Investigations of the CCSSM Based on News Stories and Media

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nhpr New Hampshire Public Radio

BDN MAINE

morning edition

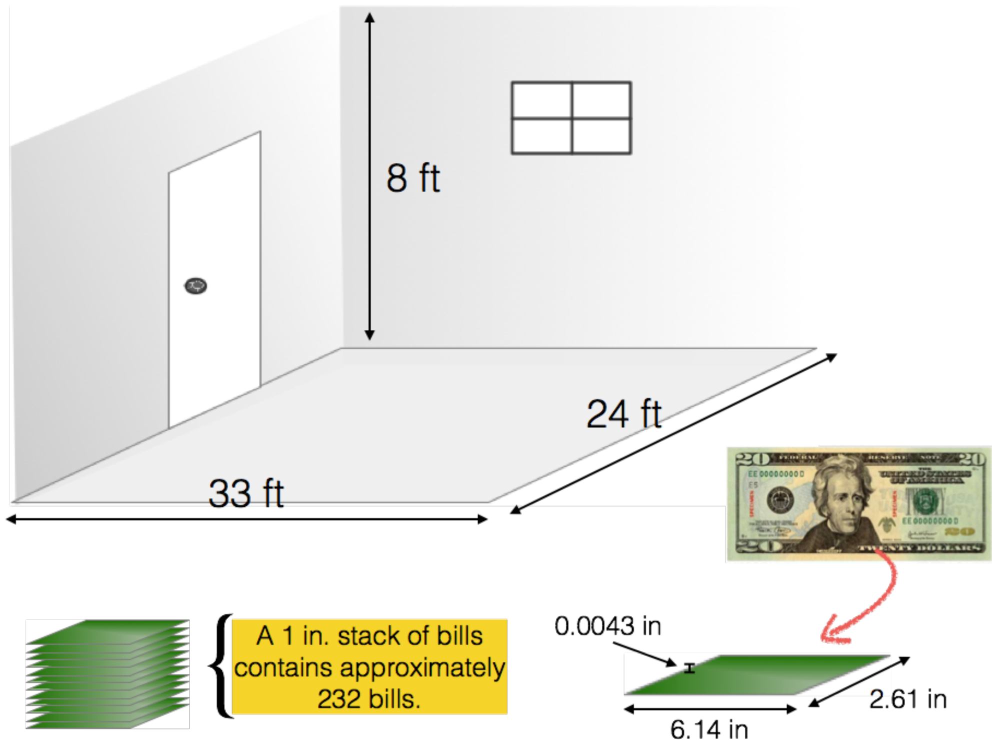
all things considered

all tech considered

NATIONAL GEOGRAPHIC

npr

Millions, Billions, and Trillions



<u>Directions:</u> Answer the following questions based on the information given above.

1. How much money will fit in this room?
2. How far would we have to extend the room to hold the national debt, approximately $18 trillion?

<u>Connections to CCSM</u>

- SMP 1 Make sense of problems and persevere in solving them.
- SMP 4 Model with mathematics.
- (Geometric Measurement and Dimension) Explain volume formulas and use them to solve problems
  1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cones…
- (Gr. 8 Geometry) Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
  9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Solutions & teaching notes

Any size room can be used for this investigation. Start by playing audio clips that make reference to millions, billions, and trillions, a sample of examples are included in the references below. Ask students to take guess at the answers before working through the computations.

1. The height of the room is 8 ft or 96 in. With 232 bills per inch there will be 232 bills per in * 96 in = 22,272 bills in a stack. Orient the bills so the longer dimension lines up with the longer dimension of the room. Along this dimension, 33 ft = 386 in, 386in ÷ 6.14 in per bill ≈ 62 bills. Along the narrower dimension, 24 ft = 288 in, 288 in ÷ 2.61 in per bill ≈ 110 bills. Thus to cover the floor 62 * 110 = 6820 bills, or 6820 stacks to fill the room. Since each stack has 22,272 bills the room contains 6820 * 22,272 = 151,895,040 bills. At $20 per bill the room holds 151,895,040 * $20 = $3,037,900,800. The room holds over $3 billion dollars.

2. Since each room holds $3,037,900,800 we will need $18,000,000,000,000 ÷ $3,037,900,800 ≈ 5925 rooms. Since each room is 33 ft long, one long room would need to be 5925 * 33 ft = 195,525 ft long or equivalently 195,525 ÷ 5280 ≈ 37.03 miles long.

Follow up by figuring a location that is about 37 miles away and show students how long this is on a map. Remind the students that about two stacks in the corner is $1 million and that 1 room bolds over $3 billion.

References

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Joyce, C. (Narrator). (2015, March 5). Jaw Fossil In Ethiopia Likely Oldest Ever Found In Human Line. *Morning Edition.* Washington DC: National Public Radio. (http://www.npr.org/2015/03/05/390717001/jaw-fossil-in-ethiopia-likely-oldest-ever-found-in-human-line)

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Whitney, E. (Narrator). (2015, January 6). 3 States Counter Obama's Proposal For Medicaid Expansion. *Morning Edition.* Washington DC: National Public Radio. (http://www.npr.org/2015/01/06/375308933/3-western-states-counter-administration-s-proposal-for-medicare-expansion)

Eat Your Veggies! Even The Ones From Fukushima

by Poncie Rutsch, NPR.org

March 5, 2015

(http://www.npr.org/blogs/thesalt/2015/03/05/390937138/eat-your-veggies-even-the-ones-from-fukushima)

Nearly four years after the Fukushima nuclear disaster, people in Japan are still hesitant to eat foods grown around the site of the accident. They worry that anything grown in the region will contain dangerous levels of radioactive elements, increasing their risk of cancer.

Sometimes, food from Fukushima will bear a photo of the farmer who grew it or a number to dial to learn more about each bag of rice or vegetables, just to ease customers' concerns.

Now there might be one more way to make customers feel confident that they aren't munching on a radioactive dinner. It's a chemical called CsTolen A, for Cesium Tolerance Enhancer.

Radioactive cesium is one of the biggest concerns following nuclear disasters. It takes a long time to decay — as much as 20 years for half of the cesium in the soil to disappear. And it dissolves in water, so plants draw it out of the soil just as they would nutrients. CsTolen A aims to block this uptake.

"The CsTolen A binds to cesium in the soil," explains Ryoung Shin, a professor at the Riken Plant Science Center in Tokyo. In a study published this week in the journal Scientific Reports, Shin and her colleagues report that this binding prevents the cesium from entering the plant.

So far, the chemical has only been tested in the lab on the model plant Arabidopsis thaliana — not on any field crops. Shin and her collaborators identified the cesium-blocking chemical from a library of commercially available compounds. After screening for the best cesium-blocking contenders, they chose five and added them to the water they used on the plant, the soil around the plant, the seeds, and the plant itself.

