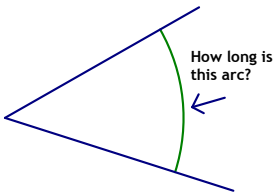
Exploring Length, Area, and Volume: Three Sequenced Hands-On Geometry Activities

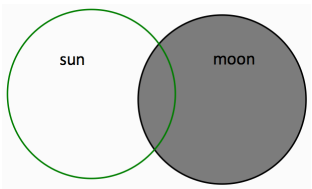
NCTM 2016 Annual Meeting
Thursday, April 14, 2016
1:00-2:15 PM
304 Moscone Convention Center
San Francisco, CA 94103

Patricia Baggett
Dept. of Mathematical Sciences
New Mexico State University
Las Cruces, NM 88003-8001
baggett@nmsu.edu

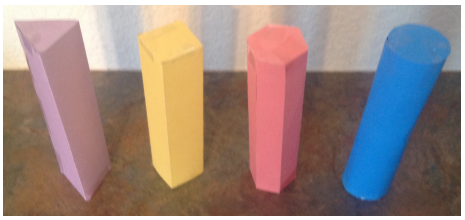
Jacqueline Lopez
Lynn Middle School
950 S. Walnut Street
Las Cruces, NM 88001
mailto:jacqlope@msn.com



Task 1
Can you find the length of this arc?



Task 2
In this eclipse, what percentage of the sun is covered by the moon?



Task 3
Can you construct 3 prisms and a cylinder, all with the same height and same base perimeter, and order them according to their volumes?

Can you measure the length of an arc, compute the percentage of the sun's disk covered by the moon in an eclipse, compare volumes of cylindrical boxes and prisms with the same heights and base perimeters? Take these mathematically linked classroom-tested tasks (and artifacts) back to your students!

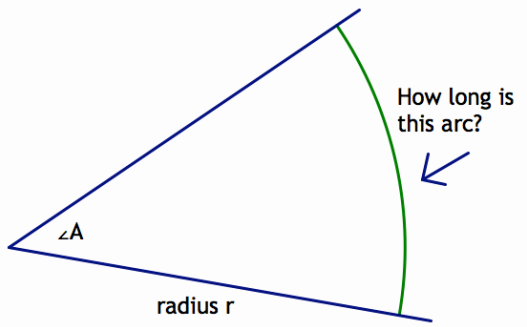
Table of Contents

Supplies you'll need:
calculator, ruler, protractor, wikki stix, compass, scissors, tape, pencil, arcs to measure (in the first unit), diagrams of eclipses of the sun

A summary of the tasks is below. For the full handout please email baggett@nmsu.edu.

First task: Measuring the length of an arc

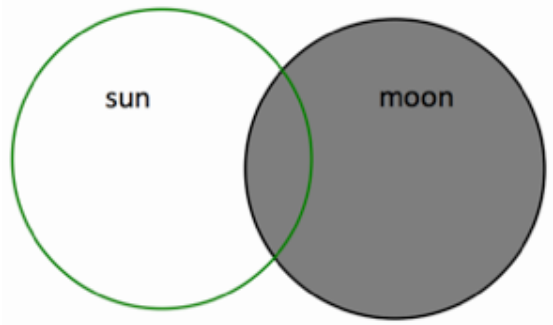
An arc is a part of the circumference of a circle. Suppose we want to find its length.



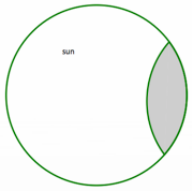
We show two ways.

1. Let A be the angle from the center of the circle to the two ends of the arc, and let r be the radius of the circle. Then the arc length is $\frac{\angle A^\circ}{360^\circ} * 2\pi r$.
2. You can measure the length using Wikki Stix!

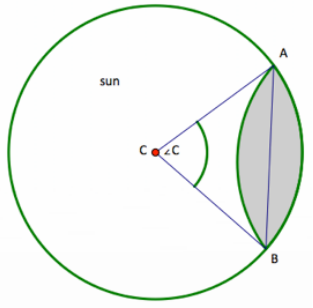
Second task: Eclipse of the sun



Our task is to find the percentage of the sun that is covered by the moon. (The disks of the moon and the sun on the sky appear to be the same size, so we will assume this in this task.) One way to look at the problem is, What percentage of the whole circle (the sun) is the grey area shown below?



Let's examine it like this:
We find the area of
the grey part, and
divide it by the area of
the whole circle. Then we
multiply the answer
by 100 to get the percentage
of the sun that is covered.



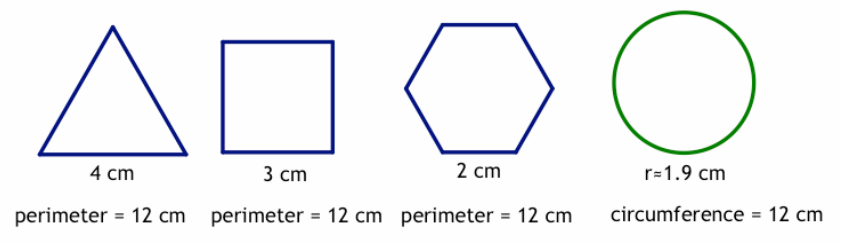
We show the solution step-by-step in the session and practice it on several diagrams of eclipses.

Third task: Constructing prisms (AKA Boxes for chocolate)
and finding their volumes (and surface areas)
(A synopsis)

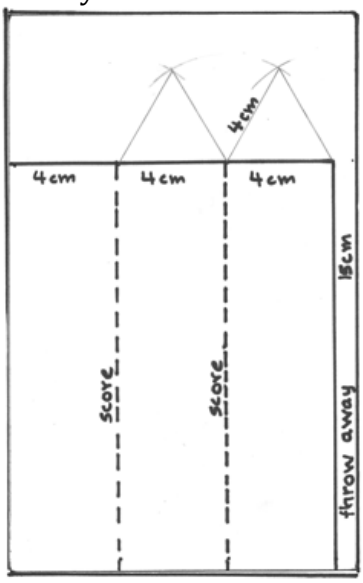
1. Design and construct four boxes. All boxes are 15 cm long, and they all have the same base perimeter of 12 cm, but the cross sections vary: equilateral triangle, square, regular hexagon, and cylinder.
2. Compare the volumes and surface areas of all four boxes. You may fill out the chart below when all computations are made.

container	area of base	lateral surf. area	total surf. area	volume	volumes in order
triangular prism		180 cm^2			
square prism		180 cm^2			
hexagonal prism		180 cm^2			
cylinder		180 cm^2			

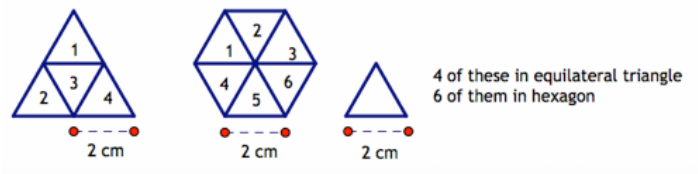
Plans for nets for all four boxes are in the handout. Here are diagrams of the tops (and bottoms) of the prisms. Notice that all the bases have the same perimeter, 12 cm.



Example: a net for the triangular prism on an 8" by 5" card



We find the area of the equilateral triangles using Heron's formula, and we notice that the area of the base of the hexagonal prism is 50% bigger than the base of the triangular prism.



Step-by-step we fill out the table above. Finally, if your students still doubt all the mathematical computations, they can check the volumes using rice. Pour it into the containers, and then into graduated cylinders!