Session 446
Friday, 4.17. 15
11:30 AM –
12:00 PM
205C (BCEC)

Helping Students Master Basic Addition Facts

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Readiness: Counting, Cardinality, Conservation

- One-to-one Correspondence
- Cardinality
- Conservation of Number

First: Counting, Cardinality, Conservation

Phases of Learning Basic Facts

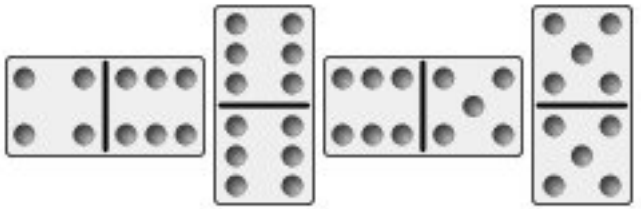
- Modeling and/or Counting All or Counting On
- Using reasoning strategies
- Instant recall

Four Number Relationships

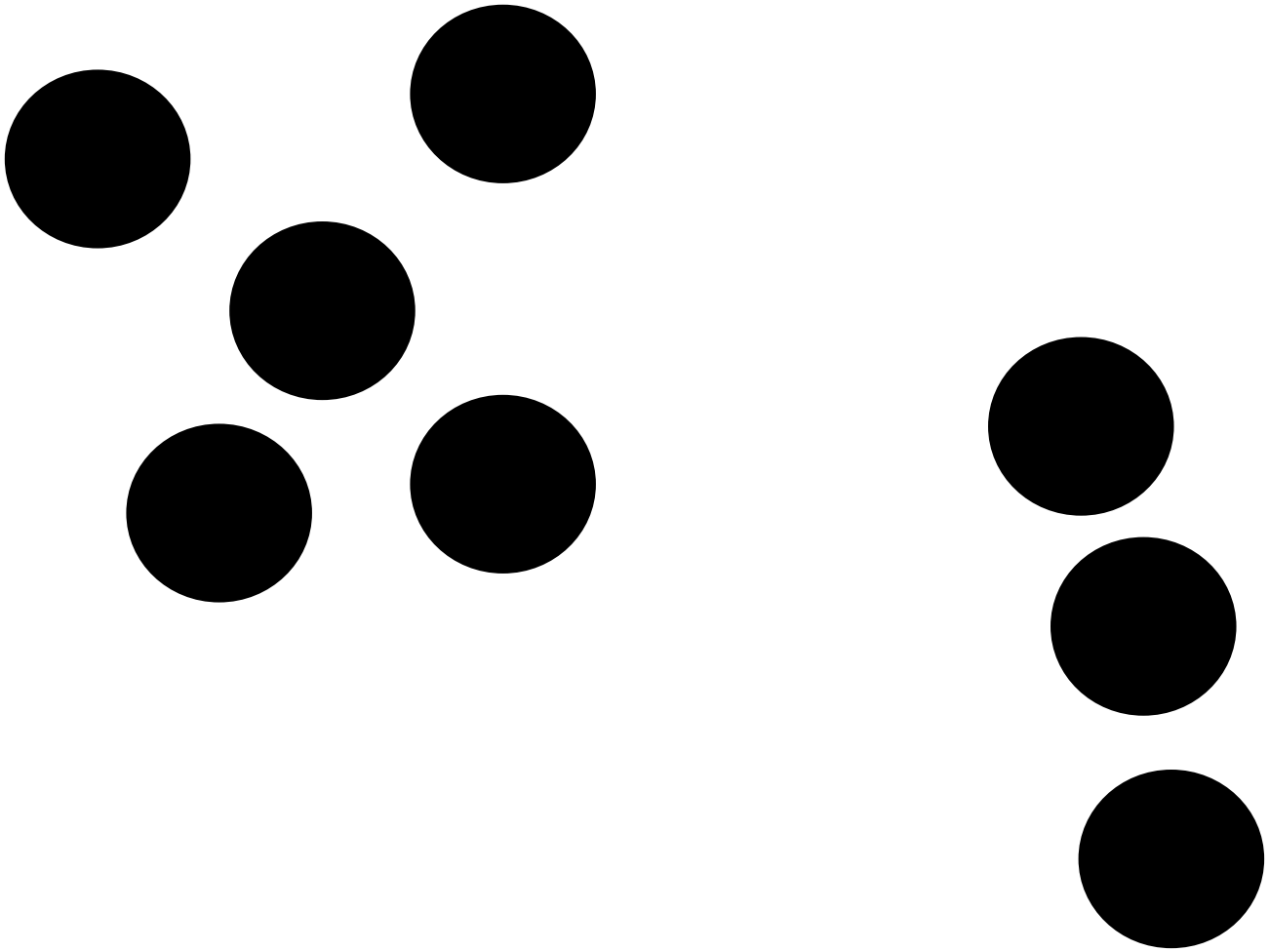
- Spatial Relationships
- Anchors of 5 and 10
- Part Part Whole
- One and Two More/One and Two Less

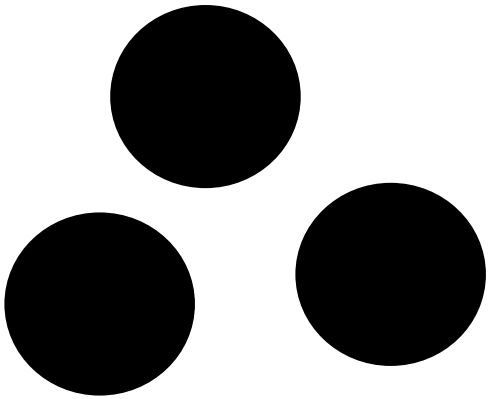
Spatial Relationships & Subitizing

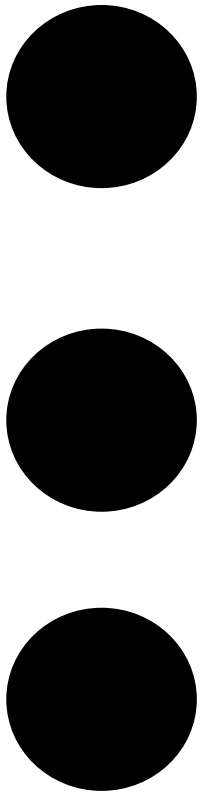
- Create Dot Cards
- Dot Card Flash
- Domino Match
  - http://www.freearcade.co Domino.flash/Domino.html

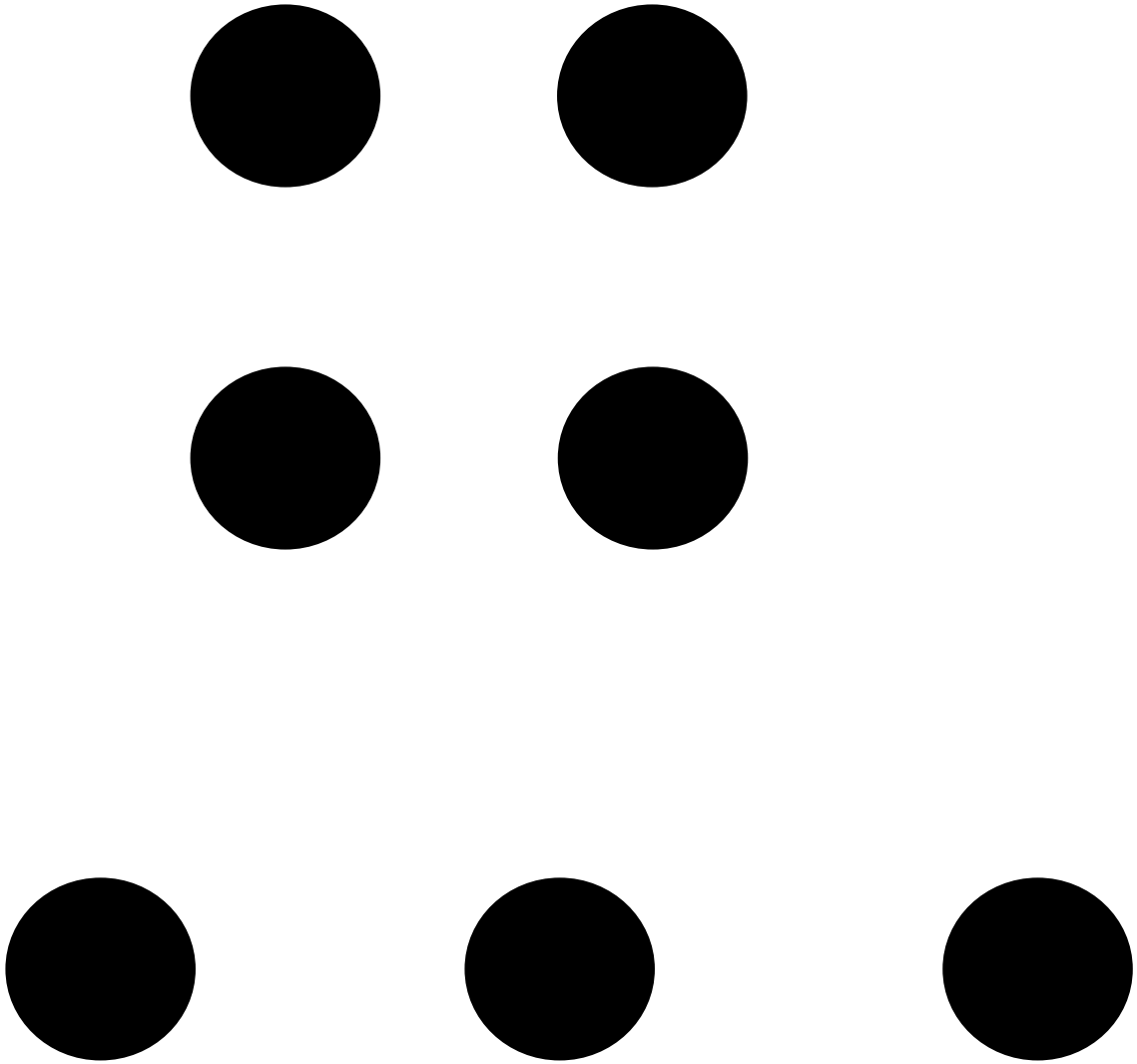


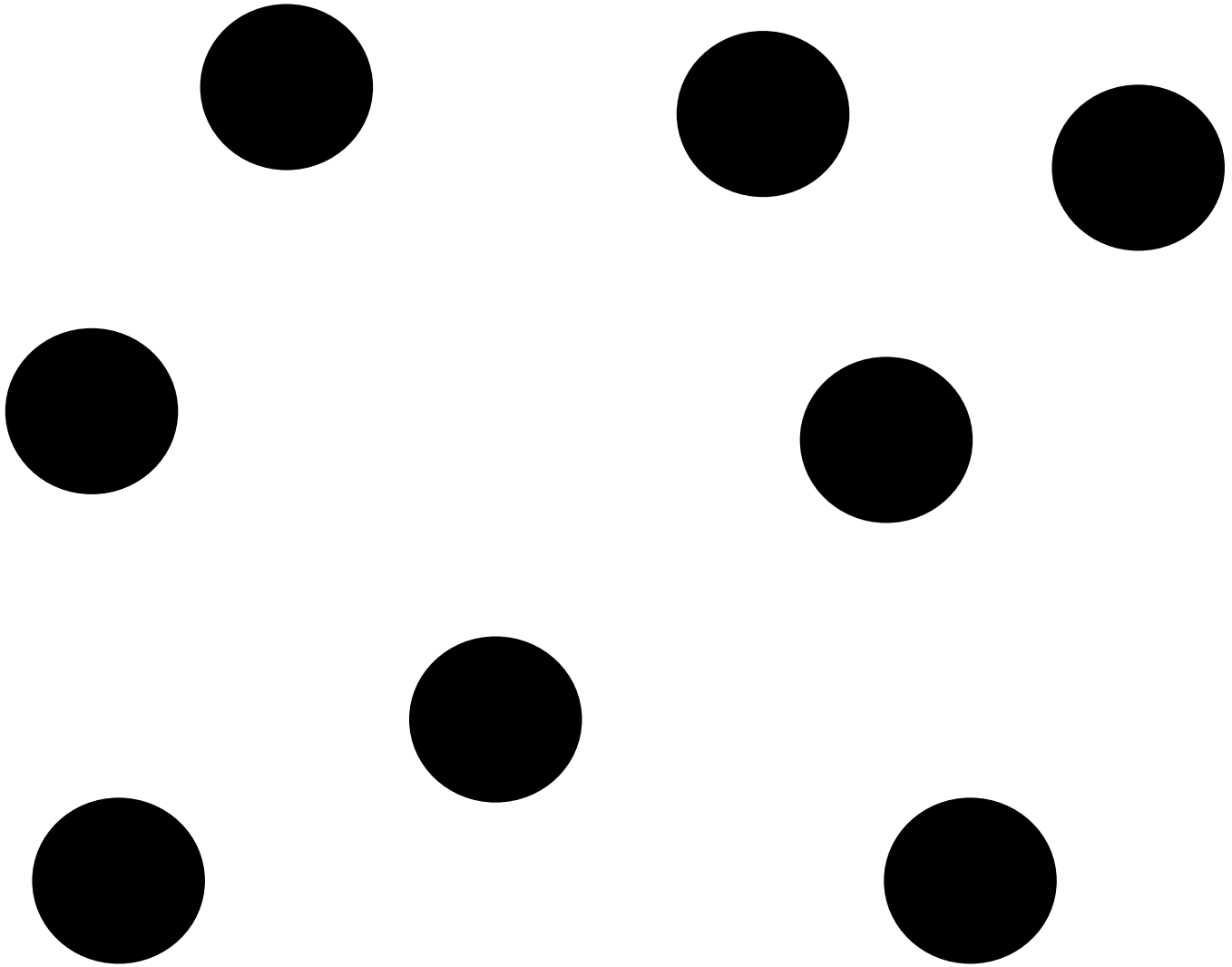
- Dice Games – Going to Boston
  - http://www.activityvillage.co.uk/going-to-boston





