Why and How to let Students Struggle? Thoughts from Research

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Principles to Action: Ensuring Mathematical Success for All

(NCTM, 2014)

Principles to Action

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking.

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Productive Struggle

"Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships."

(NCTM, 2014, p. 48)

Agenda

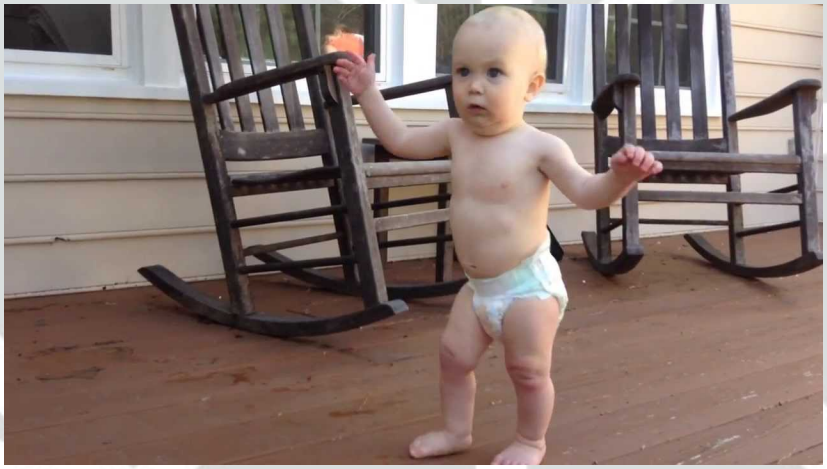
What is Productive Struggle?

Benefits of Students Productively Struggling

Ways of Supporting Productive Struggle

What is struggle?

Struggle should be less about frustration.



And more about trying or doing something new.

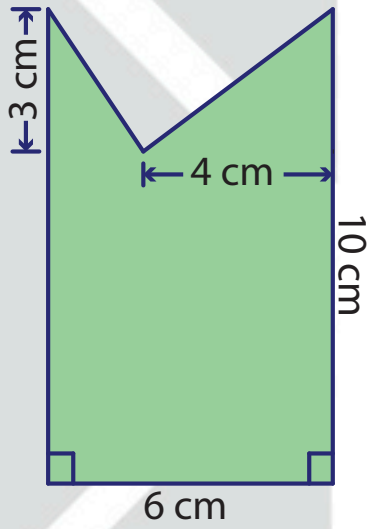
What is Struggle?

Struggle occurs when a student does a task or problem

- that initially seems beyond their capability.
- for which there is no clear path to a solution.
- that is challenging but is within the student's reach (Hiebert and Wearne, 2013).

Would this Task Generate Struggle?

Context: A class of middle school student who know how to find areas of rectangles and triangles.



The geometric shape at the left is made up of rectangles and triangles. Describe how you would find the total area of the shape and use that method to find the area.

What Makes Struggle Productive?

- Is the mathematics or task within reach?
- Is the mathematics central?
- Are the students employing sense-making?

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What Makes Struggle Productive?

- Is the mathematics or task within reach?

Within reach means "within the zone of proximal development" (Vygotsky, 1978)

  - Students may not be able to solve independently
  - Students can solve with assistance of peers or questions from teacher

What Makes Struggle Productive?

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What Makes Struggle Productive?

- Is the mathematics or task within reach?
- Is the mathematics central?

To be central, the mathematics with which the students are wrestling must be directly related to the mathematical goals of the lesson or unit?

Is this Struggle Productive?

In a middle school lesson on proportional reasoning, a student divides 7 by 12 and gets $0.508\overline{3}$ instead of $0.58\overline{3}$.

Is this struggle productive?

How do I support at this point?

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What Makes Struggle Productive?

