Scale Factors – Measure UP! & Measure Twice, Plot Once!

Important Teacher Note: One of the powerful features of TI-Nspire is the document feature. This lesson, as are all six of the lessons in the TODOS-TI project, is written as if the students are creating all of the pages in the Nspire document (the tns file). Teachers can, if they so choose, create the student documents in advance and build in all, some, or none of the technology depending on the many issues that go into planning lessons. With TI-Nspire Navigator technology, those documents can be sent to the students in advance.

* Use key mathematical vocabulary
☺ Use ELD Strategy
◆ Use TI-Nspire Navigator

OVERVIEW	Students will investigate the concept of proportionality by comparing English and Metric measures for weight and length. They will "discover" the scale factors that define each relationship by using tables, graphs, and equations. Time required: One 50-minute class period.
OJECTIVES Mathematics	Students will be able to: ▪ Compare ounces and grams in both **table** and **scatterplot representations** and discover a **linear relationship.** ▪ Measure in both inches and centimeters. ▪ Look at the graph of the class points and find the **equation of a line** that goes through most points.
Language Development	▪ Respond to open-ended and divergent questions using appropriate mathematical language. ▪ Use language to process, construct and deliver their comprehension of mathematics in multiple modalities (i.e. written, oral, receptive, and productive skills). ▪ Learn key vocabulary by using it in context. ▪ Engage in mathematical discourse in the classroom.
Technology	▪ Communicate using the Navigator system. ▪ Enter pairs of numbers in lists on the TI-Nspire handheld. ▪ Setup a scatterplot on a Data and Statistics page. ▪ Create equations using the form Y=*m*X where *m* is the **slope.**
NCTM FOCAL POINTS	**Numbers & Operations** and **Algebra** and **Geometry**: Developing an understanding of and applying proportionality, including similarity ▪ Solve problems about similar objects by using scale factors that relate corresponding lengths of the objects OR by using the fact that relationships of length within an object are preserved in similar objects.

	▪ Connections – Students apply their work on proportionality to measurement in different contexts, including converting among different units of measurement.

COMMON CORE STATE STANDARDS	CCSS.Math.Content.6.RP.1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."* CCSS.Math.Content.7.RP.2. Recognize and represent proportional relationships between quantities. ▪ CCSS.Math.Content.7.RP.2a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. ▪ CCSS.Math.Content.7.RP.2b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. ▪ CCSS.Math.Content.7.RP.2c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*. ▪ CCSS.Math.Content.7.RP.2c. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.
PRIOR KNOWLEDGE	Students are able to: ▪ Use everyday experiences with weights to examine proportional relationships between units. ▪ Measure length in both inches and centimeters with a ruler. ▪ Relate to out-of-school experiences using measurement techniques.
ENGLISH LANGUAGE DEVELOPMENT STRATEGIES	In this lesson, focus on these strategies to support English language learners oral and written receptive and productive use of language: ▪ Access prior knowledge. ▪ Encourage mathematical talk. ▪ Ask students probing questions to clarify and draw out their thinking. ▪ Have students share and justify their reasoning and process they used to solve the problem. ▪ Provide opportunities for students to work individually, pair-share, and in small and whole groups.

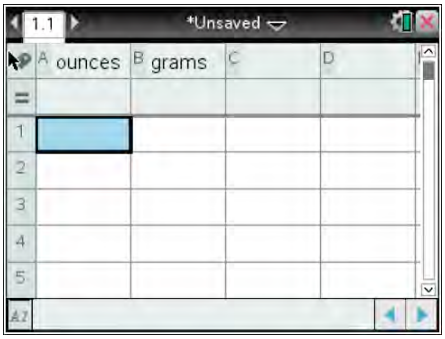
	▪ Technology: Screen Capture to focus students' discussions.

