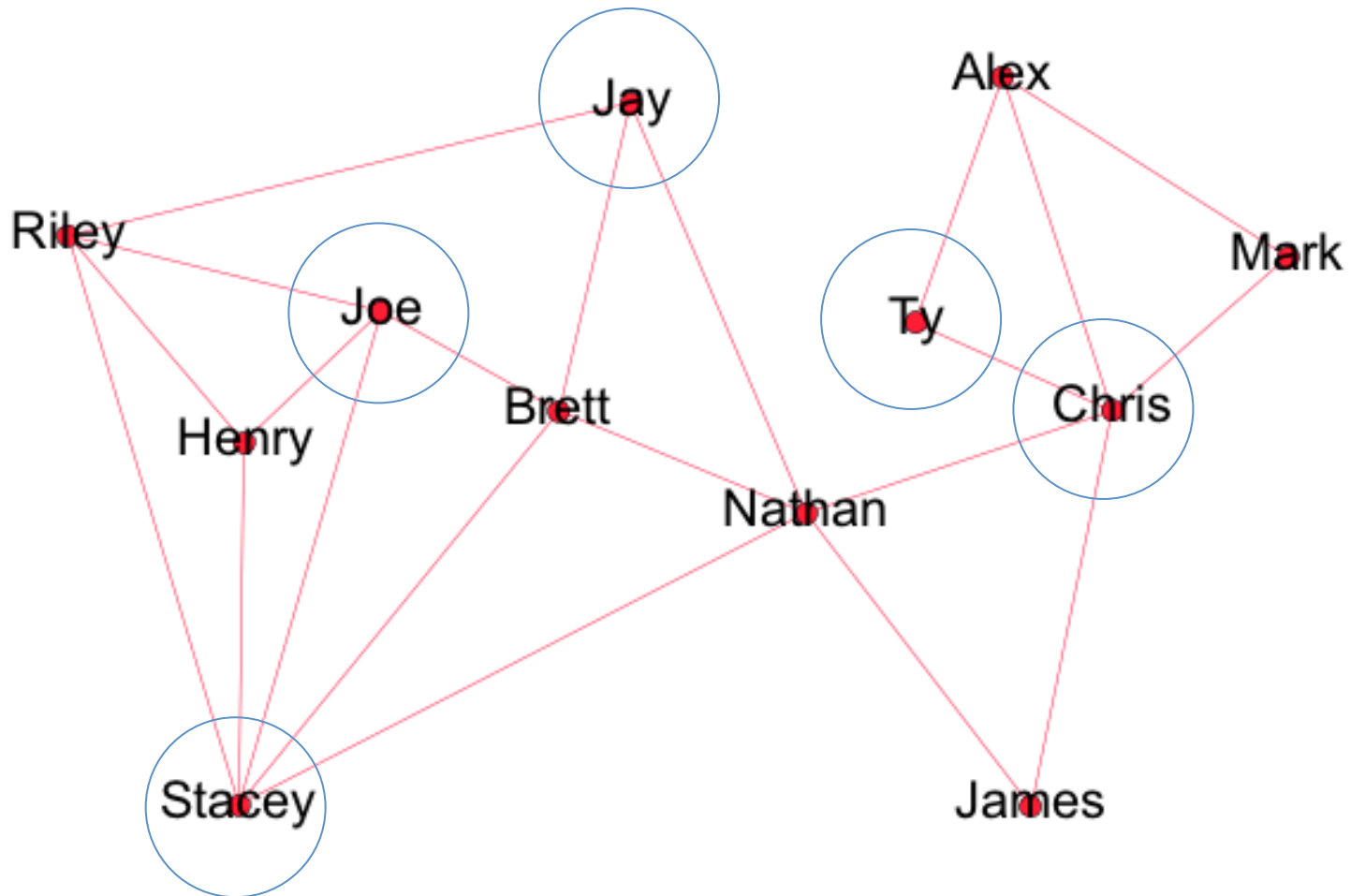


The high-degree nodes may not be enough – they don't reach everyone.

- What group of individuals could post something so that everyone either posted it had at least half of their friends posting it?