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"students expend effort to make sense of mathematics, to figure something out that is not immediately apparent"

(Hiebert and Grouws, 2007, p. 387)

Unproductive Struggle – Some Indicators and Causes

Indicators

- High frustration
- Unwilling to persist

Causes

- Problem not within reach
- Norms – students not accustomed to cognitive work

Supporting in Moments of Frustration

What **NOT** to do

- Lower the cognitive demand
  - ✓ Task becomes routinized by specifying procedures
  - ✓ Shift from concepts and understanding to correctness or completeness

(Stein, Smith, Henningsen, & Silver, 2009)

- Step in to rescue
  - ✓ Lack of immediate success does NOT mean the teacher has failed.
  - ✓ Communicates to students that they are not capable.

Supporting in Moments of Frustration

What to do

- Pick tasks at the right level

(Vygotsky, 1978)

- Maintain high expectations

(Kilpatrick, Swafford, and Findell, 2001)

Benefits of Students Productively Struggling

- Sense of Accomplishment
- Knowledge and Understanding
- High Achievement
- Improved Achievement
- Mastery and Long-term Retention

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Sense of Accomplishment

"A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on mind and character for a lifetime"

(Polya 1945, p. 17)

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Knowledge and Understanding

Dewey (1929) claims that knowledge is the result of

- Answering a question
- Disposing of a difficulty
- Clearing up confusion
- Making coherence out of an inconsistency
- Mastering a perplexity

Knowledge and Understanding

Hiebert and Werne (2003) claim that Mathematical Understandings are

- developed by
- enriched by
- extended by

persistence in problem solving

(i.e. productive struggle)

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High Achievement

The TIMSS study examined videos of mathematics lessons among high achieving countries.

Stigler and Hiebert (2009) found that

- problem types varied but
- "the *engagement of students in active struggle with core mathematics concepts and procedures*" that was common among high achieving countries was not evident in the United States. (p. 34)

High Achievement

In 2007, Hiebert and Grouws did a review of research studies on how teaching affects learning.

They found two critical features

- Focus on important concepts.
- "the engagement of students in struggling or wrestling with important mathematical ideas" (p. 387)

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Improved Achievement

Fixed Mindset

- "that intelligence or math and science ability is simply a fixed trait"

Growth Mindset

- "abilities can be developed"
- See intelligence as "malleable"

Futility of Effort

Utility of Effort

(Blackwell, Trzesniewski, and Dweck 2007, p. 247; Dweck, 2008)

