Providing Students with the Power to Prove

Mark A. Creager Indiana University macreage@Indiana.edu
Mike Daiga Indiana University mdaiga@Indiana.edu

Outline

1. Discuss what is proof and why it is important for instruction.
2. Cover some of the key findings about students understanding of proof
3. Work on some proving activities we have done with our students and
    1. Describe what makes the activities helpful
    2. What aspects of our work have helped shape our students understanding of proof

Why proof?

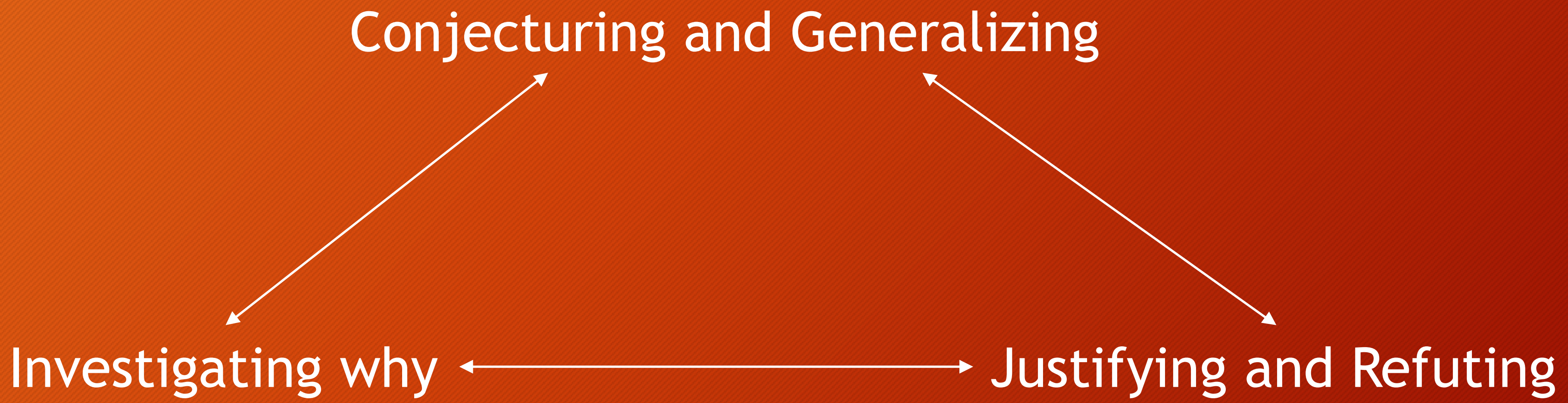
- "By the end of secondary school, students should be able to understand and produce mathematical proofs … and should appreciate the value of such arguments" (NCTM, 2000, p. 56).
- Proof is a way to establish truth, build knowledge, illuminate and synthesize relationships.
- Proof can be an great way to differentiate for high-achieving students.
- Proof is primary feature of assessment at the collegiate level.

What is proof?

- It is probably easier to describe what is not a proof.
- A typical response from pre-service elementary teachers that we work with is that proof is the two-column things they did in geometry.

Proof is a special form of argument that results from reasoning

A Model of Mathematical Reasoning



(Lannin, Ellis, & Elliott, 2011)

Proof in Curricula – What are the issues with this?

12. **Developing Proof** Complete the following proof by filling in the blanks. See Proble

Given: $\angle 1 \cong \angle 3$

Prove: $\angle 6 \cong \angle 4$

Statements	Reasons
1) $\angle 1 \cong \angle 3$	1) Given
2) $\angle 3 \cong \angle 6$	2) a. ?
3) b. ?	3) Transitive Property of Congruence
4) $\angle 1 \cong \angle 4$	4) c. ?
5) $\angle 6 \cong \angle 4$	5) d. ?

- It only allows for 1 portion of mathematical reasoning (verifying & refuting) and it really only allows for 1 part of that.
- It assumes students understand and appreciate proof.

Research on students' understanding of proof

1. Many students accept examples as verification
2. Many students do not accept deductive proofs as verification
3. Many students do not accept counterexamples as refutation
4. Students accept flawed deductive proofs
5. Many students accept arguments on bases other than logical coherence
6. Most students cannot write correct proofs.

(Reid & Knipping, 2010)

Framework for Proof Development

- Level 0: No justification given
- Level 1: An appeal to external authority or rote procedures
- Level 2: Naïve reasoning, usually with incorrect conclusions
- Level 3: Inductive reasoning (examples, experiments or empirical demonstrations)
- Level 3.5: Inductive reasoning (use of generic, extreme, or strategic examples)
- Level 4: Transition to formal reasoning (elements of formal reasoning but without precision)
- Level 5 Formal reasoning (acceptable to a mathematician)

(Simon & Blume, 1996; Quinn, 2009)

Activity 1: Finding Area on a Coordinate Plane (from Driscoll, 2007)

- Two vertices of a triangle are located at (0, 6) and (0, 12). The area of the triangle is 12 square units. What are all possible positions for the third vertex?