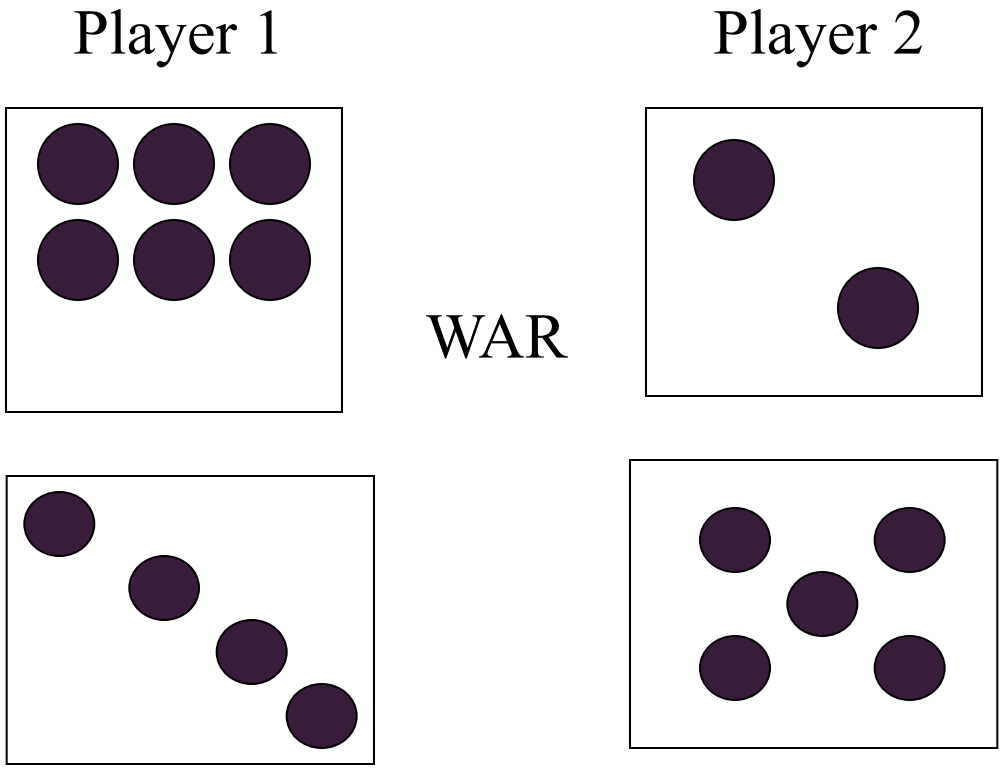






More Dot Card Activities

- Trains from 0 to 9 and back again
- War/Double War
- Difference War



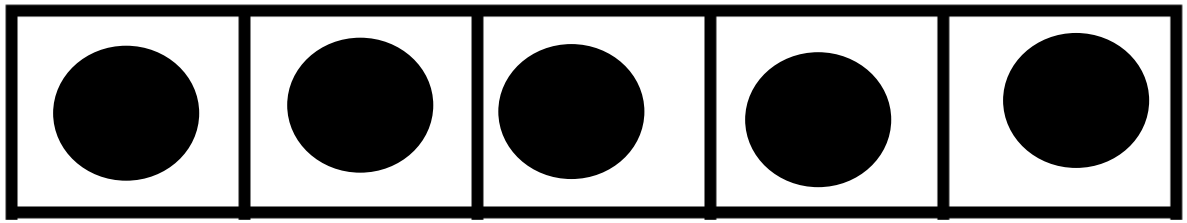
Dot Card Video – start at 3:54

Anchors of 5 and 10

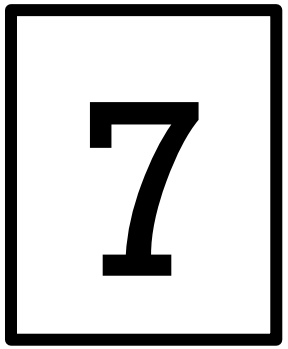
- Five Frames
- Five-And Game
- Ten Frame Cards
- Crazy Mixed Up Numbers
- Towers of 10
- Ten Frame Flash (there and missing)

Anchors of 5

- 5 Frame Flash
- 5 and More

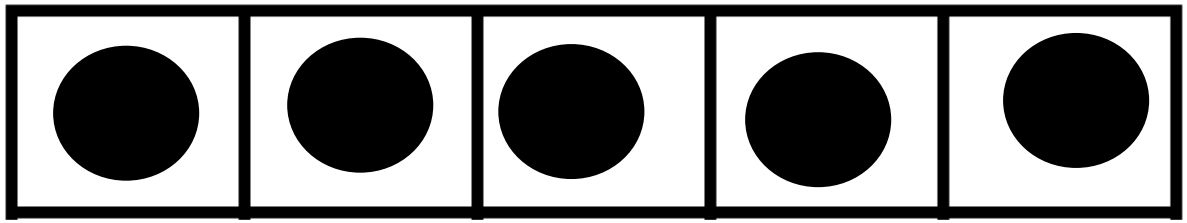


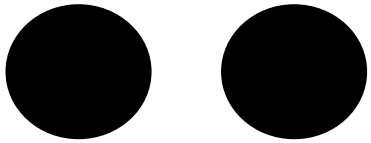
Deck of 6-10 Cards



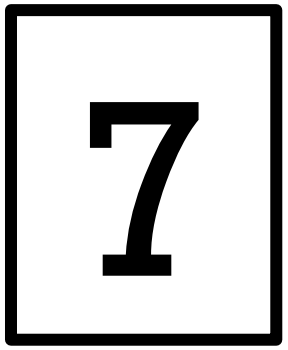
Anchors of 5

- 5 Frame Flash
- 5 and More





Deck of 6-10 Cards



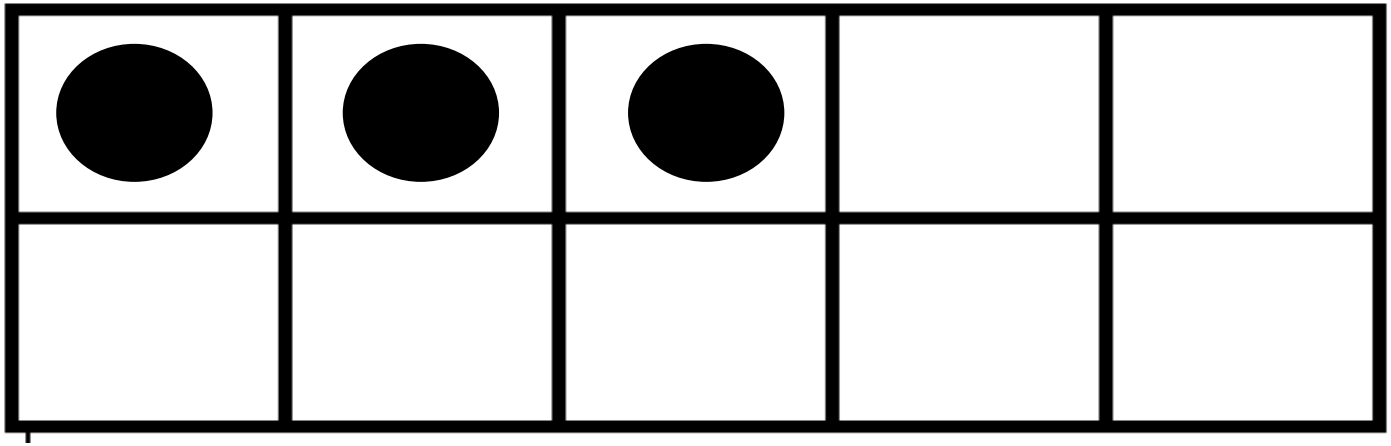
“5 and 2 make 7”

Anchors of 10

- Ten Frame Flash
- Ten Frame Flash – What's Missing?

Anchors of 10

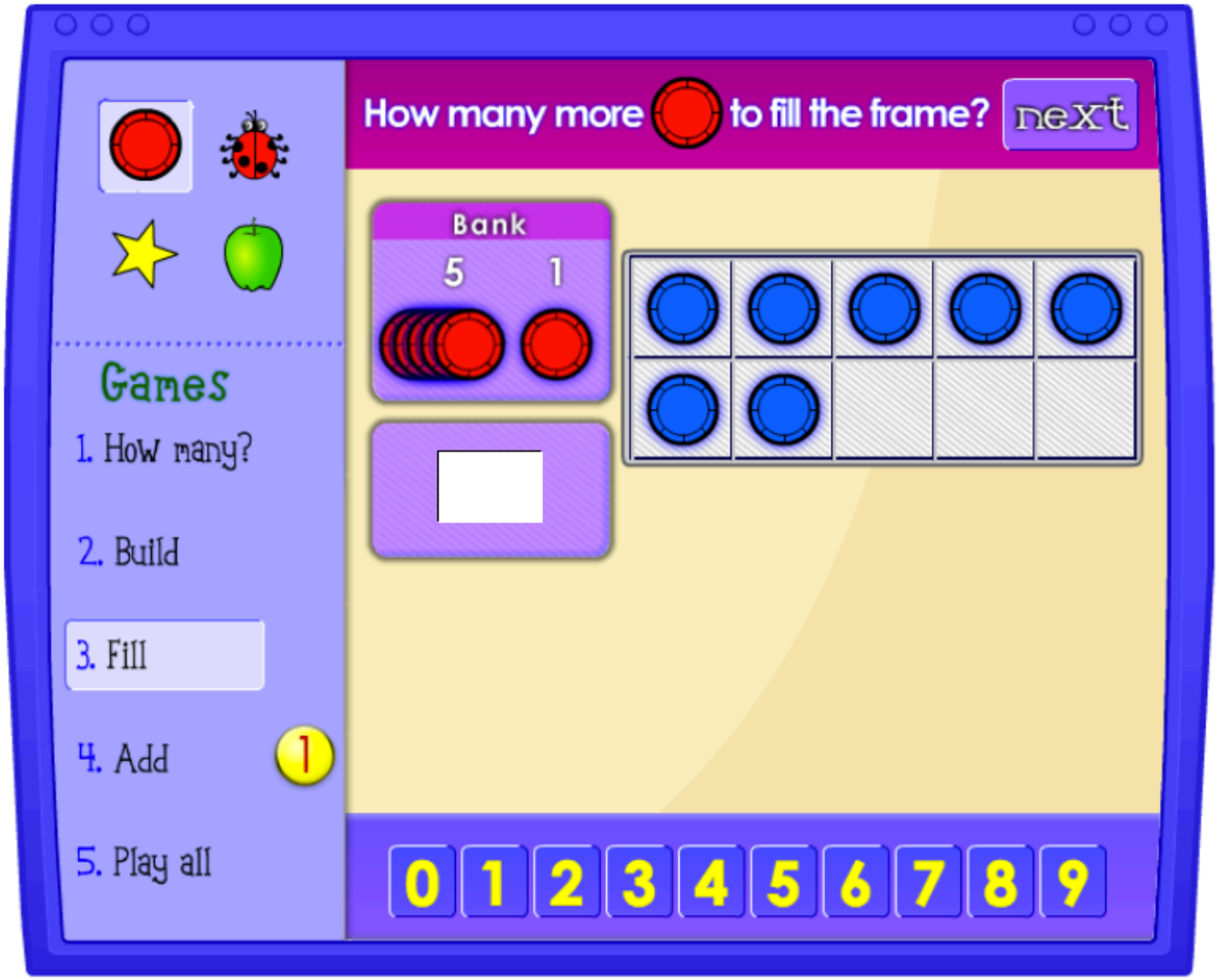
- Crazy Mixed Up Numbers
- How do students change their ten frame?