Then they tested the plant tissue for cesium accumulation. They found that the only radioactive cesium they could detect in the plants was normal, from background levels present in the atmosphere.

The study was conducted in a plant that doesn't produce food, but the researchers are confident that the chemical will work on other kinds of plants, too. Because the chemical does its job before it enters the plant, Shin says it shouldn't matter what kind of plant is growing in the soil. Plus, CsTolen A is available commercially, meaning that it is relatively easy to get and distribute.

Of course, Shin notes, it's still too early to start applying this chemical to the fields around Fukushima. There are many layers of government regulation standing between Fukushima farmers and CsTolen A. And researchers aren't 100 percent sure that the chemical has no impact on human health, because they haven't tested it on people yet. But Shin says it looks promising: Since CsTolen A should stay in the soil and never enter the plant, it should never enter the human body, either.

Even now, there are still farmers who haven't returned to their land because many areas around Fukushima remain restricted. After prohibiting anyone from entering a radius of about 12 miles from the plant, the government started to let residents back in last spring. Shin hopes that in the future, CsTolen A will help get more people within the restricted zone back on their land and back to farming.

As for Fukushima foods grown outside that circle of concern, extensive testing shows they're just fine to eat, a group of researchers reported last week.

The researchers analyzed 900,000 food samples, including tea, beef, fruits, mushrooms, and vegetables grown and raised in the Fukushima region from 2011 to 2014. Their testing revealed that radiation in these foods had returned to pre-accident levels. Even so, consumers are still shying away from foods with the Fukushima label.

Shin says it's too bad, because the produce from the region is very high quality and tasty.

"Fukushima used to be a very famous place for agricultural products," she says. "There is plenty of produce, but people don't want [it]."

Rutsch, P. (2015, March 5). Eat Your Veggies! Even The Ones From Fukushima. Retrieved from http://www.npr.org/blogs/thesalt/2015/03/05/390937138/eat-your-veggies-even-the-ones-from-fukushima.

Directions: Please read the article on the previous page and use it to answer the questions that follow.

1. According to the article, why is cesium in soil a problem?

According to the US Environmental protection agency[1], the half-life of cesium-137 is 30 years. [Cesium-137 is the isotope of cesium being referred to in the article.] The level radioactivity of an isotope is measured in units called **becquerel** (symbol **Bq**). Blomberg.com[2] reported that the prescribed safe limit in Japan for radioactive cesium-137 in vegetables is 500 Bq/kg. Additionally Bloomberg.com reported a sample of sample of spinach collected from Hitachi, 60 miles south of Fukushima, contained 1,931 Bq/kg of cesium-137.
To understand these issues better the following questions will lead to the derivation of a formula to predict the level of radioactivity as a function of the number of half-lives, and a second formula to predict the radioactive level as a function to time.

2. The table shows the radioactivity level if there are 2 Bq initially, and generically A_0 Bq initially, after 0, 1, 2, 3, or 4 half-lives. Find the missing values and complete the table.

# of half-lives	Radioactivity of Cesium (Bq)	Radioactivity of Cesium (Bq)
0	2	A_0
1		
2		
3		
4		

3. Find a formula that can be used to determine radioactive level of cesium-137 after *n* half-lives for the case when the level is A_0 Bq initially. To find a formula look for patterns in the values you found in the table and try to generalize them.

4. Find a formula that can be used to determine the radioactive level of cesium-137 after *n* half-lives for the case when it is 2 Bq initially. Compute the level that will remain after $n = 6$ half-lives (180 years).

5. Use the formula you found to find the radioactive level of cesium-137 after 10 years, 17 years, and 79 years. When the initial amount 100 BQ Show your work.

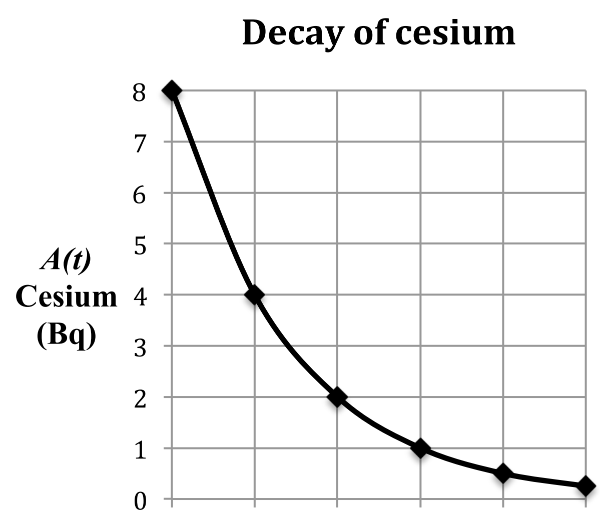
[1] http://www.epa.gov/radiation/radionuclides/cesium.html
[2] http://www.bloomberg.com/news/articles/2011-03-21/japan-sets-safe-limits-for-consuming-radiation-contaminated-food-table-

6. Write a formula that can be used to compute the radioactive level of cesium-137 if the independent variable is *t*, time in years. Explain why your new formula works.

7. How many years should it take for the level of radioactive cesium-137 for spinach in Hitachi to drop below the accepted level? Explain how you arrived at your answer.

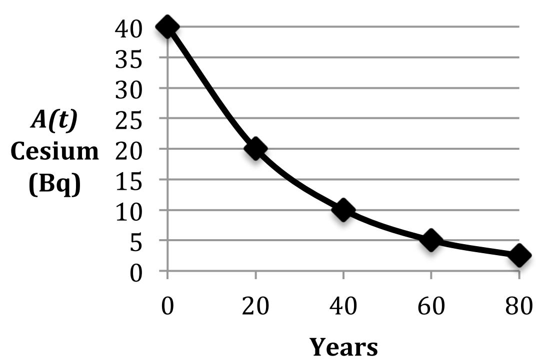
8. According to the Huffington Post[3], the level of cesium-137 in a fence outside the reactor measured 830 Bq/liter and the regulatory limit for humans is 90 Bq/liter. How long will it take for this level to reduce so as to be safe for humans?

9. The graph to the right shows *A* over an interval but the *x*-axis is not labeled. Can the *x*-axis represent *n*, *t*, both, or neither? Explain your reasoning.



10. The table and graph below show the decay of cesium-137 with 120 Bq over an 120-year period. The incomplete table shows 30-year time intervals. For each interval compute the average change in the radioactive level of cesium per year over the 30-year period. Notice how the average is different for each period. How do these values explain the shape of the graph? Explain.

