GAIMME
Mathematical Modeling for High School

NCTM 2016 Annual Meeting & Exposition

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GAIMME

Guidelines for Assessment and Instruction in Mathematical Modeling Education

a joint effort by SIAM and COMAP

Full report available for FREE from:

http://www.comap.com/Free/GAIMME/index.html

http://www.siam.org/reports/gaimme.php

Primary Audience

Teachers
(including Teacher Educators)

Additional Audiences

Test Writers
Policy Makers

Goals of GAIMME

- to paint a clearer picture of mathematical modeling <u>as a process</u>
- to help teachers incorporate the practice of mathematical modeling into classrooms

Structure of GAIMME Report

- What is Modeling?
- Modeling in Grades K-8
- Modeling in Grades 9-12
- Modeling at the Undergraduate Level
- Appendix of Resources
- Appendices of Modeling Exemplars

GAIMME is not

- a collection of lesson plans
- a complete set of classroom assessments

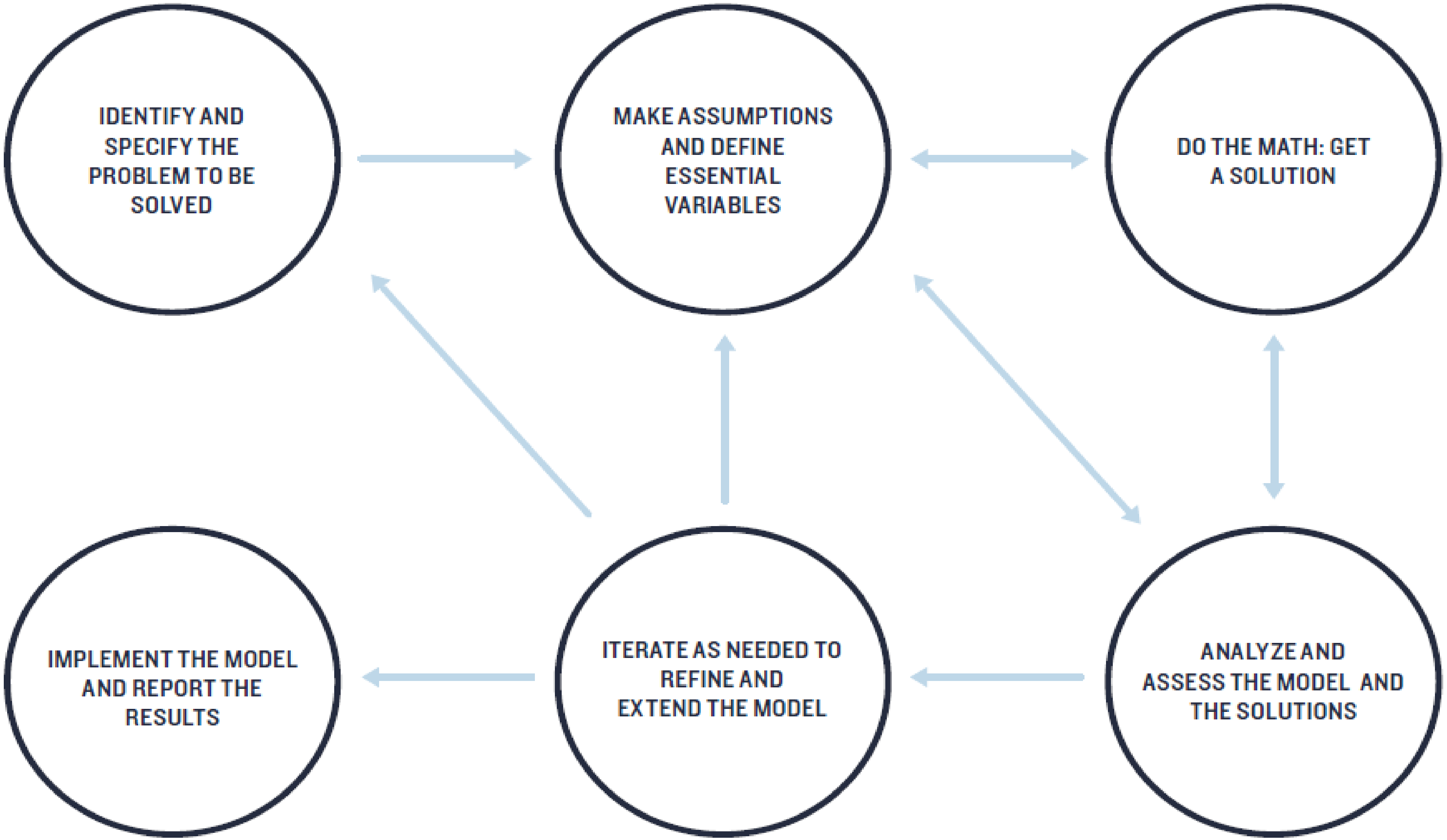
GAIMME does provide

- insight into what mathematical modeling is and isn’t
- practical advice on how to teach modeling

Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

Mathematical Modeling is a Process

- Identify the Problem.
- Make Assumptions and Identify Variables.
- Do the Math.
- Analyze and Assess the Solution.
- Iterate.
- Implement the Model.


IDENTIFY AND SPECIFY THE PROBLEM TO BE SOLVED
MAKE ASSUMPTIONS AND DEFINE ESSENTIAL VARIABLES
DO THE MATH: GET A SOLUTION
IMPLEMENT THE MODEL AND REPORT THE RESULTS
ITERATE AS NEEDED TO REFINE AND EXTEND THE MODEL
ANALYZE AND ASSESS THE MODEL AND THE SOLUTIONS

Pay-offs <u>within</u> math from including modeling

- motivate new content: topics and techniques
- reinforce concepts and illustrate application
- demonstrate the connectedness of mathematics
- broaden students' views of mathematics

Throughout human history, other fields and disciplines have drawn on mathematical ideas and methods. The important place mathematics has in current pre-college curricula is a consequence of its power to explain and give insight into all areas of life.