MATERIALS	▪ TI-Nspire handhelds, TI-Nspire Navigator system ▪ Activity sheets for each student ▪ A scale that weighs in ounces ▪ A scale that weighs in grams ▪ Ruler marked with both inches and centimeters
LESSON: TEACHER NOTES Introduce Explore	Student worksheets begin on Page 9. The numbers in the Teacher Notes correspond to the numbers in the student worksheets. Part I: Scale Factors – Measure UP! ☺ Frontload key mathematical vocabulary *Relationship* *Ounces* *Grams* *Line* *Outlier* *Table* *Scatterplot* *Linear relationship* *Slope* In this section, students enter data in a list, and see a linear proportional relationship when converting from ounces to grams. 1. Distribute the TI-Nspire handhelds and worksheet with sample packaging. 2. Students login to the TI-Nspire Navigator system. 3. ◆ Quick Poll (Open Response - Explanation) ☺ (Access prior knowledge) Ask the students: What is the **relationship** between the measurements in ounces and in grams? Then look at the Poll Summary results and discuss the students' answers. Address any major misconceptions (for example, 1 gram is heavier than 1 ounce). 4. Have students choose 7 of the 12 products from the list. Use the weight scales and record the measurements in the table on the worksheet. 5. ◆ **Quick Poll** (Open Response)**:** If you draw a graph showing each pair of measurements plotted as a point on the x-y plane, what will

you see?

Look for answers that have the word **line** and discuss. Validate other answers that show understanding and/or creativity. Ask students why they entered those responses.

6. Insert a List and Spreadsheet page into your TI-Nspire handheld:
   a. Label Columns A and B as shown below:



   b. Enter the numbers from your handout into the appropriate Nspire columns.

7. Setup a **scatterplot** to compare ounces on the x-axis to grams on the y-axis by inserting a Data and Statistics page on your TI-Nspire.
   a. Press TAB to move the cursor to the bottom of the screen as shown below and choose **ounces** for the x-axis and press ENTER.

   b. Press TAB again to move the cursor to the y-axis and select **grams** for it and press ENTER.

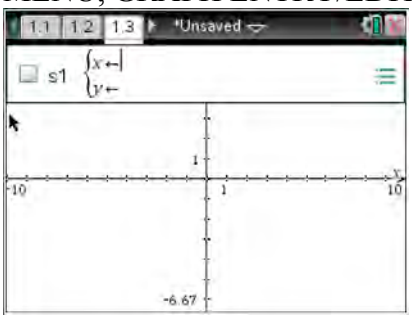
9. Once they have all done this click on Screen Capture to verify that all students have successfully entered the data and have similar **scatterplots**.

☺ (Mathematical talk): Have students compare their data and discuss similarities, differences, oddities among themselves.

10. *Leave the screen capture window open,* then send a ◆ Quick Poll (Open Response - Explanation): What is one thing that is the same in all of these graphs? Minimize the Quick Poll screen so students

Introduce Part 2	can see the graphs again. After two minutes click on the Quick Poll Student Data box to start a discussion. ☺ (Teacher questioning/Justify their reasoning) Follow with these questions: a. What are some of the differences between the graphs? b. Are there any points that are **outliers**? This question is NOT trivial –spend time talking about what outliers mean!!! c. Do the points that are far from the others stand out the most? d. Is there a point that doesn't fit on the line? What might have happened? Note: The two 2 ounce packages with different gram measurement should cause some controversy. Teacher Summary: The graph shows a **linear relationship** between measurements in ounces and appears to measure in grams. The **line** passes through the origin, and the **slope** of the line is approximately 28.3 (and it is no coincidence that 1 ounce = 28.3 grams!). At this point, students do not need to internalize these concepts, however begin modeling for them ways of talking about lines, slopes, rates of change, conversions, etc. The data points do not make a perfect line, and this should also be an interesting discussion starter. **Part 2: Measure Twice, Plot Once** In this section, students will measure their own data point, contribute it as part of a class data set, and define the relationship as the m in Y=*m*X. 1. ◆ Students should be logged in to TI-Nspire Navigator. 2. ◆ Quick Poll (Multiple Choice A to C): The length in centimeters will be…? Use the Poll Summary tab to review results. Have a class discussion. 3. Instruct students to use a straightedge (ruler) to draw a line segment in the box on their worksheet. Students are to measure their line segment twice: once using the inches side of their ruler, and once using centimeters. ☺ (Think/Pair-Share, Mathematical talk) The teacher may need to support students in how to use the ruler to measure length or they may want to ask a neighbor for help if they need assistance measuring the segment. Record these measurements on the worksheet. 4. Setup to receive a list from each student: a. Create a Quick Poll using "list(s)" as the type of question.