- 3, 5, 6, 2, 9, 5

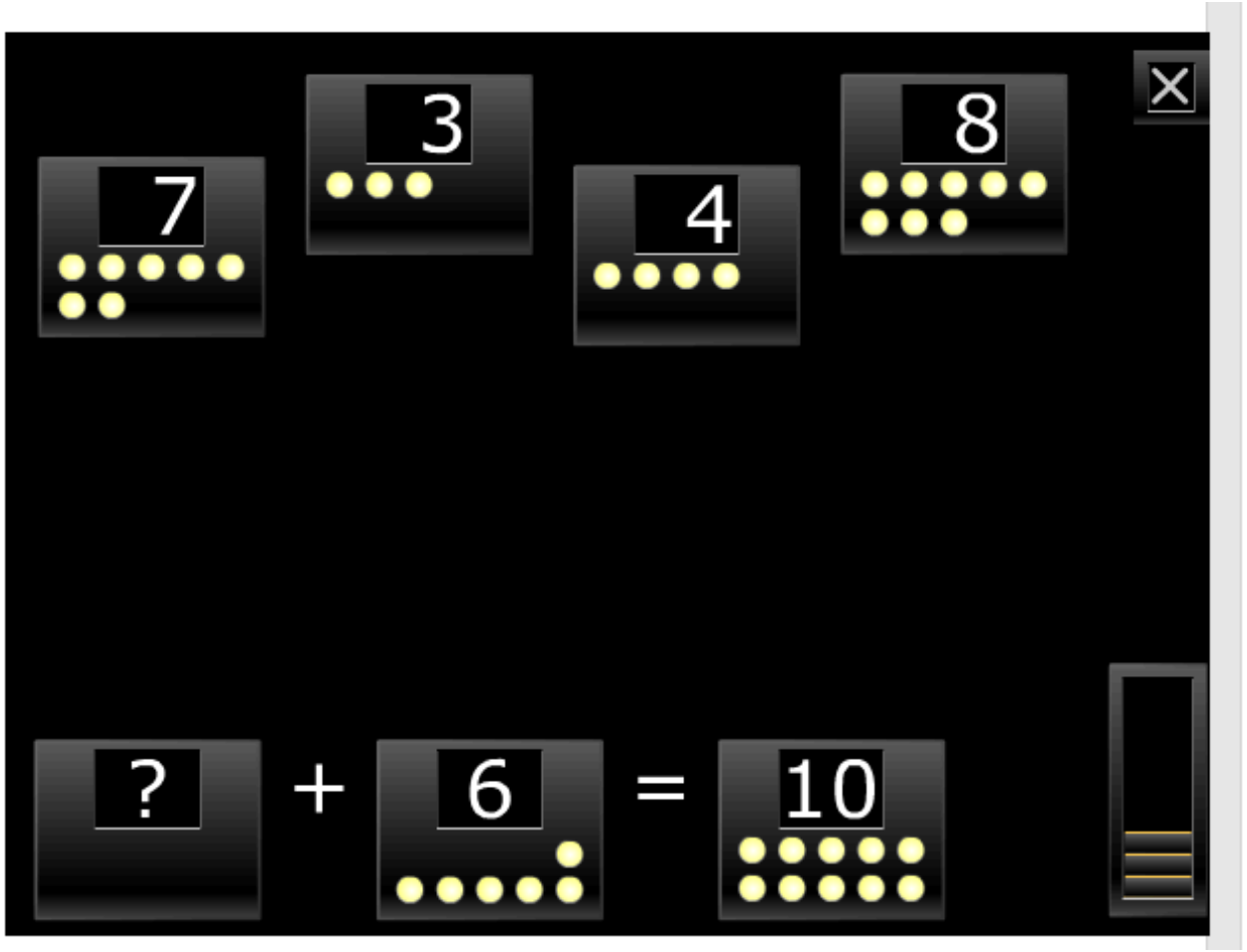
NCTM Illuminations

http://illuminations.nctm.org/Activity.aspx?id=3565



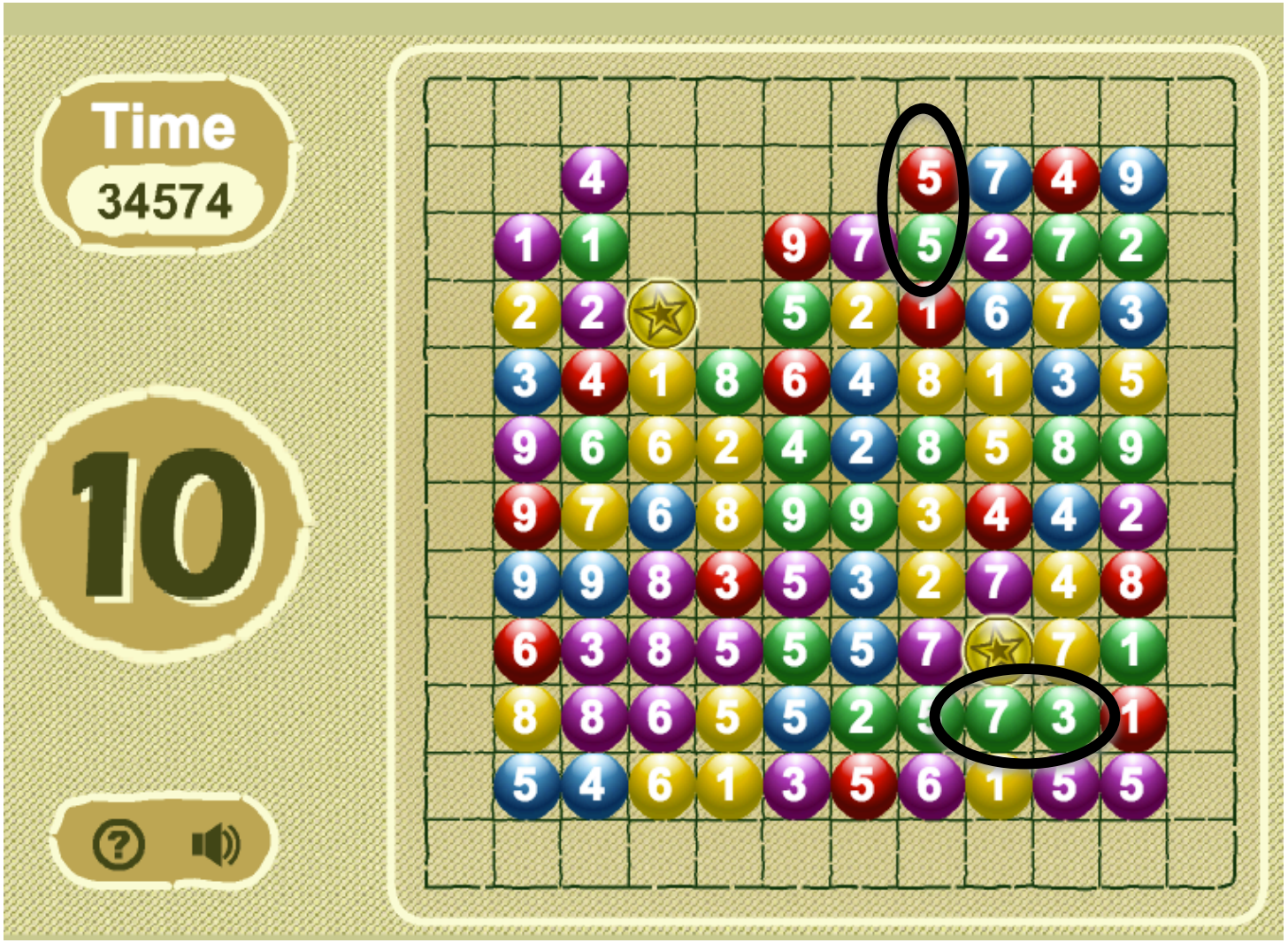
Complements of 10

http://www.classbraingames.com/2009/12/complements-of-ten-an-interactive-math-game/



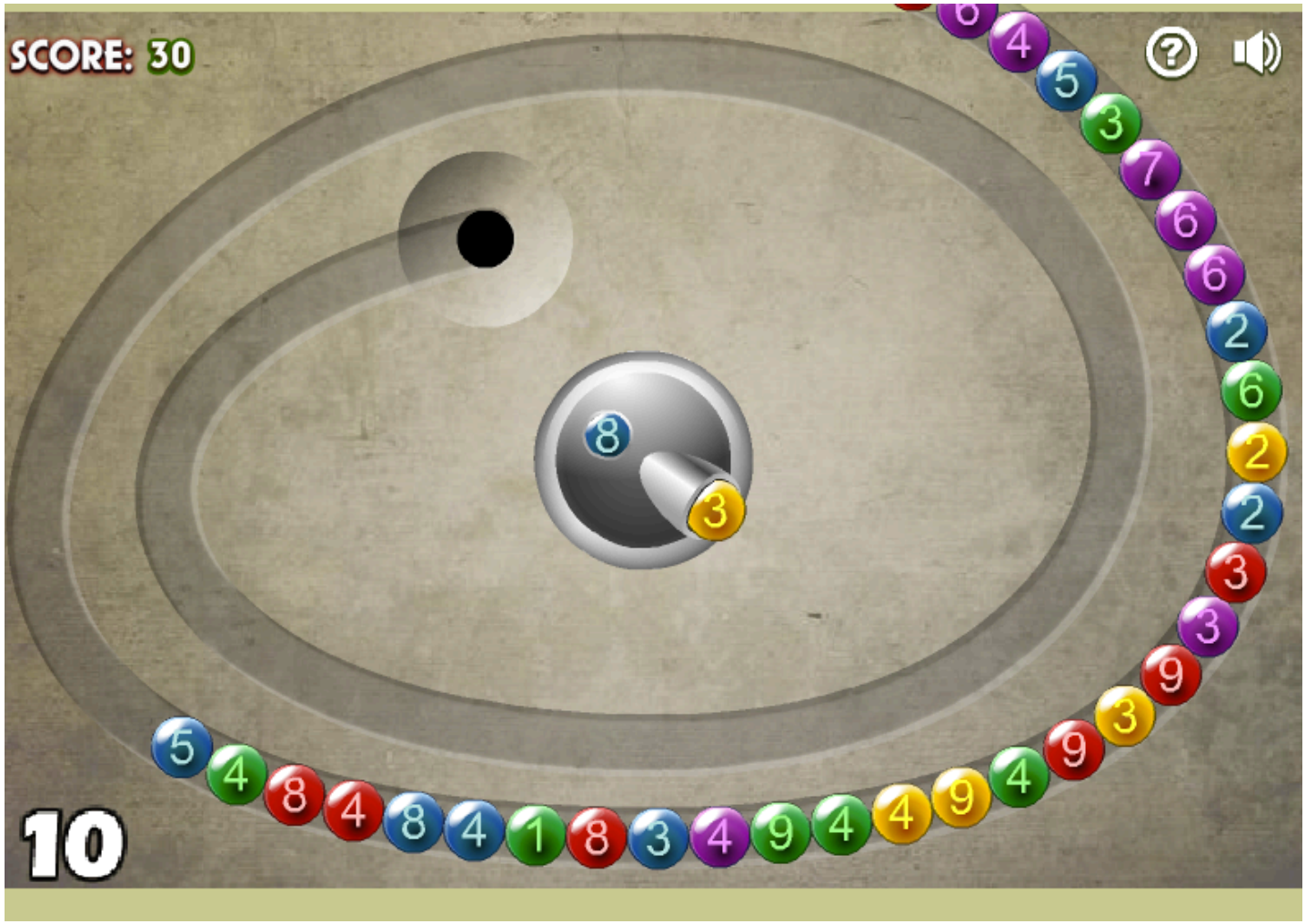
Number Twins: 10

http://www.coolmath-games.com/0-number-twins/10



Math Lines 10

http://www.coolmath-games.com/0-math-lines/10



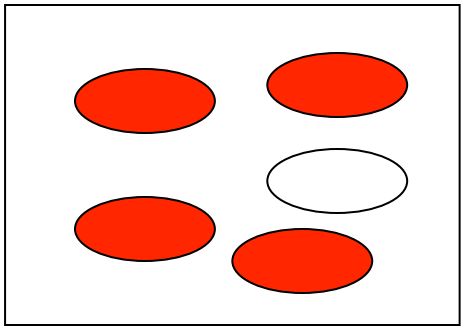
Ten Frame Video – start at 0:00

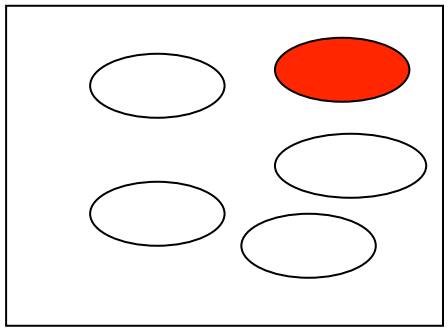
Part Part Whole

- 2-Color Counter Shake Up
- Build it in All Different Ways
- Missing Part Cards (spatial)
- Make a copy

