Linking Number Sense to Algebraic Thinking

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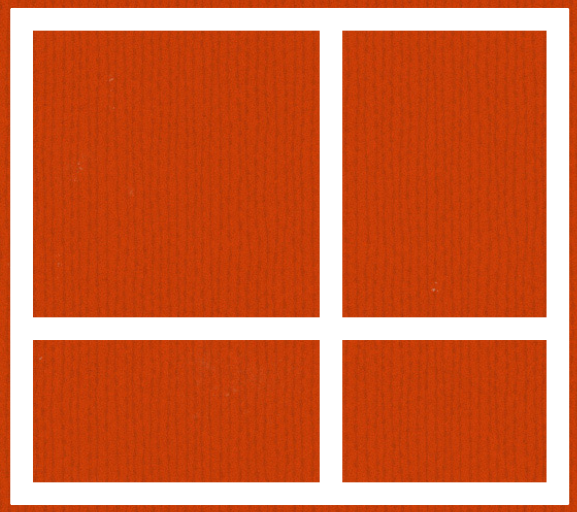
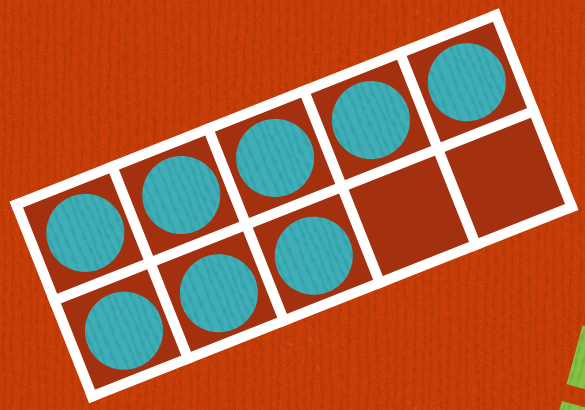
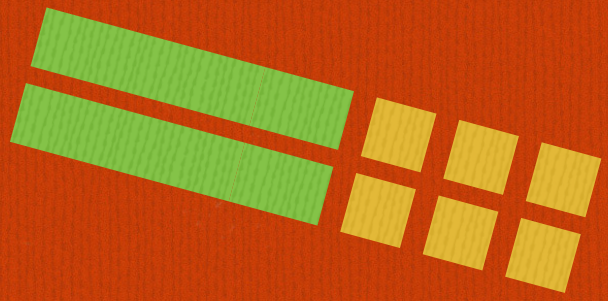
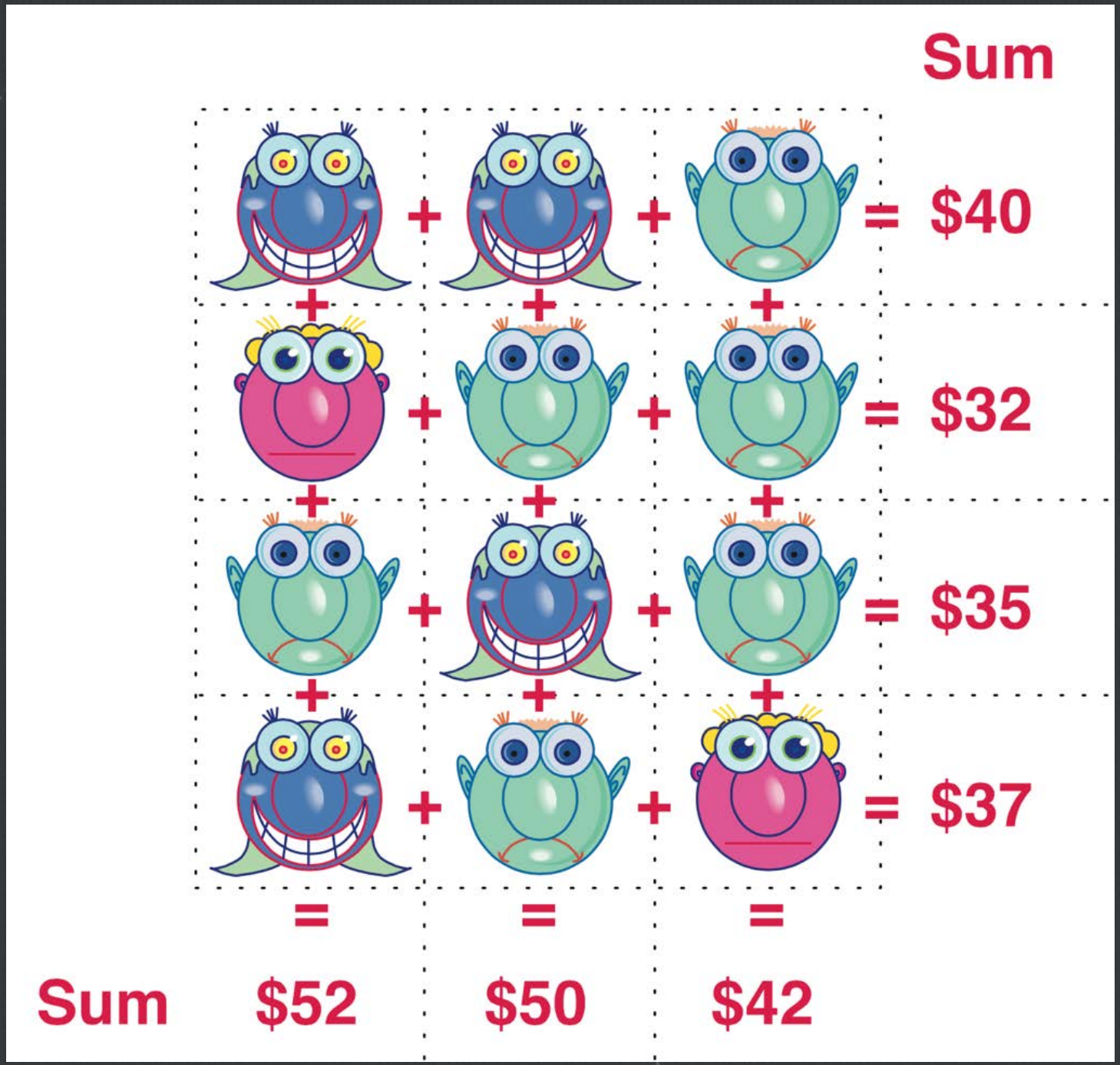