Whatever trajectory a student's life takes after high school, being able to think flexibly, to make realistic assumptions about what may happen when the future is uncertain, and to understand how conclusions are reached based on prior knowledge are very valuable life tools.

Characteristics of Modeling

- Modeling is open-ended and messy.
- Modeling requires making genuine choices.
- Modeling does not happen in isolation.

How do you make the transition from standard textbook problems to the full modeling cycle without becoming totally overwhelmed?

Principles for Teachers

- Start small.
- Scaffold initial experiences with leading questions and class discussion.
- Use common, everyday experiences to motivate the use of mathematics.
- Use bite-sized modeling scenarios that require only one or two components of a full modeling cycle.
- Share your goals and instructional practices with parents and administrators.

General modeling principles

- It is usually easier to develop useful models by starting with a simplified version of a situation than with one that is closer to reality. The first model is rarely the final model.

- Ask, “What question are we trying to answer? How can we ‘measure’ that?”

General modeling principles

- Pay attention to what you "want." If you need a number, make up a value, but note what you did. That number may become a variable later.

- Be conscious of decisions/assumptions.

- Ask, "What if?" What would happen if (pick a number or assumption) changed?

Modeling is open-ended and messy

It requires an open mind both from students and from their teachers and a willingness to explore, to fail and regroup, to revisit and improve, and to reflect.

Free Throws

In a basketball game near the start of the season, a particular player drove for the basket and was fouled. As the player approached the line, the announcer stated that he was making 78% of his free throws. The player proceeded to miss the first shot and make the second. Later in the game, the same player was fouled again. This time, the announcer stated that he was making 76% of his free throws.

How many free throws had the player attempted and how many he had made at this point in the season?