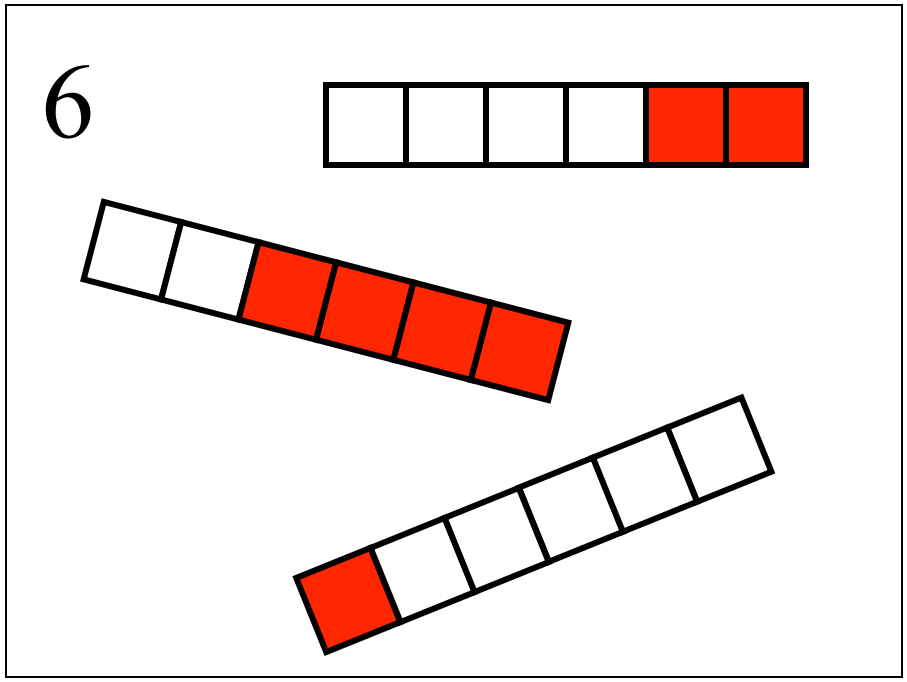
Part-Part-Whole

2-Color Shake Up

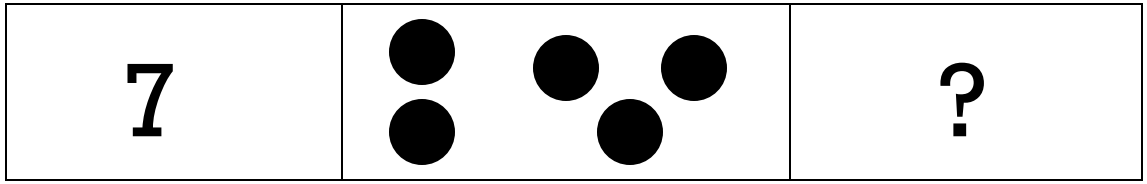




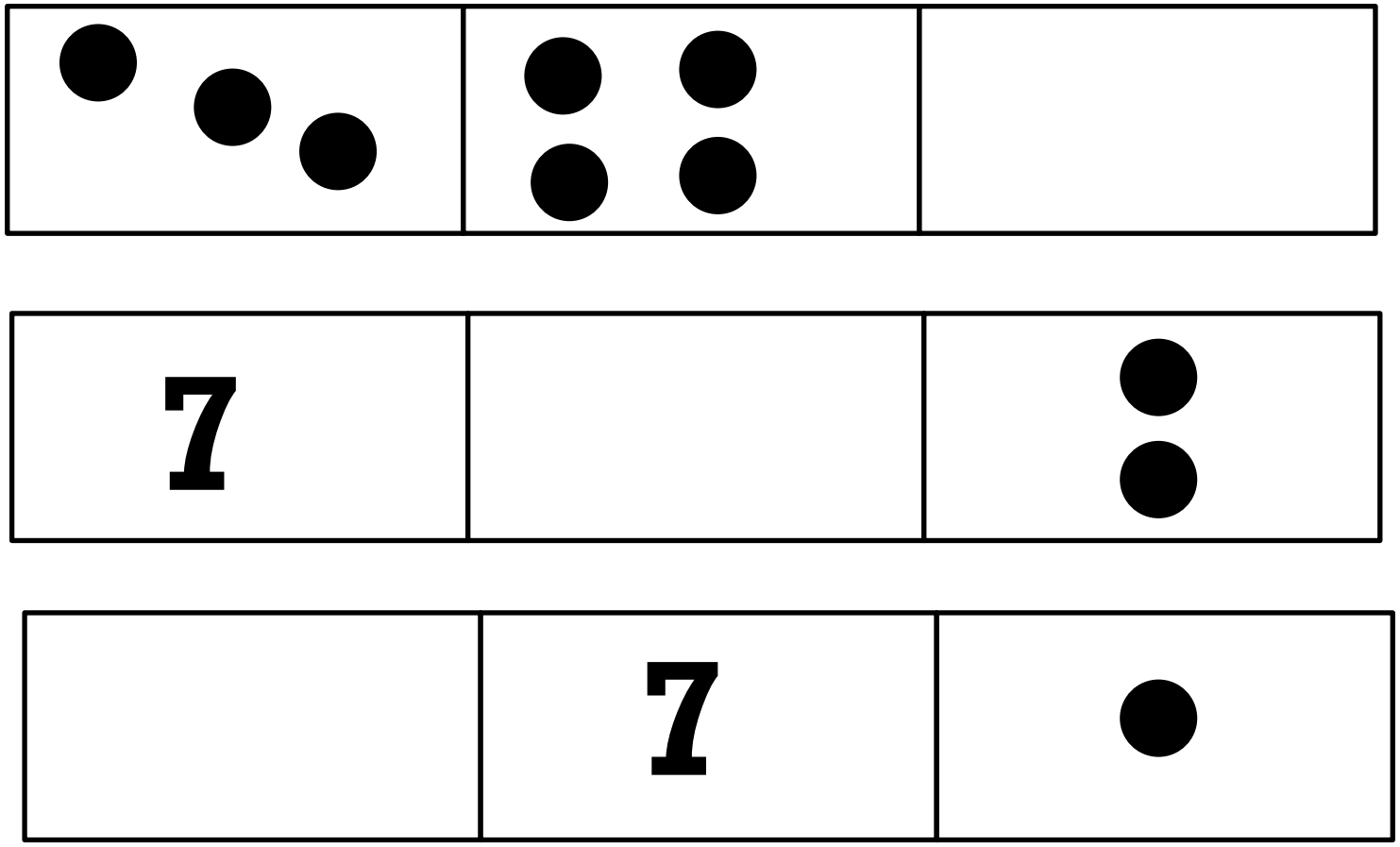
Build it in all Different Ways



Missing Part Cards

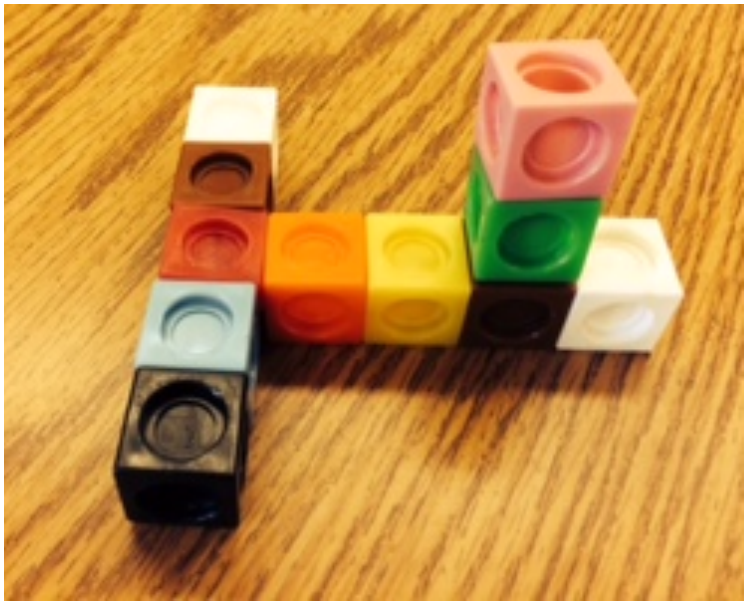


Sample Missing Part Card for Part-Part-Whole



Part Part Whole

- Make a copy (don't touch)
- Write an equation (number sentence)



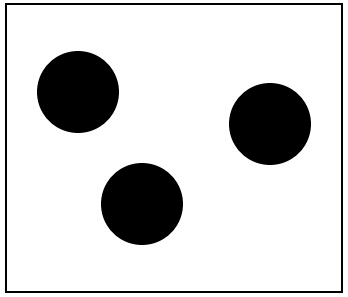
- 5+ 4 +2 = 11 or 1 + 3 + 2 + 5 =11

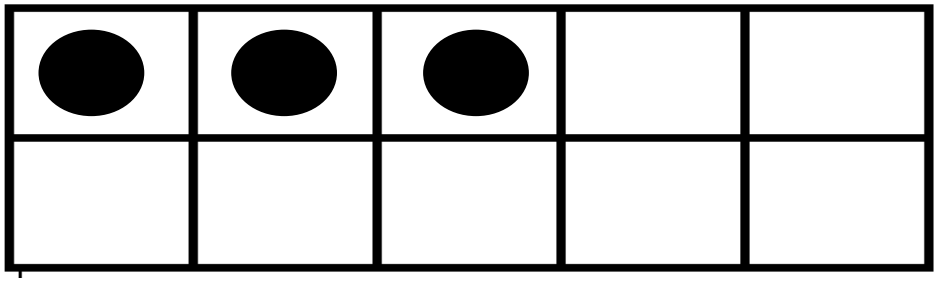
One and Two More, One and Two Less

- Dot Card Flash
- Ten Frame Flash
- Domino Trains

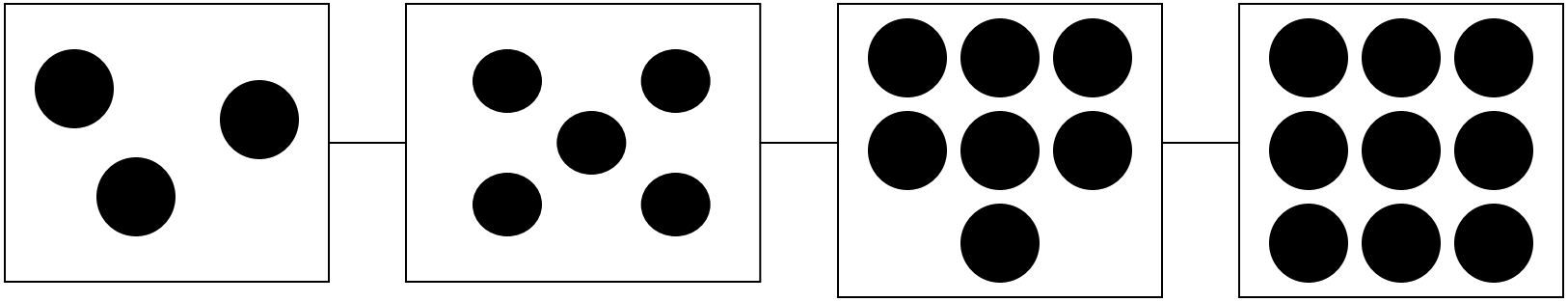
One and Two More, One and Two Less

- Dot Card Flash/Ten Frame Flash
  - One more, two more, one less, two less





- Dot Card Trains
  - One more, two more, one less, two less



Ongoing: Understanding the Meaning of Addition and Subtraction (CCSSM)

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

Number Relationship Strategies and the 14 "Hard Facts"

- Complete Addition Fact Strategy Table from Number Relationships
- "Hard Facts" (Addends Make Ten)
- Master Addition Facts First– Then Subtraction Facts

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Key
1-2 (one/two more)
S (spatial)
10 (ten fact)
D (doubles)
10+ (ten plus)
MT (make ten double ten frame)

Basic Facts - Addition

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Key

1-2 (one/ two more)

S (spatial)

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MT (make ten double ten frame)

Basic Facts - Addition

	1	2	3	4	5	6	7	8	9	10
1										
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Basic Facts - Addition

	1	2	3	4	5	6	7	8	9	10
1										
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8										
9										
10										

Key
1-2 (one/two more)
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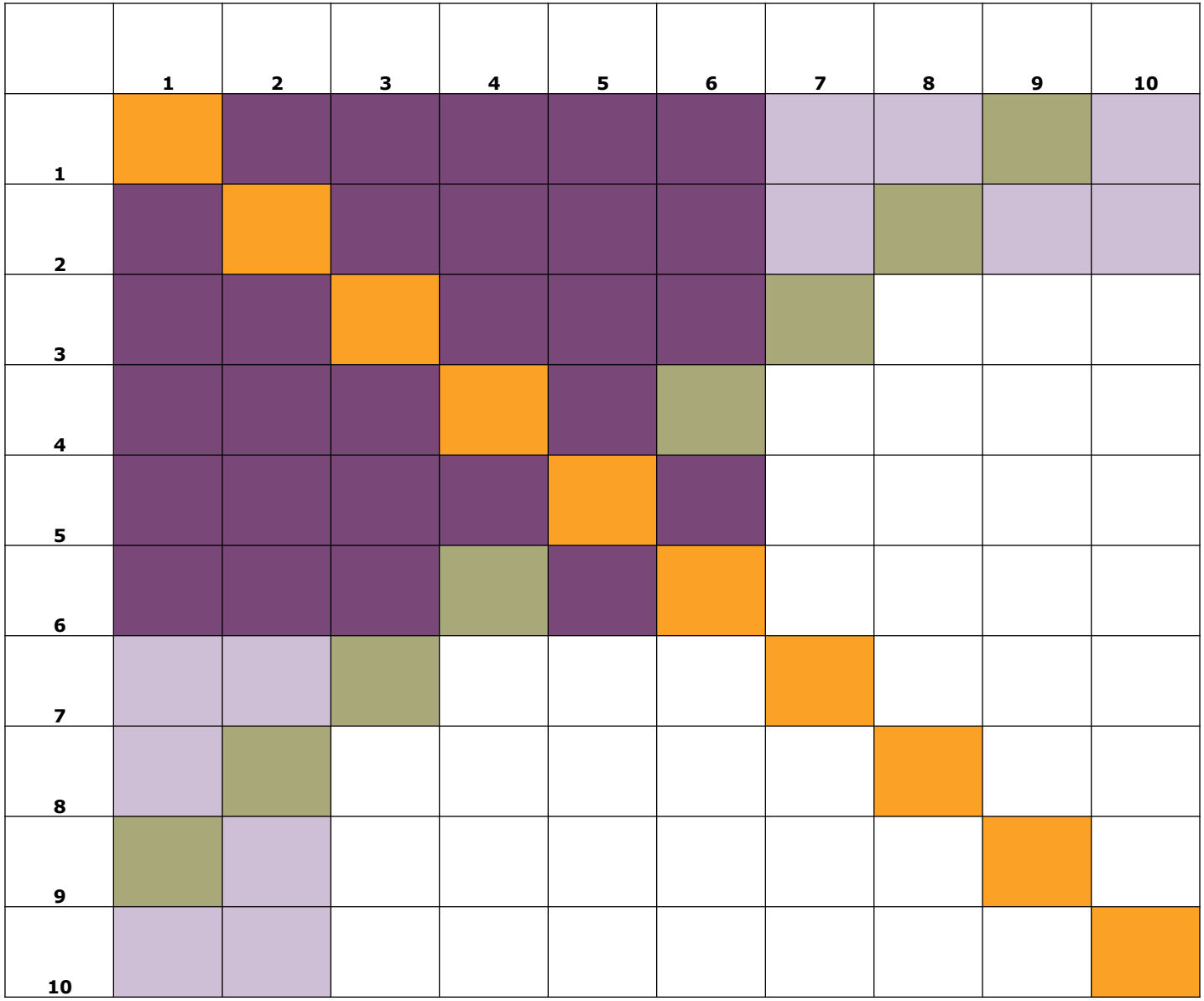
Basic Facts - Addition