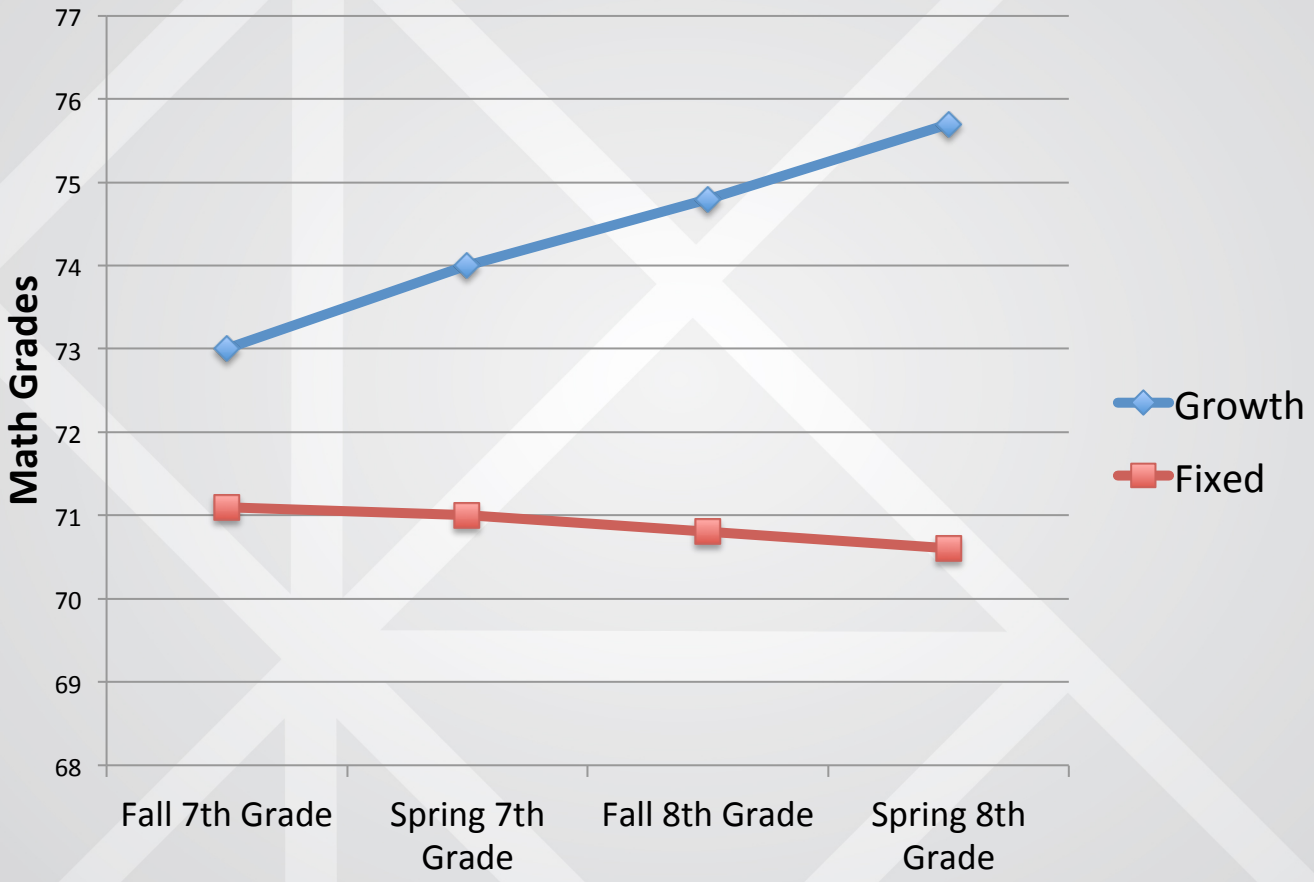
Improved Achievement

How I see the connections.

Supporting productive struggle facilitates a growth mindset.