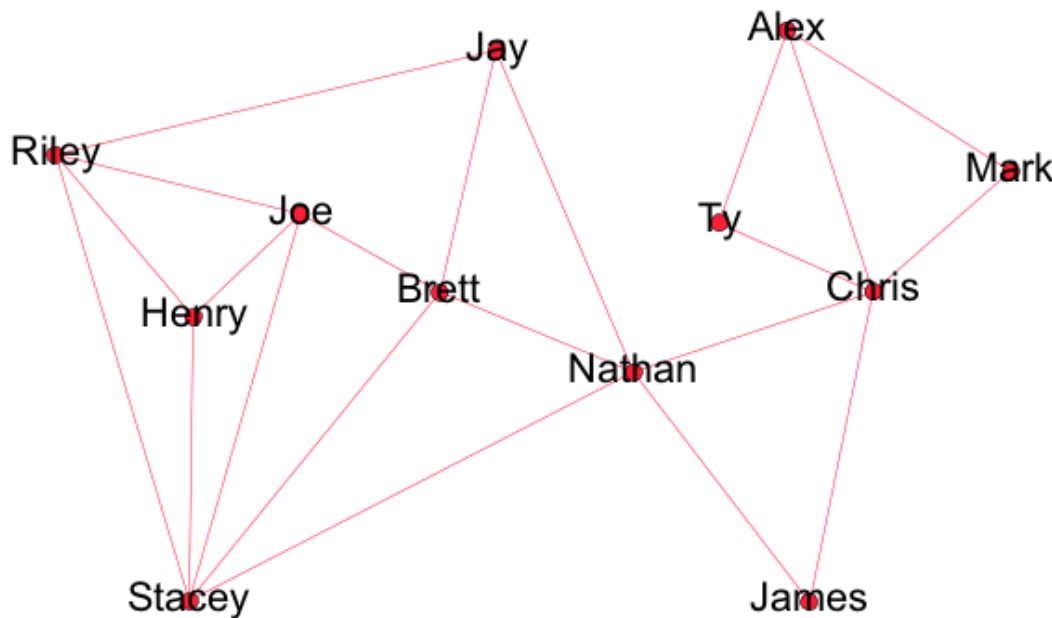
In this graph, we can accomplish it with 5 of the 12 nodes. In certain networks, a small number of influential nodes can give the illusion of a majority.





We can also quantify the *clustering coefficient* of an individual node.

- The probability that two contacts of a randomly chosen person have contact with each other

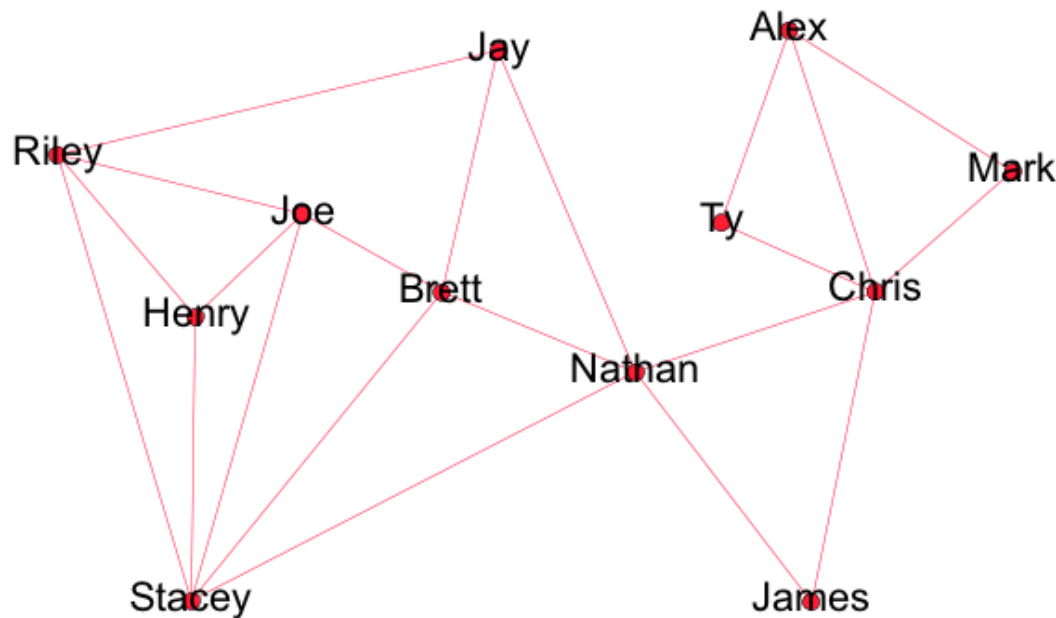


Let  $A$  be the set of nodes adjacent to  $v$ , and  $n(A)$  be the number of nodes in  $A$ .

