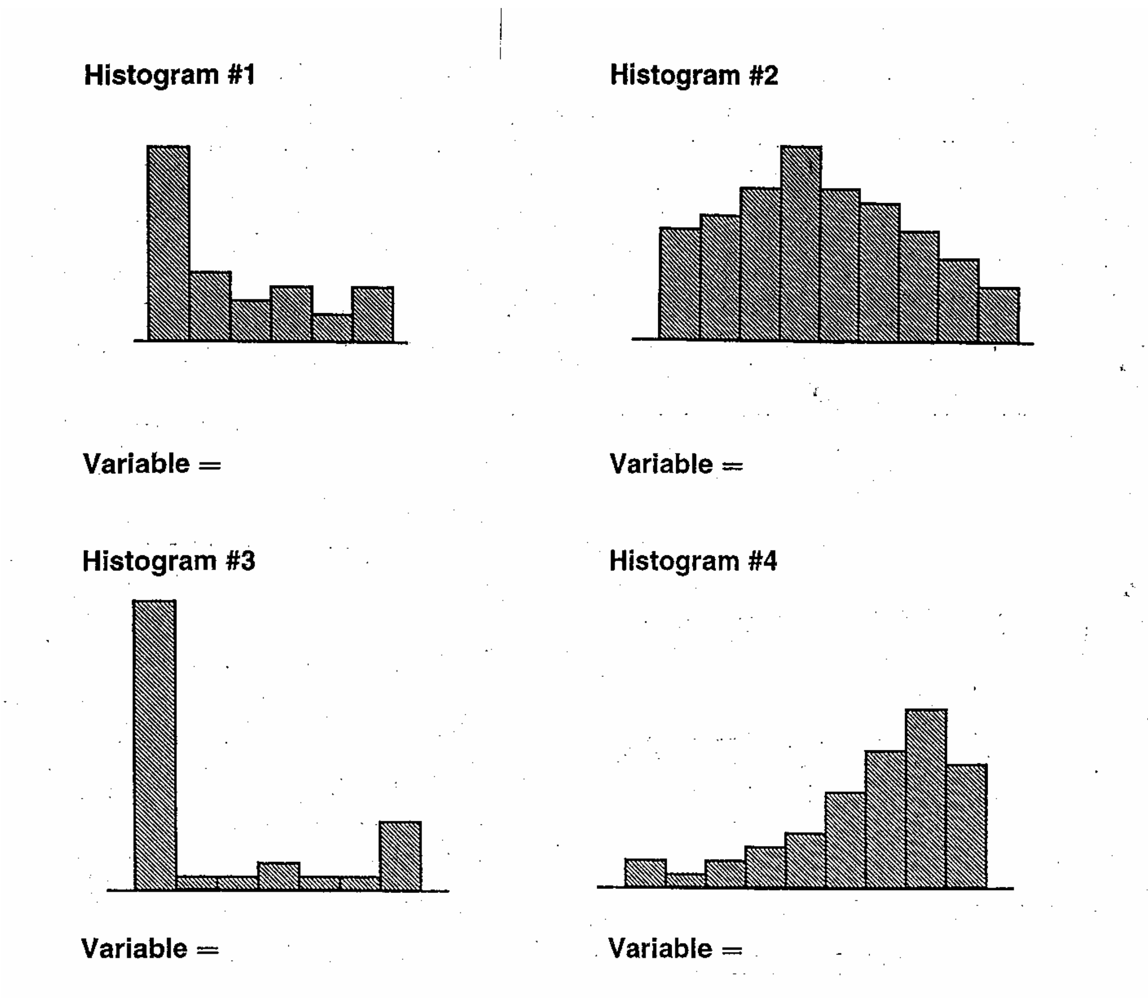
Matching Histograms to Data

1. Consider the following four histograms. Listed below are five variables.
    a. Determine which variable best matches the following histograms.
    b. Justify your reasoning.
    c. Label the histogram with a proper frequency unit and the appropriate quality or quantity being measured.

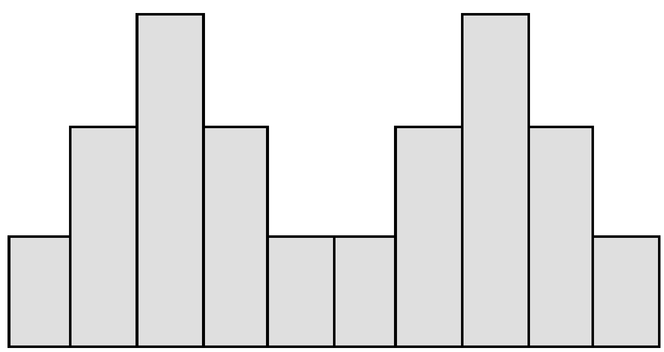


Variables:

a. Ages of citizens of the United States In 2012.
b. Miles of coastline for each of the 50 United States.
c. Average gas mileage of cars in United States in 2012.
d. Age at death for citizens of the United States in 2012.
e. Number of miles traveled to work, that is, the commuting distance, for employed adults that work in the same city in which they live.