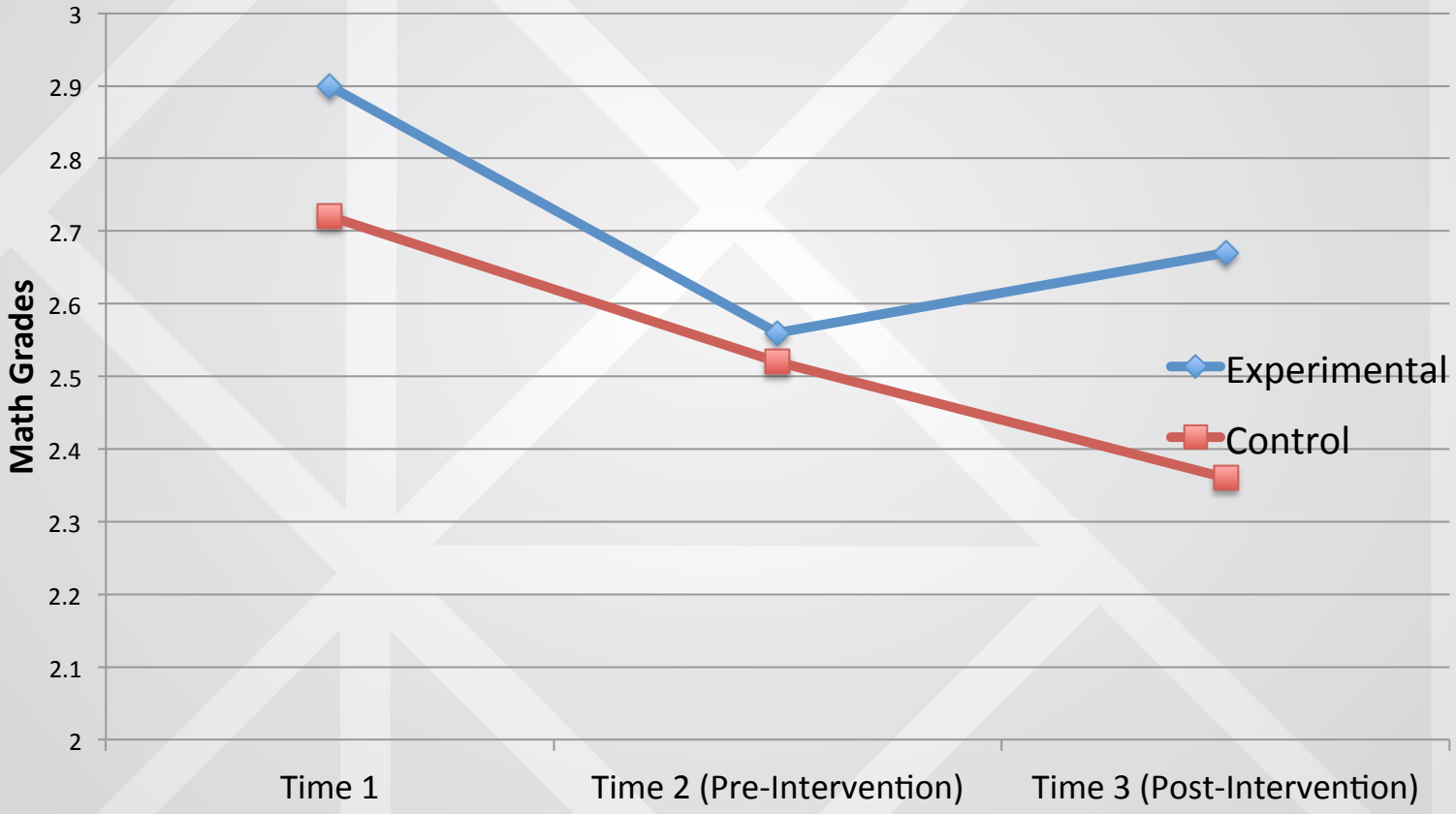
Growth mindset has been linked to improved achievement.

Improved Achievement



(Blackwell, Trzesniewski, and Dweck 2007, p. 251)

Improved Achievement



(Blackwell, Trzesniewski, and Dweck 2007, p. 257)

Improved (High) Achievement

The beliefs among students and their parents in the United States is that success in mathematics is based on natural ability – a fixed mindset.

(Stevenson and Stigler, 1992)

How many times have you heard a parent say, "I wasn't very good at math either" as justification for their child's struggles in math class?

Improved (High) Achievement

The beliefs among students and their parents in Asian countries (Japan, China, Taiwan) is that mathematical success is based on effort – a growth mindset.

(Stevenson and Stigler, 1992)

Asian parents more commonly say, my child needs to work harder if they are not performing well in a mathematics class.

