Problems Worthy of your Effort and your Students' Engagement

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NCTM and Problem Solving:
A persistent and consistent theme

For over 35 years the National Council of Teachers of Mathematics has promoted value and importance of engaging our students in Mathematical Problem Solving, Pre-K – College.

A quick trip through NCTM history reveals....

Recommendations for School Mathematics in the 1980's.

Recommendation 1.

"Problem Solving must be the focus of school mathematics"

--from *An Agenda for Action (NCTM, 1980)*

NCTM Standards in the 1990's

- Standard 1

Mathematics as Problem Solving.

"The mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can use, with increasing confidence, problem solving approaches to investigate and understand mathematical content."

--*Curriculum and Evaluation Standards (NCTM, 1989)*

NCTM Standards in the 2000's

The Problem Solving Standard

"Instructional Programs Pre-K – 12 should enable all students to:

- Build new mathematics knowledge through problem solving
- Apply and adapt a variety of strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving. "

--*Principles and Standards for School Mathematics (NCTM, 2000)*

The NCTM *Reasoning and Sense Making* Initiative

Reasoning—the process of drawing conclusions based on evidence or previous knowledge and/or prior assumptions.

Sense Making—developing an understanding of mathematics concepts in a situation or context.

-- from *Focus on Reasoning and Sense Making*

(NCTM, 2009)

My NCTM President's monthly messages

The NCTM monthly President's column in *Summing UP* provided a forum for me to discuss vision and policy issues with the members

But it didn't provide a forum for me to actually 'do mathematics' with NCTM members

The birth of "Problem to Ponder"

- To engage NCTM members in mathematics, I created a new monthly President's column "Problem to Ponder" (PTP)
- Each month PTP posed a new mathematics problem for Council members to try out with their students
- Each PTP's was continued for three months—each column contained:

i) A brand new PTP

ii) Initial ponderings about the previous month's PTP; and

iii) Extended ponderings on the PTP from two months back.

(It's all about persevering!)

Today’s plan

- Revisit a few of those “Problems to Ponder” with you
- Give you a chance to start thinking about approaches, how to begin, share notices and wonders about them
- I chose problems that I hope are ‘Worthy of your Effort’, and of your students’ effort

A Problem is ‘Worthy of Effort’ if:

- It has *multiple entry points—accessible*
- It has mathematical punch and power -- involves mathematical *connections*, *structure, big ideas, problem solving heuristics*
- Requires students to *justify* their thinking
- Opens the door for *conjectures, extensions, generalizations*
- Taps multiple Effective Teaching Practices from NCTM’s *Principles to Actions*

Mathematics Teaching Practices Prominent in Tasks Worthy of Effort

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

--from *Principles to Actions* (NCTM, 2014)

Today's Mathematical Habits of Interaction

- Private Think Time
- Elbow pair share—Notices, Wonders, Ways to begin…
- Go Round once at Table
- Table Broadcasts— "We noticed, we wondered, we conjectured, we tried…"

PTP 1. The Grouchy Customer

Where Would You Sit in Your Neighborhood Café?

* In a neighborhood café there are 10 seats in a row at the counter. Each morning, customers enter the café for their morning coffee. They don't really want to have a conversation, so they prefer not to sit next to one another at the counter.

* Two people enter the café when it opens. How many different ways can these two customers sit at the counter so that they are not next to each other?

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Your Initial thoughts, notices, wonders, approaches…

Some Possible Initial Ponderings…

- Draw a picture
- Be organized in counting
- Hold one customer in a seat and move the other one…
- Look for patterns as we makes lists and count

Holding one customer in the first seat…

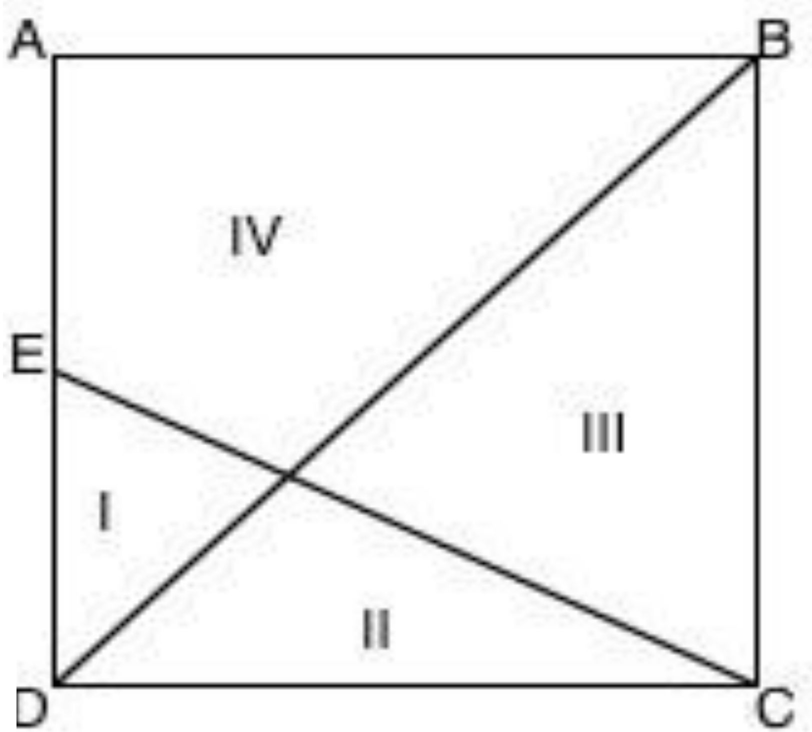
X	___	X	___	___	___	___	___	___	___	
X	___	___	X	___	___	___	___	___	___	
X	___	___	___	X	___	___	___	___	___	
X	___	___	___	___	X	___	___	___	___	
X	___	___	___	___	___	X	___	___	___	(8 ways)
X	___	___	___	___	___	___	X	___	___	
X	___	___	___	___	___	___	___	X	___	
X	___	___	___	___	___	___	___	___	X	