Activity 2: Geometric Conjectures

The statements below are examples of conjectures our students have created while exploring geometry. They may or may not be true. Decide if they are true. If you think they are, try to prove them. If you they are false, where did the student go wrong? Can the conjecture be saved?

1. At least one of the diagonals of a quadrilateral cut the area in half
2. A median of a triangle creates two smaller triangles with equal areas.

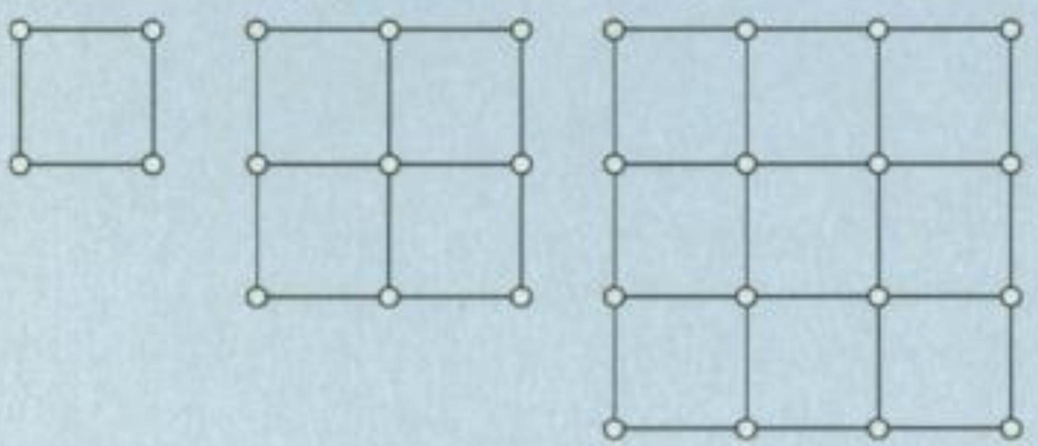
Activity 3: Equilateral Triangles (from Ellis, Beida, & Knuth, 2012)

- The ratio of the areas of an equilateral triangle and the triangle created by the midsegments of the equilateral triangle is ¼. What happens if we use a point that is different from midpoint? For example what if the point is 1/3 or ¼ of the distance, what happens to the ratio of the areas of the two triangles? What if the point is 1/n?

Activity 4:

Question: How many toothpicks does each drawing need?

(a) 1×1 (b) 2×2 (c) 3×3 (d) 4×4 (e) 10×10



(a) (b) (c)

Source: Adapted from Heid (1995, p. 145)

Is there a general rule that will give you the number of toothpicks if you know the dimensions of the square?

Can you prove that conjecture?

Aspects of Tasks that are Useful for Proof

Content Aspects

- The truth of the claim is not obvious
- Relevant content is required in a proof
- Different contents are connected

Motivational Aspects

- They are student generated

Aspects of Instruction that are Useful for Proof

- Emphasize exploration and explanation.
  - They're different things.
  - Encourage informal reasoning and technology
  - Emphasize "why is true true" versus "show that is true"
- Group work is a must!
- Help students see the big picture versus see the next step (proof outlines).
- Re-examine proofs to add sophistication.
- Have students create their own criteria for proof - "proof rubrics".

Resources

Driscoll, M. (2007). *Fostering geometric thinking: A guide for teachers, grades 5-10*. Heinemann: Portsmouth, NH.

Ellis, A. B., Beida, K., & Knuth, E. J. (2012). *Developing essential understanding of proof and proving grades 9-12*. National Council of Teachers of Mathematics: Reston, VA.

Heid, K. M. (1996). *Curriculum and evaluation standards for school mathematics: Addenda series grades 9-12: Algebra in a technological world*. NCTM: Reston, VA.

Lannin, J., Ellis, A. B., Elliott, R. (2011). *Developing essential understanding of mathematical reasoning for teaching mathematics prekindergarten-grade 8*. National Council of Teachers of Mathematics: Reston, VA.

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Reid, D. A., & Knipping, C. (2010). *Proof in Mathematics education: research, learning, and teaching*. Sense Publishers: Rotterdam, Netherlands.

Simon, M., & Blume, G. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior, 15*, pp. 3-31.

Quinn, A. K. (2009). Count on number theory to inspire proof. *The Mathematics Teacher, 103*(4), pp. 298-304.

Questions?

Thank you!

Our presentation can be found be downloaded from the conference site