PARTITIONING: The Missing Link in Building Fraction Knowledge and Confidence

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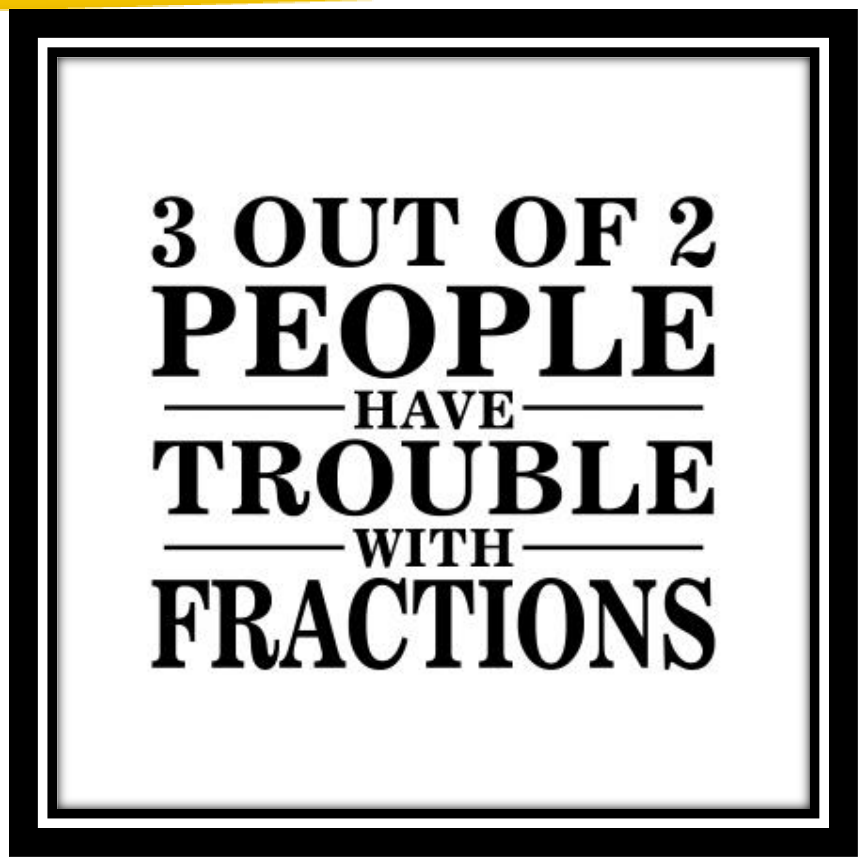
NCTM, 2015

Thanks for being here at 8:00 a.m. on a SATURDAY!!!!!!!



WHY Do Our Kids Struggle With Fractions?

- Only 50% of 8th-grade students successfully arranged 2/7, 1/12, and 5/8 from least to greatest (NAEP)
- Fewer than 30% of 17-year-olds correctly translated 0.029 as 29/1000 (NAEP)
- Only 24% of 13-17 year olds identified 2 as the estimate sum for 12/13 + 7/8, while a greater percentage identified 19 or 21 as the estimated sum. (NAEP)
- More than half the population of adults are not proportional thinkers. (Van de Walle)

What Do YOUR Students Know?

34

3
—
4

1. **Parts of Wholes** or Parts of Sets
2. **Quotient** (sharing equal portions)
3. **Ratio** (Comparing 2 quantities)
4. **Operator** (stretching/shrinking the magnitude of a number
5. **Measure** (Length on a number line created by partitioning units into subunits).

Understanding the Problem

Even before they come to school young children exhibit an awareness of fraction names such as **half** and a **fourth**.

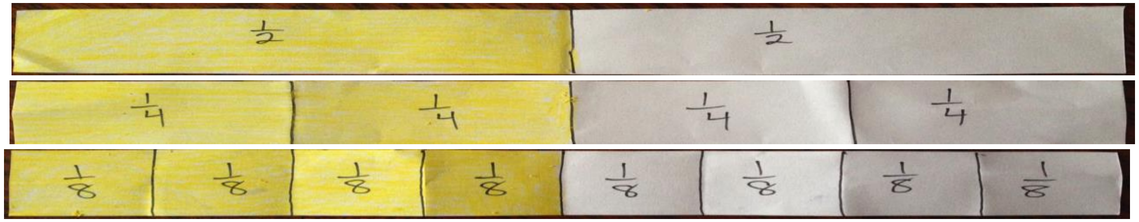
HOWEVER . . . .

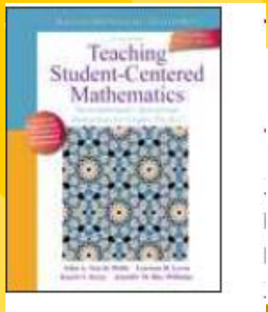
To our ears, this sounds like children understand the relationships inherent in fraction representations . . but, they are simply describing well-known objects.

.

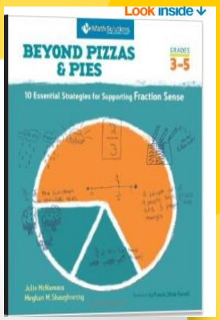
KEY IDEAS . . . . .

- That equal parts are involved,
- The number of parts names the unit,
- As the number of parts of a given whole are increased, the size of the parts (or shares) get smaller.





No one knows better than YOU . . .

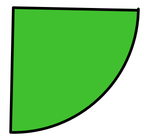


The way we've been teaching fractions is NOT serving students:

- **Introducing fraction symbol without an understanding of what it MEANS**
- **Computation taught as rules/algorithms – before understanding HOW and WHY they work!**

Results in Major Misconceptions

- Fractions are pieces or “shapes”

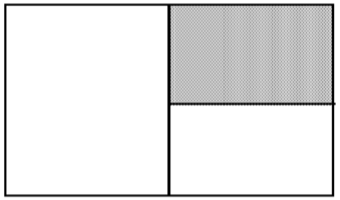


“A fourth is a pizza shape.”

- Fractions are always smaller than a whole

“4/3? That’s impossible!”

- Fraction values are determined by counting parts



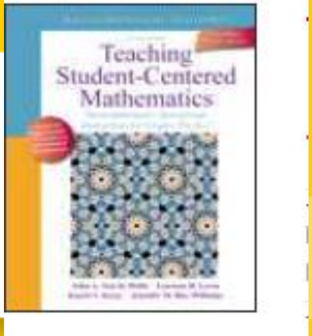
“It’s 1/3 because 1 part out of 3 parts is shaded.”

Sound Familiar?

