Mathematics Teaching Practices: A Framework for Working with Future Teachers

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Agenda

- Context of the work
- Goal of the work
- The Mathematics Teaching Practices as a framework
- The framework in the methods class
- The framework in the student teaching experience
- Challenges to successfully implementing the framework
- Revisions/Future work

Context of the work

Indiana University of PA

- largest state-owned university in PA
- ~15,000 undergraduate and graduate students
- located in western PA, 70 miles east of Pittsburgh

Context of the work

Mathematics Department

- 28 tenure track faculty
  - ✓ 17 mathematicians
  - ✓ 5 statisticians
  - ✓ 6 mathematics educators

Context of the work

The students with whom Math Ed. Faculty work

- PreK-4/Special Education Majors
- Middle Level (4-8) Education Majors
- Secondary Mathematics Education Majors
- M.Ed. in Mathematics Education grad. students

Context of the work

The students who are the focus of this work

- Middle Level Mathematics Education Majors
  - ✓ *mathematics methods course
  - ✓ *student teaching supervision of those who specialize in mathematics

Goal of the work

Engage pre-service middle level mathematics teachers in learning about and applying the following:

The 'best practices' for teaching mathematics are a set of related practices that promote active engagement of students in problem solving.

The Methods Course

- Focus on teaching mathematics content **through** problem solving

(Van de Walle, 2014)

The Methods Course

- Content addressed (aligned with PA Core Standards for Mathematics at grades 4-8)
  - ✓ Whole number operations –mult. and div.
  - ✓ Rational numbers
    - Fractions and fraction operations
    - Decimals and percents
    - Integers and integer operations
  - ✓ Proportional reasoning
  - ✓ Algebraic reasoning
  - ✓ Geometry and measurement
  - ✓ Probability and statistics

From Principles to Actions

- *Specific, research-based teaching practices essential for a high-quality mathematics education for all students combined with core principles to build a successful mathematics program at all levels*. (NCTM, 2014)

- Includes: **eight research-based essential Mathematics Teaching Practices**

The Mathematics Teaching Practices

1) Establish Mathematics Goals to Focus Learning
2) Implement Tasks That Promote Reasoning and Problem Solving
3) Use and Connect Mathematical Representations
4) Facilitate Meaningful Mathematical Discourse
5) Pose Purposeful Questions
6) Build Procedural Fluency from Conceptual Understanding
7) Support Productive Struggle in Learning Mathematics
8) Elicit and Use Evidence of Student Thinking

The Teaching Practices chosen for this work

- Implement Tasks That Promote Reasoning and Problem Solving
- Establish Mathematics Goals to Focus Learning
- Pose Purposeful Questions
- Facilitate Meaningful Mathematical Discourse

From Teaching Practices to a Framework

1. Select 'problem-based' tasks that allow students to reason and problem solve.
2. Set goals for implementing the task
3. Implement the task through posing purposeful questions
4. Implement the task by facilitating meaningful mathematical discourse

1. Select problem-based tasks

- What are problem-based tasks?
- What does problem solving look like and sound like?
- Selecting tasks problem-based tasks

1. Select problem-based tasks

- What are problem-based tasks?
  - ✓ Solve and discuss the “Algebraic Expressions Task” and the “Counting Cubes Task*.”
  - ✓ Discuss the similarities and differences between the “Algebraic Expressions Task” and the “Counting Cubes Task*”.
  - ✓ Discuss opportunities to learn mathematics by engaging in each task.
  - ✓ Generate a list of characteristics of ‘problem-based tasks’