Extensions of Grouchy Customer PTP

- What if there were *three* grouchy customers in the café first thing in the morning. How many ways could they seat themselves so none of them had to sit next to another?
- What if the café had 12 seats instead of 10? Or had 15 seats? Or *N* seats? *N* seats and *P* customers?
- What if the café had a circular counter?
- What if we are concerned about the order in which the customers are sitting?

PTP2: **Comparing Regions in a Square**

- In the figure below, quadrilateral *ABCD* is a square, and *E* is the midpoint of the side *AD*.
- How do the areas of regions I, II, III, and IV compare?



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Some Possible Initial Ponderings

- Some students start solving this problem by noticing that triangular regions I and III are similar figures (AAA), and since their sides are in a ratio of 1:2, the areas are in a ratio of 1:4 .
- Others note that triangular regions I and II have the same altitude. And since base *ED* is half of base *DC*, region I has half the area of region II.

Extensions of Regions in a Square

- What if E were at the 1/3 point?
- The 1/4th point? The 1/nth point?
- What if the the figure were a rectangle instead of a square?

Love this problem, especially because algebraic approaches, though possible, are *really* messy!

PTP 3: **Looking Squarely at the Difference!**

- Here’s a “kick back and enjoy” Problem to Ponder.

Which whole numbers can be expressed as the difference of two perfect squares?

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Your Initial thoughts, notices, wonders, approaches…

Some Possible Initial Ponderings…

- Students sometimes begin problems like these by making a list. The list might start out rather haphazardly, then lead to a more organized list which can lead to conjectures.

For example, suppose that we made a list like the following:

Generating cases systematically…

$2^2 - 1^2 = 3$					
$3^2 - 1^2 = 8$	$3^2 - 2^2 = 5$				
$4^2 - 1^2 = 15$	$4^2 - 2^2 = 12$	$4^2 - 3^2 = 7$			
$5^2 - 1^2 = 24$	$5^2 - 2^2 = 21$	$5^2 - 3^2 = 16$	$5^2 - 4^2 = 9$		
$6^2 - 1^2 = 35$	$6^2 - 2^2 = 32$	$6^2 - 3^2 = 27$	$6^2 - 4^2 = 20$	$6^2 - 5^2 = 11$	
$7^2 - 1^2 = 48$	$7^2 - 2^2 = 45$	$7^2 - 3^2 = 40$	$7^2 - 4^2 = 33$	$7^2 - 5^2 = 24$	$7^2 - 6^2 = 13$ ….

Some Notices and Wonders from our organized list…

- We might notice that the top diagonal consists of all the odd numbers.
- We note the next diagonal includes all the multiples of 4, and we can get 4 from $2^2 - 0^2$.
- Also, we notice that some whole numbers can be expressed as the difference of two perfect square whole numbers in more than one way,

  like $5^2 - 1^2 = 24 = 7^2 - 5^2$, and $4^2 - 1^2 = 15 = 8^2 - 7^2$.
- We might wonder about which even numbers we can get…

Other representations of Square Differences

- We can represent the differences of perfect squares visually on grids.
- In each case, a shaded perfect square grid is cut out from the corner of a larger perfect square grid, leaving a shape that represents the difference of two perfect square whole numbers.

Pull up Dif of Squares Pict

Three Reads

- 1) Read for the story line in the problem

- 2) Read for quantities, numbers, measurements in the problem

- 3) Read for the question (or, identify some questions that could be asked or answered)

PTP 4: The Jacobean Locks Problem

- In medieval times, the inhabitants of a remote village decided to put a number of locks on a giant chest to protect the village valuables from marauding thieves.
- For additional security, the villagers created a enough locks and keys so that no two people from the village had enough keys to open the chest, but amongst them any group of three people always had enough keys to open all the locks on the chest.

Under Lock and Key

The problem:

How many locks, and how many keys, are needed to insure that no two people can open the chest, but any three people can?

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Initial Ponderings…

- It depends on the size of the village..
- Could we find a solution for some special cases, say 3 person village, 4 person village…

(Bring up diagram for three person village…)

Four Person Village…

- Call up chart from separate word file….
- N person village? How many locks, how many keys?

But our work is also about the Math itself!!

I enjoyed all the interaction from NCTM members on the PTP's throughout my presidential term.

We must not neglect the mathematics amid all the hubbub of the policy and politics of mathematics education.

After all, Mathematics is the 'meat' of what we do, the reason we are in this profession.

Final remarks

- Thank you for coming
- Thank you for your support and communication with me while I was your NCTM President
- May all your Issues be manageable, and may all your mathematical ponderings bring delight, to both you and your students.

— Mike

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Links to Todays P2P's Below

https://www.nctm.org/P2P-cafe.aspx

https://www.nctm.org/P2P-Regions.aspx

https://www.nctm.org/P2P-SquareDifferences.aspx

https://www.nctm.org/P2P-Lock.aspx