

Wicked Cool Algebra Tools: Making Sense of Patterns & Functions

David C. Webb
University of Colorado Boulder
Freudenthal Institute US
FiUS@colorado.edu
www.fius.org

Annual Meeting of the National Council of Teachers of Mathematics
Boston, MA
April 18, 2015

The materials in this handout are selections from *Mathematics in Context* resources:
Algebra Tools, Comparing Quantities, & Expressions and Formulas

Please contact Paul Ridgway (mic@eb.com or 800-621-3900 x7049)
at Encyclopaedia Britannica for questions about these and 40+ units
designed using principles of Realistic Mathematics Education

Going Backwards

Pat and Kris are playing a game. One player writes down an arrow string and the output (answer) but not the input (starting number). The other player has to determine the input.

Here are Pat's arrow string and output.

$$\underline{\quad} \xrightarrow{+4} \underline{\quad} \xrightarrow{\times 10} \underline{\quad} \xrightarrow{-2} \underline{\quad} \xrightarrow{\div 2} 29$$

10. a. What should Kris give as the input? Explain how you found this number.
- b. One student found an answer for Kris by using a **reverse arrow string**. What number should go above each of the reversed arrows?

$$\underline{\quad} \longleftarrow \underline{\quad} \longleftarrow \underline{\quad} \longleftarrow \underline{\quad} \longleftarrow 29$$

When it was her turn, Kris wrote:

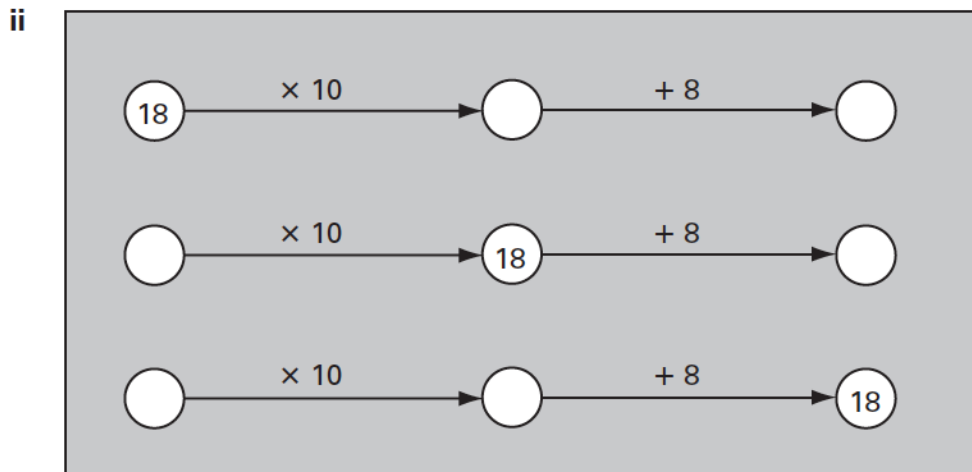
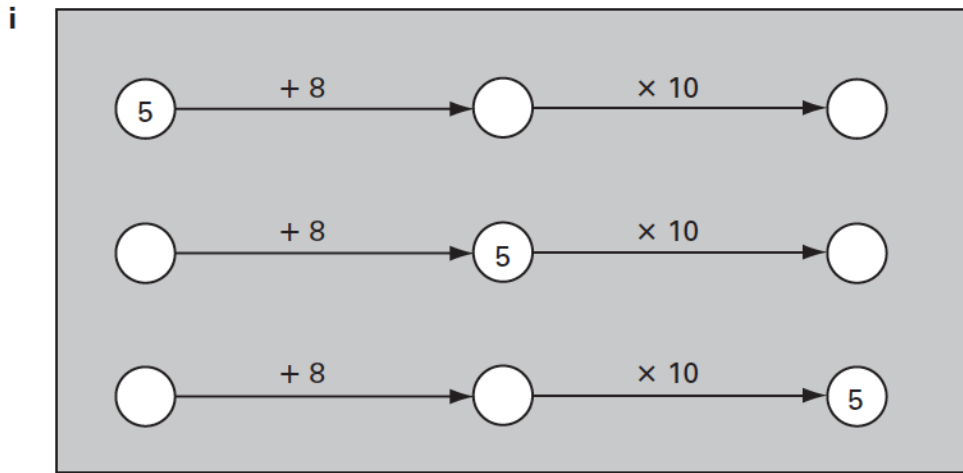
$$\underline{\quad} \xrightarrow{+3} \underline{\quad} \xrightarrow{+6} \underline{\quad} \xrightarrow{+5} \underline{\quad} \xrightarrow{-2} 6$$

11. a. What will Pat give as the input? Explain how you found this number.
- b. Write the reverse arrow string that can be used to find the input.