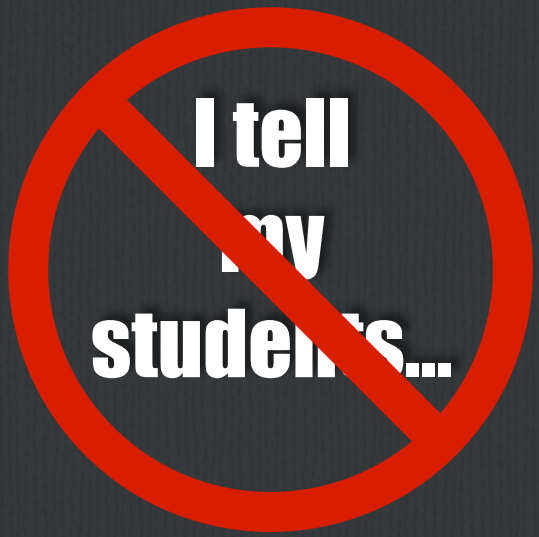




figurethis.org
(NCTM)

What's the Big Idea?

- Many SMPs, but let's focus on
  - "Look for and make use of Structure"
- Sense-making
  - Who's doing the mathematics?
  - Multiple strategies - thinking!
- Convince me!
  - Convince US!
- Who's this session for?
- Beginning the thinking

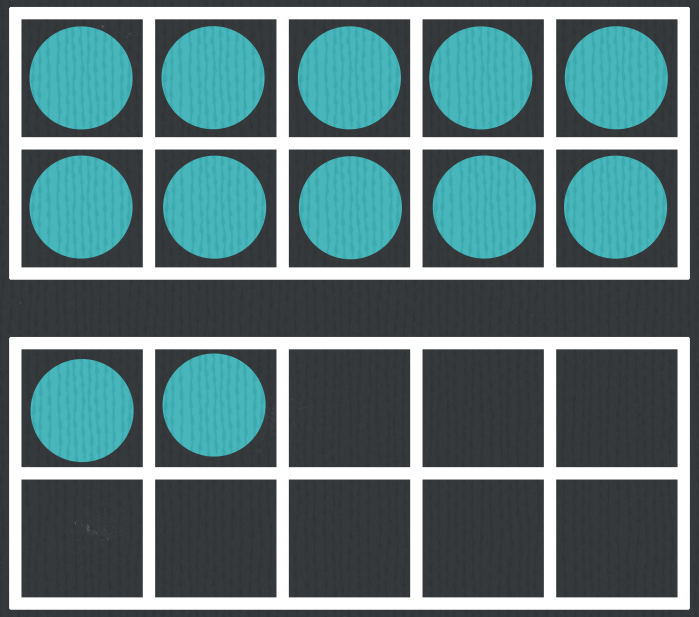


What do you notice?
What do you wonder?