Years	radioactive level of cesium (Bq)
0	120
30	60
60	30
90	15
120	7.5



Years	Average change per year (Bq/yr)
0 - 20	
20 - 40	
40 - 60	
60 - 80	

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[3] http://www.huffingtonpost.com/2013/10/10/radiation-fukushima-high_n_4075066.html

Connections to CCSM

- **SMP 1** Make sense of problems and persevere in solving them.
- **SMP 4** Model with mathematics.
- **SMP 5** Use appropriate tools strategically.
- **(Functions - Interpreting Functions)** Understand the concept of a function and use function notation
  1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- **(Functions - Interpreting Functions)** Interpret functions that arise in applications in terms of the context
  6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **(Functions - building functions)** Build a function that models a relationship between two quantities
  1. Write a function that describes a relationship between two quantities.
  a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

Solutions & teaching notes

1. Two primary reasons are it is radioactive and thus causes cancer, and it has a half-life of roughly 20 years. [Note the article incorrectly reports the half-life of cesium-137 as 20 years instead of 30.]
2.

# of half-lives	Radioactivity of Cesium (Bq)	Radioactivity of Cesium (Bq)
0	2	A_0
1	1	$\frac{A_0}{2}$
2	0.5	$\frac{A_0}{4}$
3	0.25	$\frac{A_0}{8}$
4	0.125	$\frac{A_0}{16}$

3. *Teaching note:* The pattern is more easily seen in the case when the initial amount is variable, A_0. Students must recognize that in each step they multiply by ½ and that in the *n*th step they have multiplied by ½ *n* times.
$$A(n) = A_0\left(\frac{1}{2}\right)^n$$
4. $A(n) = 2\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{n-1} = 2^{1-n}$
$$A(6) = 2\left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^5 = 2^{-4} = \frac{1}{16} = 0.0625 \text{ Bq}$$

5. Let $n = \frac{1}{3}, \frac{17}{10}$, and $\frac{79}{30}$.

   $$A\left(\frac{1}{3}\right) = 100\left(\frac{1}{2}\right)^{\frac{1}{3}} = 79.4 \text{ Bq}$$
   $$A\left(\frac{17}{30}\right) = 100\left(\frac{1}{2}\right)^{\frac{17}{30}} = 67.51 \text{ Bq}$$
   $$A\left(\frac{79}{30}\right) = 100\left(\frac{1}{2}\right)^{\frac{79}{30}} = 16.1 \text{ Bq}$$

6. *Teaching note*: This problem is a generalization of the previous one; one hint could be to look back at that problem.

   $$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{30}}$$

7. Using a calculator students can find the solution of $500 = 1931\left(\frac{1}{2}\right)^{\frac{t}{30}}$, $t \approx 58.5$ years.

8. $90 = 830\left(\frac{1}{2}\right)^{\frac{t}{30}}$, $t \approx 96.2$ years.

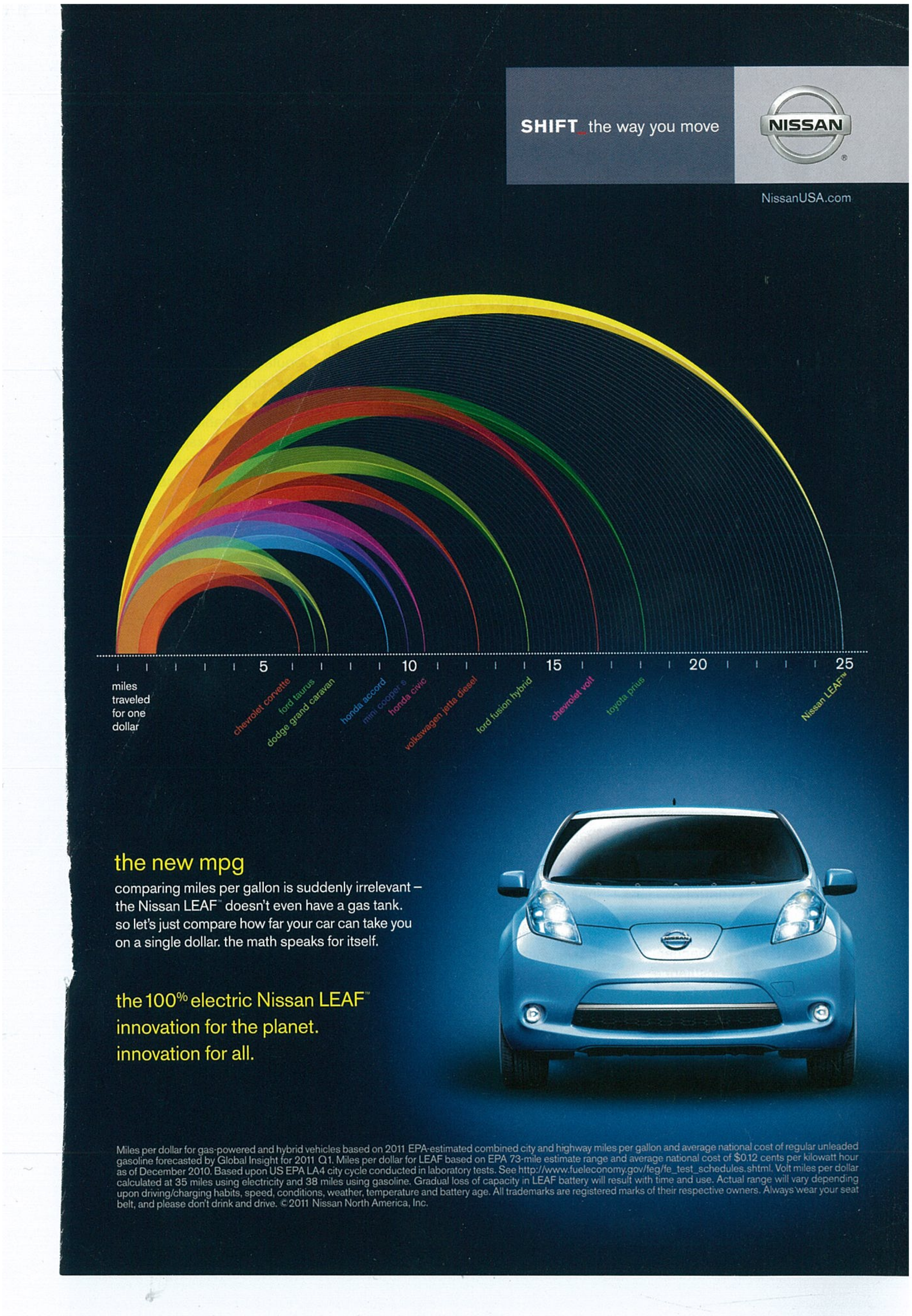
9. The axes can represent n, the number of half-lives, or t, the number of years. In the former case the first tick mark after 0 is 1 corresponding to the first half-life. Notice the value of $A(t)$ is halved form 0 to 1. In the later the first tick mark would be 30 years.