Define Variables:

Let x represent the number of attempts before the first foul shot of this game.

Let y represent the number of those shots that were successful.

Do the Math:

$$\frac{y}{x} = 0.78$$

$$\frac{y+1}{x+2} = 0.76$$

Solve . . .

$$x = 26$$

$$y = 20.28$$

Rounding gives:

$$x = 26$$

$$y = 20$$

But . . .

$$\frac{20}{26} = 0.7692$$

which does NOT round to 0.78.

Hmmmmm . . . Identify the Problem

“. . . to explore, to fail and regroup,
to revisit and improve, and to reflect”

not:

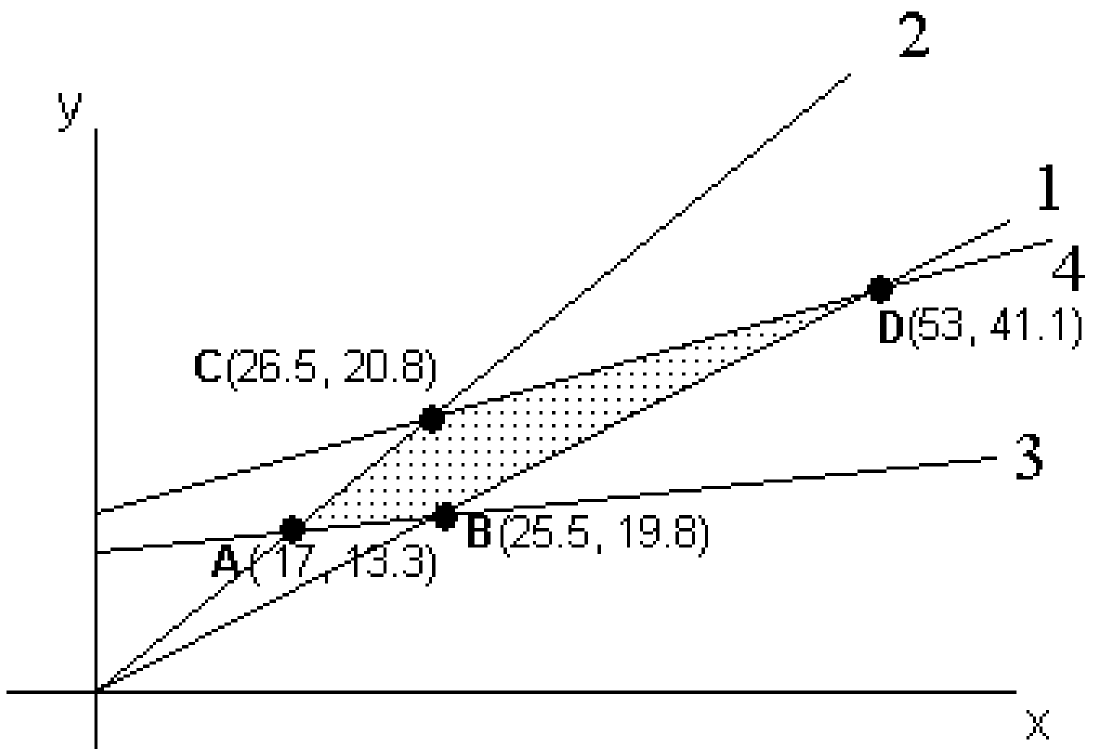
$$\frac{y}{x} = 0.78$$

$$\frac{y+1}{x+2} = 0.76$$

but:

$$0.775 \leq \frac{y}{x} \leq 0.785$$

$$0.755 \leq \frac{y+1}{x+2} \leq 0.765$$


y
x
1
2
3
4
C(26.5, 20.8)
D(53, 41.1)
B(25.5, 19.8)
A(17 , 13.3)

Modeling requires making genuine choices.

When multiple factors matter, how do you balance their influence?

Which Computer?