28 Expressions and Formulas

From the Mathematics in Context unit Expressions and Formulas
© 2013 Encyclopædia Britannica, Inc.

- Fill in the missing numbers.



Look at these two formulas.

$$A = 10 \times n + 8$$

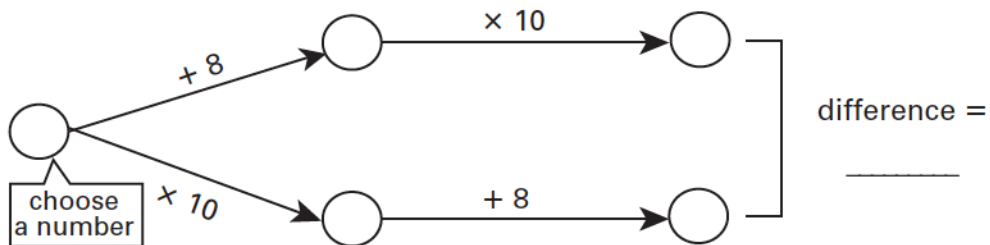
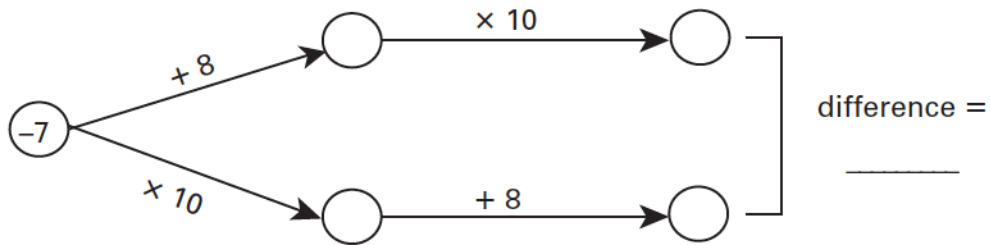
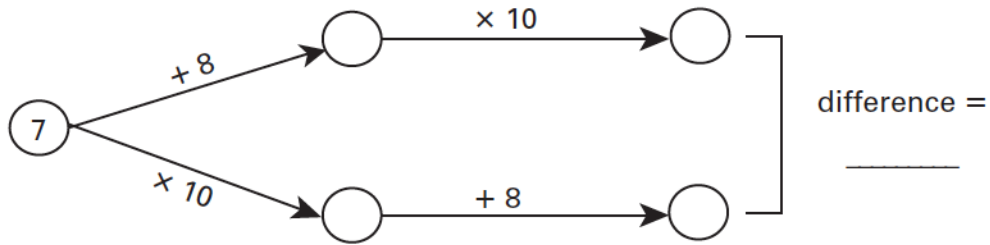
$$B = 10 \times (n + 8)$$

- Which of the two formulas corresponds to the arrow string?

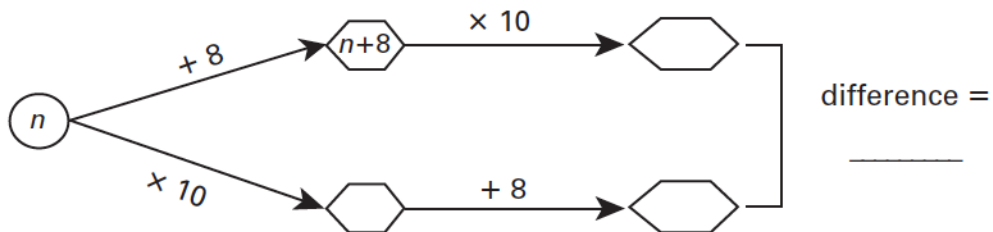


Arrow Strings (2)

- Fill in the missing numbers.