10. *Teaching note*: The goal of this problem is to have students think about how change impacts the shape of a graph making connections between the table and the graph.

    The magnitude of change decreases causing the graph to flatten out.

Years	Average change per year (Bq/yr)
0 - 30	2
30 -60	1
60 - 90	0.5
90 - 120	0.25

Misrepresenting data with circles



This advertisement appeared in *National Geographic* in May 2011 on p. 9.

Directions: Use the advertisement on the previous page and to answer the questions that follow.

1. Graphs give a visual means to compare numerical quantities. In this graph what quantities are being compared?
2. What conclusion does the advertiser want you to reach?
3. How many times bigger is the Nissan Leaf's miles per dollar than the Mini Cooper S? How do you know this?
4. Chances are you answered the previous question by comparing the values you found along the horizontal axis for these two vehicles. This can be visualized on the graph as comparing the length of two line segments. Another way is to compare the Nissan Leaf and the Mini Cooper S is to compare the area of the semi-circles used in the graph to represent each car's performance. Without performing any calculations, do you believe the Leaf's miles per dollar be bigger by the same factor as you found in problem 3? Why or why not?
5. Armed with your hypothesis from problem 4), compare the miles traveled per dollar for the Leaf and Mini Cooper S using the areas of the corresponding semi-circles in the ad. How many times bigger is the Nissan Leaf's semi-circle than the mini cooper S?
6. In questions 3 and 5 you made comparisons of the miles traveled per dollar for the Leaf and the Mini Cooper S. Why are the different?
7. What are the implications of your results for interpretations of the data presented?

Connections to CCSM

- **SMP 2** Reason abstractly and quantitatively.
- **SMP 3** Construct viable arguments and critique the reasoning of others.
- **(Algebra - Creating Equations)** Create equations that describe numbers or relationships
  1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **(Functions - Linear, Quadratic, and Exponential Models)** Construct and compare linear, quadratic, and exponential models and solve problems
  3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
- **(Functions - Linear, Quadratic, and Exponential Models)** Construct and compare linear, quadratic, and exponential models and solve problems
  3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Solutions & teaching notes

1. In the graph, the quantities being compared are the average miles traveled per dollar for different cars [*mpd*].

2. The advertiser wants you to conclude that the Nissan Leaf's *mpd* is much greater than those to which it is compared.

3. The Nissan's Leaf travels 2.5 times as many *mpd* as the Mini Cooper S on average:

   25 *mpd*/10 *mpd* = 2.5

4. Answers will vary. One likely answer is that the Leaf's *mpd* will be bigger by the same factor because the lengths (of the diameters) stay the same. Another possibility is that the Leaf's *mpd* will be bigger by a factor greater than 2.5 because the semi-circle for the Leaf appears visually to be several times larger than the semi-circle for the Mini Cooper S.

5. The Nissan's Leaf is 6.25 times larger than the Mini-Cooper S:

   For the Leaf, with diameter 25 units: $A = \frac{1}{2}\pi(12.5)^2 = 78.125\pi$ units^2

   For the Mini Cooper S, with diameter 10 units: $A = \frac{1}{2}\pi(5)^2 = 12.5\pi$ units^2

   Comparing the size of the semicircles: the Leaf is 78.125π/12.5π = 6.25 times larger.

6. In question 5 the lengths of the radii are squared. To see the effect notice that if the radius of a circle is tripled the new circle is $3^2 = 9$ times bigger than the original. In this example, the radius for the Leaf is 2.5 times bigger so the area of the corresponding semi-circle is $(2.5)^2 = 6.25$ times bigger.

7. The semi-circles misrepresent the relationship between the *mpd* of the Leaf and the *mpd* of the Mini Cooper S making it look visually 2.5 times bigger than it actually is.

This activity has been accepted to appear in the Media Clips department of *Mathematics Teacher*.

Granite Geek: No Two Snowflakes Are Alike, And Here's The Math To Prove It

by Brady Carlson, NHPR.org
February 24, 2015
(http://nhpr.org/post/granite-geek-no-two-snowflakes-are-alike-and-heres-math-prove-it)

…What a couple researchers did was they calculated all the ways that a snowflake can grow …they conclude that the number of possible snowflake shapes is 10 raised to the 10 raised to the 13 power, which is a one followed by 10 trillion zeroes - a number so much larger than any conceivable anything that could exist in the entire universe combined that you just can't even begin to think about it. They then tried to estimate how many total snowflakes have fallen in the history of the world, and they came to the conclusion that only - and I love the word only - 10 to the 40th power, so a one followed by 40 zeroes. …Now 10 to the 40th power is, again, a number so staggeringly monstrous that nothing humans or the entire solar system would ever encompass a number that large. But despite that fact, 10 to the 40th is so much smaller than 10 to the 10th to the 13th power that they conclude that the odds of any two snowflakes having been formed exactly the same are zero. So, therefore, yes: each snowflake is unique.

Directions: Please read the article above and use it to answer the questions that follow.

1. The article states that "10 raised to the 10 raised to the 13 power…is a one followed by 10 trillion zeroes." Explain why this is true.

2. How is the probability that two snowflakes are the same determined?

3. The probability that two snow flakes are the same is not exactly 0, what is it?

4. The author tries to express how large 10 to the 40^{th} power is. To get a sense of how big, suppose the earth is a sphere we could fill with sand, how many grains of sand would fit? (Assume the diameter of the earth is 12,756 km, the diameter of a grain of sand is 0.5 mm, and the sand fills the earth completely.)

5. How does your answer from 4 compare with the "staggeringly monstrous" 10 to the 40^{th}?

Connections to CCSM

- SMP 2 Reason abstractly and quantitatively.
- SMP 3 Construct viable arguments and critique the reasoning of others.
- (Number and Quantity - The Real Number System) Extend the properties of exponents to rational exponents.
  2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Solutions & teaching notes

1. 10 raised to the *n*th power is equivalent to 1 with n zeroes after it.
   $10^{13} = 10{,}000{,}000{,}000{,}000$ = 10 trillion and so $10^{10^{13}} = 10^{10{,}000{,}000{,}000{,}000}$ which is 1 with 10 trillion zeroes after it.

2. P(two snowflakes are the same) = $\frac{number\ of\ snowflakes\ that\ have\ fallen\ in\ the\ history\ of\ the\ world}{number\ of\ possible\ snowflake\ shapes}$
   Note this assumes that all the possible shapes are equally likely and occur at random. For more information see the article referenced in the story.