Improved (High) Achievement

Teach students about growth mindset

Students will see utility of effort

Students will be more willing to productively struggle

Productively struggling will influence mathematical achievement

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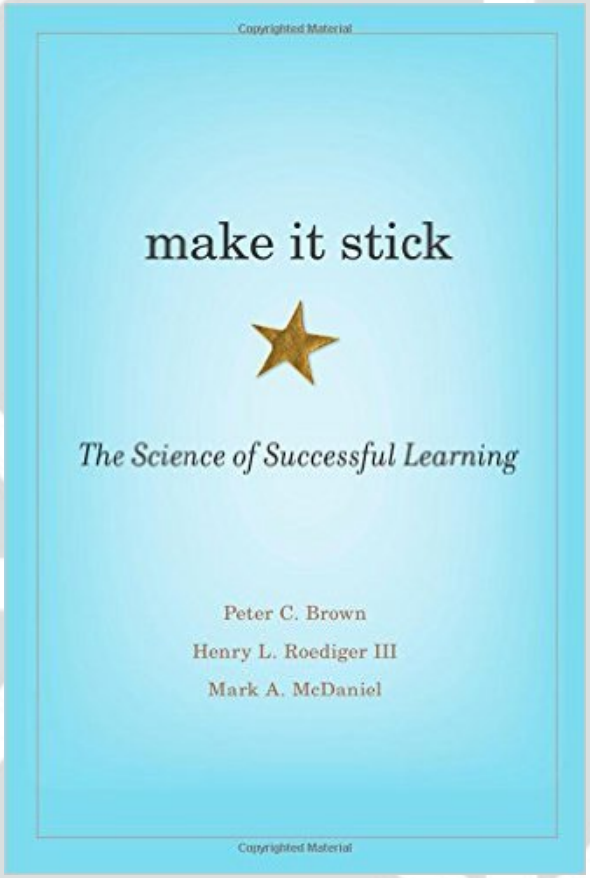
Mastery and Long-term Retention

The authors of Make It Stick contrast

- Massed practice
  - ✓ Solve many of the same type of problem in one problem set.
  - ✓ Struggle -> execution of the procedure
  - ✓ Deciding the procedure is not a struggle.

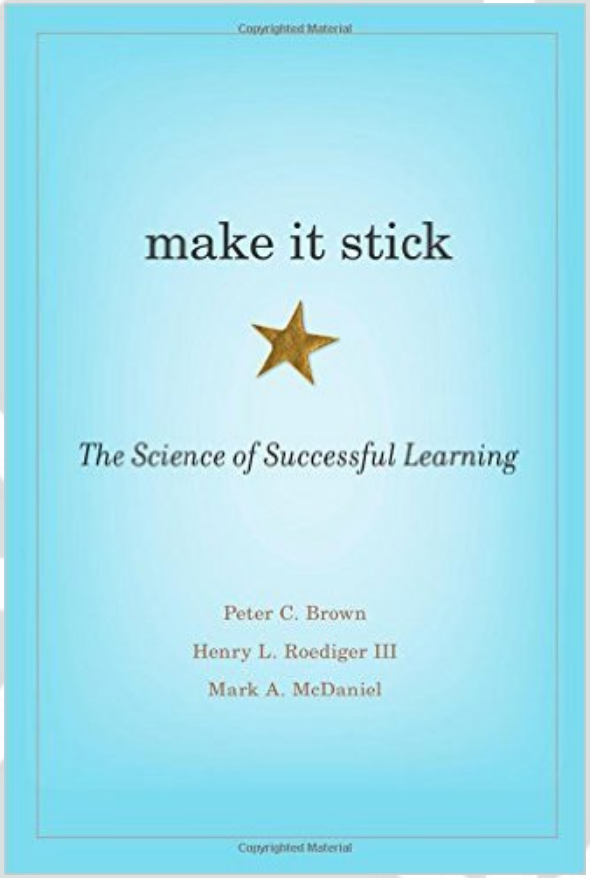
- Interleaved practice
  - ✓ Solve different types of problems in one problem set.
  - ✓ Struggle -> deciding which procedure to use



(Brown, Roediger, and McDaniel, 2014)

Mastery and Long-term Retention

Brown, Roediger and McDaniel claim that "research shows unequivocally that mastery and long-term retention are much better if you interleave practice than if you mass it."