"3/12 is bigger than 3/8 because 12 is bigger than 8"

$$\frac{2}{5}+\frac{3}{8}=\frac{5}{13}$$

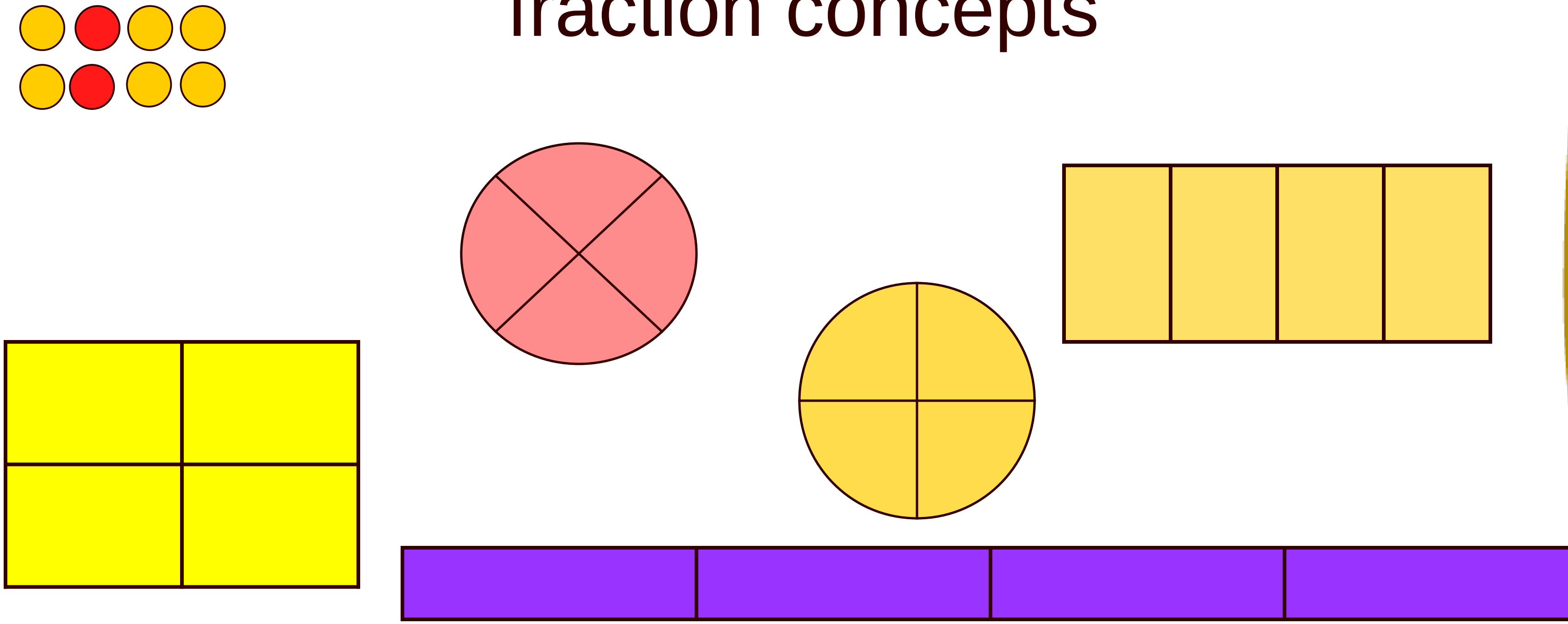
Big Ideas



1. For students to really understand fractions, they must experience fractions across many constructs: part of a whole, ratios, division, measure, operator.
2. Three categories of models exist for working with fractions—area, length, & set.
3. PARTITIONING and iterating are ways for students to understand the meaning of fractions, especially numerator and denominator.
1. Students need many experiences estimating with fractions (benchmark fractions)

Partitioning . . .

KEY to understanding and generalizing fraction concepts



Counting and coloring parts of someone else's model is next to **useless** - students need to be actively involved in making and naming their own fraction models.

How many of you are in a COMMON CORE state?

At what grade level do we begin teaching about fractions?

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

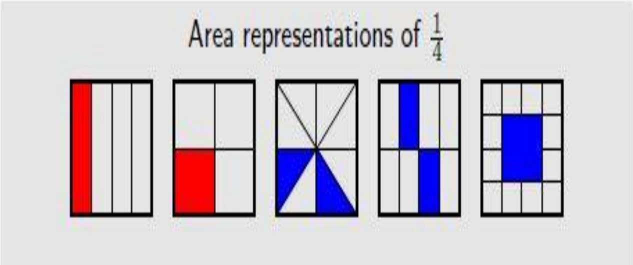
Grade 1 – Geometry

- Partition circles and rectangles into two and four equal shares, describe halves, fourths,....

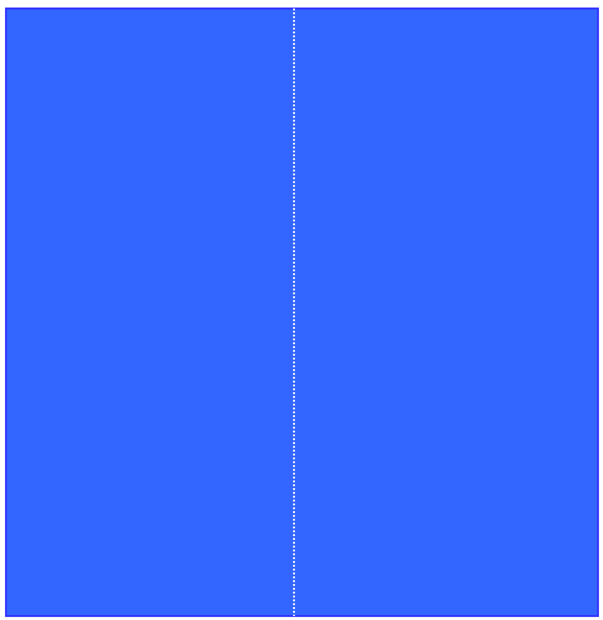
Grade 2 - Geometry

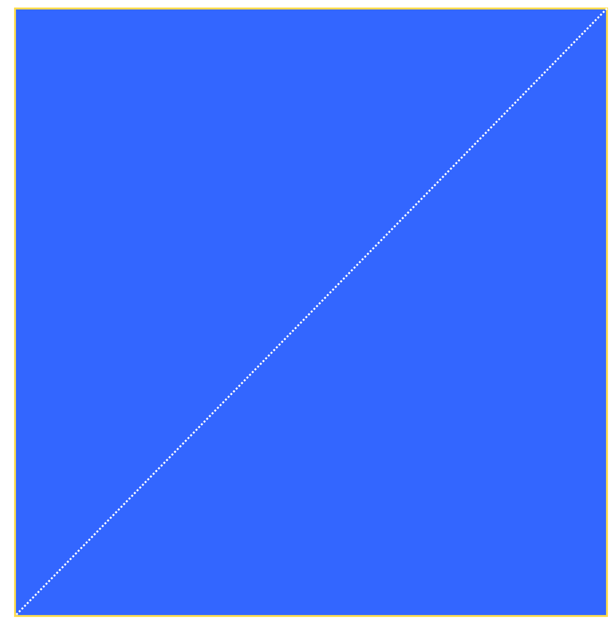
- Partition circles and rectangles into two, three, or four equal shares, describe halves, thirds, s, ... Describe the whole halves, three thirds, four s. Recognize that equal shares need not have the same shape.