The school board has decided that every mathematics classroom will be able to purchase a computer for demonstrations in the classroom. Your teacher has asked for your help to determine which computer to buy. The class finds a *Consumer's Tips* column that rates the different computers from which the teacher can pick. The consumer guide rates the computers from 0 to 10 on Performance and Ease of Use. A score of (0, 0) is terrible performance and very difficult to use, while a score of (10, 10) is a perfect computer.

Devise a method for the school board to use to rank potential computers for purchase.

Computer	Performance	Ease of Use
A	6.4	8.5
B	7.3	7.5
C	9.3	3.8
D	8.8	6.0
E	7.3	6.0
F	5.5	9.7

Table 1: *Consumer's Tips* Ratings

Suggested criteria?

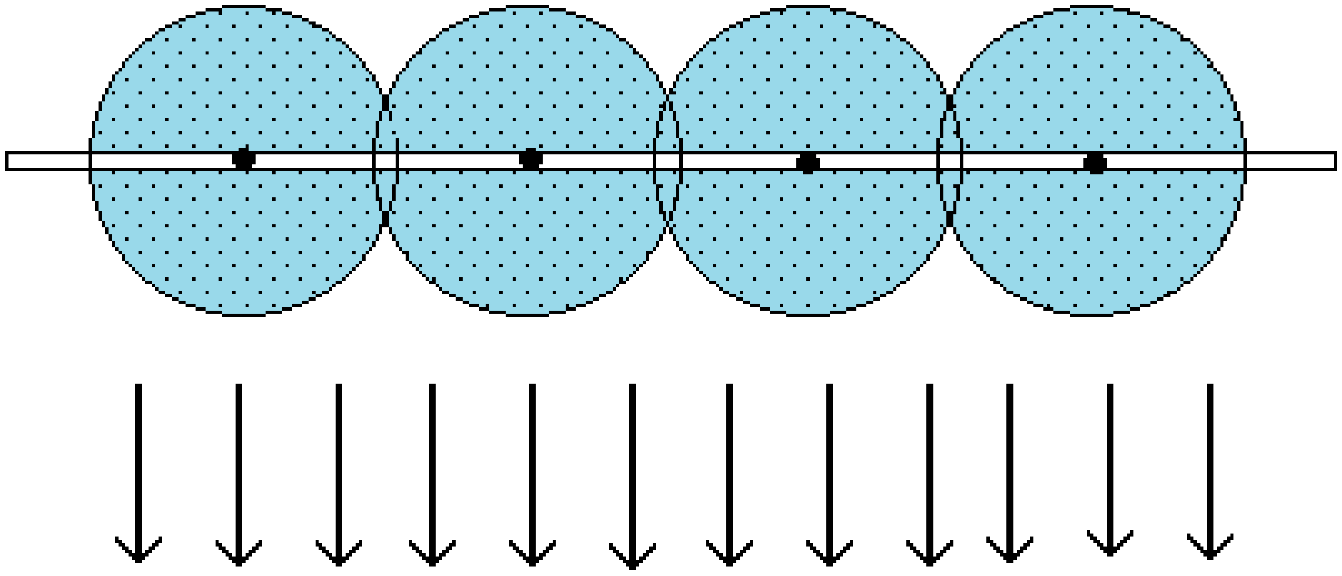
Alternate representations?

Modeling does not happen in isolation.

In the US Department of Labor document, *Soft Skills Pay the Bills*, the ability to work as a member of a team is listed third in the skills necessary in today's and tomorrow's job market.

The Irrigation Problem

A linear irrigation system consists of a long water pipe set on wheels that keep it above the level of the plants. Nozzles are placed along the pipe, and each nozzle sprays water in a circular region. The entire system moves slowly down the field at a constant speed, watering the plants beneath as it moves. You have 300 feet of pipe and 6 nozzles available. The nozzles deliver a relatively uniform spray to a circular region 50 feet in radius.