	b. Start the poll and click on the Student Data box that will show the data as the students submit it. c. After stopping the poll, right click on the table and choose the option *Send the Data to New Document* d. Save that document. You may wish to edit it such as changing the names of the columns. Save it. Send this new document to the students so that they will have all of the data. 5. ◆ Quick Poll (Open Response - Explanation): How would you describe our set of data points? After several minutes, display and discuss the results ☺ (Teacher questioning, Mathematical talk) Follow with these questions: ▪ Discuss with your partner: What patterns do you notice in this graph? ▪ Are there any points that stand out? If a student submitted his/her coordinates reversed, the point will not be on the "line". Hover over the point and look at the *x* and *y* coordinates. What is different?) ▪ Is there a **relationship** between inches and centimeters? What might it be??? Some students may "know" that the conversion factor from inches to centimeters is 1 in ≈ 2.54 cm. If a student brings this to the discussion, ask where in the graph do we see this? One idea to share with the students is for every unit you move to the right, the line goes up 2.54 units. ▪ What is the meaning of the point (0,0) on this linear graph? 6. After a short discussion about the shape of the points, show the students that they can plot a line on the graph by pushing MENU, ANALYZE, PLOT FUNCTION and typing in an equation. If they are not pleased with the fit, they can move there cursor to the equation and double click on it to change it. Possibly a better option would be MENU, ANALZE, ADD MOVABLE LINE. They can then click and move the line until they are satisfied with the fit. Screen Capture can be used to see how well students are fitting equations to the data. 7. Many of them will have an *m* value around 2.54. This is not an accident!!! a. Why do you think many of the numbers are so similar? b. What is the significance of this number? [1 inch ≈ 2.54 cm]
Close/Assess	8. Demonstrate how the **line** seems to contain the answers to ALL of

	the inch-to-centimeter problems. If Abdi drew a line 8 inches long, about how long would it be in centimeters? ▪ Have students use their cursor to point to the place on the line where X = 8 and then look at the Y-axis to determine the result. ▪ What is the centimeter measure of Vitaly's 10-inch line? ▪ How many centimeters is Sophia's line that is 9 inches? 9. Write a paragraph to explain what you learned about proportions and scale factors by doing these activities.
ASSESSMENT IDEAS	▪ Formative assessment is integrated throughout the lesson using Screen Capture and Quick Poll. ▪ Looking at the answers the student recorded on the worksheet will provide more assessment opportunities. ▪ A short Learning Check can be created to review the concepts developed throughout this lesson. ▪ Actively monitor students' language development during small group and whole class discussions.
EXTENSIONS	1. Students measure a line in the classroom (top of blackboard, door frame) in feet (use a yardstick). ▪ Convert the measure to both inches and centimeters. ▪ Insert a List and Spreadsheet page into an Nspire document. ▪ Label Column A as feet, Column B as Inches, and Column C as centimeters and enter the appropriate measurement in each. ▪ Insert a Graphs page. ▪ Change the entry line from Function to Scatterplot by selecting MENU, GRAPH ENTRY/EDIT, SCATTER PLOT.  ▪ With the cursor by the "x" in the entry line as shown above, push the VAR key and select Feet and then TAB. ▪ With the cursor by the "y" in the entry line, push the VAR key and select INCHES and then ENTER ▪ Press TAB to bring the graph entry line back. It will now show s2 instead of s1. ▪ With the cursor by the "x" in the entry line as shown above, push the VAR key and select Feet and then TAB. ▪ With the cursor by the "y" in the entry line, push the VAR key and select CENTIMETERS and then ENTER

	▪ Press MENU, WINDOW/ZOOM, ZOOM-DATA ▪ Discuss the significance of the **SLOPE** of each line. 2. Find other pairs of measurement units that have conversion factors. What do we measure with these units? What is the conversion factor? Give an example of the proportional relationships.
TEACHER REFLECTION	▪ Did I address each of the content objectives effectively? ▪ Did I use all of the targeted ELD strategies? ▪ Did I provide ample opportunity to build language? ▪ Were all of the students active participants in the mathematics discussions? ▪ Who did the thinking during the lesson?