Key verb here is DESCRIBE

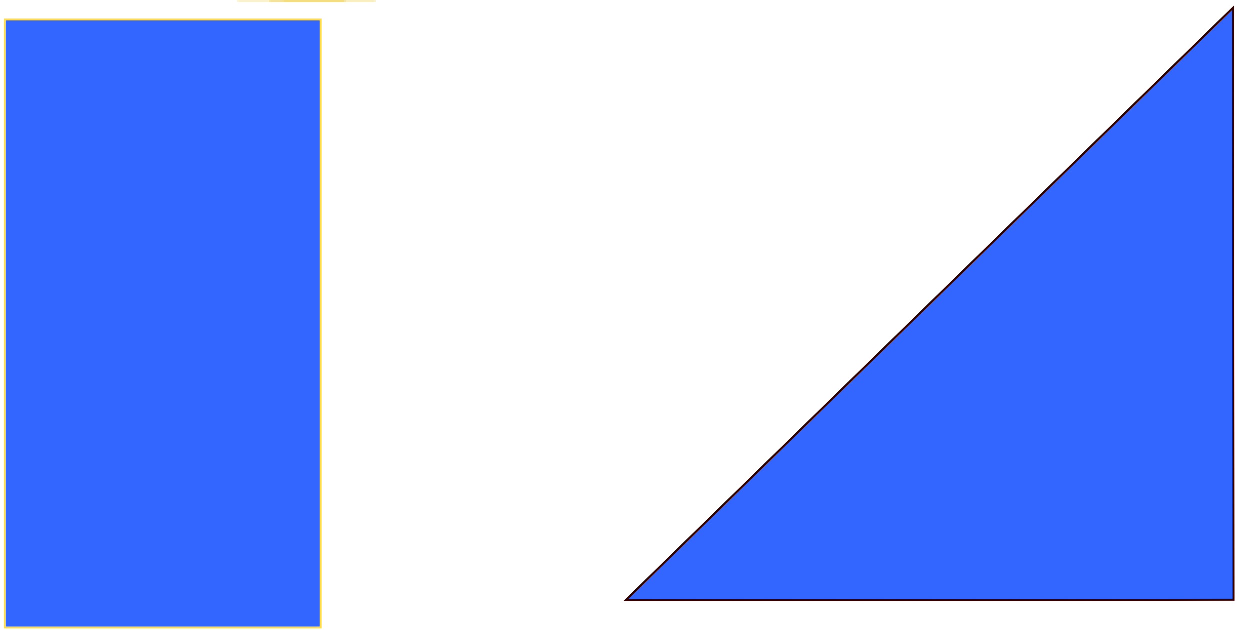


Paper Folding:
"Halving Strategy"





Both Shapes Are ***One Half***



How are they different?
How are they the same?

Developing Fraction Sense

It's ALL about the questions you ask!

- Not about the SHAPE, but "regional relationships", what they notice about the "amount" as they make more folds, etc. etc. Questions that help them make generalizations!

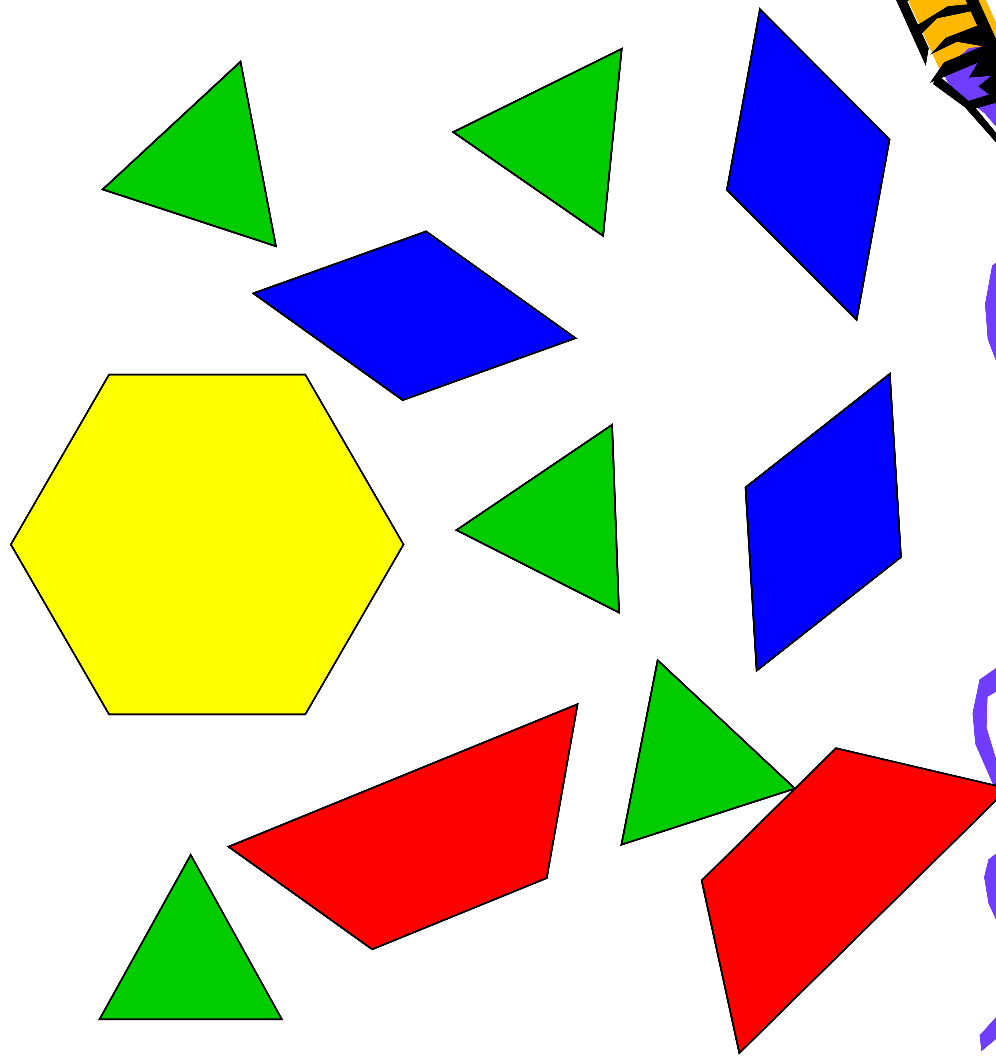
Fractions are NOT shapes!

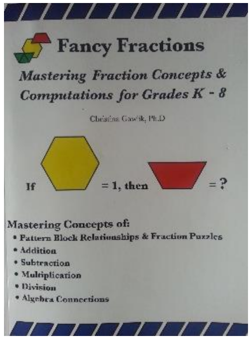
Regional Relationships

Region Relationships

Determine the fractional relationship of the pattern block pieces if the **hexagon** is the WHOLE

- Trapezoid?
- Rhombus?
- Triangle?





Regional Relationships

1. How many green triangles are in one blue rhombus ?
2. How many green triangles are in one red trapezoid ?
3. How many green triangles are in one yellow hexagon ?
4. How many blue rhombuses are in one yellow hexagon ?
5. How many red trapezoids are in one yellow hexagon ?

Grade 3 – N&O Fractions

- Develop understanding of fractions as numbers.

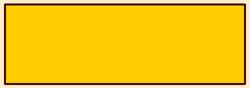
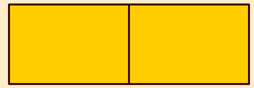
Grade 4 – N&O Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions and compare fractions.