$$C(v) = \frac{\begin{array}{c} \text{\# of edges in} \\ \text{subgraph made up} \\ \text{of pts in A} \end{array}}{\begin{array}{c} \text{\# of edges in a} \\ \text{complete} \\ \text{subgraph with } n(A) \\ \text{nodes} \end{array}}$$

We can also quantify the *clustering coefficient* of an individual node.

- The probability that two contacts of a randomly chosen person have contact with each other



For example, if  $v=Joe$ ,  
 $A = \{Riley, Henry, Stacey, Brett\}$ , so  $n(A)=4$ .

$$C(Joe)=4/6=2/3$$

So, if Joe's posts are reposted by a friend, they can quickly give an illusion of majority in his group.

# Other Cool Things To Do

Much of what we've just done also applies to disease spread. You can model the spread of flu through your class.

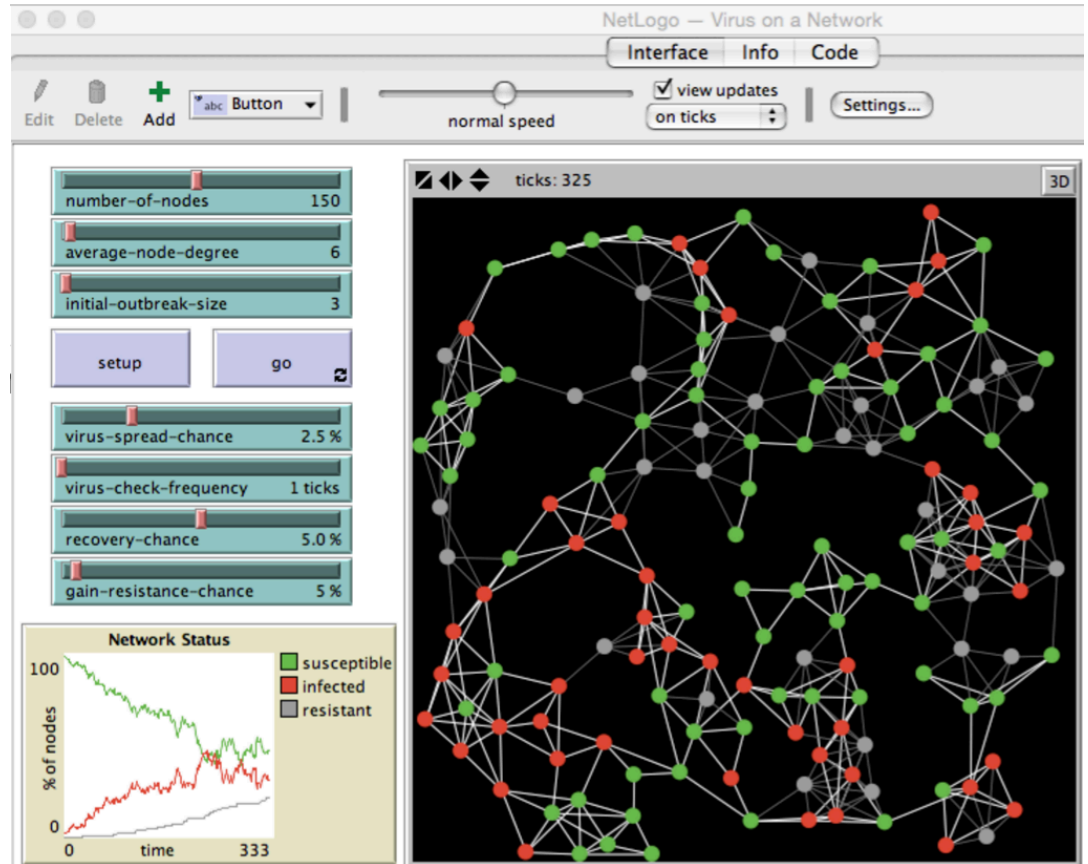
- Who should be vaccinated in order limit the spread most effectively? (degree distribution)
- What networks are vulnerable? (mean shortest path)

check out Angela & George Shiflet, "Getting the 'edge' on the next flu pandemic: We should'a 'node' better". Published on shodor.org.