How far apart should the nozzles be placed to produce the most uniform distribution of water on a rectangular field 300 feet wide?

Make Assumptions and Define Variables

- definition of uniformity for one head
- spacing of sprinkler heads
- location of sprinkler head
- location of wetness measurement
- definition of wetness at selected location
- consideration of overlap
- measure of uniformity

Do the Math

- calculate wetness in terms of location
- calculate uniformity across field
- vary spacing and compare for solution

The Elevator Problem

source: *Solving Real Problems with Mathematics*, *Volume 2.* Walton and Davidson. The Spode Group.

Memo #1
From: Your Boss
To: You
Re: Late Arrivals

I have received numerous complaints that large numbers of our employees are reaching their offices well after 9:00 a.m. due to the inability of the present three elevators to cope with the rush at the start of the day. In the present financial situation it is impossible to consider installing any extra elevators or increasing the capacity of existing ones above the current ten persons. Please investigate and let me have some possible solutions to the problem with an indication of their various advantages and disadvantages.

Memo #2
From: You
To: Your Assistant
Re: Late Arrivals

Can you find out:

1. How long the elevators take to get between floors and how long they stop for?

2. How many people from each floor use the lift in the morning?

3. How many people were late this morning?

Memo #3
From: Your Assistant
To: You
Re: Answers to your questions

1. The elevators appear to take 5 seconds between each floor,
   an extra 15 seconds for each stop, and
   another 5 seconds if the doors have to reopen.

   It also seems to take about 25 seconds for the elevator to fill on the ground floor.

2. The number of workers on each floor are:

Floor	G	1st	2nd	3rd	4th	5th
Number	0	60	60	60	60	60

3. About 60 people were late today.

Memo #4
From: You
To: Your Boss
Re: Solution to the problem with advantages and disadvantages.

?

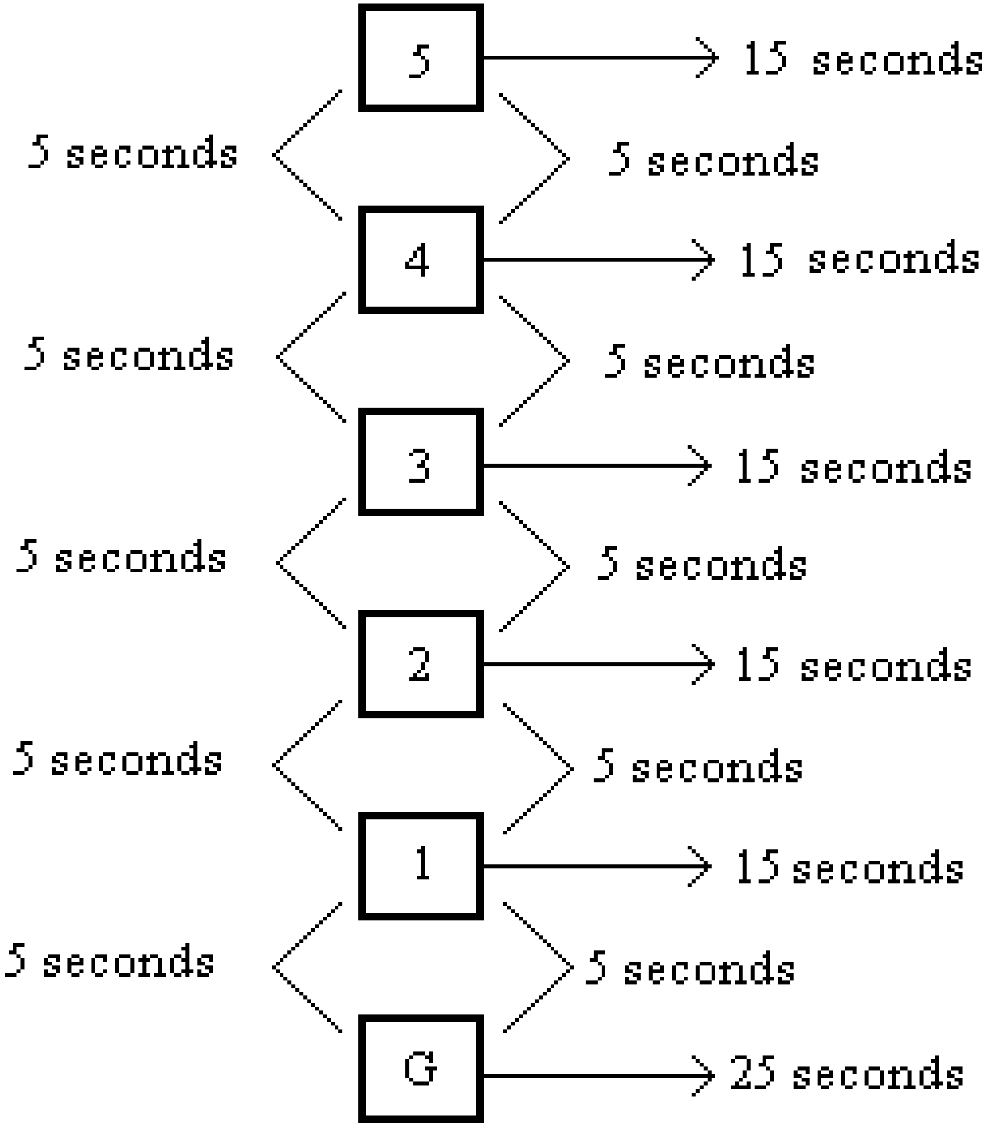
Assumption 1: We assume that at some time prior to 9:00, employees begin arriving, and, once the arrival process starts, there is a steady stream of employees waiting to take the elevators.