Grade 5 – N&O Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

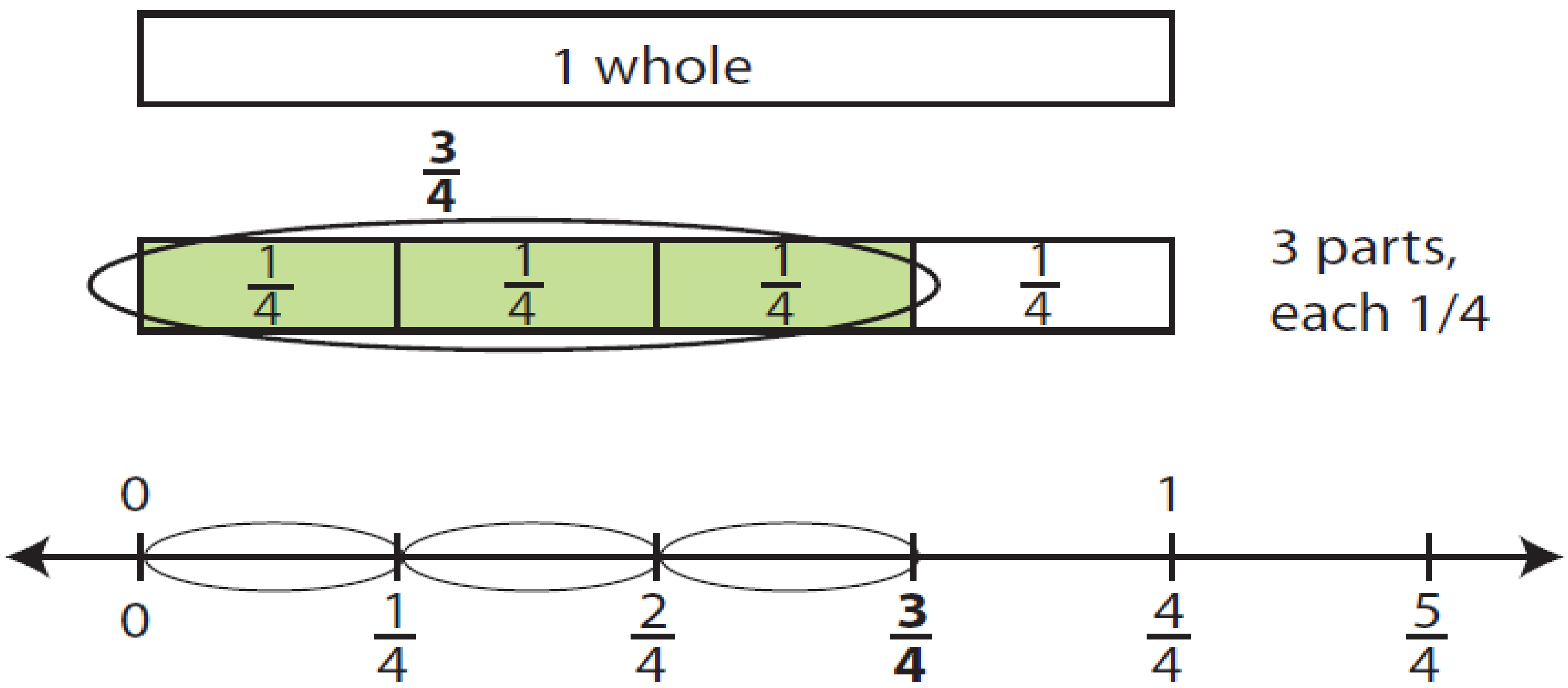
3rd Grade Unit Fractions

One whole	Fractional parts	One part	Unit fraction
	halves 		$\frac{1}{2}$
	fourths		
	eighths		
	thirds		
	sixths		

Students complete poster with partner and discuss thinking:

- **What patterns do you notice in your poster?**
- **How does the numerator RELATE to the “part”.**
- **How does the denominator RELATE to the part?**

Unit Fractions
"Connecting" Region Model to Linear Model



Fraction Greater than 1

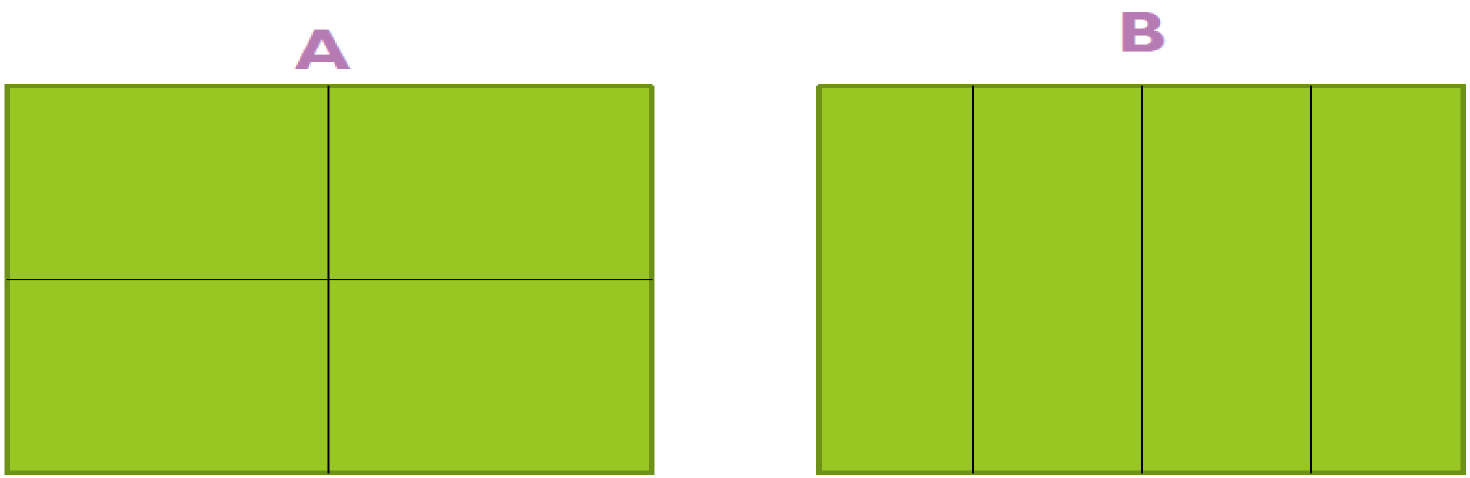
Partitioning Progression

- K-2: Regions (Area Model)
- 3: Regions and number lines (Linear)
- 4: Strengthen number lines; regions beginning to fade
- 5: Number lines

Being able to partition into any number of parts is the significant link between multiplication, division, and fractions, are reciprocal with geometric understanding.

Task: *Equipartitioning* Mini-Whiteboards

Reasoning about Equal Size



MP #3 Construct viable arguments and critique the reasoning of others

MP #5 Use appropriate tools strategically

Extending this Reasoning "Thirding" Paper Folding

Fold a piece of paper into 3 equal parts

Compare your "partitions" with your partner. (Notice similarities/differences)

Fold your paper again into thirds. (Discuss what you are noticing about the partitions. What patterns are you seeing?)

GENERALIZATION?

"As the number of parts increase they get smaller"

Equivalent Fractions Comparing Tasks

Focus on Reasoning

Teams of 4:

1. Fold paper in half, color one half of the paper yellow. Recorder writes ½ on a recording sheet to show the part of the paper that is yellow.
2. Pass the fraction model to your team member on the right.
3. Fold paper back in half and then in half again and open it.