3. P(two snowflakes are the same) = $\frac{10^{40}}{10^{10^{13}}} = \frac{1}{10^{(10^{13}-40)}} \approx \frac{1}{10^{10^{13}}}$

4. $V_{grain\ of\ sand} = \left(\frac{4}{3}\right)\pi\left(\frac{0.5}{2}mm\right)^3 = (6.545\times10^{-2}\ mm^3)\times\left(\frac{1\ m}{10^3 mm}\right)^3 = 6.545\times10^{-11}\ m^3$

   $$V_{earth} = \left(\frac{4}{3}\right)\pi\left(\frac{12{,}576}{2}km\right)^3 = (1.041\times10^{12}\ km^3)\times\left(\frac{1000m}{1km}\right)^3 = 1.041\times10^{21}\ m^3$$

   The number of grains of sand needed is

   $$\frac{V_{earth}}{V_{grain\ of\ sand}} = \frac{1.041\times10^{21}\ m^3}{6.545\times10^{-11}\ m^3} = 0.1591\times10^{32}$$
   $$= 1.591\times10^{31}\ grains\ of\ sand\ to\ fill\ the\ earth$$

5. 10^{40} is $\frac{10^{40}}{1.591\times10^{31}} = 6.2\times10^{8}$ = 620,000,000 times greater than the number of grains of sand needed to fill the earth. In other words, 620,000,000 earths filled with sand are needed to have 10^{40} grains of sand.

Is $87 billion a "drop in the bucket"?

In class we listened to the audio clip "Perspective: Iraq Funding Request" from NPR's Morning Edition on October 1, 2003 . If you want to find it on the web it is at:

http://www.npr.org/templates/rundowns/rundown.php?prgId=3&prgDate=10-1-2003

Early in the clip the announcer, Bob Edwards, says "...Hard to visualize what 87 billion dollars really means" and the interviewee Kevin Hassett says "[87 Billion dollars is a] drop in the bucket in the federal budget. It's a small amount of money really relative to the amount of money that this country generates."

Directions: Your task is to answer the question, is it fair to compare $87 billion as a portion of the federal budget with a drop in a bucket? Give a thorough explanation for your answer; be sure it is supported by appropriate mathematics.

Relevant information:

- According to the OMB [Office of Management and Budget] the for the fiscal year 2004 was $2,140,000,000,000 or $2.140 trillion dollars [1]
- Using a small dropper I found 110 drops in a teaspoon.
- Assume your bucket is a standard 5 gallon bucket.

[1] http://w3.access.gpo.gov/usbudget/fy2004/budget.html

For homework:

The following clip aired NPR's Morning Edition on May 2, 2013[2]:

RENEE MONTAGNE, HOST: Our last word in business today, is austerity at the French presidential palace. President François Aland has already enacted several cost-cutting measures since being elected last year.

DAVID GREENE, HOST: He's cut a fleet of presidential and government cars and reduced ministerial salaries, and now he's raiding the wine cellars for which the presidential palace is famous.

MONTAGNE The palace will auction off 1,200 bottles of its finest wines - some worth more than $3,000. It hopes to raise over $300,000. The proceeds will be reinvested in more modest wines for the presidential cellar. Clearly no self-respecting French president would have an empty wine cellar.

GREENE: That would never happen. After buying those cheaper wines, the money left over will go to the state budget. For the first two months of this year, France's budget deficit was $35 billion, so whatever money is saved with this wine decision, it'll really just be a drop in the bucket.

That's the business news on MORNING EDITION from NPR News. I'm David Greene.

MONTAGNE: And I'm Renee Montagne.

Directions: Determine whether or not this comparison with a drop in the bucket is a fair one.

Connections to CCSM

- **SMP 1** Make sense of problems and persevere in solving them.
- **SMP 2** Reason abstractly and quantitatively.
- **SMP 3** Construct viable arguments and critique the reasoning of others.
- **SMP 4** Model with mathematics.
- **SMP 5** Use appropriate tools strategically.

Solutions & teaching notes

For a discussion of solutions to this problem see the attached article:

Donovan, J.E., II. (2006). Making Math Real Effectively: Two Examples Based on Current Events and Research-Based Advice on Using Them. *New York State Mathematics Teachers' Journal*, 56(3), 120-123, 126.

2 http://www.npr.org/2013/05/02/180491240/the-last-word-in-business

The above ad the appeared in the *Bangor Daily News* on February 7, 2003. Notice that two prices are given for the carpet. Which of these prices is the better buy? Why? *Give an explanation for your answer.*

Retirement Planning

The advertisement below appeared in the March 15, 2004 issue of Newsweek.

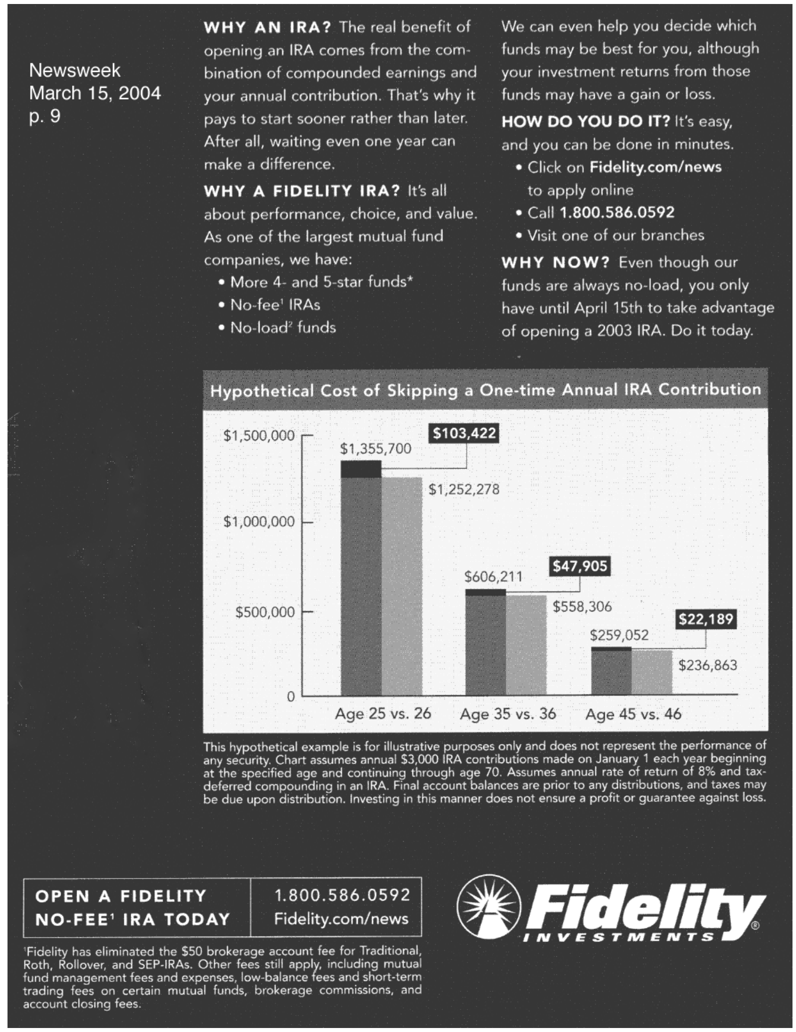
1. Use Excel to verify the results of the hypothetical examples given.
2. In Excel, create a bar graph similar to the one shown in the ad.
3. An annuity is an investment account into which regular periodic contributions are made. If you invest P dollars at the beginning of each year and earn an annual rate of i at the end of each year, at the end of t years you will have $A(t)$ where