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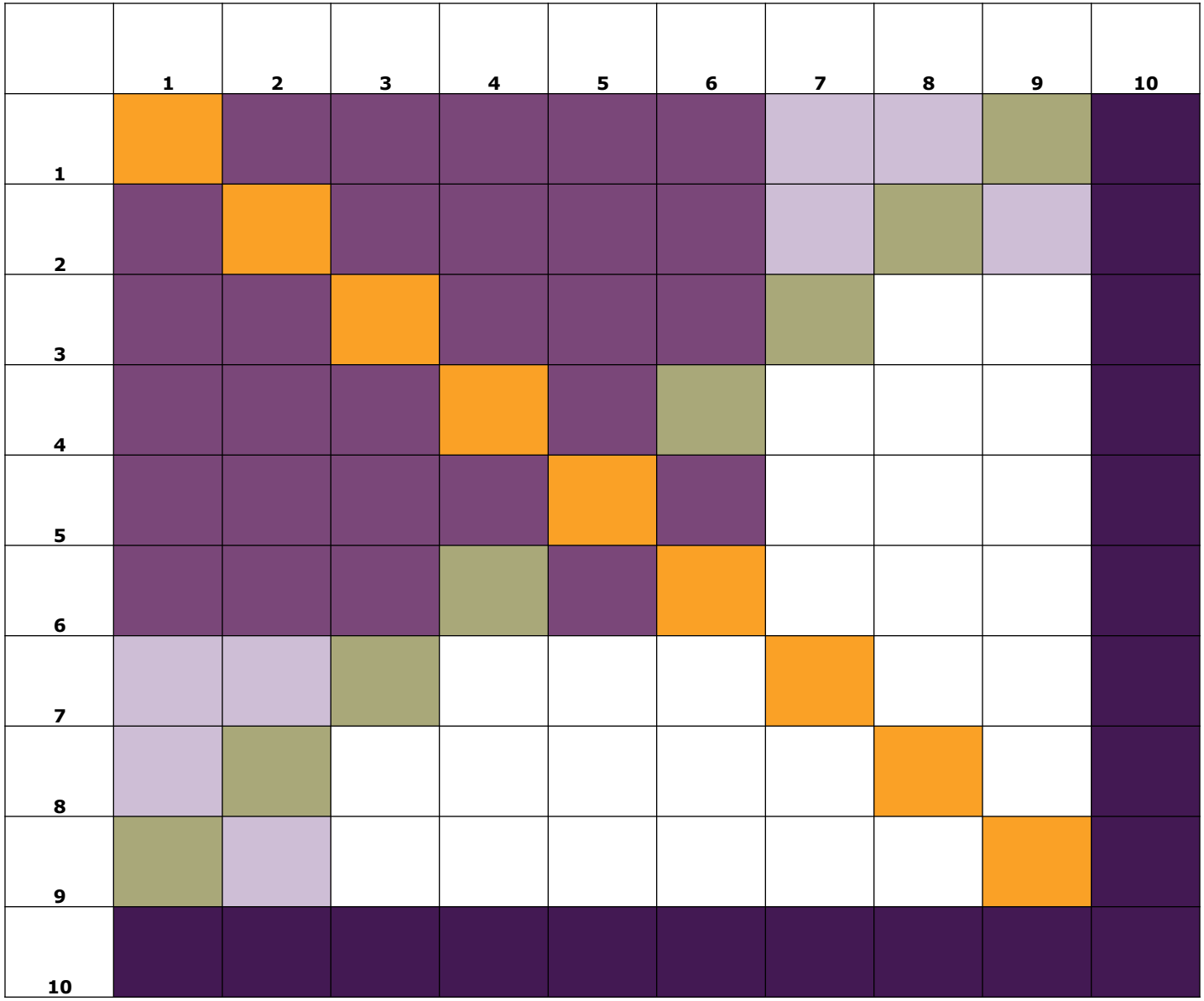
Basic Facts - Addition



Key
1-2 (one/two more)
S (spatial)
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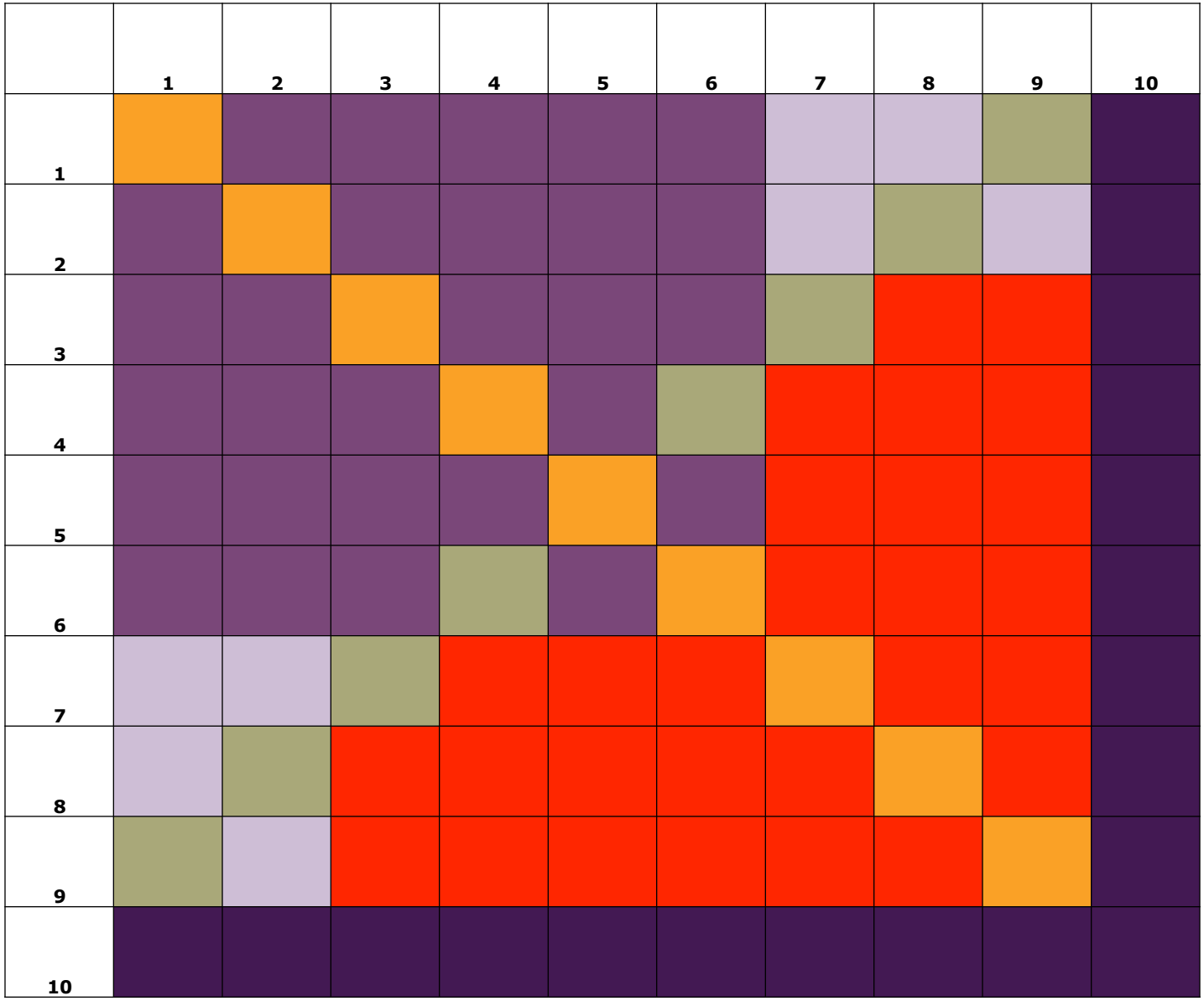
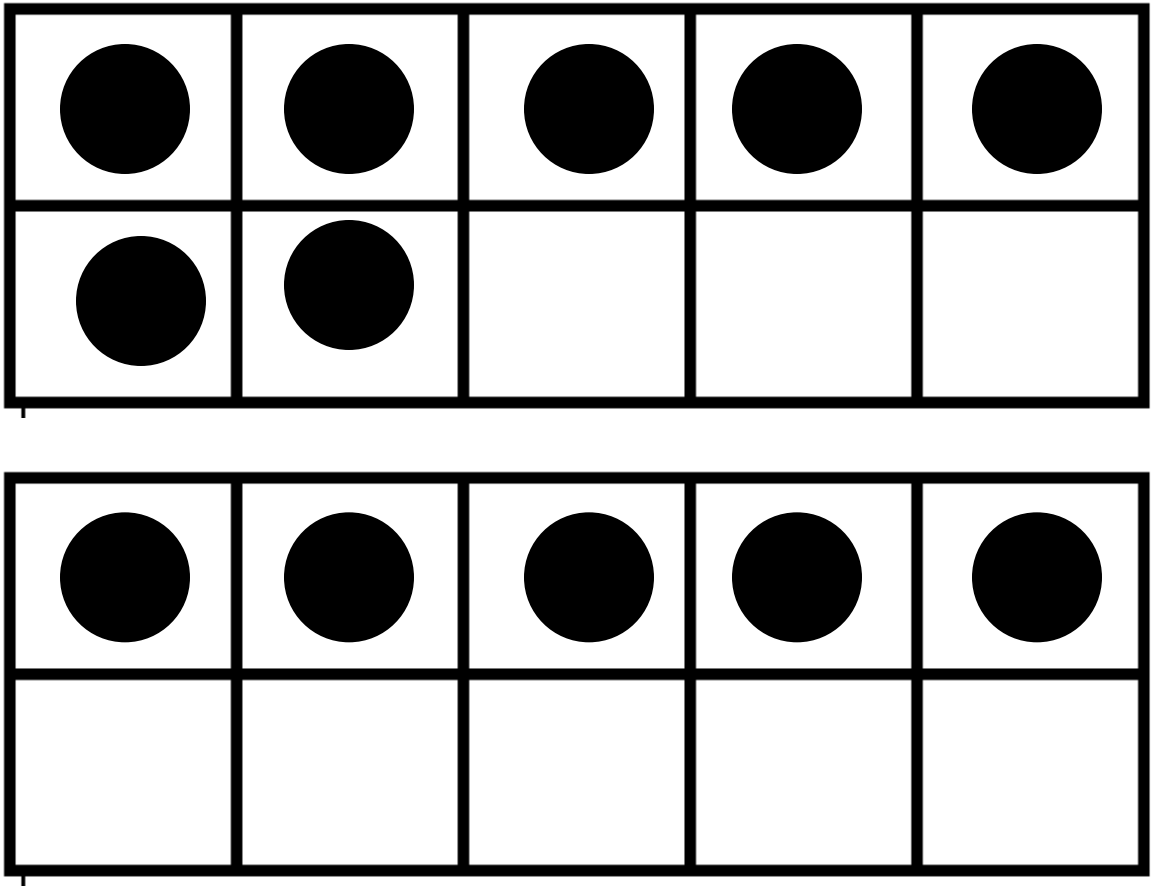
Basic Facts - Addition



Key
1-2 (one/two more)
S (spatial)
10 (ten fact)
D (doubles)
10+ (ten plus)
MT (make ten double ten frame)

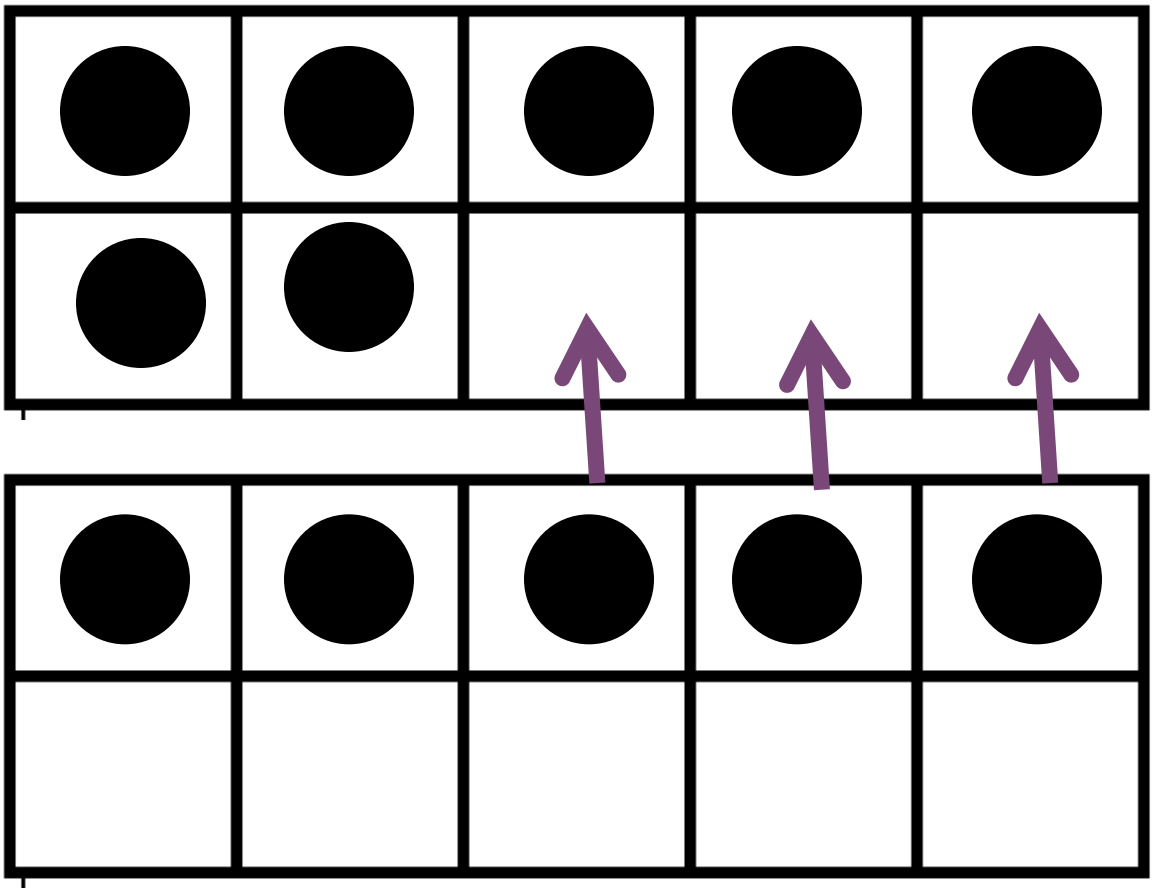
28 Facts Remain, Commutative Property, 14 Facts for the Double Ten Frame!!