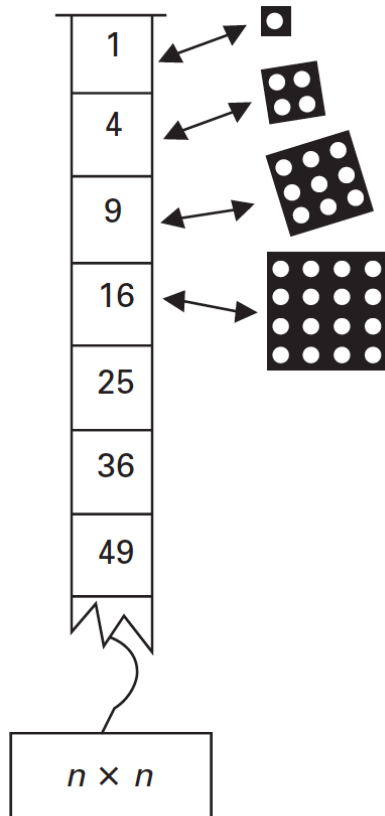
- Fill in the missing expressions.



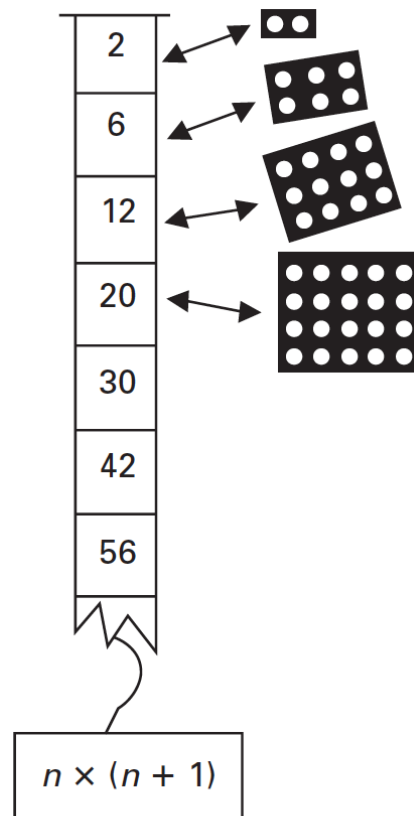


Strips and Dots (1)

square numbers



rectangular numbers



The expression for the square numbers is usually written as n^2 and read as ***n squared***.

- Compare both strips. What strip can you add to the left one to get the right one?

The expressions $n \times (n + 1)$ and $n^2 + n$ are equivalent.

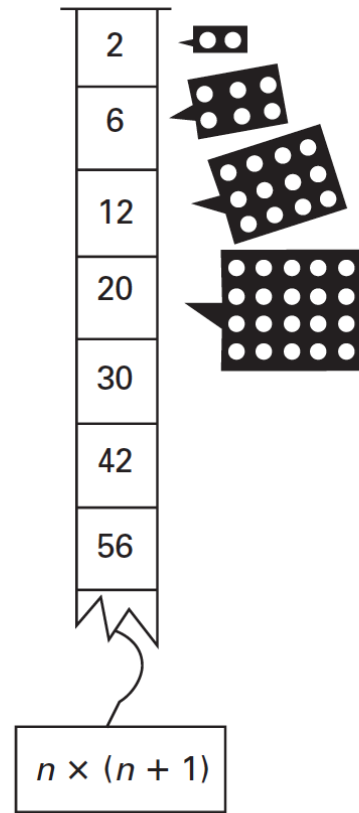
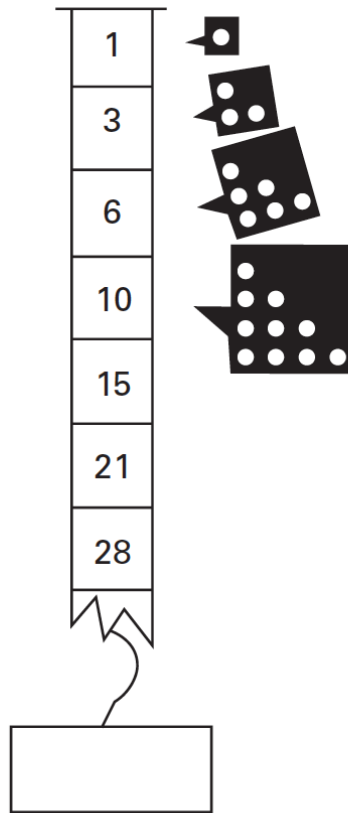
- How can you explain this using dot patterns?