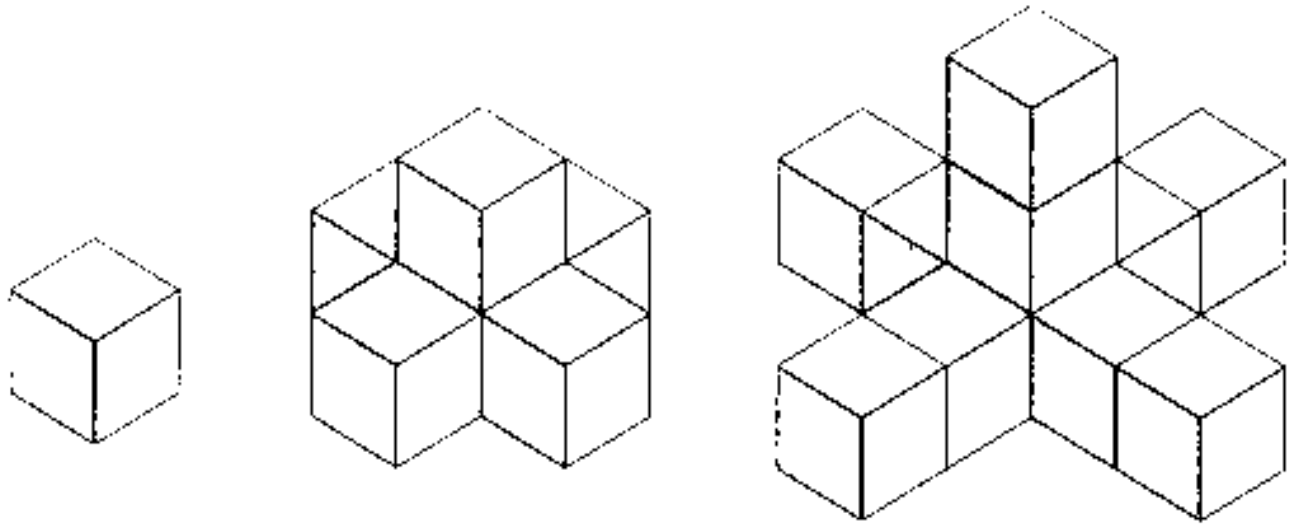
**Principles to Actions Toolkit*

Algebraic Expressions Task

The number of cubes for each step of a particular pattern can be represented by the expression "3n + 2".

- How many cubes would be in the first step?
- How many cubes would be in the 25th step?
- Which step of the pattern contains 92 cubes?

Counting Cubes Task



1. Describe a pattern you see in the cube buildings.
2. Use your pattern to write an expression for the number of cubes in the nth building.
3. Use your expression to find the number of cubes in the 5th building. Check your results by constructing the 5th building and counting the cubes.
4. Look for a different pattern in the buildings. Describe the pattern and use it to write a different expression for the number of cubes in the nth building.

Adapted from "Counting Cubes", Lappan, Fey, Fitzgerald, Friel, & Phillips (2004*). Connected MathematicsTM, Say it with symbols: Algebraic reasoning* [Teacher's Edition]. Glenview, IL: Pearson Prentice Hall.

Characteristics of Problem-Based Tasks

- Solution is not known in advance
- Have multiple entry points
- Require cognitive effort of all students
- Require justifications and explanations for answers and methods
- Encourage use of multiple approaches and strategies
- Encourage multiple representations
- Require a discussion of mathematics

(Principles to Actions, 2014) and (Van de Walle, et. al, 2014)

1. Select problem-based tasks

- What is problem solving? What does problem solving 'look' and 'sound' like?
  - ✓ Watch the video, "The Case of Peter Dubno,*"from the *Principles to Actions Toolkit*.
  - ✓ Discuss student actions and teacher actions that show or promote problem solving using evidence from the video and/or transcript*.

**Principles to Actions Toolkit*

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Principles to Actions Professional Learning Toolkit

