

Magical Makeover! Strategies for Content Rigor, Relevance, Richness

Burst Session 136

Thursday April 14 11:30 – 12:00

Golden Gate C2, Marriott

@normabgordon @CueThink

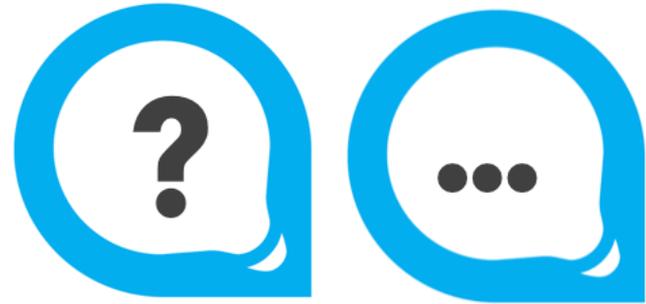
2016 NCTM ANNUAL MEETING
& EXPOSITION
April 13-16 • San Francisco

Building a Bridge
to Student Success

$$f(x) = a(x-h)^2 + k$$

BIG QUESTION

What raises the rigor, relevance and richness of application problems in math classrooms?



JARGON

Rigorous, relevant and rich content (or math problems)

(<http://www.sciencegeek.net/lingo.html>) →

“We will aggregate hands-on problem-solving across cognitive and affective domains.”

“We will deploy intuitive systems in authentic, real-world scenarios.”

“We will deliver innovative enduring understandings via self-reflection.”

NOT JARGON

Multiple entry points

Multiple pathways

Opportunity for conflict, argument, critiquing

“constructive controversy” (@ddmeyer, #NCSM16)

WHEN LEARNING IS HAPPENING STUDENTS ARE...

Questioning

Listening

Discussing

Justifying

Reasoning

Visualizing

Imagining

Reflecting

Analyzing

Experimenting

Testing

Evaluating

Demonstrating

Organizing

Assessing

Relating

Remembering

Reviewing

Practicing

Explaining

Inventing

Hypothesizing

Articulating

Applying

Persisting

Concentrating

Watching

Risking

Engaging

Puzzling

Collaborating

Patterning

Checking

Refining

Proving

Predicting

Selection of the most popular responses by more than 3000 primary and secondary school teachers

Source: http://bit.ly/NCETM_MathsMattersReport

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01 Make sense & persevere



02 Reason abstractly & quantitatively



03 Construct arguments & critique



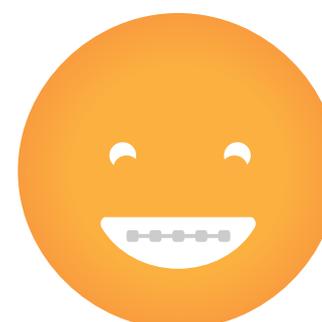
04 Model with mathematics



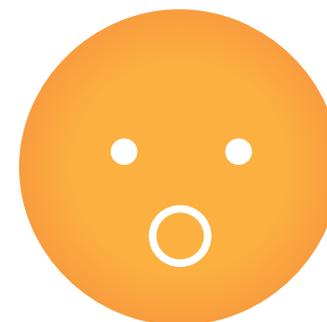
05 Use appropriate tools strategically



06 Attend to precision



07 Look for and make use of structure



08 Use repeated reasoning

MATHEMATICS TEACHING PRACTICES

FOCUS

Establish mathematics goals to focus learning

RIGOR

Implement tasks that promote reasoning and problem solving

MODEL

Use and connect mathematical representations

DIALOG

Facilitate meaningful mathematical discourse

INQUIRY

Present engaging, thought-provoking tasks

RETENTION

Build procedural fluency from conceptual understanding

EMPOWERMENT

Support productive struggle in learning

INFORM

Elicit and use evidence of student thinking



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NCTM EDITORIAL

Learning experiences that involve rigor ...	Experiences that do not involve rigor ...
challenge students	are more “difficult,” with no purpose (for example, adding 7ths and 15ths without a real context)
require effort and tenacity by students	require minimal effort
focus on quality (rich tasks)	focus on quantity (more pages to do)
include entry points and extensions for all students	are offered only to gifted students
are not always tidy, and can have multiple paths to possible solutions	are scripted, with a neat path to a solution
provide connections among mathematical ideas	do not connect to other mathematical ideas
contain rich mathematics that is relevant to students	contain routine procedures with little relevance
develop strategic and flexible thinking	follow a rote procedure
encourage reasoning and sense making	require memorization of rules and procedures without understanding
expect students to be actively involved in their own learning	often involve teachers doing the work while students watch

ACHIEVE THE CORE: RIGOR

In major topics pursue: conceptual understanding, procedural skill and fluency, and application with equal intensity.

Conceptual understanding

Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

Application

Students use math flexibly for applications in problem-solving contexts.

FOCUS LENS FOR THIS BURST



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REFLECTION QUESTIONS

Does the problem

- have important useful mathematics embedded in it?
- foster higher level thinking, problem solving and mathematical reasoning?
- invite a productive struggle using different or multiple strategies and representations?
- encourage students to consider “what if?”
- provide avenues for tiered modifications without compromising the mathematical learning objective(s)?