Strips and Dots (2)

triangular numbers

rectangular numbers



© Encyclopaedia Britannica, Inc. This page may be reproduced for classroom use.

Compare the triangular numbers with the rectangular numbers.

- What expression fits the strip of triangular numbers?
- Give one (or more) equivalent expressions.

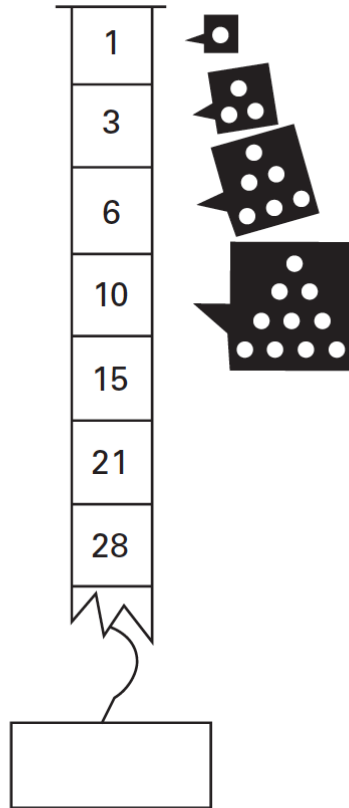
Using the expression for triangular numbers, you can calculate the sum of the first hundred positive whole numbers.

$$1 + 2 + 3 + 4 + 5 + \dots + 98 + 99 + 100 = \underline{\hspace{2cm}}$$

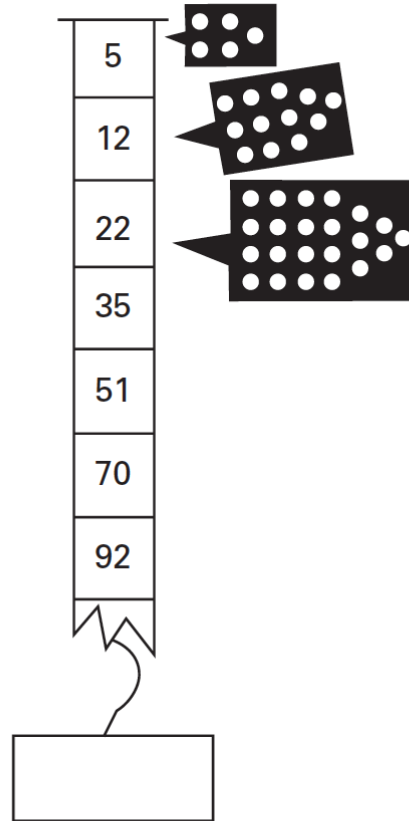


Strips and Dots (3)