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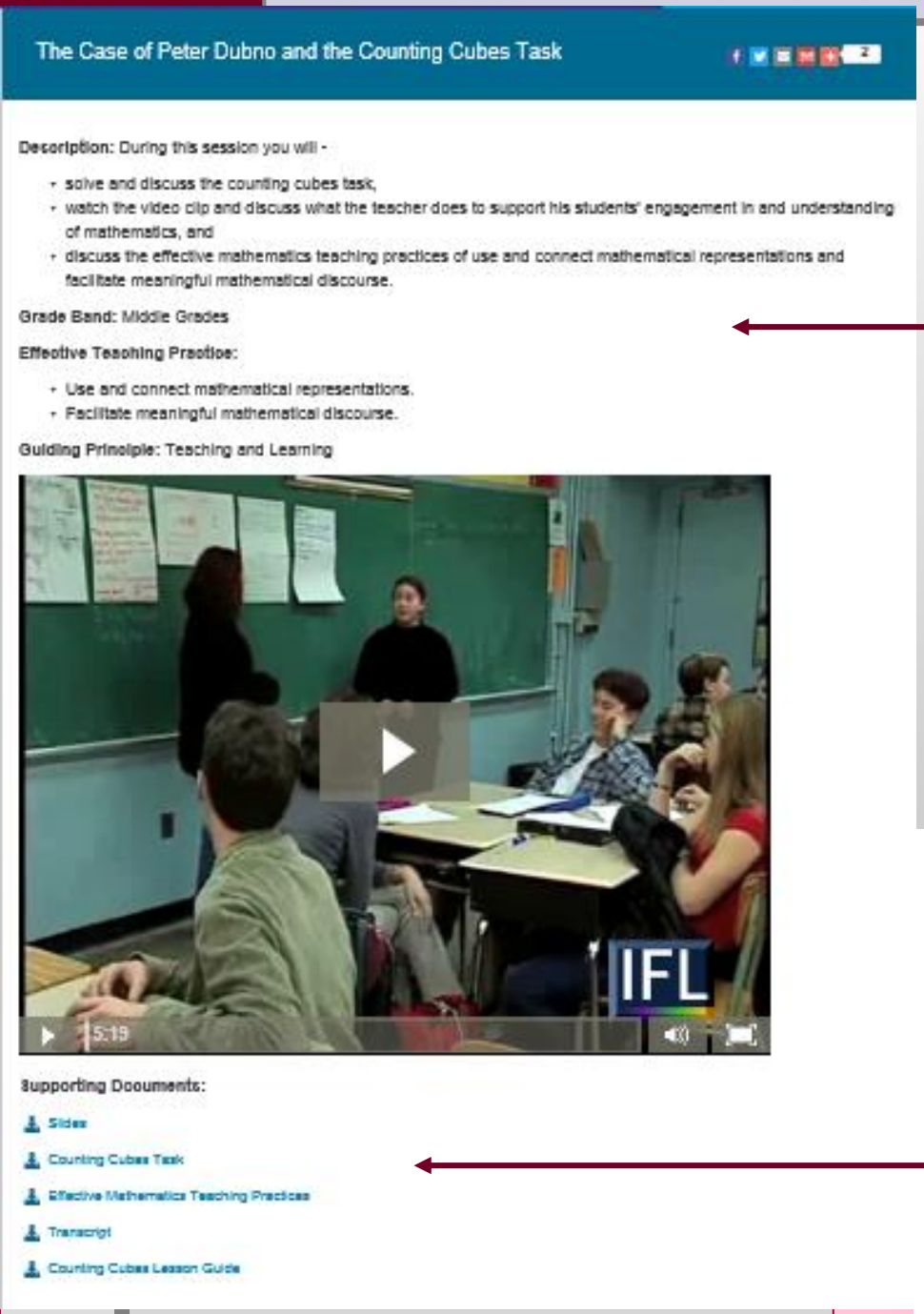
Mary Lo
Log out

These grade-band specific professional learning modules are focused on the Effective Teaching Practices and Guiding Principles from *Principles to Actions: Ensuring Mathematical Success for All*.

Presentation, presenter notes, and required materials are provided in each module to support professional learning with teachers through analyzing artifacts of teaching (e.g., mathematical tasks, narrative and video cases, student work samples, vignettes) and abstracting from the specific examples general ideas about how to effectively support student learning.

The Teaching and Learning Modules were developed in collaboration with the Institute for Learning (IFL) at the University of Pittsburgh.

Learning modules are available exclusively to NCTM members. Limited open examples are provided for each grade level and are denoted on modules marked **(EXAMPLE)**.



Description: During this session you will -

- solve and discuss the counting cubes task,
- watch the video clip and discuss what the teacher does to support his students' engagement in and understanding of mathematics, and
- discuss the effective mathematics teaching practices of use and connect mathematical representations and facilitate meaningful mathematical discourse.

Grade Band: Middle Grades

Effective Teaching Practice:

- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.

Guiding Principle: Teaching and Learning

Supporting Documents:

- Slides
- Counting Cubes Task
- Effective Mathematics Teaching Practices
- Transcript
- Counting Cubes Lesson Guide

1. Select problem-based tasks

Select a problem-based task for a fraction lesson and for the Problem-Based Tasks Project.