$$\begin{array}{r} 7 \\ +5 \\ \hline \end{array}$$

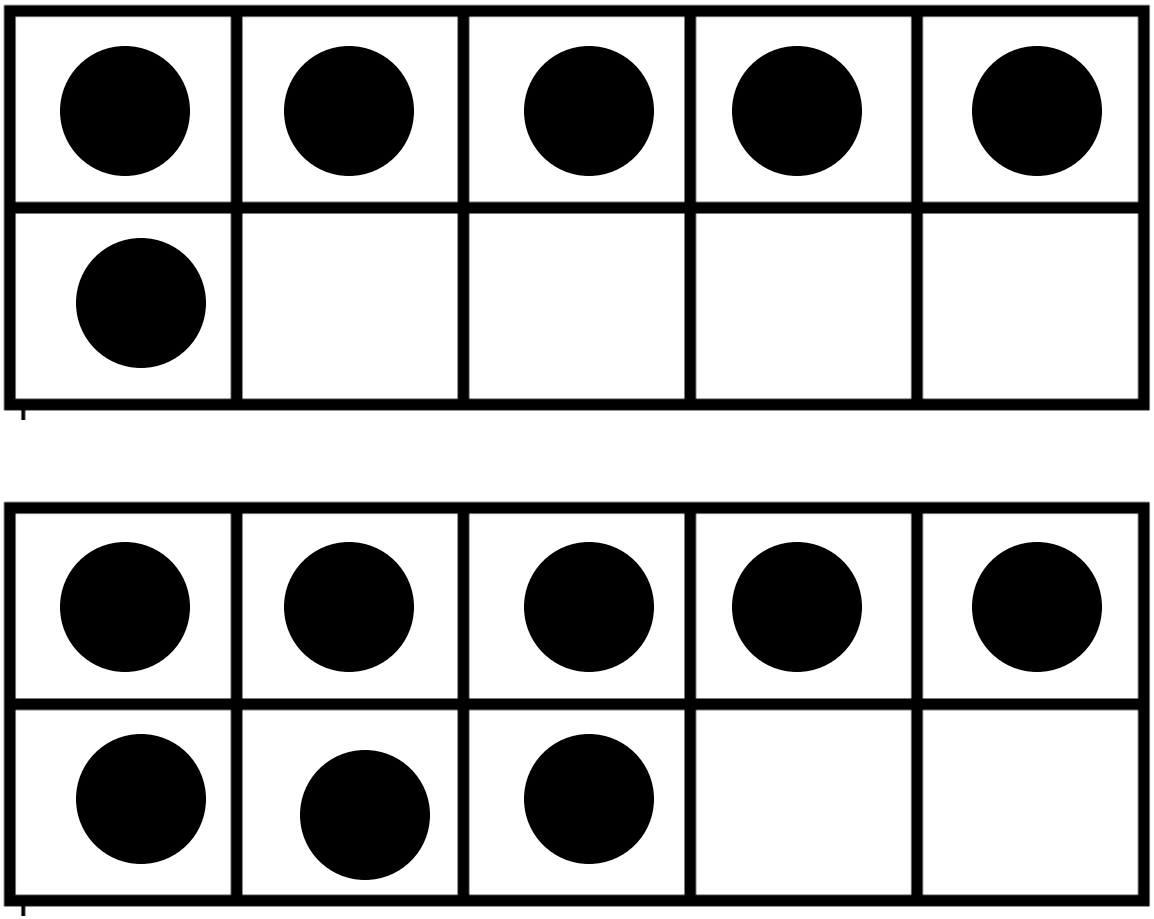


7

+5



$$\begin{array}{r} 6 \\ +\,8 \\ \hline \end{array}$$



8 + 4

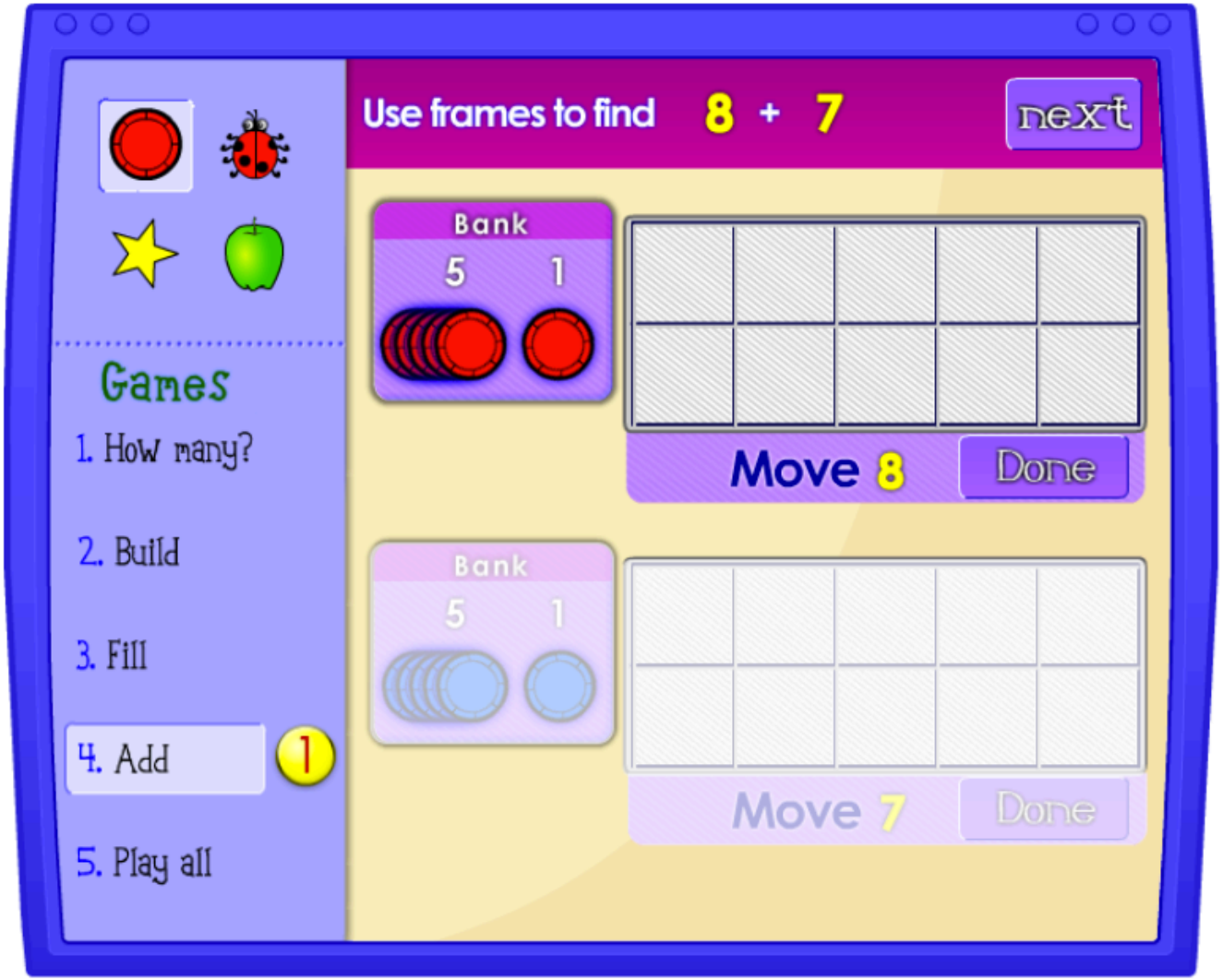
8 + 4

Think:

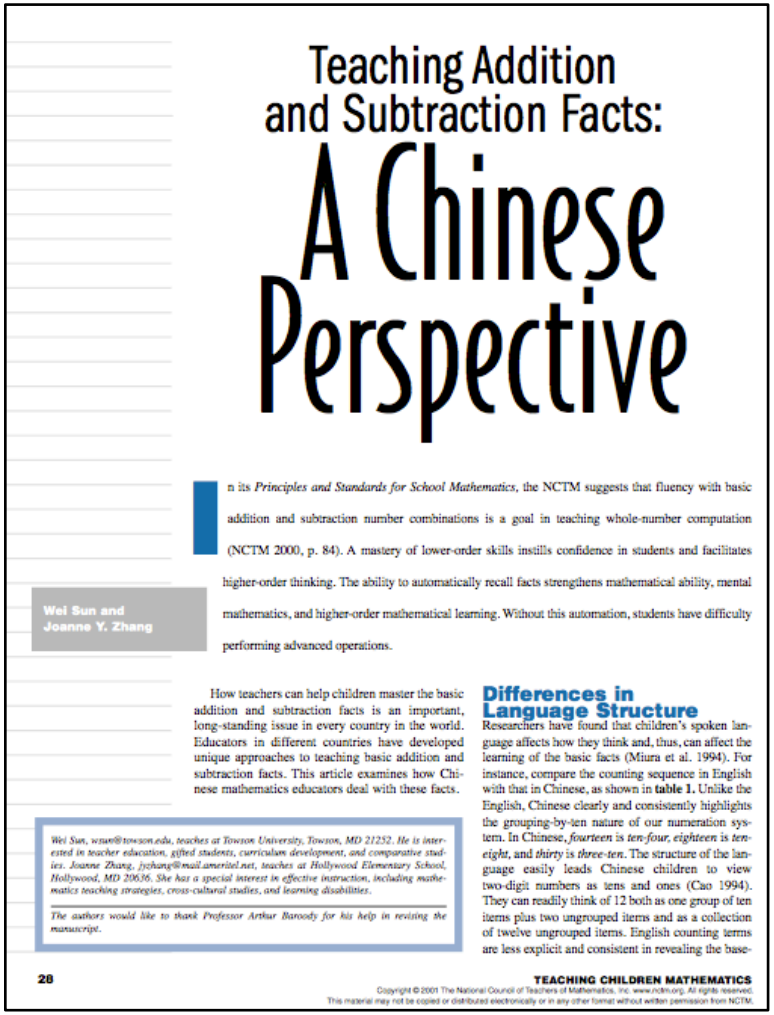
- What number in the ten frame?
- How many more to make 10?
- Decompose
- Sum

NCTM Illuminations

http://illuminations.nctm.org/Activity.aspx?id=3565



Build Place Value Understanding for Numbers from 11-20



Teaching Addition and Subtraction Facts: A Chinese Perspective

In its *Principles and Standards for School Mathematics*, the NCTM suggests that fluency with basic addition and subtraction number combinations is a goal in teaching whole-number computation (NCTM 2000, p. 84). A mastery of lower-order skills instills confidence in students and facilitates higher-order thinking. The ability to automatically recall facts strengthens mathematical ability, mental mathematics, and higher-order mathematical learning. Without this automation, students have difficulty performing advanced operations.

Wei Sun and Joanne Y. Zhang

How teachers can help children master the basic addition and subtraction facts is an important, long-standing issue in every country in the world. Educators in different countries have developed unique approaches to teaching basic addition and subtraction facts. This article examines how Chinese mathematics educators deal with these facts.

Wei Sun, wsun@towson.edu, teaches at Towson University, Towson, MD 21252. He is interested in teacher education, gifted students, curriculum development, and comparative studies. Joanne Zhang, jychang@mail.ameritel.net, teaches at Hollywood Elementary School, Hollywood, MD 20636. She has a special interest in effective instruction, including mathematics teaching strategies, cross-cultural studies, and learning disabilities.

The authors would like to thank Professor Arthur Baroody for his help in revising the manuscript.

Differences in Language Structure

Researchers have found that children's spoken language affects how they think and, thus, can affect the learning of the basic facts (Miura et al. 1994). For instance, compare the counting sequence in English with that in Chinese, as shown in **table 1.** Unlike the English, Chinese clearly and consistently highlights the grouping-by-ten nature of our numeration system. In Chinese, *fourteen* is *ten-four*, *eighteen* is *ten-eight*, and *thirty* is *three-ten*. The structure of the language easily leads Chinese children to view two-digit numbers as tens and ones (Cao 1994). They can readily think of 12 both as one group of ten items plus two ungrouped items and as a collection of twelve ungrouped items. English counting terms are less explicit and consistent in revealing the base-

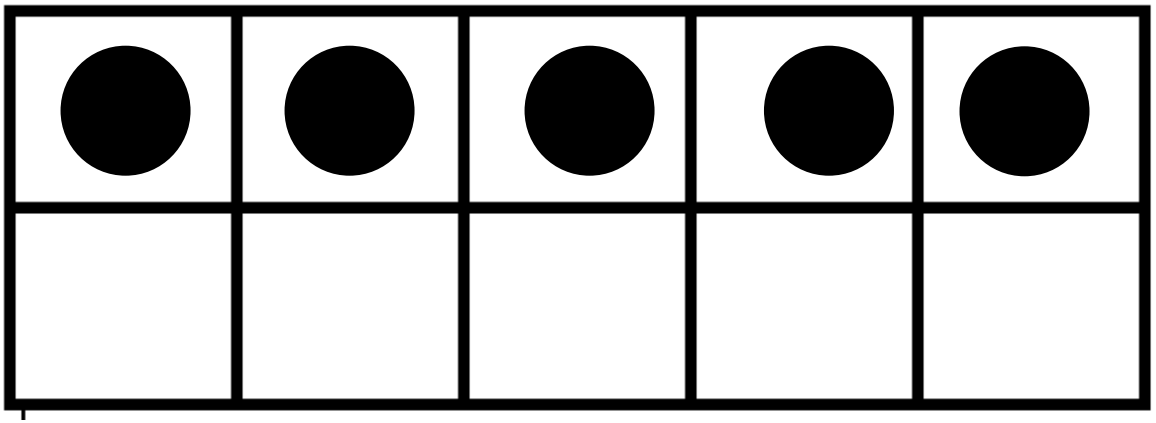
Master Addition Facts **BEFORE** Subtraction Facts

13 – 5 = ?

- Think Addition with Anchor of 10:

5 + ? = 13

5 + **5** = 10 and 10 + **3** = 13



13 – 5 = ?