triangular numbers



pentagonal numbers

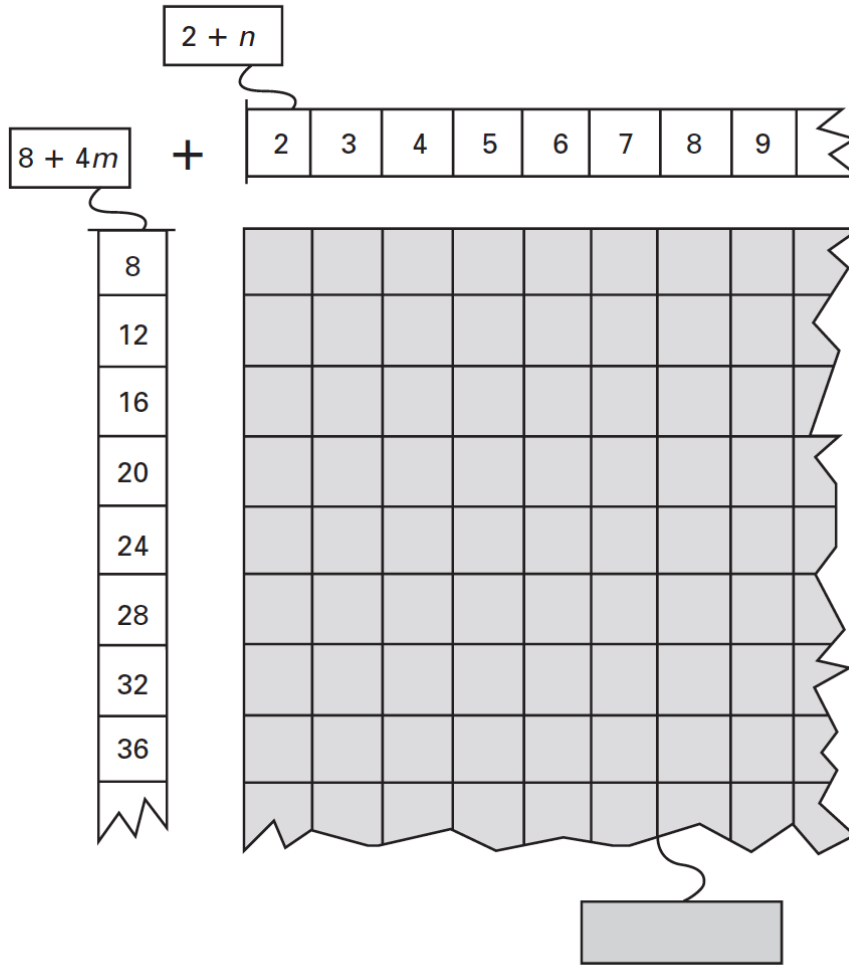


Compare the numbers of both strips.

- Which pentagonal number is right after 92?
- Find an expression that represents the sequence of pentagonal numbers.



Strips and Charts (2)



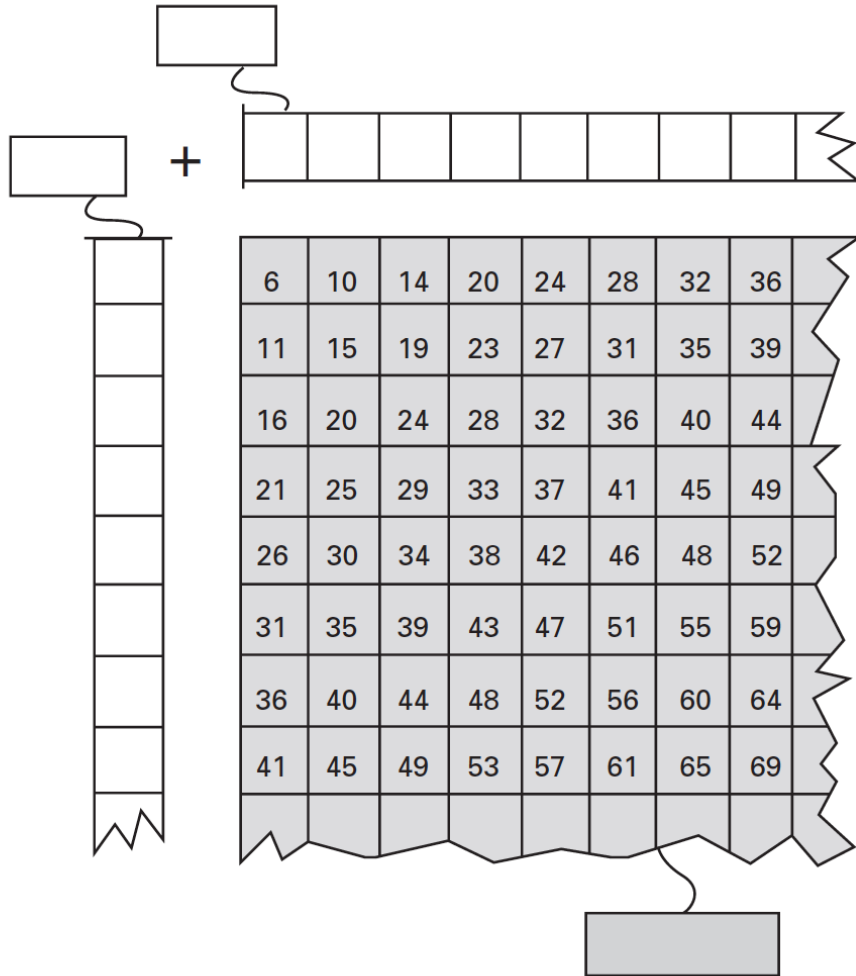
© Encyclopædia Britannica, Inc. This page may be reproduced for classroom use.

