Teaching Mathematics in Inclusive Classrooms in Five *Fairly* Easy Steps

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Learning Outcomes

1. Share practical Special Education teaching strategies that have been tailored to mathematics classes for use in helping ALL students make progress on achieving mathematics standards.
2. Provide explicit examples of differentiated teaching strategies that incorporate the Mathematical Practices in order to meet the needs of students with diverse learning styles.
3. Attendees will gain an understanding of techniques for helping ALL students experience success with meaningful learning tasks.

……In Five Fairly Easy Steps

1. Understanding Teaching v Learning Styles
2. Why and How Do You Differentiate?
3. Questioning
4. Explicit Instruction
5. Practical Applications

My Mathematics Glyph

A glyph is a pictogram. Using the paper plate provided and follow the instructions to create a picture that best describes you.

Teaching Style versus Learning Style

- What do we know about teaching styles?
- What do we know about learning?
- What do we know about different learning styles of students?

Why Differentiate?

- Common Core State Standards (CCSS)
  - Content Standards and Mathematical Practices
- High expectations for ***all*** students
- Diversity in the classroom –inter and intra-individual
- New cognitive research on human learning
- Rapid Societal and technological change
- 21st century skills

The Differentiated Classroom

- Climate
- Knowing the learner
- Assessing the learner
- Adjustable assignments
- Instructional strategies
- Curriculum approaches

Mathematical Practices

- *Make sense of problems and persevere in solving them.*
- *Reason abstractly and quantitatively.*
- *Construct viable arguments and critique the reasoning of others.*
- *Model with mathematics.*
- *Use appropriate tools strategically.*
- *Attend to precision.*
- *Look for and make use of structure.*
- *Look for and express regularity in repeated reasoning.*

Sample problem from Pennsylvania System of School Assessment (PSSA) Grade ???

A technician tests batteries for a battery manufacturer several times each week. She determines that the number of defective batteries is proportional to the number of batteries tested. The table below shows the numbers of batteries the technician tested at two different times during week 1 and the number of defective batteries she found each time.

Battery Test Results for Week 1

Number Tested	Number Defective
160	4
600	15

Between week 1 and week 2, the battery manufacturer changed its process. The number of defective batteries is still proportional to the number of batteries tested, but the constant of proportionality is greater. The technician tested 480 batteries during week 2 and found that 18 were defective. By what percent did the constant of proportionality increase?

A. $33\frac{1}{3}\%$

B. 50%

C. $66\frac{2}{3}\%$

D. 125%

Common Core State Standards

Grade 7 RP
Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.
Standard 7. RP.
Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
Grade 7.RP.A.2.B
Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

The Role of Questioning

So why use questioning?

- to help students identify their thinking process
- to see connections between ideas
- to build new understanding

Teachers need to make sure that they keep the learning goal and the big mathematical idea in mind. Plan effective questions when preparing lessons.

MP-1 Make sense of problems and persevere in solving them

Teachers:

Provide time and facilities for discussion of problem solutions

Students:

Are actively involved in problem solving

Questions to consider:

For Teachers:

- *What might be the math learning goal?*
- *What might students need to solve the problem?*

For Students:

- *Do you understand the question?/Can you restate it another way?*
- *Have you solved similar problems?*
- *What is your plan for solving the problem?*
- *Does your answer make sense?*

MP-2 Reason abstractly and quantitatively

Teachers:

Provide a range of representations, problem situations, a variety of solutions tracks

Students:

Use varied representations and approaches when solving problems

Questions to consider:

For Teachers:

- *What diagrams or other representations might students use in solving the problem*

For Students:

- *Can you think of a number sentence (equation) to match the story (situation)?*
- *What do the numbers in the number sentence mean?*
- *How are the facts in the problem related to one another?*

MP-3 Construct viable arguments and critique the reasoning of others

Teachers:

Provide opportunities for students to share, and listen to each others conclusions

Students:

Understand and use prior learning in constructing arguments.

Questions to consider:

For Teachers:

- *How might we structure class discourse of student strategies that focus on key ideas?*

For Students:

- *What does your answer mean?*
- *How can you be sure your answer is correct?*
- *How are solution strategies similar or different?*

MP-4 Model with mathematics

Teachers:

Provide a variety of contexts for students to apply the mathematics learned.