Topic	Resources	Rubrics
• Introduction to lesson planning (my way) • Select a fraction task and plan part 1 of a lesson • Problem-Based Task Project	• *Planning a Playground* task and lesson • "Thinking Through a Lesson," MTMS article • Van de Walle, et. al , Chapter 3	• Selecting and Setting Up the Task rubric • Problem-Based Task rubric

Lesson Plan Rubric 1 – Selecting and Setting Up the Task

Lesson Plan 1 Rubric

Criteria	Possible Points	Points Earned
HEADING	1	
RATIONALE AND BACKGROUND		
○ The task is problem based: ▪ The solution is not known in advance to students. ▪ The task has multiple entry points for students. ▪ The task can be solved in multiple ways and/or uses multiple representations ▪ The task requires justifications and explanations for answers and methods. ▪ The task requires discussion by students. ▪ The task requires cognitive effort and productive struggle by students.	3	

Selecting and Setting Up the Task

○ The task is problem based:
- The solution is not known in advance to students.
- The task has multiple entry points for students.
- The task can be solved in multiple ways and/or uses multiple representations
- The task requires justifications and explanations for answers and methods.
- The task requires discussion by students.
- The task requires cognitive effort by students.

2. Setting goals for implementing the task

- What are goals? What is the difference between goals and objectives?
  - Examine goals of “Counting Cubes” lesson
  - Discuss difference between goals and objectives.
  - Examine goals of “Planning a Playground.”
  - Write mathematical goals for fraction lesson (see Lesson Plan 1 rubric)

2. Setting goals for implementing the task

- Goals of "Counting Cubes" lesson

Expressions can be used to model linear relationships that occur in real-world or mathematical contexts. The term(s) in an expression can be related directly to the situation that is being modeled.

- Goals of "Planning a Playground."

Equivalent fractions can be used to make parts of fractions the same size. Addition and subtraction of fractions is the same as joining and separating parts of fractions with the same whole.

Lesson Plan Rubric 1 – Selecting and Setting Up the Task

- The goals:
  - Goal statement describes the mathematics concept students will understand more deeply as a result of the lesson. The goal is a statement ABOUT MATHEMATICS.
  - Goal is aligned with the standards and objectives.

RATIONAL		
○ The task is problem based: ▪ The solution is not known in a ▪ The task has multiple entry po ▪ The task can be solved in mul ▪ The task requires justificatio ▪ The task requires discussion b ▪ The task requires cognitive eff		
○ The goals: ▪ Goal statement describes the mathematics concept students will understand more deeply as a result of the lesson. The goal is a statement ABOUT MATHEMATICS. ▪ Goal is aligned with the standards and objectives.	2	
○ Prior knowledge necessary for students to engage in the task is described	1	
LESSON STANDARDS AND OBJECTIVES		
○ Objectives: ▪ Describe what the students will be doing during the lesson to work towards the goal ▪ Aligned with the goal of the lesson ▪ Aligned with the PA Standard, Assessment Anchor, and Eligible Content	1	
○ PA Core Standard (number and description), Assessment Anchor and Eligible Content (number and description) are listed.	1	
○ Standards for Mathematical Practice are listed and appropriate for the given task	1	
MATERIALS/RESOURCES/EXPECTATIONS		
○ Copy of problem-based task, with citation, is included	1	
○ Multiple solutions to the task are included, including potential incorrect solutions	3	
○ Description of how students will work is included.	1	

Selecting and Setting Up the Task

3. Implementing the task through posing purposeful questions:

- Review the Teaching Channel Video, “A Passion for Fractions” at www.teachingchannel.org.
- In the transcript, highlight the questions asked by teacher. Characterize the questions.
- Read and discuss “Questioning Our Pattern of Questioning” - MTMS Article
- Read and discuss the “Coin Circulation Task” case from *Principles to Actions.*
- Discuss opportunities to learn mathematics in each classroom.