2. For the variable you did not use in question one, sketch a histogram that would represent that data. Be sure to properly label your histogram and include a scale along the x-axis.

3. Consider the following histogram. Come up with two different variables that could describe the histogram. Explain your reasoning for why those variables are suitable.



Matching Histograms- Teacher's notes

Content and Practice Standards:

- CCSS.MATH.CONTENT.HSS.ID.A.1: Represent data with plots on the real number line (dot plots, histograms, and box plots).
- CCSS.MATH.CONTENT.HSS.ID.A.3: Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
- CCSS.MATH.PRACTICE.MP2: Reason abstractly and quantitatively.
- CCSS.MATH.PRACTICE.MP3: Construct viable arguments and critique the reasoning of others.

Prior Knowledge:

- Students work with histograms in 6th grade and comparing distributions in 7th grade, so they may have some familiarity with this topic. Students need to have a general understanding of a histogram.

Learning Outcomes:

- Students will be able to match histograms to variables by determining logical distributions associated with variables and comparing the overall spread of each histogram.
- Students will be able to produce their own histograms based on their own interpretation of variables and defend their reasoning.

Ideas for Implementation:

- To begin this task, it might be helpful to first review distributions and spread before starting (normal distribution, skewed left/right, unimodal, bimodal).
- This task is intended to be performed in groups where students have the opportunity to discuss their ideas.
- Whole class discussions are also needed for groups to share their work and critique the reasoning of others.

Solutions:

1. Possible Solutions: Histogram 1 = Variable e
Histogram 2 = Variable a
Histogram 3 = Variable b
Histogram 4 = Variable d

Expect discussion about Histograms 1 and 3 since these are both skewed and students might struggle. The best match for Histogram 3 is miles of coastline for each of the 50 United States however; many will argue that histogram 1 could also be miles of coastline for each of the

50 United States. A point to be brought up is that 28 states have 0 miles of coastline, which is more than half of the states. If you look at Histogram 1, less than half of the data is in the first bin. Be open for discussion for both answers.

2. Students should be making a graph for average gas mileage for a car in 2012. Different answers are acceptable for question 2. Remind students that hybrid cars could produce a second peak or a skewed right diagram.

3. Question 3 leads students to think about real life situations with a statistical mindset and consider distributions of real life data. Students must analyze how the data is distributed and come up with their own variables to match. This question is good for students to discuss in groups. Have students with different answers share and discuss. Possible answers that students might have:

Height of students (mode for girls and boys)
Time spent getting ready in the morning (similarly, mode for girls and boys)
Amount of money students spend on clothes
Test Grades of students
Price of cell phones (phones are either really cheap-old flip phones, or really expensive-iPhones, smartphones)
Less likely for students to say but possible:
Size/weight of a specific type of animal (i.e. cats: domestic will be very small, whereas lions, panthers, etc. will be much larger, there is no real in between. A large domestic cat might be the same size as a wildcat kitten; these may fall in the middle bins of the histogram)
Time spent at the gym in a week by students (most people either go to the gym or not)

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Strategic Cop

A statistically-minded state trooper wondered if the speed distributions are similar for cars traveling northbound and for cars traveling southbound on an interstate highway. The state trooper uses a radar gun to measure the speed of all northbound cars and all southbound cars passing a particular location during a two-minute period. Here are the state trooper's results:

Northbound Cars	Southbound Cars
60 62 62 63 63	55 56 57 57 58
63 64 64 64 65	60 61 61 62 63
65 65 65 66 66	64 65 65 67 67
67 68 70 83 42	68 68 68 68 71
69 73 86 70 67	50 75 64 66 70
71 63 75 61 70	65 66 77
64 64	

1. Represent each of the two distributions.

2. Is the speed limit the same on both sides? Explain.

3. What do you think the speed limit is on each side? Why?

4. Based on your analysis, what speed should have gotten tickets?

Strategic Cop- Teacher Notes

Content and Practice Standards:

- CCSS.Math.Content.HSS-ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).
- CCSS.Math.Content.HSS-ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
- CCSS.Math.Content.HSS-ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).\
- CCSS.Math.Practice.MP1 Make sense of problems and persevere in solving them.
- CCSS.Math.Practice.MP3 Construct viable arguments and critique the reasoning of others
- CCSS.Math.Practice.MP4 Model with mathematics.

Prior Knowledge:

- Students work with comparing distributions in 7th grade and with representing data in 6th grade. You may want to remind students about the different types of representations for data.

Learning Outcomes:

- Students will be able to compare two distributions and construct arguments about these distributions in their real world context.