Students:

Apply mathematics learned to problems they solve and reflect on the results.

Questions to consider:

For Teachers:

- *In what ways can we ask students to interpret models?*

For Students:

- *What number sentence(equation) describes the problem?*
- *How are the numbers in the problem connected?*
- *Is your answer reasonable?*
- *What does your solution represent?*

MP-5 Use appropriate tools strategically

Teachers:

Use appropriate tools instructionally to strengthen the development of mathematical understanding.

Students:

Use tools to deepen understanding.

Questions to consider:

For teachers:

- *How can tools be made available for students?*
- *What are the advantages and limitations of specific tools?*

For students:

- *What tools can help you solve this problem?*
- *Which tool is the most useful for this problem? Why is that your choice?*

MP-6 Attend to precision

Teachers:

Emphasize the importance of mathematical vocabulary and model precise communication.

Students:

Based on the problem, attend to accuracy and efficiency.

Questions to consider:

For Teachers:

- How can you reinforce the meaning of symbols (=, <, >)
- How can you orchestrate opportunities for students to talk and write about math?

For Students:

- What do the symbols that you used mean?
- What units of measure are you using?
- Explain what [term for the lesson] means?

MP-7 Look for and make use of structure

Teachers:

Provide time for applying and discussing properties.

Students:

Look for, develop, and generalize arithmetic expressions.

Questions to consider:

For Teachers:

- *How might you facilitate discussion about math properties?*

For Students:

- *What do you notice about the solutions you have just completed?*

MP-8 Look for and express regularity in repeated reasoning.

Teachers:

Models and encourages students to look for and discuss regularity in reasoning.

Students:

Use repeated applications to generalize properties.

Questions to consider:

For Teachers:

- *What kind of problems will draw attention to repetition?*

For Students:

- Are there shortcuts for solving similar problems?

Sample problem from Pennsylvania System of School Assessment (PSSA) Grade ???

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D. 125%

Sample problem:

1. Introduce the problem. Allow “think time.”
   - What is this problem about?
     Think about ways you might solve the problem and then share with a partner.
   - What is your plan for solving the problem?
   - What do we know? What do we not know?
   - What are your ideas for starting this problem?

     Give plenty of wait time. Tell the students you don’t want answers yet.

Sample problem:

2. Collect ideas on the board/overhead.
    - Does anyone have any other ideas?
    - Does anyone have comments/thoughts about some of the ideas shared?

Do not comment on specific ideas at this point even if they ask.

Sample problem:

3. Student work time.

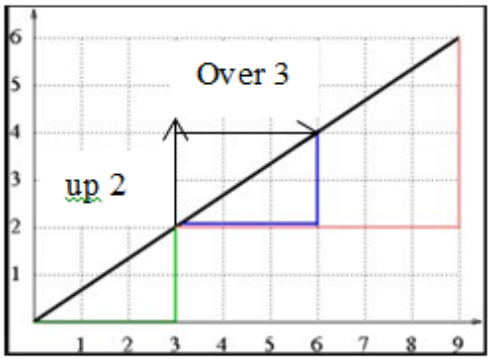
Allow students time to engage with the problem. When they are for help provide helpful hints—guidance rather than technical help.

- Which way did you decide to use or start the problem? Why?
- What have you got so far? How did you do that?
- What worked? What didn't work?
- Why do you think it is not working out?

Explicit Instruction Transcript

Joann: The problem mentions proportions and so when I think about proportions I think about ratios and rate of change. I know rate of change leads to slope. But I am not making the connection between slope and this problem. An example of slope to me would be the pitch of a roof. Where does that fit in here?

Edel: The pitch of a roof is the vertical rise over its horizontal span—using math vocabulary we would say "rise over run." This is how we would describe the slope of a line.



So the slope of this line would be 2/3.

Joann: But what about an equation? What does slope mean?

Explicit Instruction Transcript

Edel: The equation representing the line above would be $y = \frac{2}{3}$.