Assumption 2: We assume that the current situation is that each elevator carries employees to all floors, necessitating stops on each floor on each trip.

Assumption 3: Since the concern is getting employees to their floor efficiently, we assume that the only use of the elevators between 8:30 and 9:00 is in getting from the ground floor to the appropriate floor for their job.

Assumption 4: Elevator doors do not re-open.

Assumption 5: Every elevator is full with 10 workers getting on at the ground floor.


5
15 seconds
5 seconds
5 seconds
4
15 seconds
5 seconds
5 seconds
3
15 seconds
5 seconds
5 seconds
2
15 seconds
5 seconds
5 seconds
1
15 seconds
5 seconds
5 seconds
G
25 seconds

After spending several class periods on the problem, one Precalculus student commented,

"Never before, in all my math experiences, had I seen a problem as open ended and varying as this one. Working on a problem like this with no obvious answer and many different options was a wholly new experience for me. This problem helped me visualize the role math could and most likely will play in my future."   (emphasis added)

Most drivers have a “usual” region in which they do most of their driving. However, gas prices may vary widely so that gas may be substantially cheaper somewhere other than within that usual region. Would it be more economical to go to a station outside the usual region to buy gas? Thus, the general question we wish to address is, “How might we determine which gas station is the most cost-efficient?”

Suppose there is a station on your normal route that sells gas for $3.00 a gallon. A station 5 miles off your route sells gas for $2.85 a gallon. Should you travel the extra distance to buy gas at that station?


$3.00
A
HOME
SCHOOL
5 MI
B
$2.85

Station A is on your normal route and sells gas for $3.00 a gallon while Station B, which is 5 miles off your normal route, sells gas for $2.85 a gallon. Your car gets 30 mpg, and your friend’s car gets only 10 mpg. Should either of you drive to Station B for gas?