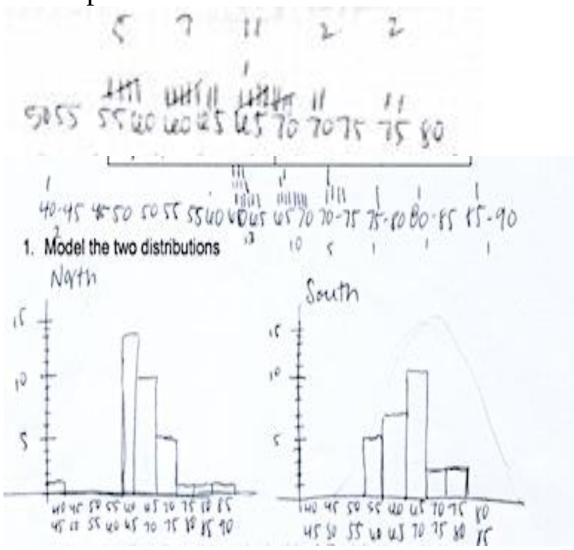
Ideas for Implementation:

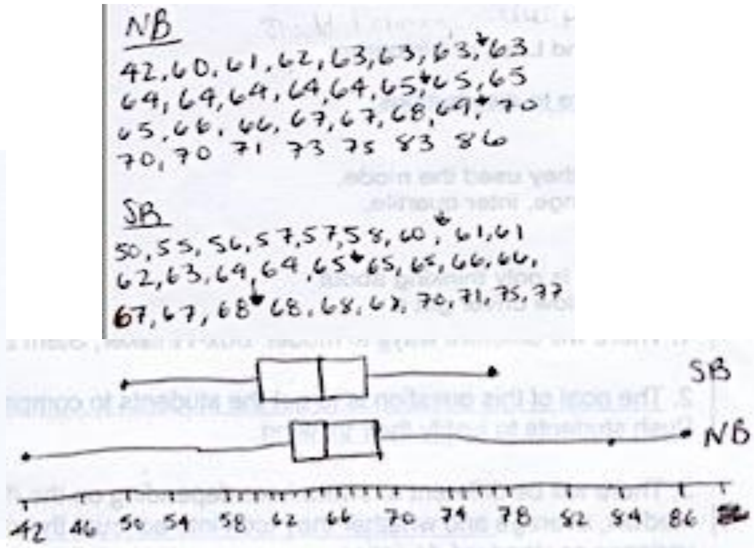
- **How to start this activity:** Talk about what an interstate is and you could even capture I-40. (If you capture the interstate make sure it is at **night** because with the amount of data points given it will not make sense to make it during the day.)
- **To close:** Have a discussion**.** Since each group may approach the task differently, there are going to be different representations. Discuss these various representations.

Solutions:

1. There will different ways to model this data such as Box-Whisker, Stem and Leaf, dot plot, or histogram. Graphs that could be misconceptions: Bar graph since it implies categorical data and Scatter plot since it requires bivariate data and this is univariate data.

Example Student work:





2. The goal of this question is to get the students to compare to distributions. Push students to justify their thinking. There are no wrong answers as long as they justify their thinking.
3. There will be different answers here depending on the if they used the mode, median, or mean and whether they took into account the range, inter quartile, variance or standard deviation.
4. This pushes students to think about outliers. If a student is only thinking about the high speeds ask them about the low speed? Does the slow driver get a ticket? (Some states ticket cars that go too slow on the interstate.)

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Error Analysis: Solving Equations

You are told to solve the equation $\frac{3x-6}{2}=9$, and then to get into groups to check your work and decide on an answer. You solve the equation and get x = 8 (the right answer). Sarah and Chris both solved the equation and got x = 4. They are telling you that since both of their answers agree, they must be right and you must be wrong. Convince them that you are right by explaining to them what each person did wrong. Their work is shown below.

Chris's work:	**Sarah's work:**
$\frac{3x-6}{2}=9$ $3x - 6 = 18$ $\underline{\quad +18 \quad +18}$ $3x + 12 =$ $x = 4$	$\frac{3x-6}{2}=9$ $\frac{3x}{2}-\frac{6}{2}=9$ $\frac{3x}{2}-3=9$ $\frac{3x}{2}=6$ $3x = 12$ $x = 4$

Explain to Chris what he did wrong and how to fix it.	Explain to Sarah what she did wrong and how to fix it.

Error Analysis: Solving Equations – Teacher Notes

Common Core Standards:

- CCSS.Math.Content.HSA.REI.1: Reasoning with equations and inequalities. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step.
- CCSS.Math.Content.HSA.REI.3: Solve linear equations and inequalities in one variable.
- CCSS.Math.Practice.MP3: Construct viable arguments and critique the reasoning of others.
- CCSS.Math.Practice.MP6: Attend to precision.

Prior Knowledge:

- This task would be used after teaching how to solve linear equations, or as a review before moving to other techniques for solving equations. Students should already be familiar (but probably not proficient) with manipulating equations.

Learning Outcomes:

- Students will be able to recognize common errors in solving equations, and to attend to precision and accuracy when they solve their own equations.