Let pick another example: $y = 3x$ where y represents the amount you spend renting a movie and x is the number of movies you rent. So the slope (or rate of change) here is 3. This means that you pay $3 for each movie rented. Therefore the slope tells us how much we pay per movie.

For our example, the constant of proportionality (or rate of change or slope) tells how many good batteries there are for each defective battery found.

Joann: I have figured out that the constant of proportionally for the first week is $\frac{1}{40}$ and for the second week is $\frac{1}{26.6}$. But how do I compare these two?

Explicit Instruction Transcript

Edel: The numerator and the denominator of a fraction need to be natural numbers (the counting numbers: 1, 2, 3, …)

So go back to the number of defective batteries for the second week and work out the constant of proportionality again.

Joann: $\frac{18}{480}$ So I need to simplify. $\frac{18}{480} = \frac{9}{240} = \frac{3}{80}$. It cannot be simplified any further.

Edel: Good. Now how can we compare $\frac{1}{40}$ and $\frac{3}{80}$?

Joann: Well $\frac{1}{40}$ is the same as $\frac{2}{80}$ so when I compare the rate for the second week has increased by 1.

How can I think about that as a percent?

Explicit Instruction Transcript

Edel: So when we want to see the percent increase, we need to compare the difference in the two amounts with the original amount.

Joann: Ok so when I compare the difference which was an increase of 1 to the original amount which was 2 I can say that 1 is 50% of 2.

Therefore the percent of increase is 50%. Is that one of the answers? *Checks to see.*

Yes!!

Sample problem:

4. Whole class discussion once most of the class has made significant progress.

    - Let's review what you have so far. No answers, just strategies.
    - What have you tried that didn't work?
    - What have you tried that seems to be successful?
    - How do you know your answer is correct?

Sample problem:

5. Whole class reporting

- What is it about [a particular strategy] that makes solving the problem easier?
- What is it about [a particular strategy] that is different? Creative?
- What ideas do you see that you could have used?

Explicit Instruction

What is it?

- Skill based with active student participation
- Integrates smaller learning units into a meaningful whole
- Tailored specifically to students' learning needs
- Constant monitoring for understanding

Elements of Explicit Math Instruction

Clear Teacher Models

- Use of clear and consistent wording
- Distinct explanations and demonstrations of smaller units of learning
- Involve students in teacher models
- Determine the appropriate amount of teacher models
- Think-alouds

Doabler, C. T., & Fien, H. (2013). Explicit math instruction: What teachers can do for teaching students with mathematics difficulties. *Intervention in School and Clinic* 48(5). 276-285. DOI: 10.1177/1053451212473151

Elements of Explicit Math Instruction

Guided Practice

- Select and sequence instructional examples
- Preteach prerequisite skills
- Verbal prompts
- Use multiple representations of math ideas (CRA)
- Monitoring of student learning
- Cummulative review

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Elements of Explicit Math Instruction

Academic Feedback

- Ongoing and timely feedback
- Corrective feedback
- Positive feedback

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Explicit Instruction in Practice

1. States clear expectations and objectives at the start of instruction
2. Starts instruction with a relatively easy instructional example
3. Uses a variety but limited number teacher models-instructional examples
4. Uses consistent wording throughout the activities
5. Provides clear demonstrations and step-by-step explanations of smaller learning units
6. Provides frequent practice opportunities
7. Uses math manipulatives to build conceptual understanding
8. Offers ongoing academic feedback
9. Provides cumulative review at the end of the third activity

Adapted from: Doabler, C. T., & Fien, H. (2013). Explicit math instruction: What teachers can do for teaching students with mathematics difficulties. *Intervention in School and Clinic* 48(5). 276-285. DOI: 10.1177/1053451212473151

Practical Applications

- Assessment - Pre assessment, formative assessment
- Adjustable Assignments
- Grouping -TAPS -TASK
- Focus Activities-Anchor/sponge activity
- Rehearsal Strategies- Rote versus elaborative
- Centers-Choice, rotation, thematic
- Projects -Structured, open-ended, adjustable
- Choice Boards -Think-tac-toe, menu, R.A.F.T., Cubing