# Disease Spread

Simulation of virus spread on a network using NetLogo

Students can edit the code to alter the network.



# Other Cool Things To Do

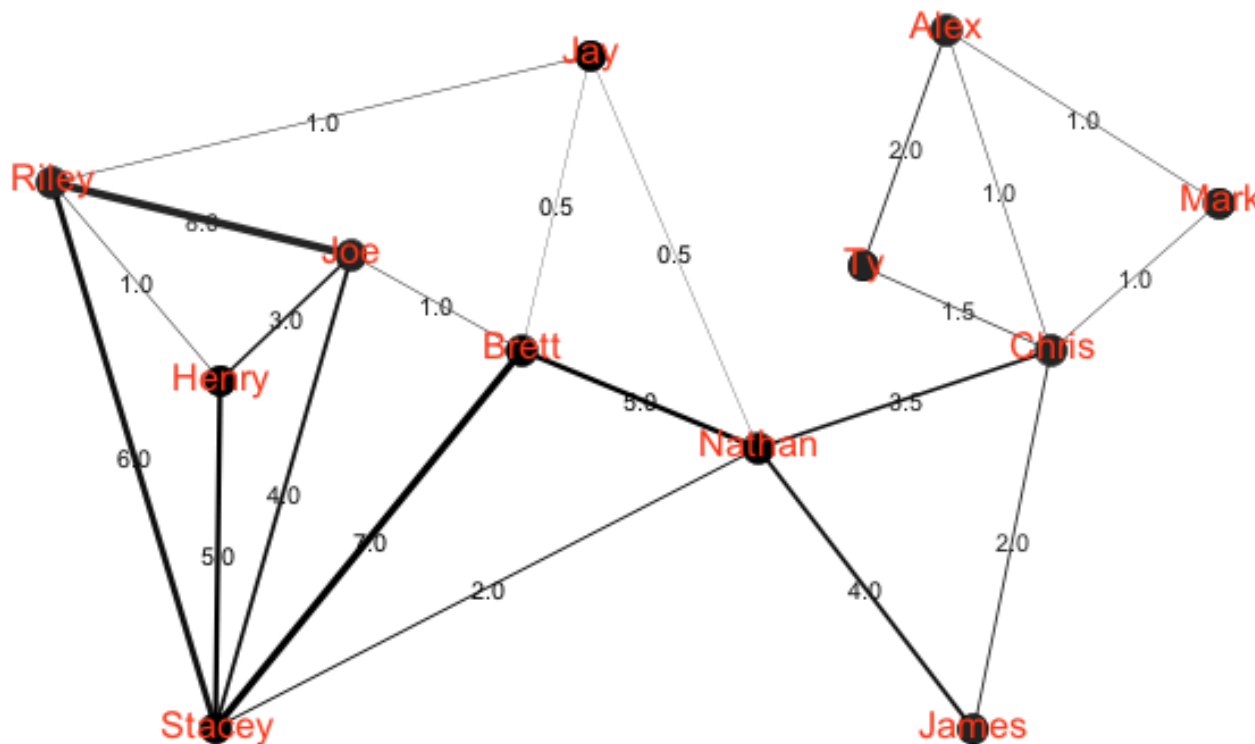
We have assumed:

- All links are equally strong.
- All individuals are equally influential.

How could we improve on that?