Lesson notes/ plan

- Begin this lesson by solving the original equation together, so that students feel confident in the answer. Then, pose the problem about the two wrong solutions.
- This works best when students work in pairs or groups of 3, so that they can discuss potential reasoning or arguments.

Extension/Possibilities:

- Similar problems may be posed for other types of equations, such as quadratic (factoring, completing the square), exponential and logarithmic, or trigonometric equations.
- This idea could also be used in presenting “student solutions” which use a graph or table.

This task was created by Latoya Clay and Michelle Cetner as part of the Noyce Project. This Project is supported by the National Science Foundation under the Grants No. DUE-0733794 and DUE-1240003 awarded to North Carolina State University. Any opinions, findings, and conclusions or recommendations expressed herein are those of the principal investigators and do not reflect the views of the National Science Foundation.

space on the shelf and are trying to decide which type of books to buy to fill that space. To help them decide, they looked at data from receipts over the last year. Out of the 2500 books purchased last year,

- 1200 children's books were purchased.
- 890 nonfiction books were purchased.
- 720 general interest books that were fiction were purchased.

Jamie says that more children's books were purchased than general interest, so they should invest in more children's books. Hunter says that more nonfiction books were purchased than fiction, so they should invest in more nonfiction books. So, together, they decided that the probability of people buying nonfiction children's books is the highest, and they was to buy those to fill their extra space.

Use the data to decide if Jamie and Hunter are making a good choice. You can use the partially filled in relative frequency table to help you.

Type	Fiction	Nonfiction	**Total**
Children's			**0.480**
General Interest	0.288		
Total		**0.356**	

Your decision: ____

What would you say to Jamie and Hunter to explain your reasoning to them? Include in your answer what Jamie and Hunter did wrong.

Error Analysis: Frequency – Teacher Notes

Common Core Standards:

- CCSS.Math.Content.HSS.ID: Summarize, represent, and interpret data on two categorical and quantitative variables.
- CCSS.Math.Content.HSS.ID.5: Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data.
- CCSS.Math.Practice.MP2: Reason abstractly and quantitatively
- CCSS.Math.Practice.MP3: Construct viable arguments and critique the reasoning of others.
- CCSS.Math.Practice.MP6: Attend to precision

Prior Knowledge:

- Students work with frequency tables in 7th grade, so they may have some familiarity with this topic, but no prior knowledge is needed outside of how to calculate percents (to use the relative frequency table).

Learning Outcomes:

- Students will be able to interpret two-way categorical data and to use a frequency table to construct a mathematical argument.

Lesson notes/ plan

- Students may need to be reminded that relative frequency tables show percentages of the total. It may be useful for students that have a difficult time conceptualizing the percentages to make a first table with counts.
- It may help students who have trouble with writing their explanations to ask them to explain to each other or to the teacher first. If one student is explaining to a second student, it may also help to have the second student take notes about what the first says.

Extension/Possibilities:

- Ask students how the table would look if there were more categories.
- Ask students to try to reconstruct a (wrong) table that Jamie and Hunter may have tried to construct.
- Ask students to find numbers (such as the total amount of books) that would not work with the rest of the data. For example, if 3000 books were bought total, it would be impossible to fill in the table.

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Cross-sections of 3D Objects

A **cross section** is a section made by a plane cutting anything transversely, especially at right angles to the longest axis.

Rewrite this in your own words: A **cross section** is _________________

____

Example: The cross sections for the banana at the right are____________

Challenge #1: Using your play-dough, create a cylinder.

Draw a picture of your cylinder.	Using your floss as an intersecting plane, cut your cylinder parallel to the bases. Draw a picture of your two pieces.	What shape is the cross-section? Draw a picture of just the cross-section.

Challenge #2: Recreate your cylinder.

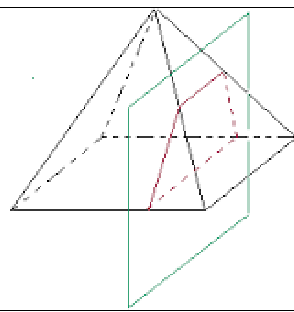
Use the floss to cut the cylinder by intersecting both bases (vertically). Draw the two pieces and the cross section.	Recreate your cylinder. Use the floss to cut the cylinder by intersecting one base and one side (diagonally). Draw the two pieces and the cross section.

Challenge #3: Using your play-dough, create a cube. Cut your cube in three different ways that result in different cross-sections. Draw and use words to describe how you cut the cube and the cross-section that that resulted for each.

Challenge #4: Using your play-dough, create a triangular prism. Cut your prism in three different ways that result in different cross-sections. Draw how you cut the cube and the cross-section that that resulted for each.

Name____________________________ **Cross-section student worksheet**

For each, fill in the table with the description of a *cross section of an object*, a picture of the *whole object*, a picture of the *cut object* showing the cross section, and a description of the *shape of the cross section*.