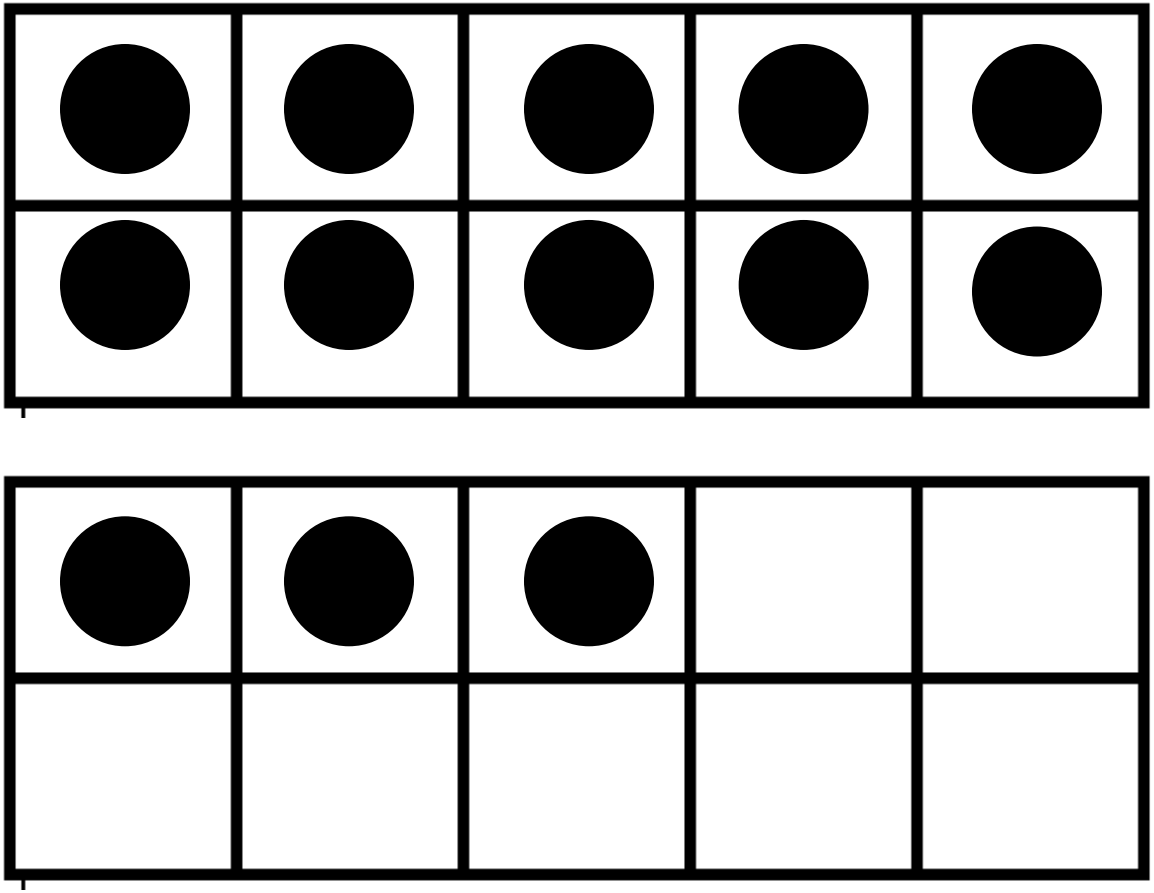
- Think Subtraction with Anchor of 10:

$$13 - 3 = 10$$

$$10 - 2 = \mathbf{8}$$

See ten frames on next slide…

13 – 5 = ?

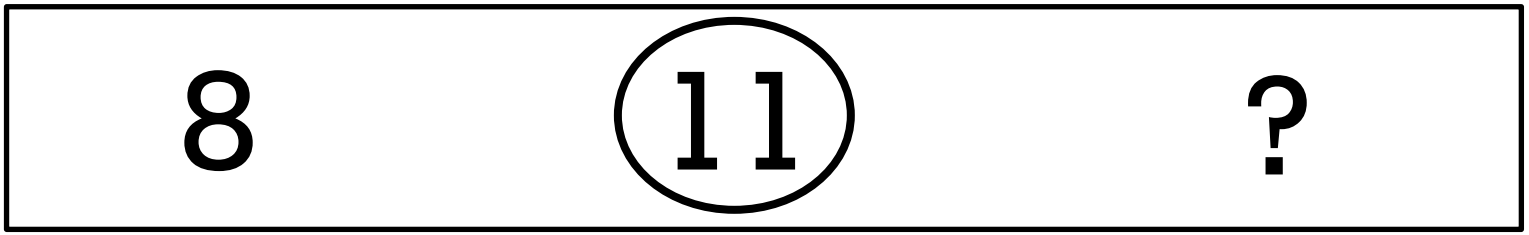


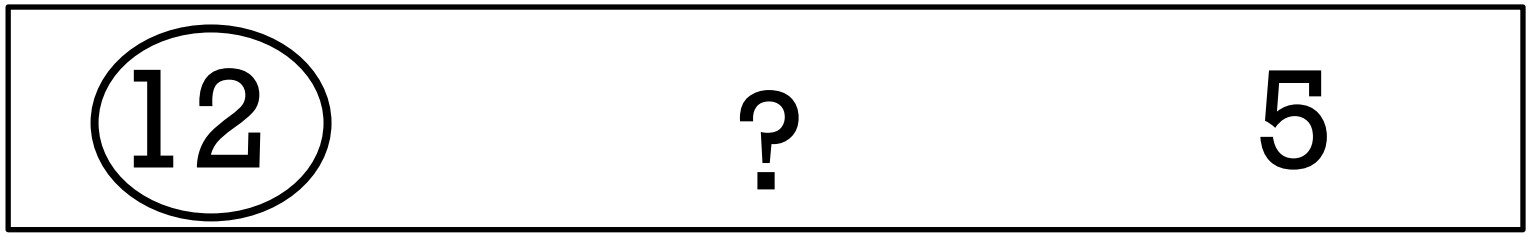
15 – 8 = ?

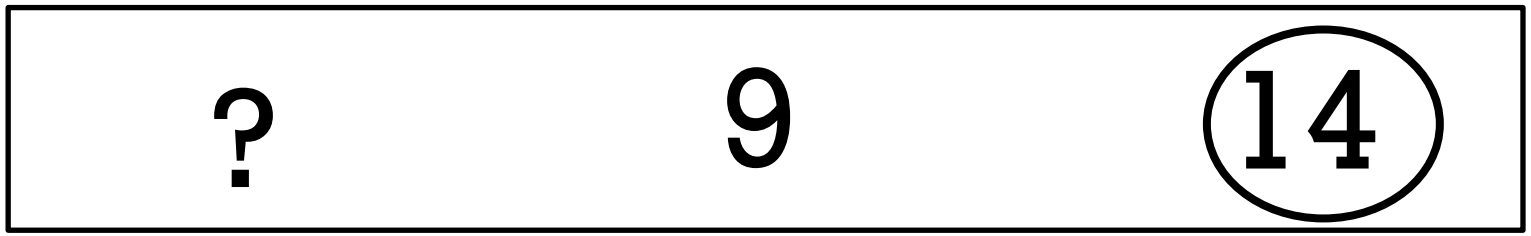
- Think Addition with Anchor of 10:

- Think Subtraction with Anchor of 10:

Missing Part Cards – 14 Hard Facts

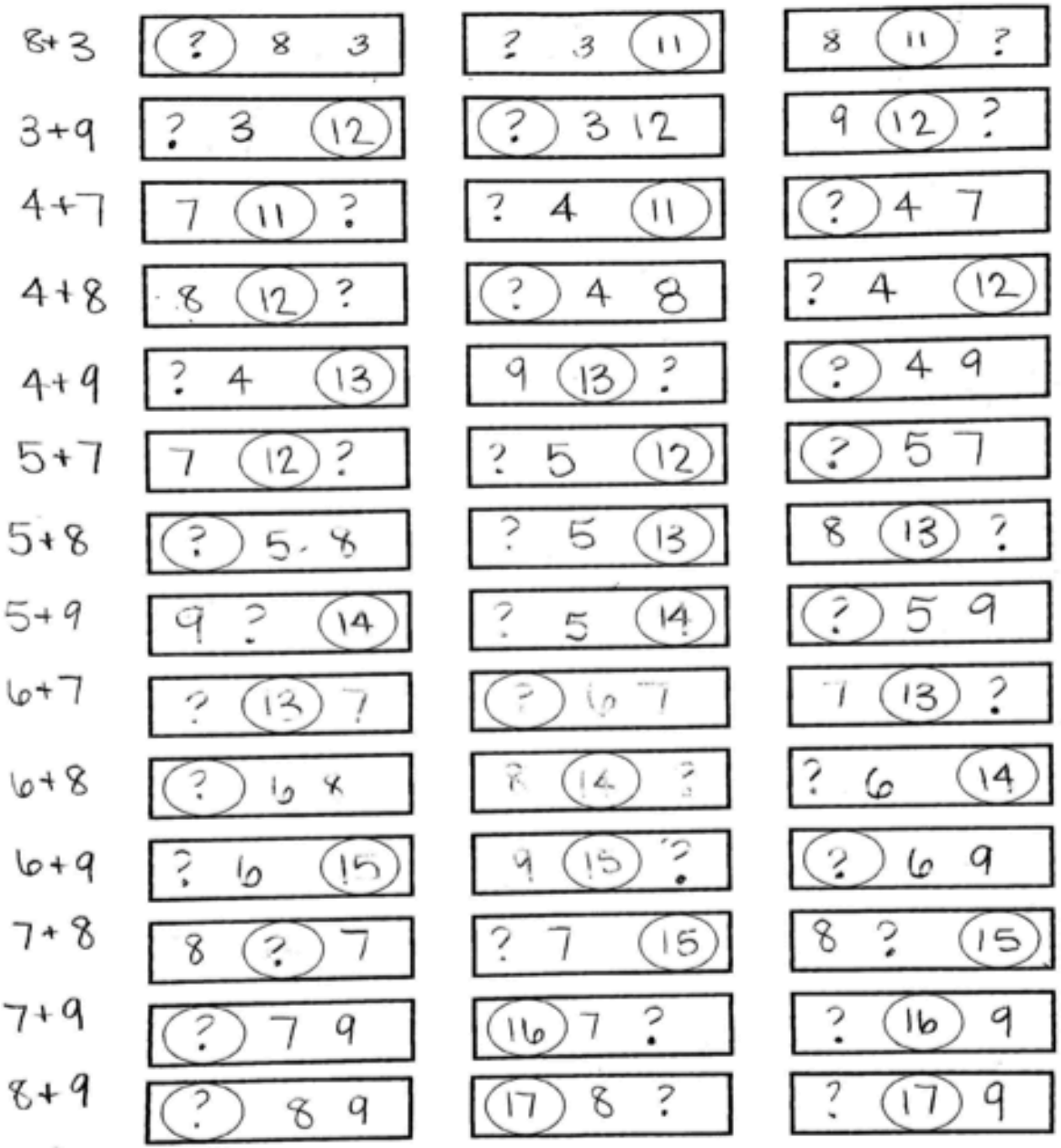




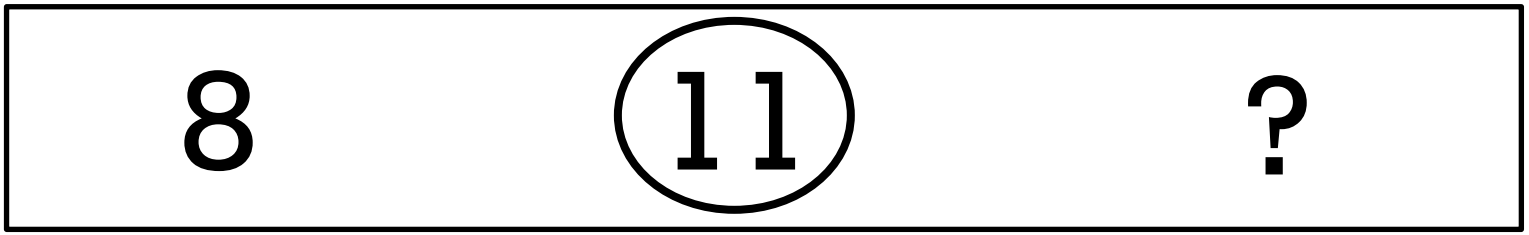


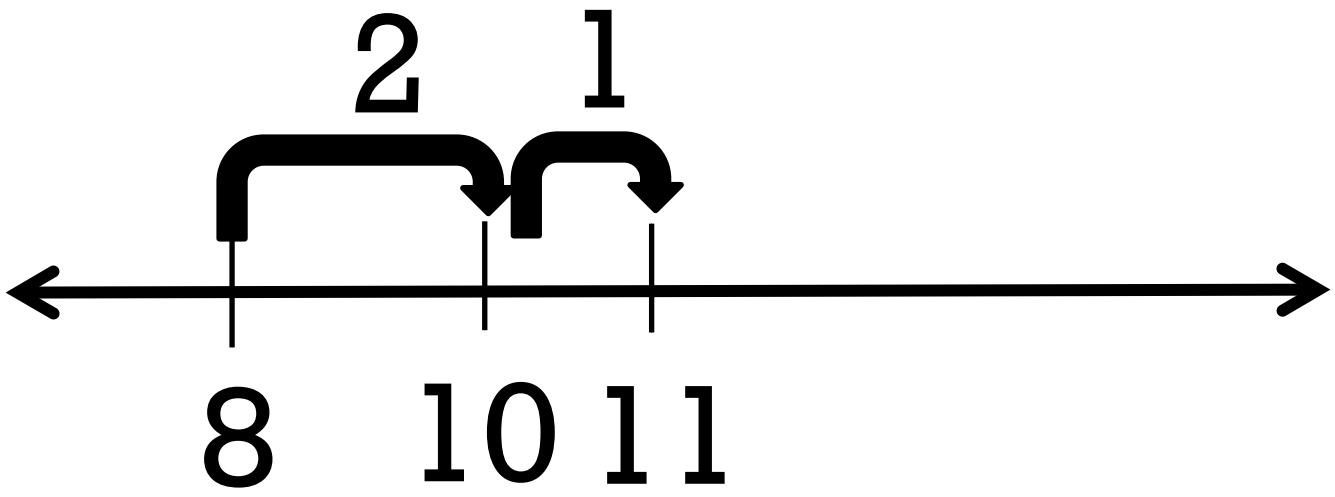
Missing Part Cards:

14 Hard Facts



Missing Part Cards – 14 Hard Facts





Number line model

Numbers 10-20 and beyond

- Addends Make Ten (with Ten Frames)

  8 +6 becomes 10 +4

- Math Solutions game: Make 20
- Ten frame flash (Numbers 11-59)

Game 9

Race to 20

Time

20 minutes

Materials

dice, 1 per pair of students

double ten-frame (Reproducible D), 1 per student

counters, 25 each of two colors per student

Extension

dice with a small sticker on one face, 1 per pair of students

double ten-frame (Reproducible D), 1 per student

counters, 25 each of two colors per student

Overview

In this game, students roll a die and place that number of counters on a double ten-frame in an effort to reach 20 first. This game builds students' understanding of landmark numbers, specifically ten and twenty. Through the use of two colors of counters, students decompose the number twenty and use number strings to compose twenty. The game also naturally lends itself to encouraging students to compare numbers. Further ideas for introducing and summarizing the game are included in the "Teacher Reflection" section. In the assessment, students use their experience with *Race to 20* to compare quantities and connect the game to number sentences.