Discuss:

What fraction of the model is now yellow?

3. Label the yellow 1/2 = 2/4 Pass the paper to the right and repeat 2 more times.

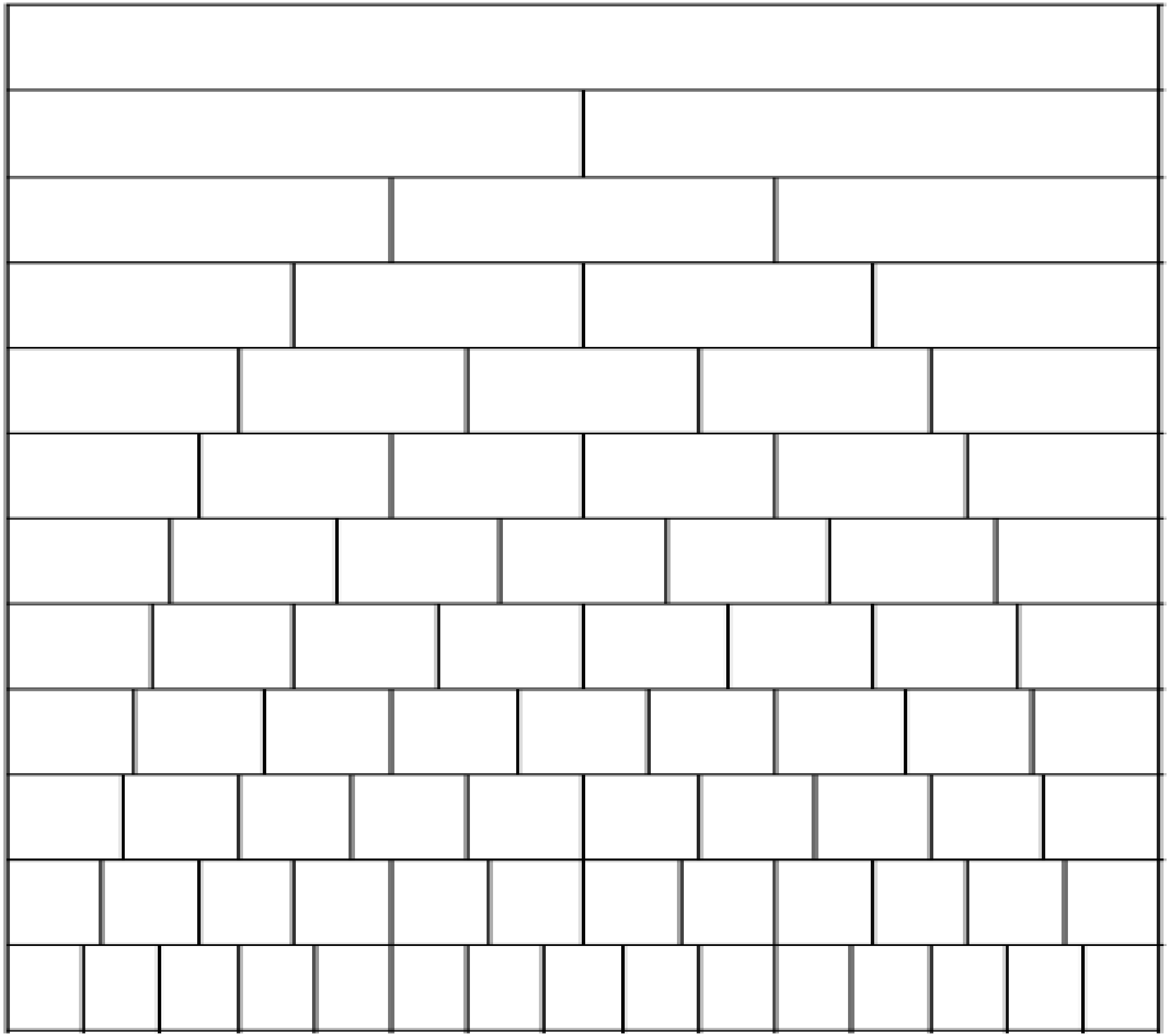
Generalizing to Support Fraction Renaming

Where the number of parts is increased (or decreased) by a certain factor, the number of parts required is increased (or decreased by the same factor).

Expecting Students to Reason and Generalize:

Eliminates the need for, and the problems caused by the inappropriate rule that WE tell them: *"what you do to the top you do to the bottom"* as students have the capacity through partitioning, to identify what is happening to the number of parts.

Directions: Roll the die twice. The first roll represents "*how many*" and the second roll represents "*how much*". Color in the amount. The winner is the person with the greatest number of wholes.



1 whole

2 halves

3 thirds

4 fourths

5 fifths

6 sixths

7 sevenths

8 eighths

9 ninths

10 tenths

12 twelfths

15 fifteenths

Partitioning Problems You Try:

A carpenter has a 10-foot piece of wood. If the carpenter cuts the wood into 2-foot length pieces, what fraction of the wood would one piece represent?

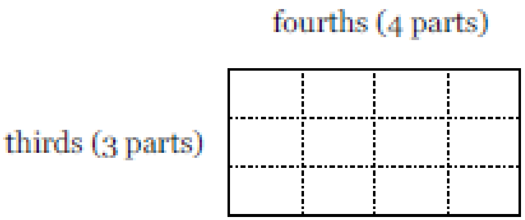
Partitioning Fractions Problems – You Try:

Debbie made brownies in a rectangular pan to share with her 3 friends. Show how Debbie might cut the brownies so that She and her friends get the same size piece of brownie (with none leftover)? If possible, show more than one solution.

Partitioning to Understand Multiplication of Fractions:

- Fold your paper into fourths.
- Turn your paper and fold it into thirds.
- Unfold your paper and discuss what you see.

Explain mathematically what is happening with each fold?



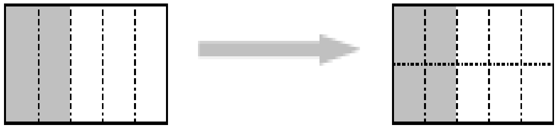
Thirds by fourths give twelfths (12 parts)

Instead of *"Do to the Top Number What you do to the Bottom Number" . . . .*

Understand the Algorithm!