#	Description	Whole object	Cut object	Cross section
1	Cutting down a tree trunk parallel to the ground			
2	Splitting a log in half lengthwise			
3				An isosceles trapezoid
4	Cutting the corner off of a cube.			

Cross-sections of 3D Objects – Teacher Notes

Class Level: Math 2 (10^{th} grade)

Common Core Standards:

- CCSS.Math.Content.HSG.GMD: Visualize relationships between two-dimensional and three-dimensional objects
- CCSS.Math.Content.HSG.GMD.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects
- CCSS.Math.Practice.MP2: Reason abstractly and quantitatively
- CCSS.Math.Practice.MP8: Look for and express regularity in repeated reasoning

Prior Knowledge:

- Students work with cross sections in 7^{th} grade, so they may have some familiarity with this topic, but no prior knowledge is needed.

Learning Outcomes:

- Students will be able to determine the relationship between 2D figures and 3D objects. They will discuss strategies for determining the shape of various cross sections of 3D objects.

Lesson notes/ plan

- Discuss the "formal" and "friendly" definitions and examples of cross-sections with students. An example "friendly" definition may be *the shape you get when cutting straight across an object.*
- Distribute play dough and string to students. Demonstrate how to cut cross-sections. Allow a couple of minutes to practice cutting a cross section together as a class.
- Give students time to create each object and cut cross-sections.
- Discuss strategies for determining the shape of cross sections for each. Strategies include:
  - Look for vertices and edges that the cross-section intersects.
  - Think about if the cross section intersects sides that are curved or straight.

Extension/Possibilities:

- Students may explore other objects, such as pyramids, cones, spheres, and other prisms.
- This may be used as a lead-in to conic sections.
- Students can be asked to create objects of revolution (e.g. the region in the first quadrant below $y = 4 – x$, revolved around the y-axis).
- This type of activity could also be used with calculus students, in which they create approximations of objects of revolution or objects of known cross-sections, and then cut the cross-sections.

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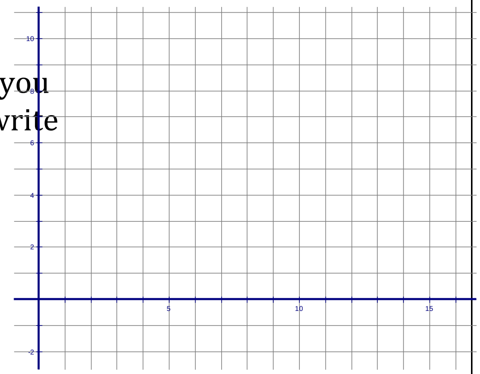
Name________________________________ CBR worksheet - linear

1. For the equation $y = (1/4)x + 1$
   a. Graph the equation at the right.
   b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

      Regression equation:____________________
      R = ________________

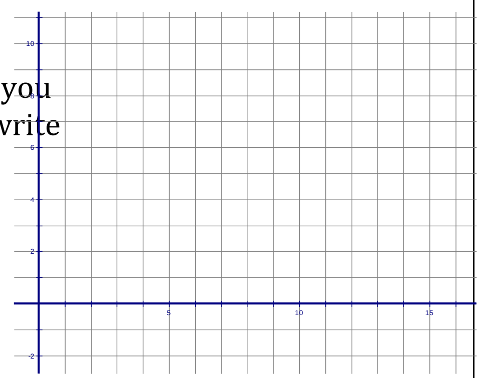
   c. Explain how to use the y-intercept and slope to know how to walk.



2. For the equation $y = 8 - (1/3)x$
   a. Graph the equation at the right.
   b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

      Regression equation:____________________
      R = ________________

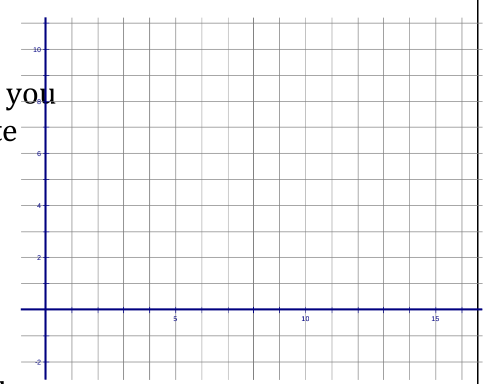
   c. Explain how to use the y-intercept and slope to know how to walk.



3. For the equation $y = (.2)x + 2$
   a. Graph the equation at the right.
   b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

   Regression equation:____________________
   R = ________________

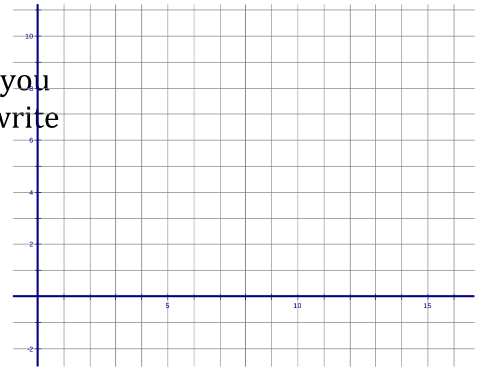
   c. Explain how to use the y-intercept and slope to know how to walk.