$$A(t)=P\times\left[\frac{(1+i)^t-(1+i)}{i}\right]$$

4. Verify the numbers in the Fidelity advertisement using this formula.
5. If someone started at age 20 and saved \$3000/year under the assumptions here, how much would they have when they retired at age 70?
6. Derive this formula. (Hint: Write out the first few terms of the calculation from the spreadsheet and look for patterns

Note: If the compounding takes place n times per year $A(t)$ is

$$A(t)=PMT\times\left[\frac{\left(1+\frac{i}{n}\right)^{nt+1}-\left(1+\frac{i}{n}\right)}{\frac{i}{n}}\right]$$



Consider the following scenario:

Two friends, Cole and Bryn, are both about to celebrate their 25th birthdays. Sitting around one cold winter day talking over a cup of coffee Bryn started to talk about retirement, "When I retire I am going to spend all my days sitting on a beach in Florida." "Yeah right," Cole said, "you will never have enough money to afford such a lifestyle." Bryn returned, "Yes I will, I have a plan to save and I am going to start on my 25th birthday. For my 25th birthday I am going to deposit \$2000 in an IRA. I am going to continue to do this for my next 9 birthdays, 10 total, and that will be all I need for retirement." Cole replied, "I too have been thinking about retirement although my plan is a little different than yours. In fact, I think my plan will give me more money to retire with." "Maybe, maybe not," Bryn replied, "what is your plan?" "Well, I am tired of being poor. I finally have a decent job and I can afford to put away \$2000 per year now but I am going to wait for 10 years to start, I am going to start on my 35th birthday. For the next 10 years I am going to see the world. Each year I am going to use the \$2000 to take a trip abroad. From age 35 to 59 I will save \$2000 a year and when we turn 60 I will have more money saved than you."

7. Assume Cole and Bryn each earn a constant interest rate for the life of their investments and that the interest compounded annually. Describe the conditions for which Cole's total will exceed Bryn's.

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Making Math Real Effectively: Two Examples Based on Current Events and Research-Based Advice on Using Them

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The mathematics tasks in which students engage–projects, problems, constructions, applications, exercises, and so on–and the materials with which they work frame and focus students' opportunities for learning mathematics in school. Tasks provide the stimulus for students to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts. Good tasks can help students to develop skills in the context of their usefulness. Tasks also convey messages about what mathematics is and what doing mathematics entails. Tasks that require students to reason and to communicate mathematically are more likely to promote their ability to solve problems and to make connections. Such tasks can illuminate mathematics as an intriguing and worthwhile domain of inquiry. (NCTM, 1991, p. 24)

The NCTM's vision of "worthwhile mathematical tasks" quoted above sounds great in theory, but how can this be put into practice to create meaningful learning opportunities in the classroom? Two ingredients for success are the context of a task and the actions of the teacher when using the task. The context of a task is important because it influences whether or not a student will engage in a task; students must engage for learning to occur. Tasks that have meaning to students' realities are likely to achieve this goal. But even if a student engages in a mathematically rich task, the learning opportunity can be undermined by the actions of the teacher. This leads to three important questions: how can engaging contexts be found, what teacher actions undermine learning, and what teacher actions support task-based learning?

An area with great potential for generating engaging contexts and mathematically rich tasks is current events. Two specific examples follow to illustrate how current events can be used as a basis for mathematical tasks. These examples provide a context to then discuss teacher actions that undermine and support learning. The first example involves a comparison of the U.S. federal budget to drops in a bucket, and the second investigates the population of an invasive species, mute swans in the Chesapeake Bay. Each example includes selected quotes from the news item, a task, mathematical analyses of the task, and an overview of the embedded mathematics.

Example 1: A drop in the bucket?

In September and October of 2003 there was much discussion and debate about the President's request for $87 billion to fund ongoing military efforts in Afghanistan and Iraq. Pundits from both sides of the issue spoke their minds in the national media. In an interview on National Public Radio [NPR], an economist discussed the magnitude of such a monetary request (Montagne, 2003). The interview began as follows:

> **Show Host:** While Congress debates the President's spending request it may be hard to visualize what 87 billion dollars really means. NPR's [interviewer] asked economist [Economist]... to put the figure into perspective.
>
> **Interviewer:** ...Senator Bird last week called this request fiscal shock and awe; that's a quote from him. Are you shocked? Are you awed?
>
> **Economist:** No, absolutely not. No, <u>it's a drop in the bucket in the federal budget</u>. It's a small amount of money really relative to the amount of money that this country generates. ...Fiscal shock is a bit of an exaggeration.

The interview lasted four and a half minutes. Many statements were made that could be the subject of mathematical analysis, but the economist's initial remarks provide a good source for a rich task.

Is it accurate to compare an $87 billion piece of the federal budget with a drop in a bucket? Why or why not? Be sure to clearly state any assumptions you make.

Although the phrase "a drop in the bucket" is just a cliché, it can be evaluated on its mathematical merit and in this case the analysis shows it to be quite an exaggeration. There are several approaches to investigate the comparison, two are given below. You may wish to put the article aside and attempt the task yourself before reading on.

One approach is to focus on the money. In the fiscal year 2004 the U.S. Government budgeted $2,229,000,000,000 or $2.229 trillion dollars ("Budget of the United States", 2004) for spending. The ratio of $87 billion to the total budget is

$$\frac{\$87{,}000{,}000{,}000}{\$2{,}229{,}000{,}000{,}000} = \frac{\$87}{\$2{,}229} \approx \frac{\$39}{\$1000} \approx \frac{\$1}{\$26}.$$

In other words, about $1 out every $26 in the budget is needed to fund the request. For this 1:26 ratio to be maintained as the ratio a single drop to a bucket full of drops, the size of the bucket would be about 26 drops, i.e. the "bucket" would be smaller than a teaspoon.