Student Handout – Scale Factors – Measure UP! Part 1

1. Login to TI-Nspire Navigator

Diego went food shopping and recorded the weight of each item he bought. Note that the weights were all given in both ounces and grams. (English and metric units)

Raisins	(2.1 oz / 59.5 g)	String Cheese	(1.75 oz/49.6 g)
Fruit Bar	(4.5 oz/ 128 g)	Crackers	(3.25 oz/92 g)
Pan Dulce	(1.55 oz/43.9 g)	Churro	(1.3 oz/37 g)
Apple	(2 oz/61 g)	Mango	(2 oz/56.7 g)
Chips	(2.5 oz/71 g)	Pretzels	(4.1 oz/116.2 g)
Granola Bar	(1.25 oz/ 35.4)	Corn chips	(1 oz/28.3 g)

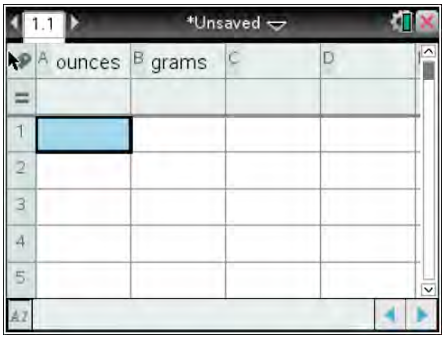
3. **Quick Poll**: What is the relationship between the measurements in ounces and the measurements in grams? Record some of your classmates' good ideas here!

4. Select 7 items from the list above and put the weights in the table below.

Weight in Ounces (oz)	Weight in grams (g)
1.	
2.	
3.	
4.	
5.	
6.	
7.	

5. Quick Poll: Make a Prediction: If you draw a graph showing each pair of measurements plotted as a point on the x-y plane, what will you see? What did your class see most often? Record here.

6. Insert a List and Spreadsheet page into your TI-Nspire handheld:
   a. Label Columns A and B as shown below:



   b. Enter the numbers from your handout into the appropriate Nspire columns.

7. Setup a **scatterplot** to compare ounces on the x-axis to grams on the y-axis by inserting a Data and Statistics page on your TI-Nspire.
   c. Press TAB to move the cursor to the bottom of the screen as shown below and choose **ounces** for the x-axis and press ENTER.

   d. Press TAB again to move the cursor to the y-axis and select **grams** for it and press ENTER.

9. Check on the big screen to see if your scatterplot looks like those of your classmates.

10. Quick Poll: What is one thing that is the SAME in all of these graphs?

What are some differences in the graphs?

Do any points seem to "stand out"? Why?

If you had to describe the "shape" of this data, what would you call it? Why?

What can we conclude from this investigation?

Student Handout – Scale Factors – Measure Twice, Plot Once! Part 2

1. Logon to Navigator again.

2. Quick Poll: Maria drew a line segment 6 ¾ inches long. Predict: If she measures the line using centimeters, the length in centimeters will be
A) less than 6 ¾ ,
B) equal to 6 ¾
C) greater than 6 ¾.

3. In the box below, draw a segment using your ruler

Measure the length of your line using inches. Length = _____ inches

Measure the length of your line using centimeters. Length = _____centimeters

4. Send your measurements to the teacher by entering your data in a Quick Poll

You will now receive the data point from ALL your classmates. Be Patient!!

5. Quick Poll: How would you describe our set of data points? Record an idea you liked from the class discussion.

6. Your next task is to determine an equation that best "fits" the data. Follow the instructions of your teacher to do that.

7. Why do you think many of the numbers are so similar?

What is the significance of this number?

8. This line can help us estimate the number of centimeters for inch lengths NOT in our lists. Suppose Abdi drew a line exactly 8 inches long. How can you use our line to find the length of Abdi's line in centimeters?

What is the measure of Abdi's line in centimeters? _____

What is the centimeter measure of Vitaly's 10-inch line? _____

How many centimeters is Sophia's line that is 9 inches? _____

9. Write a paragraph below to explain what you learned about proportions and scale factors by doing these activities.