CUETHINK CRITERIA

- Provides an opportunity to articulate routine and non-routine problem solving processes; in many instances the actual “question” could be open-ended allowing for multiple solutions.
- Provides opportunity for extension or modifications for accessibility without compromising the mathematical learning objective.
- Is relevant and interesting to students.
- Is aligned to the Common Core State Standards.

MAKEOVER PRACTICE

BEFORE

Cleo wants to make guacamole dip for a party she is going to. Mike will be at the party too. Avocados (needed to make guacamole) are on sale for \$0.85 and Cleo has \$11.25. How many avocados will she be able to buy?

EMPOWERMENT – create opportunity for all to engage

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AFTER

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AFTER

Cleo needs avocados to make guacamole for a party. Avocados are on sale. How many avocados can she buy?

RIGOR and INQUIRY

AFTER

Here are two story problems. Explain how you can use your answer to EITHER one to answer the other.

- Cleo needs avocados to make guacamole for a party. Avocados are on sale for \$0.85 and Cleo has \$11.25 to spend. How many avocados can she buy?
- Mike's class raised \$112.50 for an afternoon at the movies. Tickets are \$8.50. How many ticket can the class purchase?

RIGOR and INQUIRY ...

What is the relationship between the digits in this number?
(e.g. 777, etc.)

How would adding a 0 to the end of a number affect the
value of the digits? (e.g. 75 becoming 750)

How do you think place value connects to other math
operations? (e.g. explore the relationship between place
value and multiplication/division)

CCSS.MATH.CONTENT.4.NBT.A.1

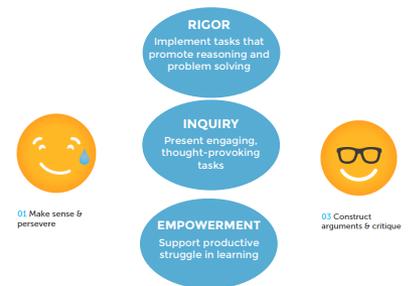
Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.

RIGOR and INQUIRY

What is the relationship between **the digits in this number?** (e.g. 777, etc.)

How would **adding a 0 to the end of a number** affect **the value of the digits?** (e.g. 75 becoming 750)

How do you think **place value** connects to other math operations? (e.g. explore the relationship between place value and multiplication/division)



MAD LIBS (?)

What is the relationship between _____ and _____?

How would [operation] affect the value of _____?

How do you think _____ connects to other math operations?

Why does _____ of _____ have _____?

Why does the _____ have _____?

Using what you know about _____ what is the _____?



**KEEP CALM
AND**

(FILL IN THE BLANKS)

YOUR TURN!



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BIRTHDAY CAKE

BEFORE

Seb is making a round cake for his sister Gabby. He has used 6 sugar roses for decoration. In between each two roses, he has put three candles. How old is Gabby?

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INQUIRY AND RIGOR

Seb is making a round cake for his sister Gabby. He has used 6 sugar roses for decoration. In between each pair of roses, he has put three candles. ~~How old is Gabby?~~

Challenge: Can Seb use this same pattern for her cake next year? Be sure to show or explain why or why not?

WORKSHOP PARTICIPANT SAMPLES (NCSM16)

Gabby is turning 6. How could her brother decorate the cake to celebrate her?

Seb is making a round cake for his sister Gabby. He used sugar roses for decoration. In between each two roses, he placed some candles. How old is Gabby?

See is making a round cake for his sister Gabby. He has used 6 sugar rises for decoration. In between each two roses, he has put three candles.

PIZZA

BEFORE

Mack ate $.25$ of his pizza and Justin ate 0.50 of his pizza. Mack says that he ate more pizza than Justin. Explain. Show your thinking by creating a model or representation.

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AFTER

Mack ate $.25$ of his pizza and Justin ate 0.50 of his pizza. Mack says that he ate more pizza than Justin. **Do you agree with Mack? Explain why or why not.** Show your thinking by creating a model or representation.



WORKSHOP PARTICIPANT SAMPLES (NCSM16)

Mack and Justin both ate some of a pizza. Mack says he ate more pizza than Justin. What are some of the amounts each of the boys could have eaten if Mack was correct?

Use a sentence frame: Mack ate $\frac{1}{2}$ of his pizza and Justin ate $\frac{1}{4}$ of his pizza. Mack says he ate more pizza than Justin. Convince me that Mack is wrong. Show your thinking by creating a model or representation.

SNOW CONES

BEFORE

Omar has a snow cone machine. It takes $\frac{2}{3}$ of a cup of ice to make a snow cone. How many full snow cones can Omar make with 4 cups of ice?

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AFTER

Omar has a snow cone machine and 5 cups of ice. It takes $\frac{2}{3}$ of a cup of ice to make a snow cone. He wants to make enough snow cones for himself and 7 friends.