3. Implementing the task through posing purposeful questions:

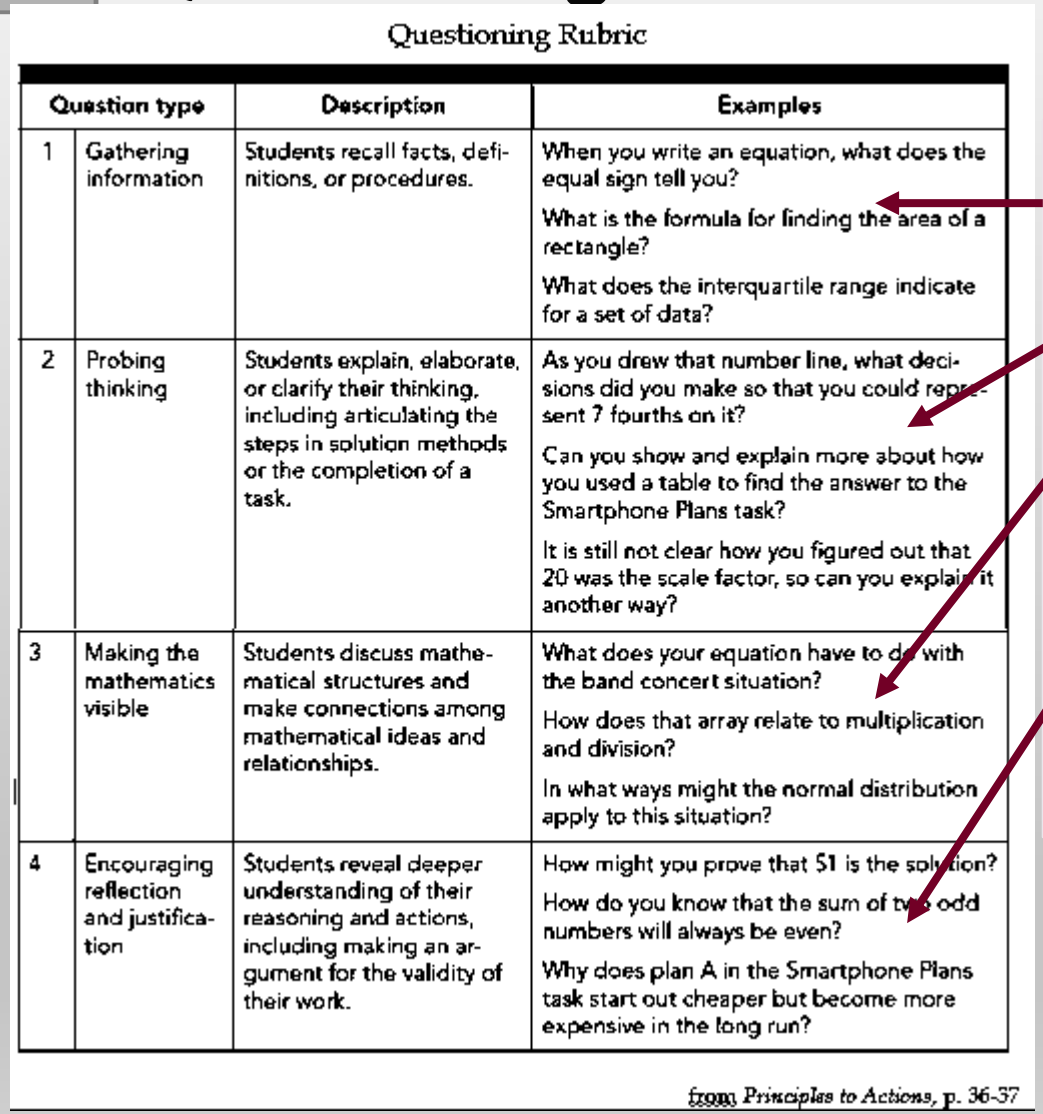
Plan and teach a lesson designed around a problem-based task with a focus on teacher questioning

Topic	Resources	Rubrics
• Select a geometry task and plan parts 1 and 2 of a lesson. Focus on including high level or 'focusing' questions. • Teach the lesson to classmates.	• *Principles to Actions* • "Thinking Through a Lesson," MTMS article • "Questioning our Pattern of Questioning" article • "Planning a Playground" Lesson Plan	• Supporting Students' Learning of Mathematics rubric • Questioning rubric

Lesson Plan Rubric 2 – Supporting Student's Learning of Mathematics

Supporting Students' Learning of Math	Lesson Body		
	○ Lesson relates to the stated goals and objectives	2	
	○ Describes in sequence the experiences that will occur during the lesson.	4	
	○ Includes errors, misconceptions, and struggles students may face as well as potential responses to students.	2	
	○ Lists potential questions to be asked during the lesson (the majority of which are high level or focusing questions) and anticipated student responses:	4	
	○ Majority of questions are focusing or high level	2	

Questioning Rubric



Questioning Rubric

Question type		Description	Examples
1	Gathering information	Students recall facts, definitions, or procedures.	When you write an equation, what does the equal sign tell you? What is the formula for finding the area of a rectangle? What does the interquartile range indicate for a set of data?
2	Probing thinking	Students explain, elaborate, or clarify their thinking, including articulating the steps in solution methods or the completion of a task.	As you drew that number line, what decisions did you make so that you could represent 7 fourths on it? Can you show and explain more about how you used a table to find the answer to the Smartphone Plans task? It is still not clear how you figured out that 20 was the scale factor, so can you explain it another way?
3	Making the mathematics visible	Students discuss mathematical structures and make connections among mathematical ideas and relationships.	What does your equation have to do with the band concert situation? How does that array relate to multiplication and division? In what ways might the normal distribution apply to this situation?
4	Encouraging reflection and justification	Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work.	How might you prove that $1 is the solution? How do you know that the sum of two odd numbers will always be even? Why does plan A in the Smartphone Plans task start out cheaper but become more expensive in the long run?