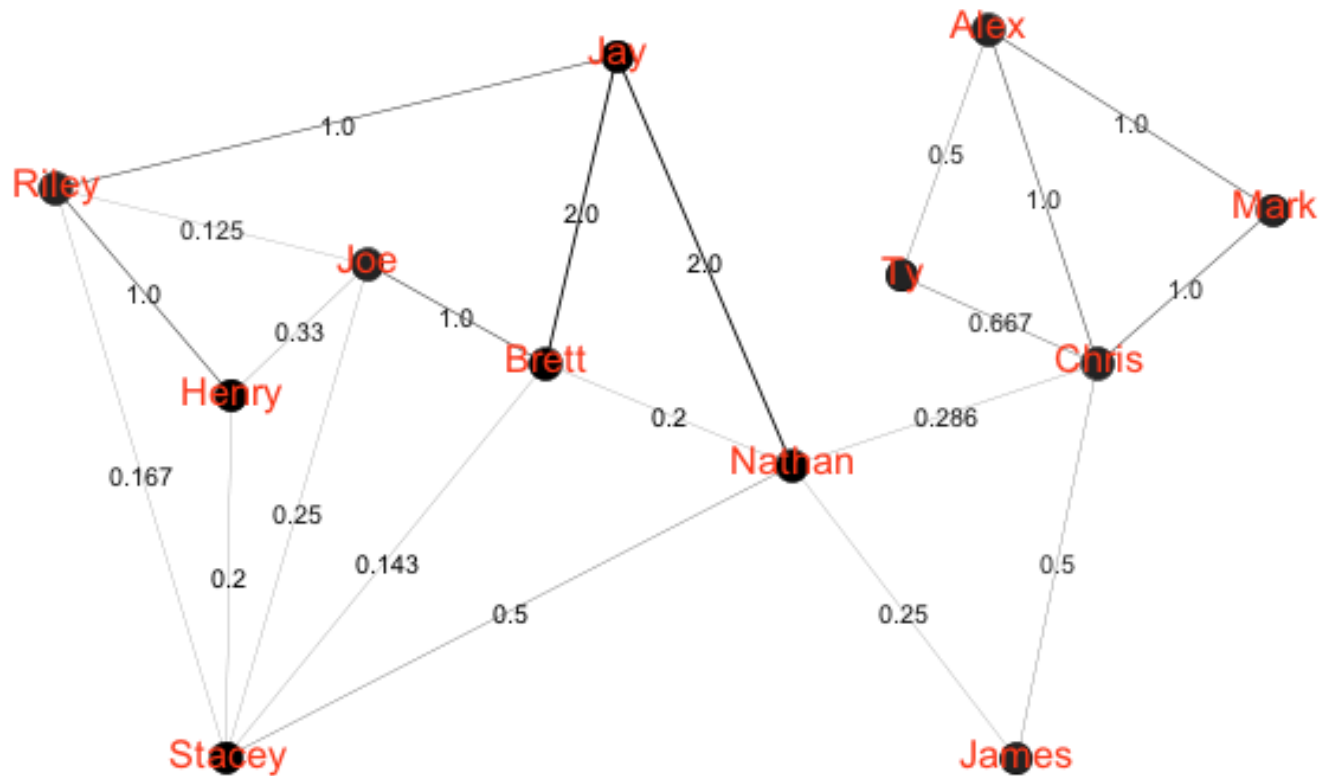
We can add weights by some measure of the strengths of the ties.

- In this case, we can measure how many times they interact, on average, per week.
- Or we could quantify it in other ways – count interactions, time spent interacting, etc



What can we say now about the way information might move through this graph?

The problem is that it's easy to find shortest paths, but maximum paths are trickier. But we can change our edge weights so strong relationships have smaller weights.



Now we could look at shortest paths (introduce Dijkstra's algorithm)

The *closeness centrality* of a vertex:

- The farness of a vertex is the sum of all the shortest paths to other vertices.
- The closeness is the reciprocal of the farness



The *betweenness centrality* for a vertex is a measure of how many shortest paths pass through the vertex.

- For each pair of vertices, find the shortest paths, and then compute the fraction that pass through the vertex in question.
- For a given vertex, add those fractions up for every pair. This is the betweenness centrality of a vertex

	Betweenness
Nathan	30.333
Chris	24.5
Stacey	12.833

# Other Cool Things To Do

## Degrees of separation

- On Twitter, almost everyone is within 5 degrees of separation
- On Facebook, the average is 4.74. 99.6% of users are separated by 5 degrees
- Wikipedia links are another good network.
  - The Oracle of Wikipedia

# Other Cool Things To Do

Weight the vertices instead of the edges

- Eigenvector Centrality
- Page Rank

On average, your friends will have more friends than you.