WORKSHOP PARTICIPANT SAMPLES (NCSM16)

Take out the "4 cups" to be more open-ended

Omar has a snow cone machine. It takes $\frac{2}{3}$ of a cup of ice to make a snow cone. Omar has 4 cups of ice.

For access (empowerment): Take out the numbers. Omar has a snow cone machine. It takes ____ cup(s) of ice to make a snow cone. How many full snow cones can Omar make with ____ cups of ice?

JUMP ROPES

BEFORE

Sally has 30.25 meters of rope. She wants to cut it into 5 equal pieces to make jump ropes for 5 students. How long will each jump rope be?

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AFTER

Sally has 12.25 meters of rope. She wants to make 5 jump ropes that she and her friends can use at recess. Jump ropes are only fun to use if they are the right length. According to the PE teacher at Sally's school, "the jump rope should be 1.6 times as long as the jumper." If Sally's tallest friend is 5 feet and two inches tall.

What should Sally do?

WORKSHOP PARTICIPANT SAMPLES (NCSM16)

She has 30.25 meters of rope. How can she divide this to create multiple jump ropes?

Sally has some rope. She wants to cut it into equal pieces to make jump ropes for some of her friends.

Remove some of the numbers: Sally has ___ feet of rope. She wants to cut it into 5 equal pieces for ____ students. How long will each jump rope be?

BALLET CLASS

BEFORE

Clara enjoys ballet class

How many minutes did Clara spend the entire month?

Show the answer in minutes and convert it to hours.

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Clara enjoys ballet class

How many minutes did Clara spend the entire month?

Show the answer in minutes and convert it to hours.

AFTER

Clara enjoys ballet class.

How much time did Clara spend practicing ballet last month?

Choose the best units for your answer and convince me why your units make the most sense.

WORKSHOP PARTICIPANT SAMPLES (NCSM16)

Clara spent 1080 min practicing ballet over the course of 15 days. Make a list of times she could have practiced each of those days.

BASEBALL

BEFORE

When a baseball is thrown or hit into the air, its height in feet after t seconds can be modeled by $\{equation\}$ where $\{variable\}$ is the initial vertical velocity of the ball in feet per second and $\{other\ variable\}$ is the ball's initial height. A player throws the ball home from a height of $\{some\ number\}$ ft with an initial vertical velocity of $\{some\ number\}$ ft/s.

The ball is caught at home plate at a height of $\{some\ number\}$ ft. $\{some\ number\}$ seconds before the ball is thrown, a runner on third base starts toward home plate at an average speed of $\{some\ number\}$ ft/s.

Does the runner reach home plate before the ball does? Explain.

EMPOWERMENT = LESS IS MORE

AFTER

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The ball is caught at home plate. at a height of $\{some\ number\}$ ft. $\{some\ number\}$ seconds

Before the ball is thrown, a runner on third base starts toward home plate. at an average speed of $\{some\ number\}$ ft/s.

Does the runner reach home plate before the ball does? Explain.

Information:

Baseball motion model: $\{equation\}$, $\{variable\}$, $\{other\ variable\}$

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The ball is caught at home plate at a height of $\{some\ number\}$ ft.

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PARTING THOUGHTS

...the cumulative effect of students' experience with instructional tasks is students' implicit development of ideas about the nature of mathematics—about whether mathematics is something they personally can make sense of, and how long and how hard they should have to work to do so.

Stein, Smith, Henningsen, & Silver, 2000

Funded by The [National Science Foundation](#), CueThink is an innovative application focused on improving critical thinking skills and math communication of students in grades 2-12. Our mission is to empower students to see challenges as opportunities.

The screenshot displays the CueThink application interface. The main window is titled "UNDERSTAND" and shows a math problem: "Your friend asks you for change for a dollar. You check your pocket and find you have slightly over a dollar in change but not exactly. What is the MOST amount of money you could have?". Below the problem, there are two columns for student input: "What do you notice?" and "What do you wonder about?". The "What do you notice?" column contains the following text: "change for a dollar.", "slightly over a dollar", "not exactly.", and "MOST amount of money". The "What do you wonder about?" column contains: "If there is more than one answer." and "How to find the coins to use". Below these columns is a section titled "Estimate your answer" with the text "99 cents? 2 dollars". To the left of the main window, there is a smaller window showing a math problem with a pie chart. The pie chart is divided into six sectors, with three sectors shaded green and three shaded blue. To the right of the main window, there is an "ANNOTATION" panel with a list of annotations and their timestamps. The annotations include: "Hard to read" (0:00), "awesome visual image of the problem!" (0:00), "Could you explain what a 'negative integer' is?" (0:12), "Your picture made it very easy for me to understand the difference between the two heights" (0:19), and "I like how you showed the -- then how that becomes addition" (0:33). The annotation panel also shows a "My plan" section with the text: "I will set up a difference problem given that distance is the same as distance".

REFERENCES

- [NCTM research brief](#)
- [NRICH article](#)
- Achieve The Core: bit.ly/Achieve_Rigor
- “Selecting and Creating Mathematical Tasks; From Research to Practice.”
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CueThink

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