- Fill in the chart. What expression fits the chart?
- What strip fits $m = 3$? What is the corresponding expression?
- Answer the same questions for $n = 0$.
- For $m = n$.
- For $m = n + 1$.



Name _____ Date _____ Class _____

Strips and Charts (3)



- What expression fits the chart?
- What strips can be used to make the chart?

© Encyclopædia Britannica, Inc. This page may be reproduced for classroom use.

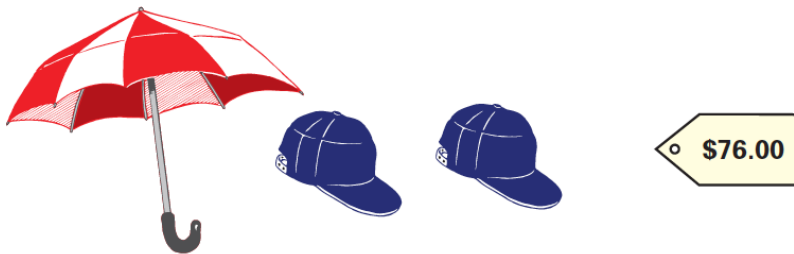
Water and Lemonade



Nikki went to the store and bought 3 bottles of water and 1 bottle of lemonade. She paid \$9. Her friend Angie bought 4 bottles of water and 2 bottles of lemonade. Angie paid \$14.

1. Using the information above, find the cost of 1 bottle of water and the cost of 1 bottle of lemonade. Explain your reasoning.

4. What is the price of one umbrella? One cap?

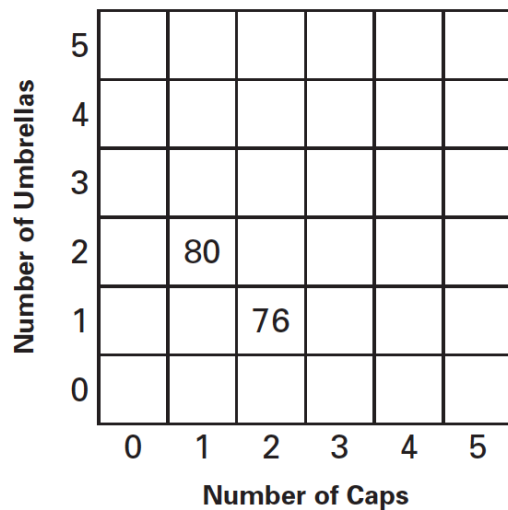


You can use a chart to solve some of these shopping problems.

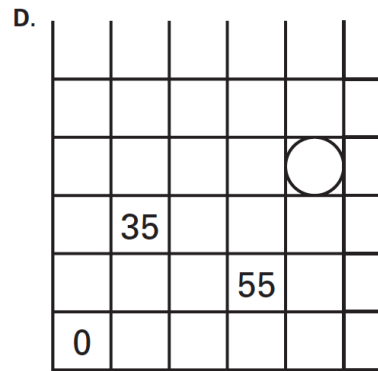
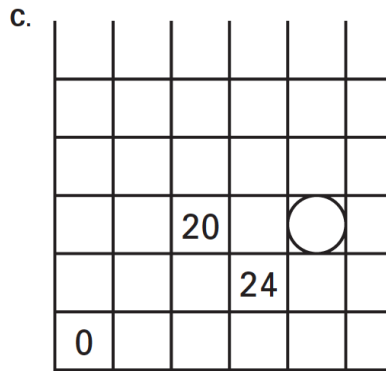
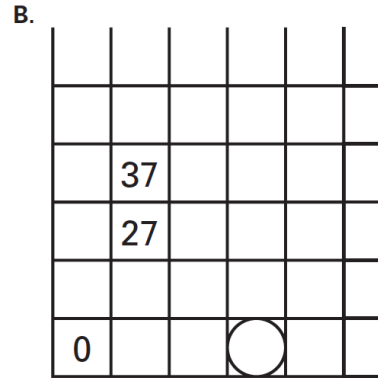
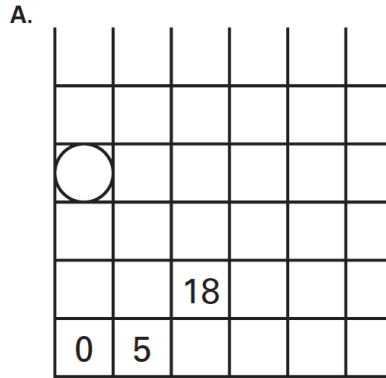
This combination chart represents the problem of the caps and the umbrellas (page 17).