- Why?

# #TheDress

February 26-27,  
2015

What color is it?





# #TheDress

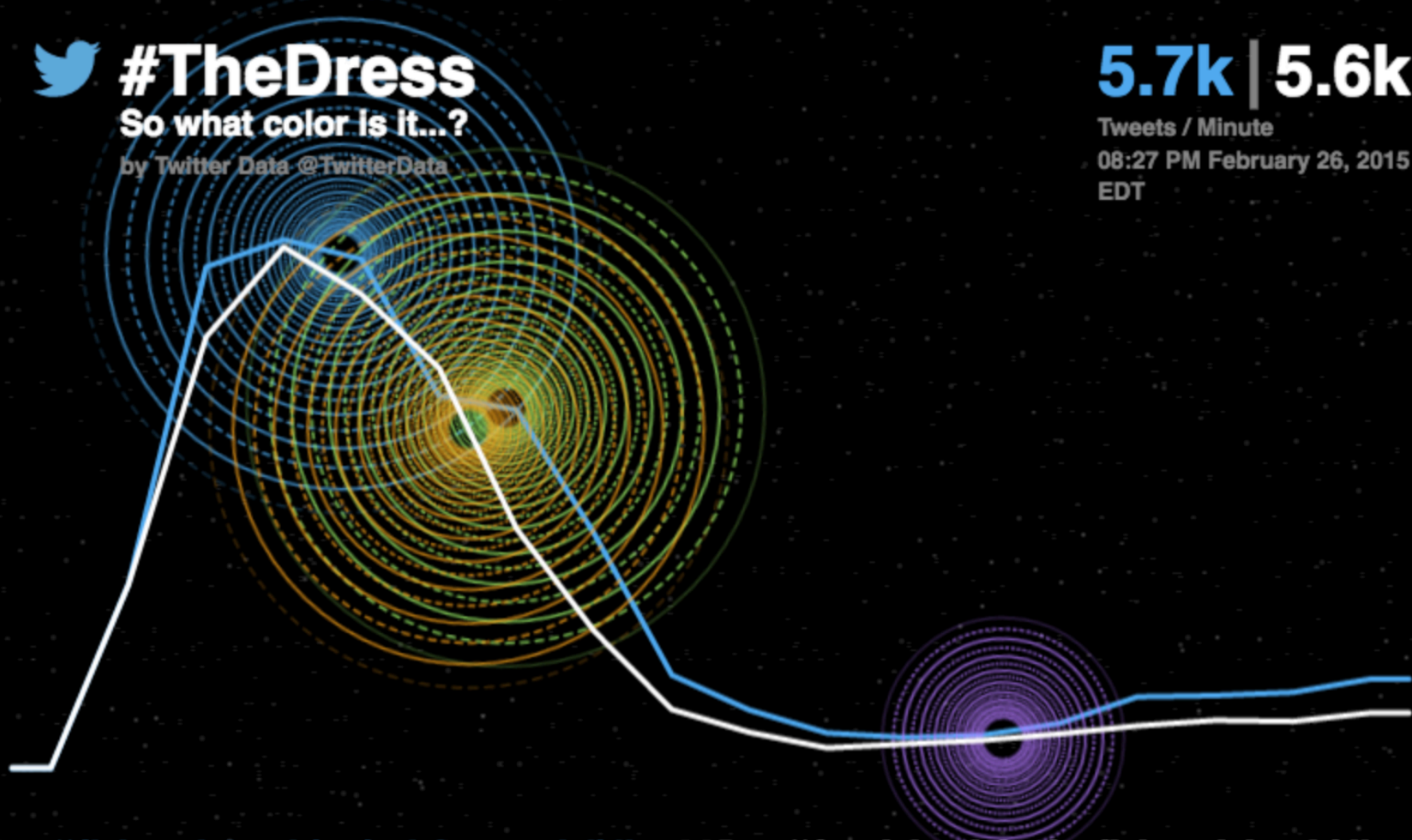
So what color is it...?