from *Principles to Actions*, p. 36-37

1. Gathering information
2. Probing thinking
3. Making the mathematics visible
4. Encouraging reflection and justification

4. Implementing the task through facilitating discourse

- Solve the "Candy Jar Task" from *Principles to Actions Toolkit*.
- Read "The Case of Mr. Donnelly" from *Principles to Actions Toolkit*.
- Discuss Mr. Donnelly's actions in implementing the task.
- Read and discuss "Orchestrating Discussions" from MTMS.
- Discuss the "Facilitating Discussion" rubric.

4. Implementing the task through facilitating discussion

Plan and teach a lesson designed around a problem-based task with a focus on discussing the mathematics

Topic	Resources	Rubrics
• Select a task and plan parts 1, 2, and 3 of a lesson. Focus on including "Facilitating Discourse." • Teach the lesson to classmates.	• *Principles to Actions* • "Orchestrating Discussion," MTMS article	• Summarizing and Assessing Understanding rubric • Facilitating Discourse rubric

Lesson Plan Rubric 3– Summarizing and Assessing Understanding

Summarizing and Assessing Understanding	○ Lists higher level questions to generate discussion and anticipated student responses.	4	
	○ Lists higher level questions to assess and advance students' understanding and anticipated students responses.	4	
	○ Majority of questions are focusing or high level.	3	
	○ Questions address the goals and objectives of the lesson.	4	
	TOTAL	15	

Facilitating Discourse Rubric

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

The Student Teaching Experience

- **7 weeks in mathematics gr. 6-8**
  - ✓ supervised by Math Ed. Faculty

- **7 weeks in gr. 4-5, any subject**
  - ✓ supervised by College of Ed. faculty OR adjunct faculty

The Student Teaching Experience

1. Set goals for implementing the task
2. Select 'problem-based' tasks that allow students to reason and problem solve.
3. Implement the task through posing purposeful questions.
4. Implement the task through facilitating meaningful mathematical discourse.

The Student Teaching Experience

1. **Set goals for implementing a mathematics task**
   - Lesson plan rubric
   - Conference
   - Reflection

The Student Teaching Experience

2. Select 'problem-based' tasks that allow students to reason and problem solve.

- Lesson plan rubric
- Problem based task rubric
- Conference
- Reflection

The Student Teaching Experience

3. Implement the task through posing purposeful questions

- Lesson plan rubric
- Observation
  - ✓ Script questions asked
  - ✓ Chart of whom questions are asked
- Questioning Rubric
- Conference
- Reflection

The Student Teaching Experience

3. Implement the task through facilitating meaningful mathematical discourse:

- Lesson plan rubric
- Observation
  - ✓ Script questions asked
  - ✓ Chart of whom questions are asked
- Facilitating discourse rubric
- Conference
- Reflection

Challenges to implementing the framework

- **State testing in Pennsylvania**
  - ✓ Tied to teacher evaluations
  - ✓ 2-3 weeks of lost instructional time for testing and "test prep" (depending on the school district)

Challenges to implementing the framework

- **Cooperating teacher/School district**
  - ✓ Teacher beliefs
    - "Our kids can't do that kind of thing."
    - "Kids in 'that class' can't do that kind of thing."
  - ✓ Time
    - "It takes too long to do those problems."
    - Some schools have classes of 40 minutes for mathematics, others have classes of 80 minutes.
  - ✓ Curriculum –
    - "It's not in our curriculum."
    - "It's not on the state test."

Challenges to implementing the framework

- **Disconnect with College of Ed. Requirements**
  - ✓ Rigor of lesson planning requirements
  - ✓ Rigor of student teaching requirements
  - ✓ WHEN the student teacher receives the math placement

Challenges to implementing the framework

- **Time in the methods class for students to teach a lesson**

Planned revisions to the work

- "Flipping" the methods class to allow more time for teaching the lesson
- More emphasis on setting goals
- More emphasis on facilitating discussion
- Revise lesson plan format

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