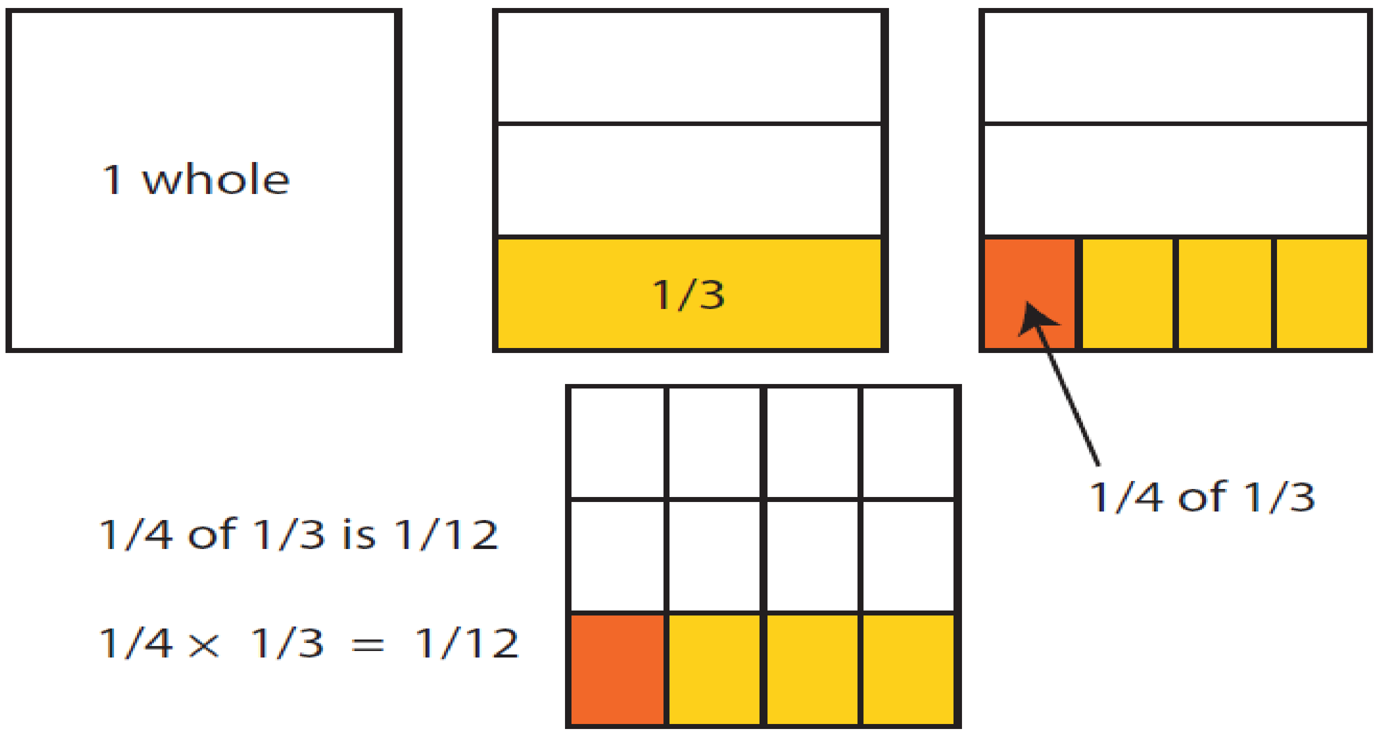
Fold strip into fifths. Then fold one time in middle.

Discuss what you see. What generalizations can you make?

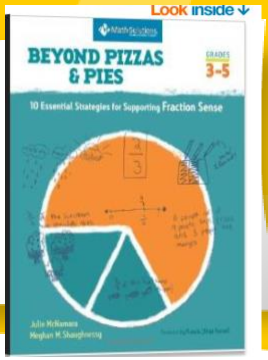


As the number of parts IN the whole is increased by a factor of 2 (or doubled) to 10 parts, the number OF parts of the whole has also increased from 2 to 4 to show 4/10.

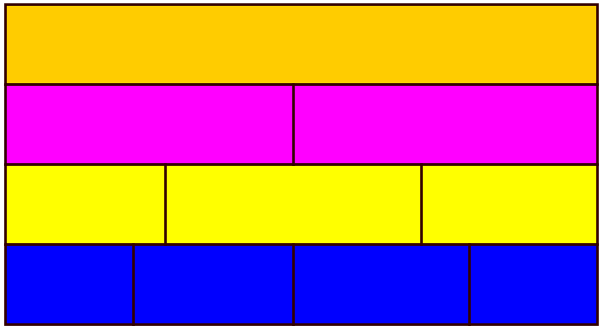
Partitioning to Represent Fraction Multiplication

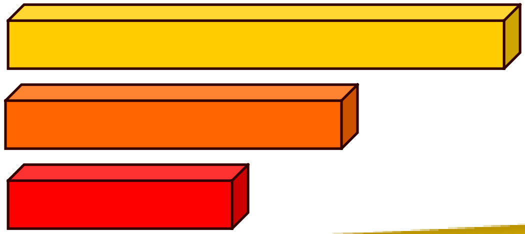


Connecting Fraction Strips & Tape Diagrams to Number Line (linear model)



Linear region is cut into smaller parts; lengths are compared.





Why Number Lines?

- Easier to divide the whole into equal parts because only length is involved.
- Addition and subtraction are much easier to model.
- Multiplication and Division are much easier to model.

Connecting Fraction Strips, Number Line to the ruler (Linear Models)

Individually: Make a **Magnified Inch** Ruler
Fold until it is partitioned into 16 equal parts.
Use your "Magnified Inch Ruler" to measure:

1. The width of your chair back
2. A purse or bag at your table

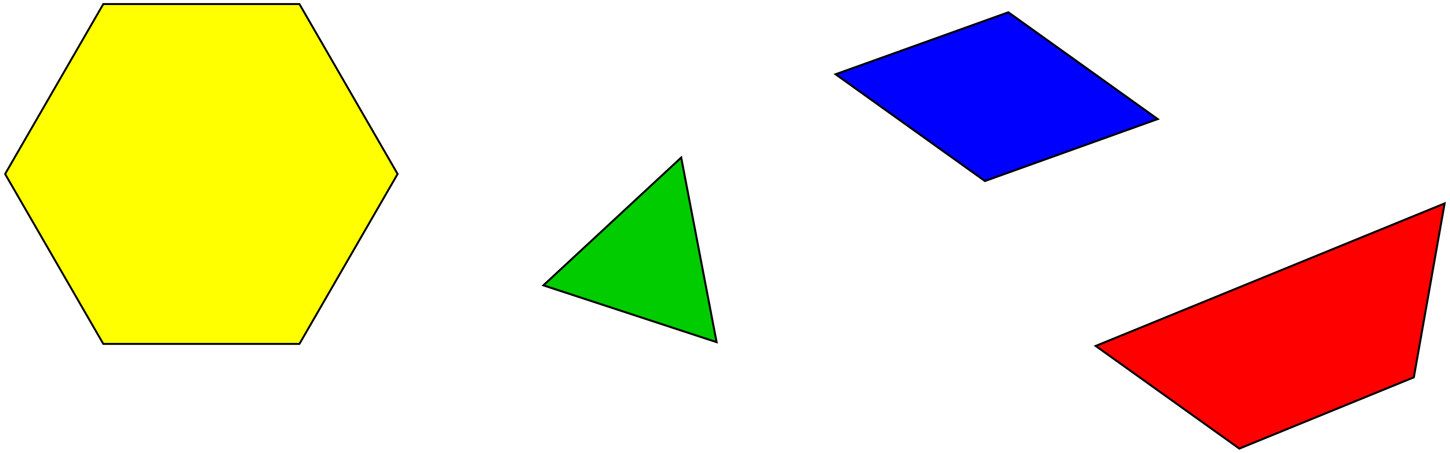
In teams measure:
3. Measure a "Gallery Participant's" height.

Be PRECISE in your measurements. Discuss *"measurements greater than 1" and how understanding fractions helps you use a ruler.*

A Thought To Ponder . . .

"What would happen if you were scared half to death twice?"

What operation?