(Brown, Roediger, and McDaniel, 2014, p. 50)

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Ways of Supporting Students in Productive Struggle

- Rethinking Teaching
- Establishing Norms – Explicitly Teach Students about Struggle
- Moving Forward with Supporting Productive Struggle

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Rethinking Teaching

Unproductive Beliefs	Productive Beliefs
When students are frustrated and do not experience success immediately it tells me that I have failed (Smith, M. 2000).	I need to help students understand that "confusion is something you go through, not a permanent state of being" (Carter 2008, p 136).
Students need direct demonstration in order to learn, so I need to provide clear explanations (Smith, J. 1996).	I need to give the students the opportunity to solve problems on their own and give them the sense of efficacy that comes with taking charge of their own learning (Polya 1945).
Parents believe that my job is to make doing mathematics easy for students because "a good teacher … helps students learn in a smooth and effortless way" (Hiebert and Wearne 2003, p. 6).	I need to challenge my students and allow them to struggle in order for them to learn mathematics deeply (Hiebert and Wearne 2003).

Rethinking Teaching

Unproductive Beliefs	Productive Beliefs
If I scaffold a task by breaking it down into small subtasks, students will learn to solve problems like the original task (Smith, M. 2000).	If I let me students struggle to figure out which strategy to use (Brown, Roediger, and McDaniel 2014) and how to break a task down, they will become confident in their approach to problems of all types.
Tests show which of my students are good at math and their potential (Dweck 2008).	A brain is like a muscle and only gets stronger when it is pushed to do difficult things. My students can, with effort, improve in mathematics. Tests do not define my students (Dweck 2008).
I shouldn't make my students struggle because they are afraid of challenges, making mistakes, and that it will reveal their intellectual limits (Dweck 2008).	My students will learn that failure isn't something to be avoided, it "is a manifestation of learning and exploration" (Catmull 2014, p. 109).

Rethinking Teaching

Unproductive Beliefs	Productive Beliefs
If my students struggle to solve problems, they will blame that teacher and parents will complain to the administrators.	Once my students are acclimated to the expectations of problem solving and productive struggle, they will appreciate the autonomy these struggles provide (Boaler 2002).
When my students are stuck, I need to step in and show them the steps so they can learn the procedure for getting answers.	Telling my students the answer does not help them learn; in fact, it stops their learning (Carter 2008).
If my students "are allowed to make errors, it's the errors that they will learn" (Brown, Roediger, and McDaniel 2014, p. 90)	If my students realize that errors are a natural part of learning, they can see mistakes as "turning points along the path to mastery" (Brown, Roediger, and McDaniel 2014, p. 91)

Ways of Supporting Students in Productive Struggle

- Rethinking Teaching
- Establishing Norms – Explicitly Teach Students about Struggle
- Moving Forward with Supporting Productive Struggle

Establishing Norms – Explicitly Teach Students about Struggle

- Mistakes are sites for learning
- Persistence has value
- Intelligence is malleable – a growth mindset
- Problem solving is a process

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Mistakes are Sites for Learning

- A common belief is that "failure is bad; failure means you didn't study or prepare; failure means you slacked off or – worse! – aren't smart enough to begin with. Thus, failure is something to be ashamed of."

(Catmull 2014, p. 108)

- Rather mistakes should be seen as "a manifestation of learning and exploration. If you aren't experiencing failure, then you are making a far worse mistake: You are being driven by desire to avoid it"

(Catmull 2014, p. 109)

Mistakes are Sites for Learning

From mistakes and “struggle does clarity emerge.”

(Catmull 2014, p. 152)

Establishing Norms – Explicitly Teach Students about Struggle

- Mistakes are sites for learning
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Persistence Has Value

"That which we persist in doing becomes easier – not that the nature of the thing is changed, but the power to do is increased."