An alternative approach is to focus on the bucket. First off, how big is a bucket? Let us assume a 5-gallon bucket. How many drops are needed to fill the bucket? An informal experiment using a children's medicine dropper yielded 110 drops in a teaspoon. This equivalence can be used to estimate the number of drops in a 5-gallon bucket as follows:

$$\frac{110\ drops}{1\ tsp} \times \frac{3\ tsp}{1\ T} \times \frac{2\ T}{1\ oz} \times \frac{128\ oz}{1\ gal} = \frac{84{,}480\ drops}{1\ gal} = \frac{422{,}400\ drops}{5\ gal}$$

In other words, a single drop is one of 422,400 needed to fill a 5-gallon bucket based on our assumptions. For this ratio to be maintained as the ratio of $87 billion to the federal budget, the federal budget would have to be $36,748,800,000,000,000 – over $36 quadrillion – which is 16,487 times our current budget. Similarly, if a request of our current budget was made so that it maintained the ratio 1:422,400 the request could only be for $5,276,989, or 0.0061% of the $87 billion request.

Mathematically this task is rich. The concepts of volume, ratio, proportion, conversion, units of measure, estimation, experimental error, and percentage are embedded within the context of large numbers. There are several follow-up questions that can be used to extend the task and have students focus on specific details. For example:

- How do the assumptions made about the size of a drop and the size of a bucket impact the results?,
- Suppose the economist really meant to compare the request to the GDP, how would this affect the comparison?, and
- What physical objects around school have sizes relative to one another that represent the ratios in the example?

Example 2: Mute Swans in the Chesapeake Bay

Over the past few years there have been many news stories about invasive species of plants and animals. For example in New York, Lakes Erie and Ontario have dealt with zebra mussels.

> To date, at least 162 nonindigenous aquatic species have colonized the Great Lakes—North America's most susceptible entry point for biological interlopers. The most economically significant aquatic invader to have hit the continent is the zebra mussel (*Dreissena polymorpha*), an import in 1988 from the Black Sea that has become an ecological and economic disaster... Inedible to most of North America's indigenous species, the mussels proliferate into massive colonies that strangle native ecosystems. They also clog water pipes of industrial plants, causing them to reduce or suspend operations. (Harder, 2002, p. 234)

A recent interview on NPR focused on invasive species (Williamson, 2003):

> **Host:** ...A federal judge ruled this week that a controversial plan to kill hundreds of mute swans on Maryland's eastern shore will be postponed until at least the end of the year. The swans are not a native species, and the state says they threaten the Chesapeake Bay's ecosystem. NPR's [Reporter] reports.
>
> **Reporter:** It sounds like a science fiction movie from the '50s--a couple of exotic animals show up in your neighborhood. Maybe they're somebody's pets. They adapt to their surroundings, food is plentiful, they have no predators. One day, they escape. They thrive in the wild. A short time later, there are four of them, then 16, before long, several hundred. Soon, they're driving native birds and animals away, but this isn't science fiction. This is happening in every state. These aliens are called an invasive species. They're not only animals, they can be plants or bugs and they have become a big problem. ...
>
> **State Official:** ...The mute swan is an exotic species from Asia. ...We currently have 3,600 of these birds on the Chesapeake in Maryland today. All of our swans are the descendents of five birds that escaped in 1962. When we look out at our 3,600 birds and the damage that they're doing to the aquatic grass beds and to the other wildlife species, it's of great concern to us. Left undisturbed, we'll have well in excess of 10,000 of these birds inside of a decade. We don't know what the capacity of the Chesapeake is for them, but the damage is going to increase until such time as there are just too many for the aquatic grass beds and then the whole population theoretically will crash some day.

The story continued for almost 6 minutes, but the comments quoted above give enough information to pose a task about population growth.

Find a model that is consistent with the given data and use it to predict the mute swan population in 20 years. Explain why your prediction is consistent with the given data. Do you think your prediction will come true? Why or why not?

Again you may wish to put the article aside and answer the question for yourself.

To begin, the reporter's hypothetical description of the growth can be used to generate a model. According to the description, the population of invasive species follows the pattern: 2, 4, 16, "and before long several hundred." Assume the numbers reflect the population level of successive generations. Since swans reproduce annually, it can further be assumed that each generation is 1 year apart. So, what is the pattern? Rewriting the list as 2, $2^2, 4^2$ suggests that the reporter may have had in mind that the growth of successive generations is the square of the previous generation, i.e.

$$Population_{Next} = (Population_{Now})^2 .$$

If so, the next few terms of the list would be: 256; 65,536; and 4,294,697,296. Such growth is untenable for many reasons. First, consider what would have to happen in the reproduction at each generation. Theoretically, the first two swans would have to produce 1 swan each on average resulting in a population of 4. In the second generation each swan would have to produce 3 swans on average to give a population of 16. In the third generation, each of the sixteen swans would have to produce 15 swans on average, and so on. In this model, the number of swans produced on average by each swan in successive generations increases exponentially. It does not seem reasonable that the ability of swans to reproduce would increase so drastically with each generation. The second reason this model is untenable is that the population growth is too rapid. After only four generations the population is over 60,000. This approach assumes a particular pattern based what the reporter said, but now that our reasoning shows the first model is faulty, let's consider another.

The original model can be modified to be more reasonable by assuming the swans average reproduction rate remains constant across generations. For example, assume that on average the swan population doubles from one generation to the next, i.e.

$$Population_{Next} = 2Population_{Now} .$$

The implications of this assumption are easily found and represented in a spreadsheet. Figure 1 shows a worksheet produced using Microsoft® Excel. The first column shows the year and the second column shows the population of that generation. The data in the table shows the population will exceed 1 million before 1980, and thus is not reasonable for the given data.

	A	B
1	Year	Population
2	1962	5
3	1963	10
4	1964	20
5	1965	40
6	1966	80
7	1967	160
8	1968	320
9	1969	640
10	1970	1,280
11	1971	2,560
12	1972	5,120
13	1973	10,240
14	1974	20,480
15	1975	40,960
16	1976	81,920
17	1977	163,840
18	1978	327,680
19	1979	655,360
20	1980	1,310,720

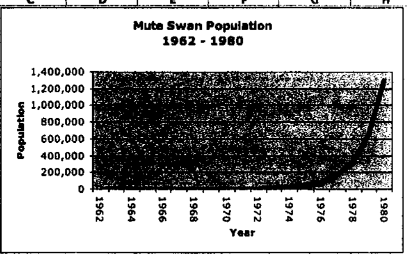

FIGURE 1: Mute swan population if the population doubles each generation.