Related Lessons

You might teach the following lessons first:

- R-4 Number Strings
- R-5 Adding Nine
- R-6 Sums One More Than Ten

Consider these lessons as a follow-up:

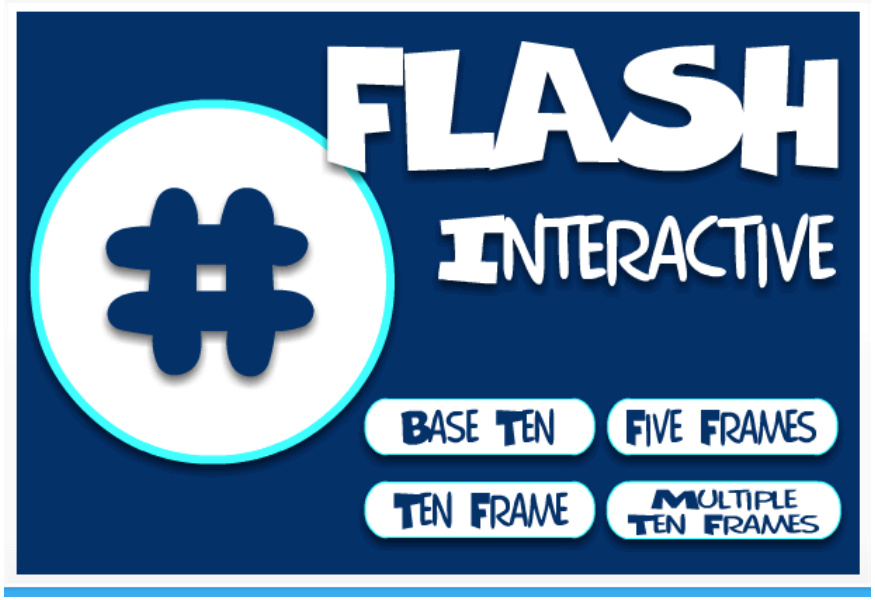
- P-1 Two-Color Counters
- P-3 Mystery Sums

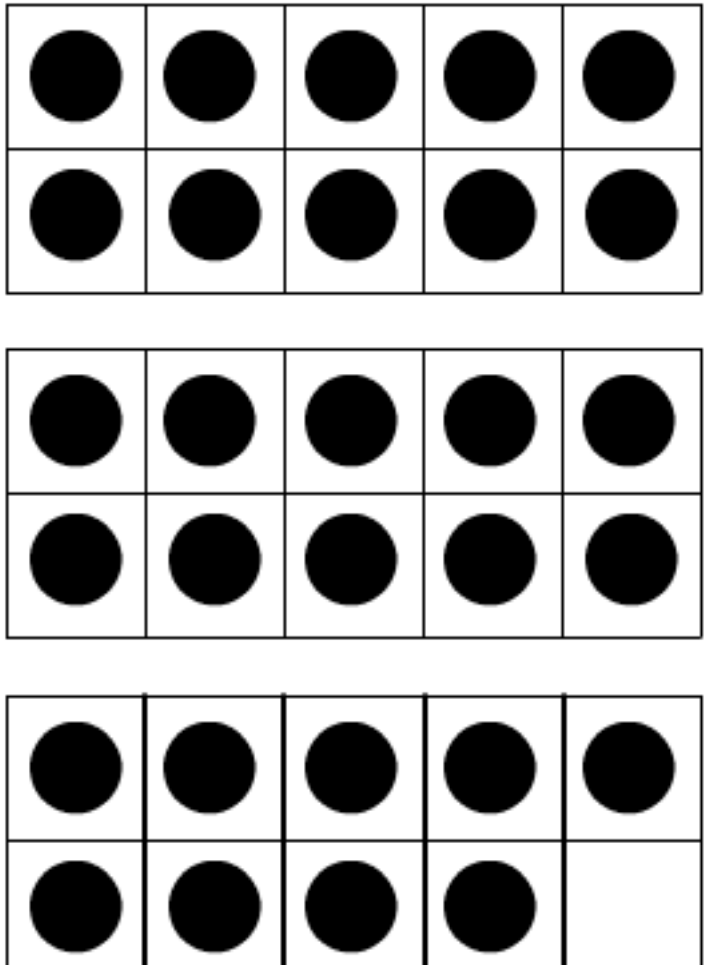
Key Questions

- How many more counters do you need to have ten?
- How many more counters do you need to have twenty?

FLASH INTERACTIVE

http://www.fuelthebrain.com/games/number-flash/





Concern - Timed Tests

- Assessment Purposes
- Doesn't help with strategies or promote instant recall

Phases of Learning Basic Facts

- Modeling and/or Counting All or Counting On
- Using reasoning strategies
- Instant recall

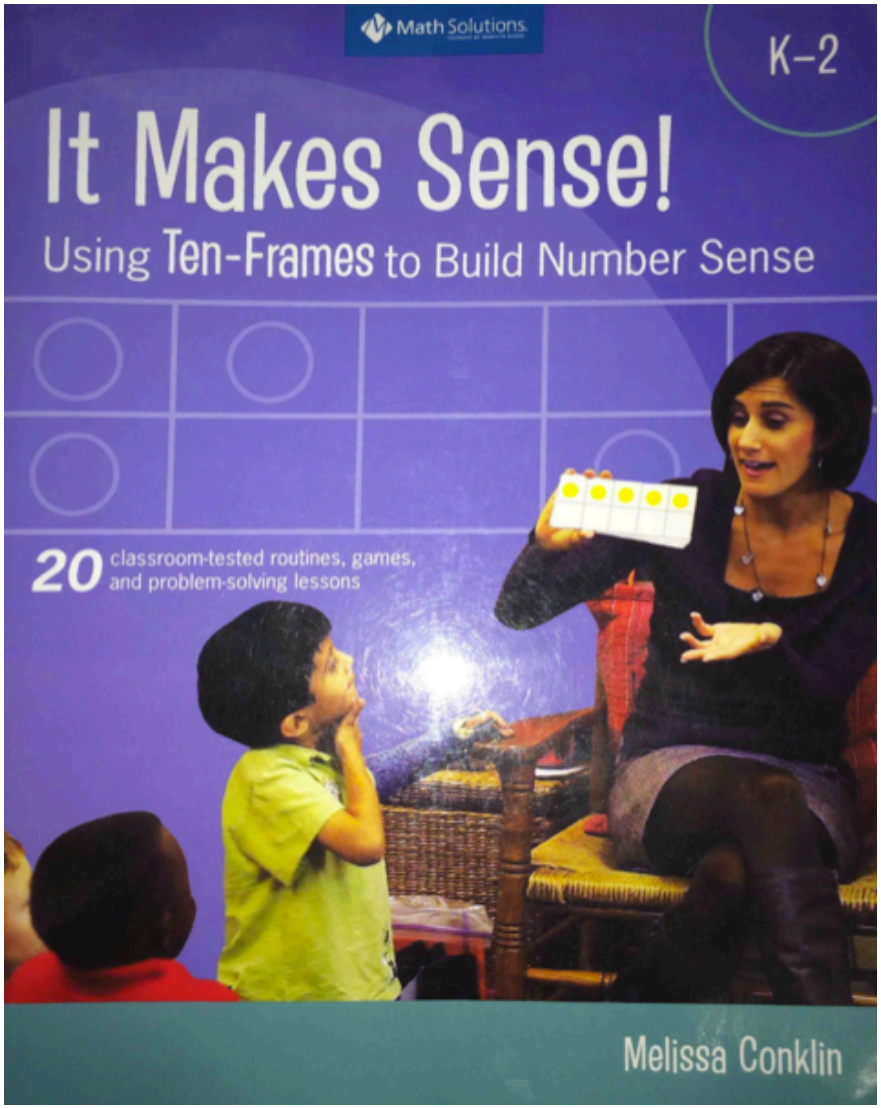
CCSSM

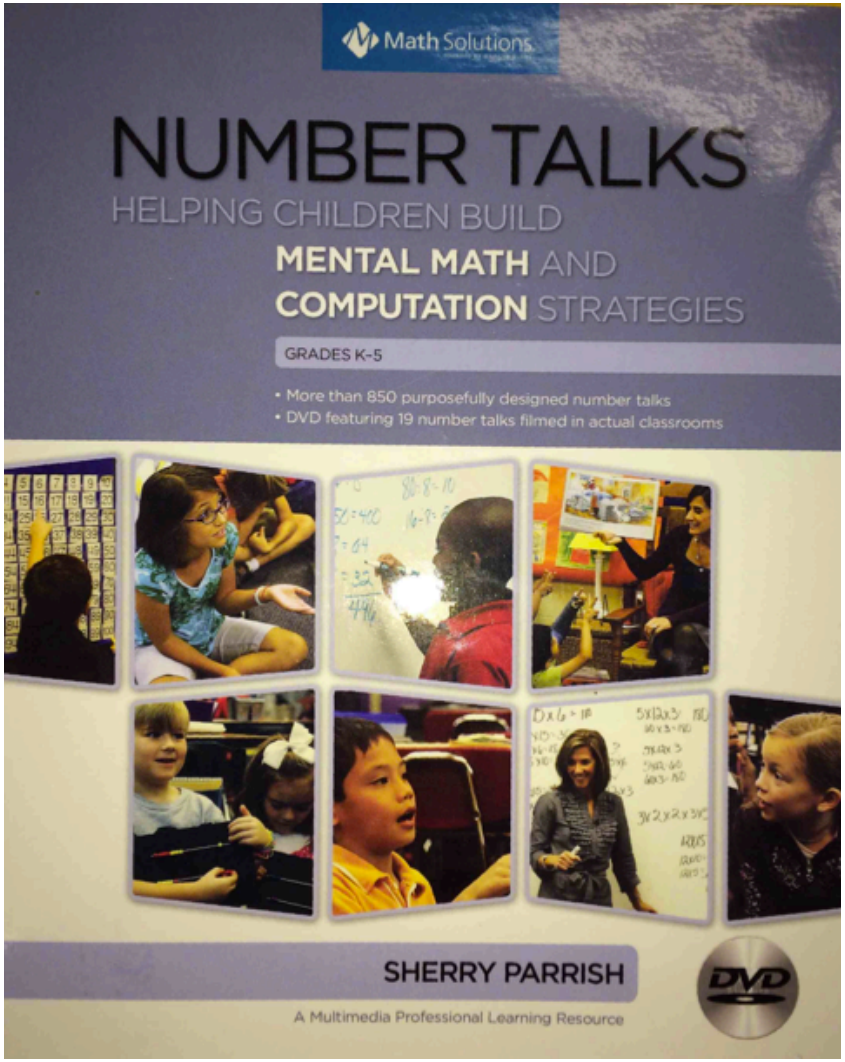
- Kindergarten: **Decompose numbers** less than or equal to 10 into pairs **in more than one way**, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
- Kindergarten: **For any number from 1 to 9, find the number that makes 10** when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- Kindergarten: **Fluently add and subtract within 5.**

CCSSM

- First Grade: Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; **making ten** (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14)**; decomposing a number leading to a ten** (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

- Second Grade: Fluently **add and subtract within 20 using mental strategies** By end of Grade 2, know from memory all sums of two one-digit numbers.


Math Solutions
K–2
It Makes Sense!
Using Ten-Frames to Build Number Sense
20 classroom-tested routines, games, and problem-solving lessons
Melissa Conklin


Math Solutions
NUMBER TALKS
HELPING CHILDREN BUILD
MENTAL MATH AND
COMPUTATION STRATEGIES
GRADES K–5
• More than 850 purposefully designed number talks
• DVD featuring 19 number talks filmed in actual classrooms
SHERRY PARRISH
A Multimedia Professional Learning Resource
DVD

Other Resources

- Bay-Williams, J. and Kling, G. Enriching Addition and Fact Mastery Through Games. Teaching Children Mathematics, November 2014, Vol. 20, Issue 4.
- Conklin, M. It Makes Sense! Using Ten-Frames to Build Number Sense. Math Solutions, 2010. (Race to 20 Game and Base Ten Riddles) http://mathsolutions.com/documents/9781935099109_lessong9.pdf
- Sun, W. and Zhang, J. Teaching Addition and Subtraction Facts: A Chinese Perspective. Teaching Children Mathematics, September 2001.
- Van de Walle, J. Elementary and Middle School Mathematics, Pearson, 2004.
- Parrish, S. Number Talks! Math Solutions, 2010.