Ralph Waldo Emmerson

"Achievement consists of never giving up… If there is no dark and dogged will, there will be no shining accomplishment; if there is no dull and determined effort, there will be no brilliant achievement"

(Confucious as quoted in Stevenson and Stigler 1992, p. 97)

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Intelligence is Malleable – A Growth Mindset

"genius often appears to be developed over time through focused, extended effort"

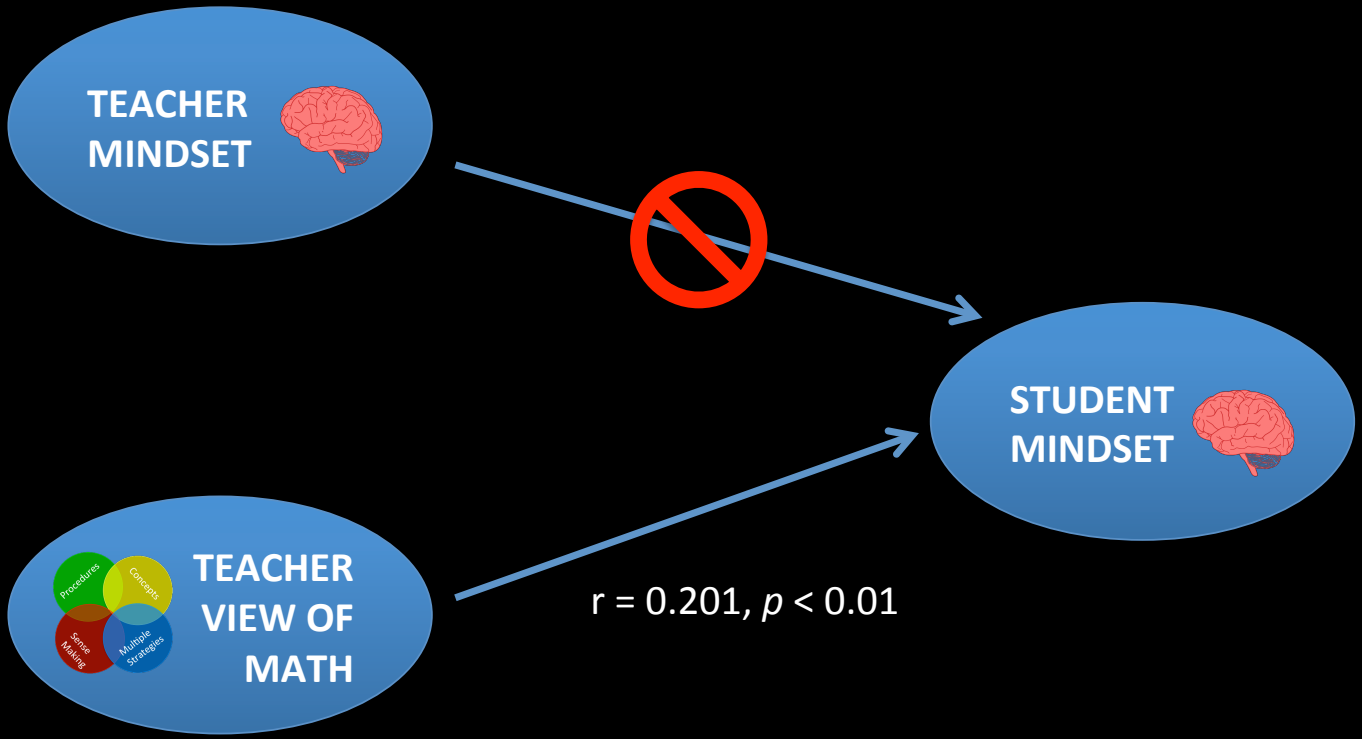
(Dweck 2008, p. 3)

The Beatles, who are considered musical geniuses, had an estimated 1200 live performances before their explosive success in 1964.

Bill Gates, who is considered a computer genius, had programmed for thousands of hours before he started his first computer software company in his early twenties.

(Gladwell, 2008)

What teacher beliefs predicts student mindset?