by Twitter Data @TwitterData

5.7k | 5.6k

Tweets / Minute

08:27 PM February 26, 2015  
EDT



“(blue black) -(white gold)” VS. “(gold white) -(blue black)”



# Other Cool Things To Do

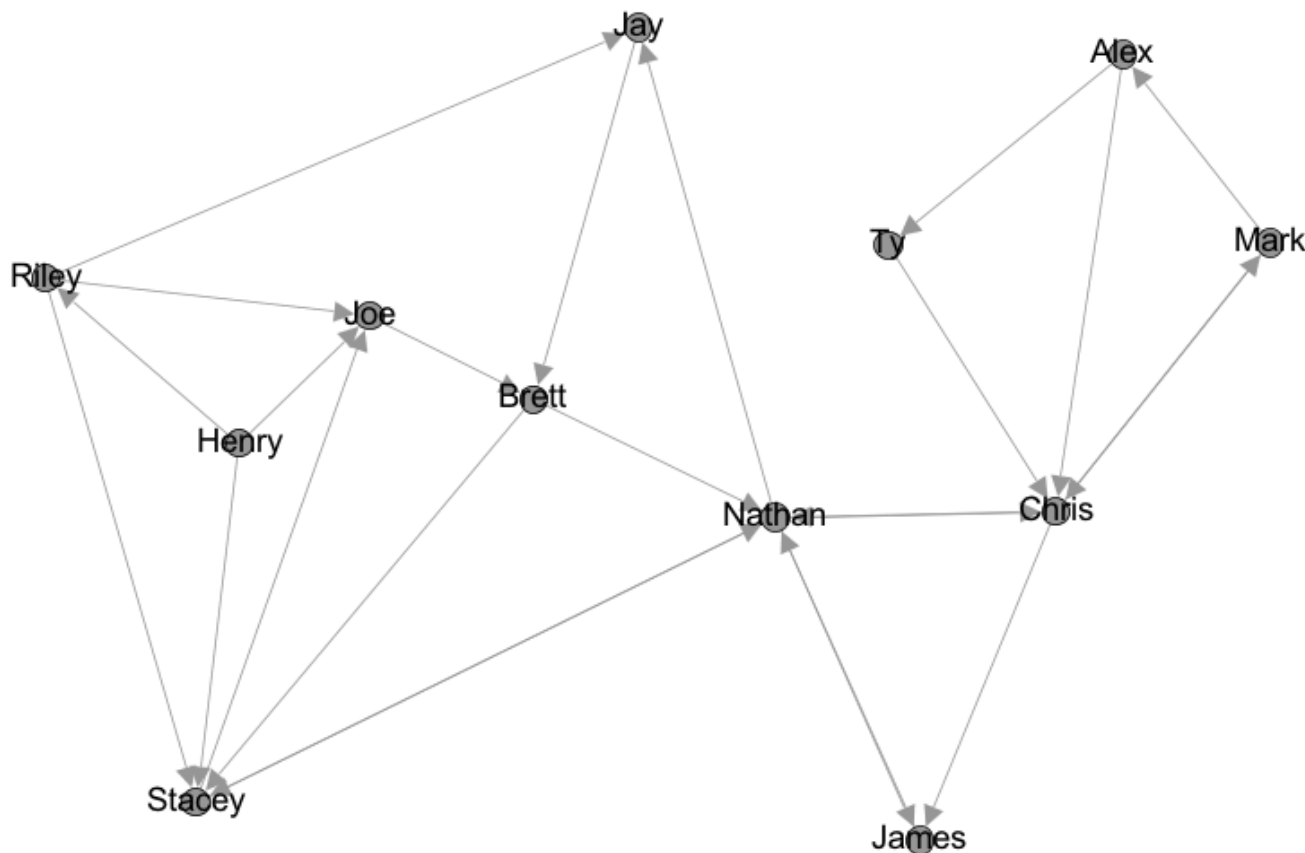
What's the difference between a Twitter network and a Facebook network?

Directionality!