4. For the equation $y = (-1/4)x + 8$
   a. Graph the equation at the right.
   b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

      Regression equation:____________________
      R = _______________

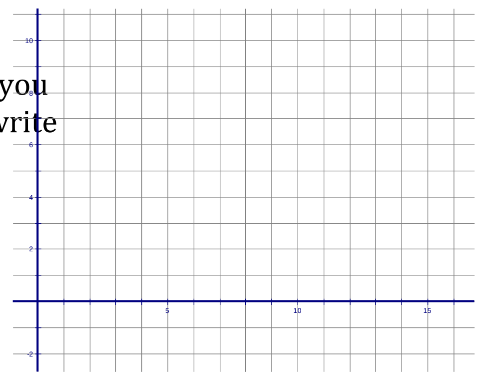
   c. Explain how to use the y-intercept and slope to know how to walk.



5. For the equation $y = (0.4)x$
   a. Graph the equation at the right.
   b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

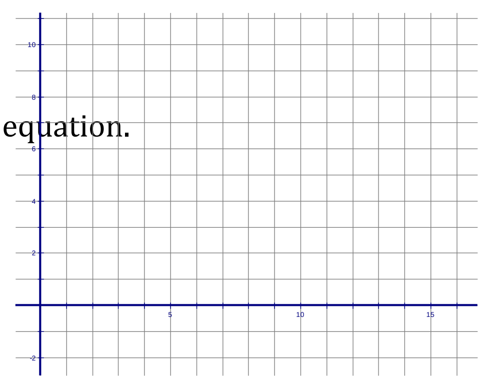
      Regression equation:____________________
      R = _______________

   c. Explain how to use the y-intercept and slope to know how to walk.



6. For the equation $y = (1.2)x - 2$
   a. Graph the equation at the right.

   b. Explain why you can't use the CBR to graph this equation.



CBR for Linear – Teacher Notes

Common Core Standards:

- CCSS.Math.Content.HSA.CED.2: Graph equations on coordinate axes with labels and scales.
- CCSS.Math.Content.HSA.REI.10: Understand that the graph of an equation in two variables is the set of all solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- CCSS.Math.Content.HSF.IF: Interpret functions that arise in application in terms of the context.
- CCSS.Math.Practice.MP2: Reason abstractly and quantitatively.
- CCSS.Math.Practice.MP1: Make sense of problems and persevere in solving them.

Prior Knowledge:

- Students work with linear functions in 8th grade, so they may have some familiarity with this topic, but no prior knowledge is needed.

Learning Outcomes:

- Students will be able to determine the relationship between slope and y-intercept in the equation of a line and their own physical movement in forming the graph. Students will understand slope as a rate and y-intercept as an initial value.

Lesson notes/ plan

- First, acquaint students with CBR equipment. This includes installing the program on calculators, operating the program, and changing parameters (I like feet instead of meters because floor tiles are often 1 foot long). If this is their first time using CBRs, there is also a "distance match" application that they can play with.
- Put students in groups. This works best will 3 – 4 students per group, but I have also used it with up to 7 students in a group and it still worked ok.
- Give out the worksheet. Challenge all groups just to do #1 and to show you when they have a good match. This helps troubleshooting so that all groups have a good handle on using the equipment before they get to the more challenging problems.
- I have also found success making this into a "game" as well. Assign points for 1st, 2nd, and 3rd place for each question for which group approximates the line the closest. However, since there is no way to really know if what they are drawing is on the calculator, when I do this, I have them show me the calculator once they draw the graph and I sign off on it for each.

Extension/Possibilities:

- This activity works well for different types of graphs. The electronic form of this document has this activity written for quadratic functions, polynomials, and domain and range.

- Students may also be asked to create different graphs from each other (e.g. one group does $y = \frac{1}{4}(x) + 1$ and the other group does $y = 3 - \frac{1}{4}(x)$). The groups can work together to graph the system, and recognize that the intersection is the time when the two people pass each other.

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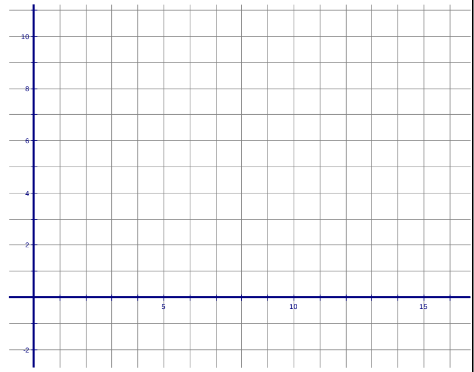
Name_________________________________ CBR worksheet

1. a. Draw the graph of a parabola that has a maximum at
(5, 8), and a y-intercept at 2.