Kathy Sun, Santa Clara University – NCTM 2016 Presentation

Belief #2: Teacher Views of Math

One-dimensional
(Skemp, 1978; Stein et al., 1996)

Procedures

Memorization

Fact

Rules

Closed

Multi-dimensional
(Stein et al., 1996; Boaler, 2006)

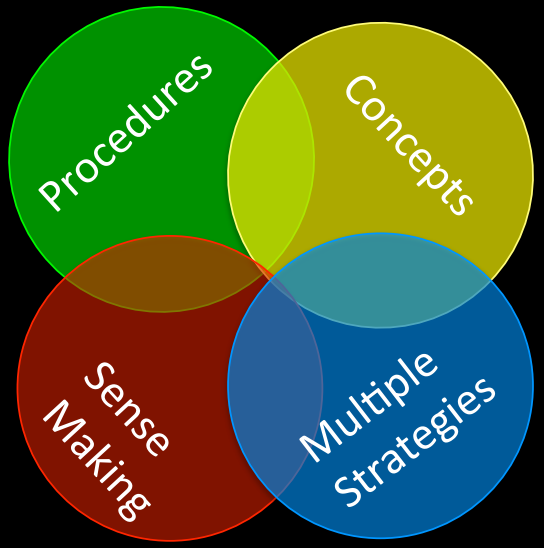
Procedures with Connections

Conceptual Understanding

Sense Making

Multiple Strategies

Open



Intelligence is Malleable – A Growth Mindset

Instead of Praising the Person	Try Praising the Process
Great Job! You must be smart with dividing fractions.	Great job! You must have worked really hard to solve those fraction division problems.
See, you *are* good with similar triangles. You got an A on your last test.	You really studied for your similar triangles test and your improvement shows it.
You got it! I told you that you were smart.	I like the way you tried all kinds of strategies on that problem until you finally got it.
You are such a good student!	I love how you stayed engaged with the problem and kept your concentration. That's great!

Establishing Norms – Explicitly Teach Students about Struggle

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Problem Solving is a Process

If students have a way of talking about the problems solving process, they are better able to articulate where and how they are stuck. Thus, enabling you to better support them.

(Carter, 2008: Warshauer, 2011)

Problem Solving is a Process

Polya's Four Steps of Problem Solving

- Understand the problem
- Devise a plan
- Carry out the plan
- Look back

(Polya, 1945)

Ways of Supporting Students in Productive Struggle

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Moving Forward with Supporting Productive Struggle

Where are we?

- Teachers and students are reoriented in their approaches to situations of struggle
- Teachers have selected complex tasks that
  - ✓ are within the reach of students,
  - ✓ are focused on central mathematics,
  - ✓ encourage sense-making
- What is the appropriate amount of scaffolding?

Moving Forward with Supporting Productive Struggle

We can frame our scaffolding using Polya's four-steps of problem solving.

- Understand the problem
  - ✓ What is the problem asking you to do?
  - ✓ Are there any terms in the problem with which you are unfamiliar?
  - ✓ Can you state the problem in your own words?

Moving Forward with Supporting Productive Struggle

- Devise a plan
  - ✓ What strategies have you considered? (e.g. solve a simpler problem, guess and test, look for a pattern, or draw a picture (Polya 1945))
  - ✓ What are the pros and cons of that strategy?
  - ✓ What are the attributes of the problem that suggested you should use the strategy you selected?

Moving Forward with Supporting Productive Struggle

- Carry out the plan
  - ✓ How did you implement the strategy you chose
  - ✓ What problems did you run into when you tried that strategy?
- Look Back
  - ✓ Does the solution make sense?
  - ✓ Do you think there is a more efficient way to solve this problem?
  - ✓ How is this problem's solution or strategy related to other problems you have solved?

Summary

What is Productive Struggle?

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  - ✓ Mistakes are sites for learning
  - ✓ Persistence has value
  - ✓ Intelligence is malleable – a growth mindset
  - ✓ Problem solving is a process
- Moving Forward with Supporting Productive Struggle
  - ✓ Scaffold using Polya's Four Steps

Questions?

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