In order to adapt the model to reflect the known mute swan population data, the model's assumptions need to be reconceptualized in terms of percentage growth. When a quantity doubles, it grows by 100% of its original value. What if instead of growing at a 100% rate, the population of swans increases at a slower rate? For example, suppose the population increases by 50% during each reproduction period, i.e.

$$Population_{Next} = 1.5Population_{Now}$$

With the spreadsheet, this change is easily accommodated. Figure 2 shows a table containing populations over the 41-year period from 1962 – 2003 for three different growth rates *r*. Through trial and error a growth rate that that fits the given data is found. The data in column D show that with an annual growth rate of 17.41% the population will be 3,605 in 2003, closely approximating the population of 3600. Extending this table (not shown in Figure 2) shows the population will exceed 10,000 in 2010 and will grow to 89,327 in 2023, i.e. in 20 years.

	A	B	C	D
1		r = 50%	r = 20%	r = 17.41%
2	Year		Population	
3	1962	5	5	5
4	1963	8	6	6
5	1964	11	7	7
6	1965	17	9	8
7	1966	25	10	10
8	1967	38	12	11
9	1968	57	15	13
10	1969	85	18	15
11	1970	128	21	18
12	1971	192	26	21
13	1972	288	31	25
14	1973	432	37	29
15	1974	649	45	34
16	1975	973	53	40
17	1976	1,460	64	47
18	1977	2,189	77	56
19	1978	3,284	92	65
20	1979	4,926	111	77
21	1980	7,389	133	90
22	1981	11,084	160	106
23	1982	16,626	192	124
24	1983	24,939	230	145
25	1984	37,409	276	171
26	1985	56,114	331	201
27	1986	84,171	397	235
28	1987	126,256	477	276
29	1988	189,384	572	325
30	1989	284,076	687	381
31	1990	426,113	824	447
32	1991	639,170	989	525
33	1992	958,755	1,187	617
34	1993	1,438,133	1,424	724
35	1994	2,157,199	1,709	850
36	1995	3,235,799	2,051	998
37	1996	4,853,699	2,461	1,172
38	1997	7,280,548	2,953	1,376
39	1998	10,920,822	3,544	1,616
40	1999	16,381,233	4,253	1,897
41	2000	24,571,850	5,103	2,227
42	2001	36,857,774	6,124	2,615
43	2002	55,286,662	7,349	3,070
44	2003	82,929,992	8,819	**3,605**

FIGURE 2: Mute swan population growth by constant percentages.

Embedded in the analysis above are several important concepts including recursion, generalization, percentage growth, exponential growth, the connections between percentage and exponential growth, mathematical modeling, and the opportunity to connect these through graphs tables, and equations. As with the previous example, follow-up questions can be posed to further students' analysis. For example:

- According to the state official, "the whole population will theoretically crash someday." Explain what this means. Modify your original model to account for the crash., and
- Find information about an invasive species that affects your community using the internet, town library, or the town offices. Find a model that is consistent with the data you find.

Using Non-routine Tasks in the Classroom

Having students complete rich mathematical tasks is not a guarantee of rich mathematical learning; teachers' actions have an impact. Based on extensive classroom observations, Stein, Grover, and Henningsen (1996) identified 7 teacher actions associated with maintaining high-level cognitive demands of tasks and 6 factors associated with the decline of high-level cognitive demand. These factors are shown in Figure 3.

Consider how the factor 1 associated with the decline could play out with the drop in the bucket example. The original problem asks: Is it accurate to compare an $87 billion piece of the federal budget is with a drop in a bucket? The cognitive requirements of this task change significantly if the statement of question includes the procedures to be performed. For example:

Compare the portion of the federal budget that $87 billion is with the portion of a bucket filled with water a drop occupies by computing (a) the ratio of a drop to a bucket (estimating the size of a drop and the bucket) and (b) the ratio of $87 billion to the U.S. budget (you can look this information up on the internet). Compare the values from (a) and (b) to see if the comparison is valid.

Compare your thinking about this version of the task to your thinking about the original task, what is the difference? The difference is mathematical thinking. Both versions include mathematical thinking, but in the original tasks students have to consider ways to make the comparison and reason about the validity of their choice. Differences between students or groups can be used to initiate class discussion.

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Factors associated with the decline of high-level cognitive tasks
1. Problematic aspects of the task become routinized (e.g. students press the teacher to reduce the complexity of the task by specifying explicit procedures or steps to be performed; the teacher "takes over" the thinking and reasoning and tells students how to do the problem.
2. The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.
3. Not enough time is provided to wrestle with the demanding aspects of the task or too much time is allowed and students drift into off-task behavior.
4. Classroom management problems prevent sustained engagement in high level cognitive activities.
5. Inappropriateness of task for a given group of students (e.g. students do not engage in high level cognitive activities due to lack of interest, motivation or prior knowledge needed to perform; task expectations not clear enough to put students in the right cognitive space.
6. Students are not held accountable for high level cognitive processes (e.g. although asked to explain their thinking, unclear or incorrect student explanations are accepted; students are given the impression that their work will not "count" toward a grade.)
Factors associated with the maintenance of high-level cognitive demands
1. Scaffolding of student reasoning and thinking.
2. Students are provided a means for monitoring their own progress.
3. Teacher or capable students model high-level performance.
4. Sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback.
5. Tasks build on students' prior knowledge
6. Teacher draws frequent connections.
7. Sufficient time to explore (not too much, not too little).

FIGURE 3: Factors associated with learning outcomes from tasks with a high-level cognitive demand.

Conclusion

Circle Time is but one "moment" in the day to engage young learners in mathematics. Other times during the day, such as free play, transitions, snack time, story time, or during other content, can offer opportunities for infusing mathematics in the early childhood classroom. Teachers can draw upon many available resources for ideas on how to effect the desired integration of mathematics during these other "moments" in the school day.

Suggested Resources

Early Childhood Mathematics by Susan Sperry Smith (2006), Pearson Education, Inc.

Reasoning for Elementary Teachers by Long and De-Tempo, (4e), Addison-Wesley Any Early Childhood Mathematics methods textbook

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Continued from page 123

Each of the factors in Figure 3 are discussed in more detail in the book *Implementing Standards-Based Mathematics Instruction: A Casebook for Professional Development* by Stein, Smith, Henningsen, and Silver (2000). The book also includes detailed classroom episodes to illustrate how these factors play out in the classroom.

Conclusion

The importance of student work on tasks that require more than routine application of skills cannot be overstated; "the mathematics tasks in which students engage frame and focus students' opportunities for learning mathematics in school" (NCTM, 1991, p. 24). Tasks based on current events do not differ from other nonroutine tasks in the opportunities they create for learning, but they differ in other very important ways: they generate interest and enthusiasm, there are inherent opportunities for interdisciplinary discussion, and students' form their own answers to the question, "When are we ever going to use this stuff anyway?"

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