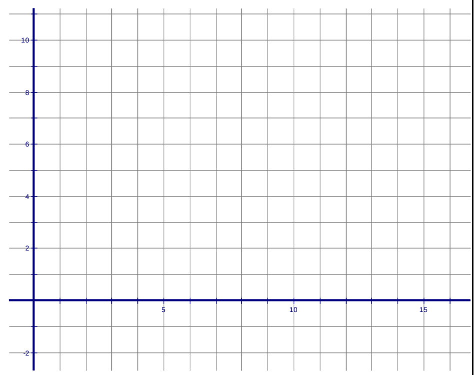
b. How does the y-intercept tell you about using the CBR?

c. How does the vertex tell you about using the CBR?

d. Use the CBR to try to make the same graph.

e. Now, use STAT-CALC-QUADREG to find the equation. Write it here.

2. a. Graph the equation $y = .2x^2 - 2x + 6$



b. Find the y-intercept.

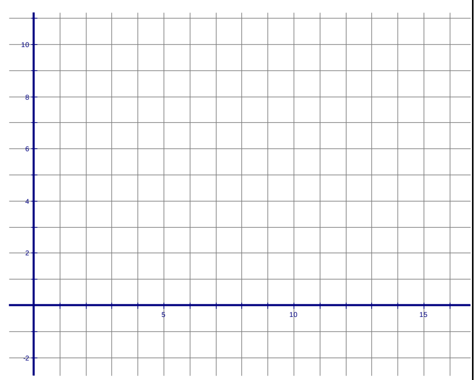
c. What does the y-intercept tell you about using the CBR to make the graph?

d. Find the vertex.

e. What does the vertex tell you about making the graph?

f. Use the CBR to try to make the graph. Tell whether you think you came close or not.

3. a. Draw the graph of a line that has a y-intercept at 1 and a slope of ¼.



b. Explain how the y-intercept tells you how to use the CBR.

c. Explain how the slope tells you how to use the CBR.

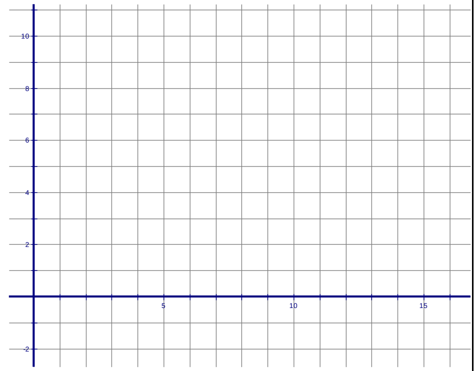
d. Use the CBR to make the graph you drew.

e. Now, use STAT-CALC-LINREG to find the equation. Write it here.

c. Write the equation of a line with y-intercept at 1 and slope ¼. Was your LINREG equation close? Why or why not?

4. For the equation $y = (-.2)x + 8$

   d. Graph the equation at the right.



b. Explain how the y-intercept tells you how to use the CBR.

c. Explain how the slope tells you how to use the CBR.

d. Use the CBR to make the graph you drew.

e. Now, use STAT-CALC-LINREG to find the equation. Write it here.

5. a. What was the difference between the graphs in #1 and 2 compared to #3 and 4?

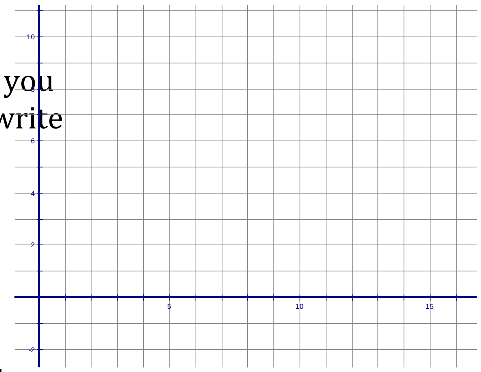
   e. For all four questions, what does the y-intercept tell you?

   f. For lines, what does the slope tell you?

   g. For parabolas, what does the vertex tell you?

Name________________________________ CBR worksheet

7. For the equation $y = (1/4)x + 1$

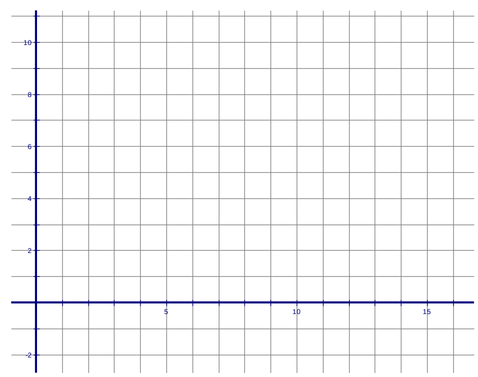


   h. Graph the equation at the right.
   i. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

      Regression equation:___________________
      R = _______________

   j. Explain how to use the y-intercept and slope to know how to walk.