$$C^* = \frac{T \cdot P}{M \cdot T - 2D}$$

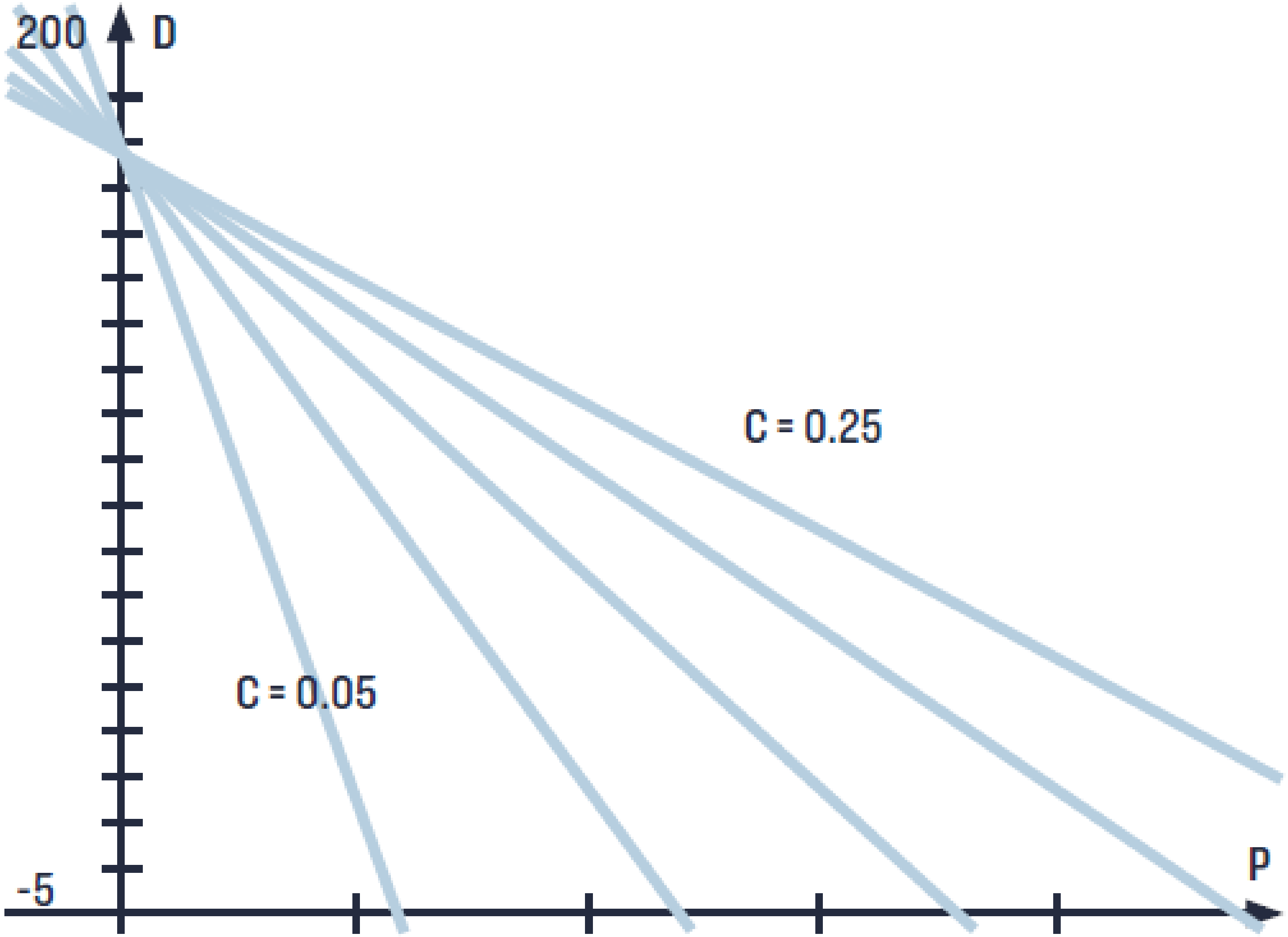
where,

T = tank size

P = price per gallon

M = mpg

D = distance to station


200
D
C = 0.25
C = 0.05
P
-5

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