8. Complete this chart on **Student Activity Sheet 4**. Then find the prices of one cap and one umbrella. Is this the same answer you found for problem 4 on page 17?

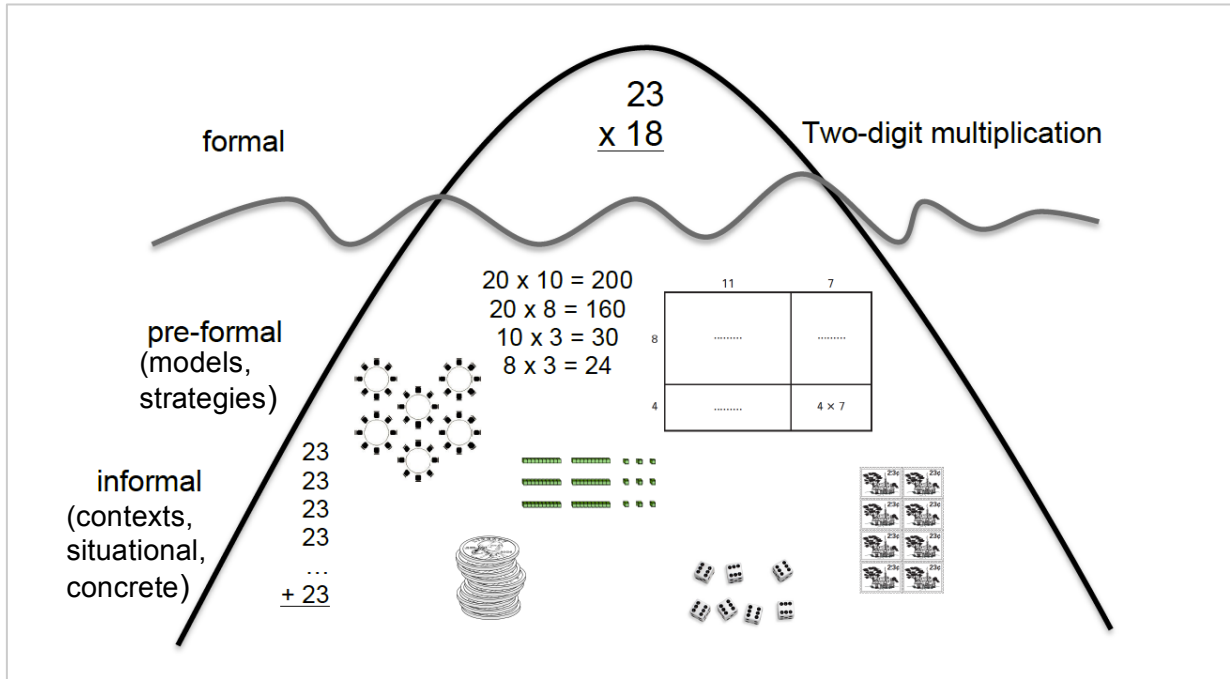
**Costs of Combinations
(in dollars)**



24. Complete the puzzles.



© Encyclopædia Britannica, Inc. This page may be reproduced for classroom use.



5th International Realistic Math Education Conference:
September 18 – 20, 2015

International Society for Design and Development in Education
Conference: Sept 22 – 25, 2015



Registration info: www.fius.org



University of Colorado
Boulder