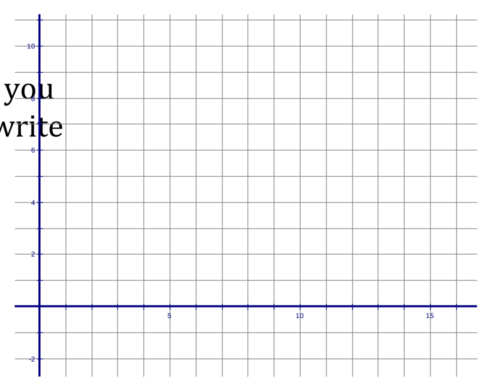
8. Repeat #1 for the equation $y = 8 - (1/3)x$



   Equation:___________________
   R = __________________

   Explain:

9. For the equation $y = 0.1(x - 5)^2 + .5$,

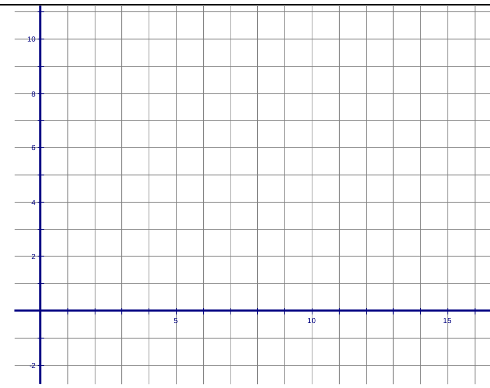


   a. Graph the equation at the right.
   b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

      Regression equation:___________________
      R^2= _______________

   c. What does the y-intercept tell you?

10. Repeat #3 for the equation $y = 0.1x^2 - 0.8x + 2$.

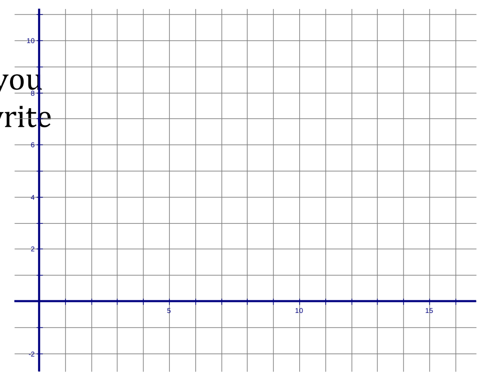


Regression equation:____________________

R^2= ________________

What does the y-intercept tell you?

11. For the equation $y = .02x^3 - .44x^2 + 2.8x + 1$,



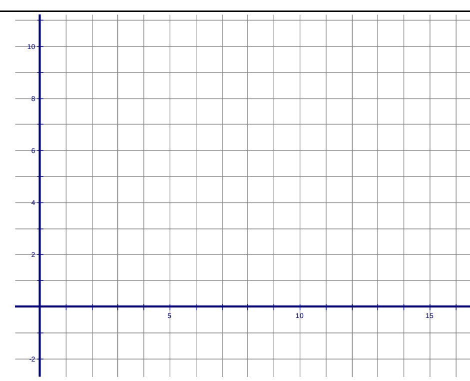
a. Graph the equation at the right.
b. Use the CBR distance sample and get as close as you Can. Graph your best picture on the graph and write the regression equation below. Also write the correlation coefficient.

   Regression equation:____________________

   R^2= ________________

c. What does the y-intercept tell you?
d. What is the relative maximum? What do you do at that point?
e. What is the relative minimum? What do you do at that point?

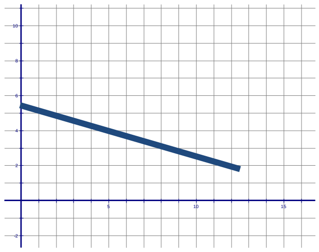
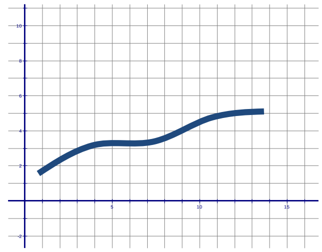
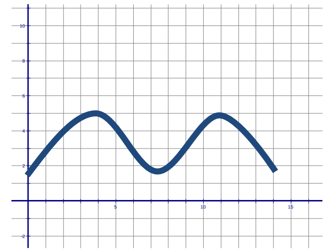
12. Repeat # 5 for $y = -.02x^3 + .44x^2 - 2.8x + 9$.



Name___________________________________ CBR worksheet

Date_____________

1. Graph the functions below using your CBR. Get me to check each off when you get it.

2. Which graph has a y-intercept at the origin?

   What does this mean physically (with your CBR)?

3. Graph the function below using your CBR. Get me to check it off when you get it.

4. Explain what the independent variable and dependent variable are for this function.

5. Tell the domain and range for the graph in #3.

6. Try to graph what is below on your CBR. Why can't you do it? Explain why this is not a function.