

Properties, Partials, and Practices

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About this session...

- Examine the properties of multiplication.
- Consider how the properties are used to develop multi-digit computation.
- Identify how the Standards for Mathematical Practice are evident in the development of computation.

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Factor Dazzle

1. Player 1 selects a number (put a chip/counter/initials on the number).
2. Player 2 finds all of the factors for that number (puts a chip/counter/initials on the number).
3. Players take turns until no more spaces are available.
4. Players add numbers covered by their chip.
5. Player with the highest sum wins.

1	2	2	2	2	2
2	3	3	3	3	3
3	4	4	4	4	5
5	5	5	6	6	7
7	8	8	9	9	10
10	11	12	13	14	15
16	18	20	21	22	24
25	26	27	28	30	32

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Factor Dazzle

Factor Dazzle is available on Calculation Nation.

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How does this picture help you see that there are lots of ways to think of 6×7 ?

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Do you see the distributive property on the multiplication chart?

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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Looking for patterns

- What do you notice when you look at rows 2, 3 and 5 on the multiplication chart?

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What do you notice about the rows?

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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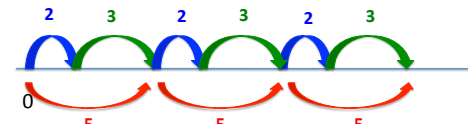
The Sum of the Products of 2x4 and 3x4 equals the product of 5x4

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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The Distributive Property on a Number Line?

- Assign a whole number jump size to the **blue jumps**.
- Then assign a whole number jump size to the **green jumps**.
- Does this always work? Why?



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Using Counters...

- How might you use counters to show that $(2 \times 2) \times 3 = 2 \times (2 \times 3)$?
- Remember it's about seeing why the values HAVE TO be the same.

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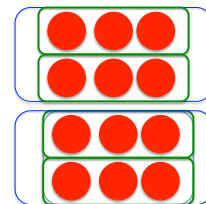
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One possibility

1 set
1 set of 3
1 set of (2 sets of 3)

2 sets of (2 sets of 3)

$2 \times (2 \times 3)$



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Another Possibility

(2 sets
 (2 sets of 2)
 (2 sets of 2) sets of 3

(2 x 2) x 3

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So What? (about properties)

What's Missing? How do you know?

- 7 x 6 is the same as (7 x 5) + (7 x 1)
- 9 x 25 is the same as (4 x 25) + (4 x 25) + (1 x 25)
- 18 x 6 is the same as 9 x 12
- 24 x 12 is the same as 6 x 48

Leith, L., Chui, K., Jones, D. (2013) Putting Essential Understanding of Multiplication and Division into Practice (3-5) Reston, VA: National Council of Teachers of Mathematics 2013

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How Might Our Students Solve These Problems?

- Neha has 11 bracelets with 6 beads. Deryn has 13 bracelets with 6 beads each? Who has more beads? How do you know?
- There is a tray of 12 cookies each weighing 4 ounces. There is a tray of 12 brownie bites each weighing 3.2 ounces. Which tray weighs more? How do you know?

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We know that 50 x 40 = 2,000. What is an easy way to find 49 x 40?

Explain your easy way.

Does this always work?
 Create 2 other examples that prove your thinking.

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Write 4 different equations to describe this array?

X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X
X	X	X	X	X	X	X	X	X

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Employing Properties to Develop Understanding of Multi-digit Computation

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Which representation of 4×6 do you think is "best"? Why?
 Which representation is "best" for 28×15 ? Why?

A: 4 groups of 6 hearts. B: A 4x6 grid of blue squares. C: $6 + 6 + 6 + 6$. D: A number line starting at 0 with 4 jumps of +6, ending at 24.

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Multiplication involving multiples of 10

- It makes sense that 4×30 ends in a 0 since $4 \times 3 \text{ tens} = 12 \text{ tens}$ (and no ones).

IT'S MUCH MORE THAN ADDING ZEROS.
 $4 \times 30 \rightarrow 4 \times (3 \times 10) \rightarrow (4 \times 3) \times 10 \rightarrow 12 \times 10$

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How many seconds are in an hour?

Notice that 60×60
 $= (6 \times 10) \times (6 \times 10)$
 $= (6 \times 6) \times (10 \times 10)$

When we multiply tens by tens our product must be in hundreds.
 And that we can make use of the associative property!

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Model Your Solution

- There are 5 lines of people for an amusement park ride. Each line has 13 people. How many people are in line altogether?

Use base ten models and/or draw a picture to show your thinking.

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13×5

There are 5 lines of people for an amusement park ride. Each line has 13 people. How many people are in line altogether?

- Concrete:** Build with tiles/blocks
- Pictorial:** Draw a representation
- Number:** What does solution look like with numbers?

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Concrete: 13×5

$10 \times 5 = 50$
 $3 \times 5 = 15$

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We were able to think of 13×5 as $(10 \times 5) + (3 \times 5)$. Which of the following would this idea also work with? How might you think of each of these?

34×7
 61×9
 6×85
 3×47

Would our models still work?
 Do they lose their efficiency?

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Mr. Kern's classroom is 35ft x 17ft. He is planning to partition it into 4 smaller rectangles for stations. What could be the sizes of the 4 stations?

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Mr. Kern's classroom is 35ft x 17ft. He is planning to partition it into 4 smaller rectangles for stations. What could be the sizes of the 4 stations?

	30	+	5
10	300		50
+			
7	210		35

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$46 \times 25?$

$(x+3) \quad (x-8)$
 $(40+6) \times (20+5)$

$800 + 200 + 120 + 30 = 1,350$

Numerical

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Oscar was at the top of a stadium looking at the parking lot. He wondered how many cars there were in a particular lot. He could see there were 48 rows of 32 cars.

How are the algorithms similar? How are they different?

Partial Products Algorithm	Partial Products Algorithm	Traditional Algorithm
48×32	48×32	48×32
16	1200	96
80	240	1440
240	80	
+ 1200	+ 16	

Smith, L., Chou, K., Jones, D. (2013) Finding Common Understanding of Multiplication and Division with Practice (3-5) Boston, MA: National Council of Teachers of Mathematics 2013

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Practice-Rich Task

Using dice or digit cards generate 3 digits. Create the largest product and the smallest product possible. Justify your products.

?	?
x	?

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Missing Factors, Missing Products

$$\begin{array}{r}
 20 + ? \\
 ? \\
 + \\
 4
 \end{array}
 \begin{array}{|c|c|}
 \hline
 & \\
 \hline
 80 & \\
 \hline
 \end{array}$$

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Does It Work With Any Factor?

Basketballs weigh 1.4 pounds. There are 6 balls on the rack. What is the total weight of the basketballs on the rack?

$$\begin{array}{r}
 ? + ? \\
 ?
 \end{array}
 \begin{array}{|c|c|}
 \hline
 & \\
 \hline
 & \\
 \hline
 \end{array}$$

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How do these ideas elicit the Standards for Mathematical Practice?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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