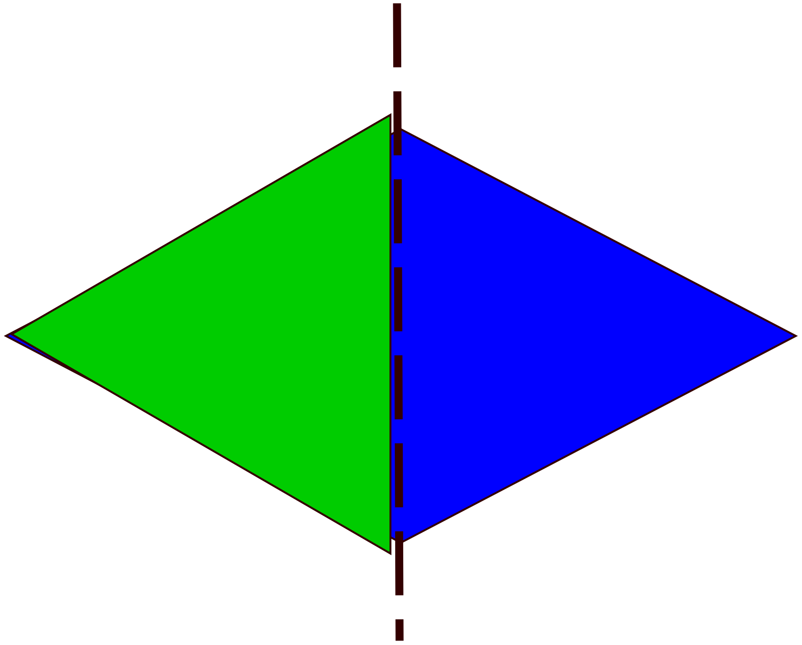
Use your Pattern Blocks to show this expression:

$$\frac{1}{2} x \frac{1}{3}$$

Shape Set Fractions

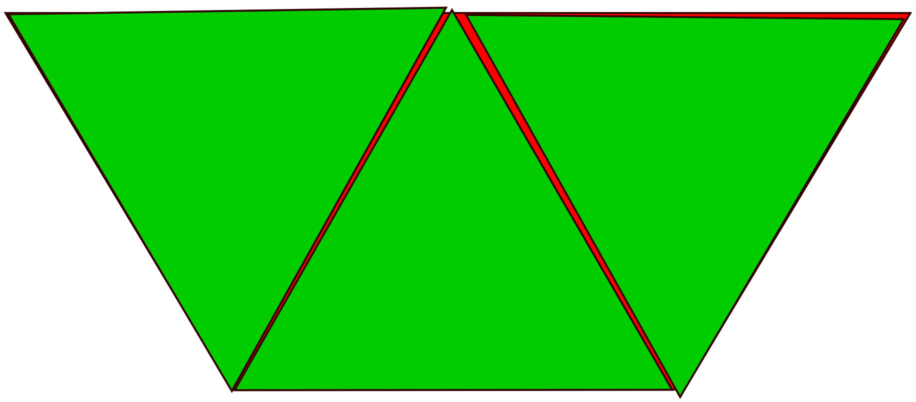
$$\frac{1}{2} \times \frac{1}{3}$$

Think about it this way: one-half **of** one-third.



$$\frac{1}{3} \times \frac{1}{2}$$

This should read one-third **of** one-half.



Division of Fractions

Partition division: “how many in each part” as opposed to “how many groups in”

With Your Elbow Partner Solve:
1) Partition the rectangles and Discuss What patterns you notice?

Modeling Division of Fractions

1) $1 \div \frac{1}{4} = 4$

How many __________are there in one?

There are ________________one fourths in one.

Related Multiplication 1 x ___-____ = 4

2) $\frac{1}{4} \div 1 = \frac{1}{4}$

How many ____________ are there in one fourth?

There is ________________ of a one in one fourth.

Related multiplication ¼ x______=1/4.

$$3 \div \frac{1}{2} = ?$$

Yours is not to reason why, just invert and multiply!

How would YOU get kids to REASON through this and make sense of the operation?

Table Task: Building Understanding

Partition mini-white boards to show thinking:

- How many one-sixths are in 2?

$$2 \div \frac{1}{6} = ?$$

- How many one-halves are in 3?

$$3 \div \frac{1}{2} = ?$$

- How many one-fifths are in 2?

$$2 \div \frac{1}{5} = ?$$

What patterns do you notice?

Generalize: How might these patterns help develop a method for dividing by fractions?

Importance of Sense-Making

- Knowing THAT an algorithm works is not the same thing as knowing WHY an algorithm works (or does not work)

- Analyzing patterns and use of contextual problems, gives meaning to the symbols for numbers, operations, and their relationships.

Could YOUR Students Do This?

Write a word problem for which $3 \div \frac{1}{2}$ would be the method of solution.

NAEP, 8th grade 13% Correct

$$\frac{1}{3} \div \frac{1}{6}$$

Making Sense of Division

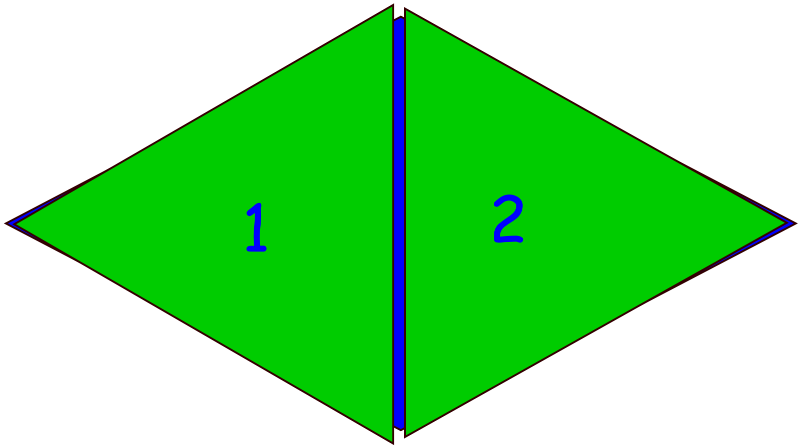
Teams:

Use your shape set to show your reasoning.

$$\frac{1}{3} \div \frac{1}{6}$$

How many one-sixths regions "**fit into**" one-third?