So, to explore a Twitter network, we have to realize that the relationships may be one way

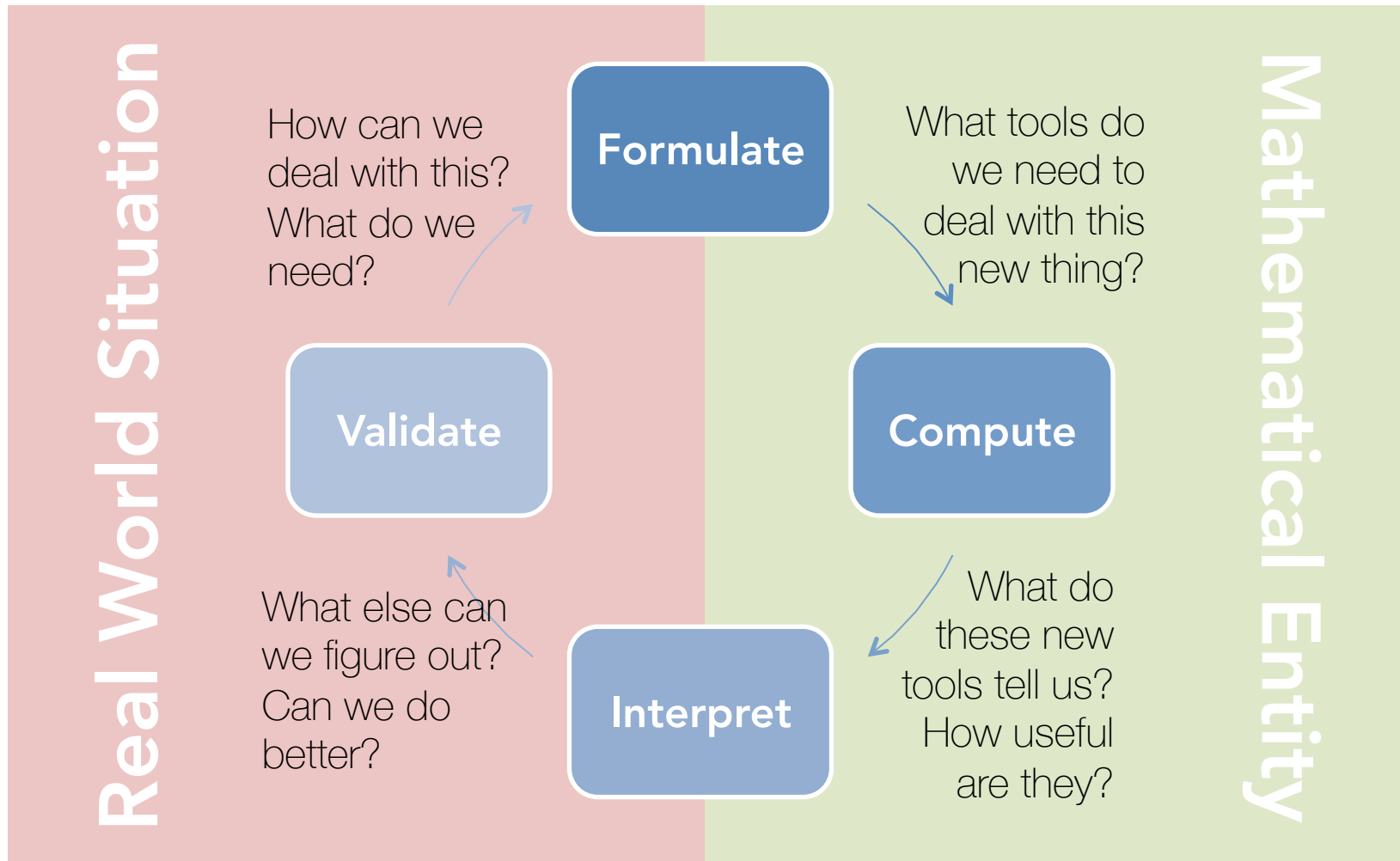
- To model this, we need a directed graph

# Directed Graphs and Twitter



Who is the most influential now?

# Social Networks: A Compelling and Useful Context for Graph Theory





# THANK YOU!

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