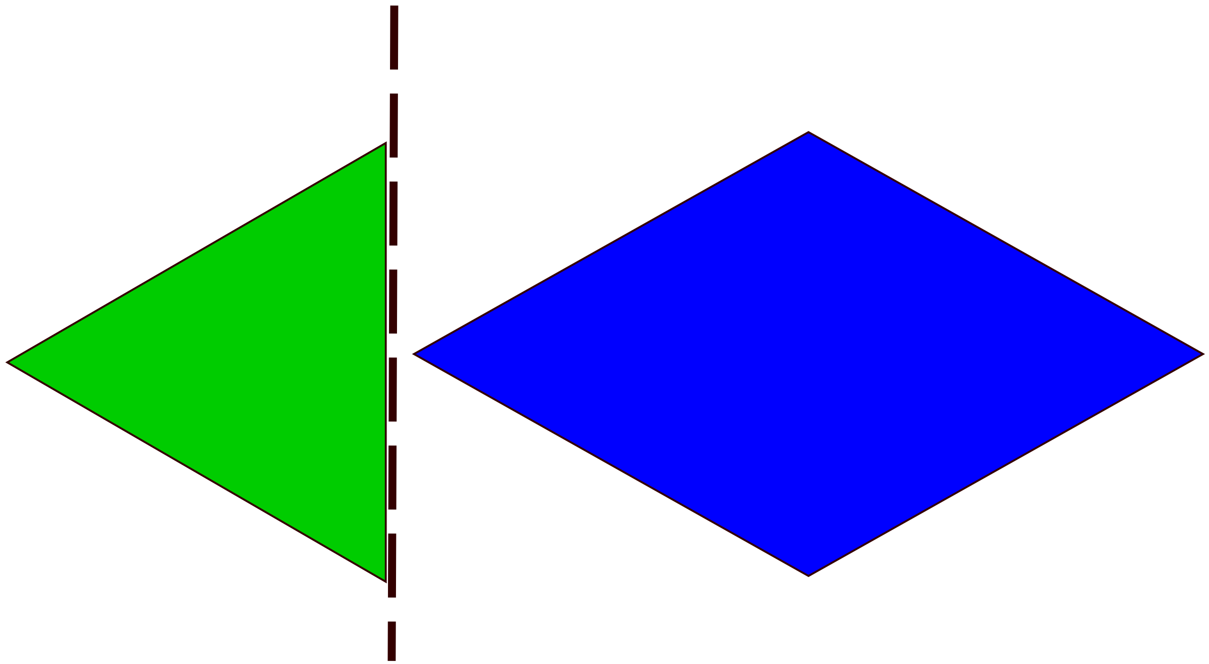
(*goes into* or *guzinta-rule*)



2 one-sixths fit into one-third...not 2 wholes.

$$\frac{1}{6} \div \frac{1}{3}$$

How many one-thirds **fit into** one-sixth?



Only half of the one-third "goes into" the one-sixth.

Making Sense of Fractions

We must go beyond
how we were taught
and teach how we wish
we had been taught.

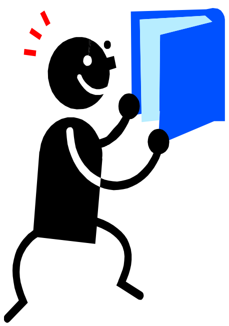
Miriam Leiva, NCTM Addenda Series, Grade 4, p. iv

Resources:

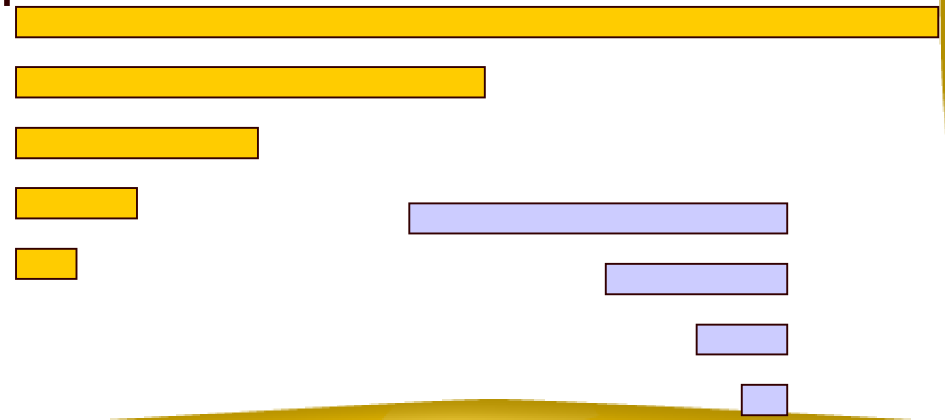
- Fancy Fractions-Mastering Fraction Concepts & Computations Grade K-8, (C. Gawlik)
- Teaching Student-Centered Mathematics (Van de Walle, Lovin, Karp, Bay-Williams)
- Beyond Pizzas & Pies (McNamara, Shaughnessy)
- *Regional Relationship* activity sheets, NCTM Illuminations
- Flip Books for CCSS-M, www.katm.org (Melisa Hancock)
- *Partitioning-The Missing Link in Building Fraction Knowledge & Confidence* (Dianne Siemon)


TEACHERS . . .
Thanks
FOR ALL YOU DO!!!!

Grade 3: Paper Folding Anchor Task
What do you notice as the number of parts increases?



1. Fold a sheet of paper in half. Repeat until it can't be folded in half again – discuss what happens to the number of parts and the size of the parts



2. Halve paper strips of different lengths, compare halves – how are they the same? How are they different?

ALWAYS Lead Kids to Generalize: The size of the part depends upon the whole and the number of parts (SMP 7 & 8)

What Patterns Do you Notice?

Make Generalizations based on these Patterns.

Patterns are a tool, not a topic. (Jason Zimba)

No. of parts	Name
1	whole
2	halves
3	thirds
4	quarters (fourths)
5	fifths
6	sixths
8	eighths
9	ninths
10	tenths
12	twelfths
15	fifteenths

Generalize: As the number of parts increases, the size of the parts gets smaller – the number of parts, names the part

Partitioning a Number Line

- Make a number line from 0 to 2.

- Put the following fractions on the number line:

$^{1}/_{6}$, $^{2}/_{6}$, $^{5}/_{6}$, $^{6}/_{6}$, $^{8}/_{6}$, $^{11}/_{6}$

Describe your process to the person sitting across from you.

Prove that the fractions are in the correct location.

Partitioning a Number Line

- Create another number line 0 to 2.
- Plot the following numbers:

  $^{1}/_{3}$, $^{3}/_{4}$, $^{4}/_{3}$, $^{2}/_{3}$, $^{3}/_{2}$

- In between each of your fractions, plot anoth… e 9 fractic…

Describe your process to the person sitting across from you.

Prove that the fractions are in the correct location.

Who is winning the race? Here is the fraction of the distance covered from the start by the racers.

Mary: $\frac{3}{4}$	**Lisa:** $\frac{1}{2}$	**Han:** $\frac{5}{6}$	**Joey:** $\frac{5}{8}$	**Jill:** $\frac{5}{9}$	**Larry:** $\frac{2}{3}$

1. Predict
   a. Who do you think is winning?
   b. Who can you rule out?

2. Explain how you decided who is winning the race.

Place each person in their approximate place along the race track:

3. More people arrive to play. Assign a fractional distance to how far they have traveled based on this information:

a) Joe is between Lisa and Joey_____________

b) Ben is between Han and Larry_____________

c) Cori is between Joey and Jill______________

"Partitive Interpretation of Division"

Greg has 1 ¼ hours to finish his three household chores. If he divides his time evenly, how many hours can he give to each?

You may use an area model, linear, or set model to show your reasoning.

Standards for

Mathematical Practice

1. Make sense of problem and persevere in solving them.
6. Attend to precision.

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.
5. Use appropriate tools strategically.

7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning

- Overarching habits of mind of a productive mathematical thinker
- Reasoning and explaining
- Modeling and using tools
- Seeing structure and generalizing

adapted from McCallum (2011)
Standards for Mathematical Practice

Fancy Fractions by Chris Gawlik

Go to:

https://www.createspace.com/3823695

Discount Code: 4RC94EA5