- ☐ We'll go through several types of numerical questions and consider them next to algebraic questions.
- ☐ How are they similar?
- ☐ How are they different?
- ☐ What are the connections?

12

$x + 2$



Opening it up

- The answer is 12. What is the question?
- The answer is $x + 2$. What is the question?
- Come up with a few for each, then try to connect them.
- But not right now :(

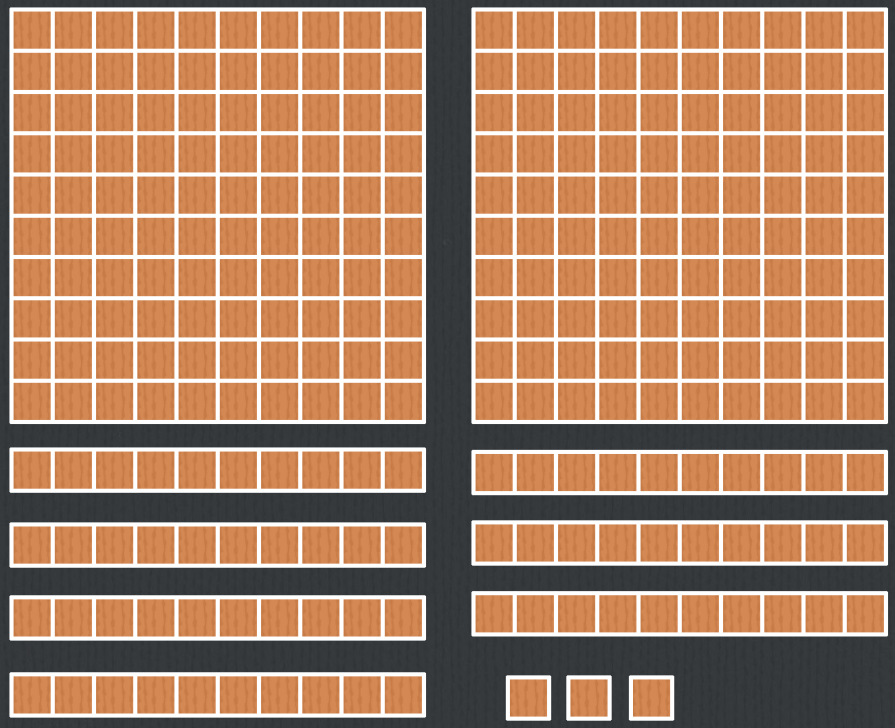
Missing Value

- Part-Part-Whole

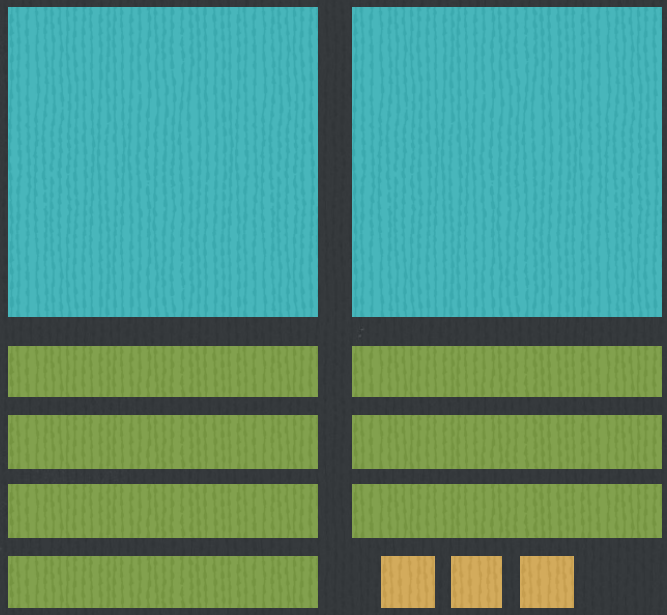
$$\square + 4 = 13$$

$$x + 4 = 13$$

273



$2x^2 + 7x + 3$

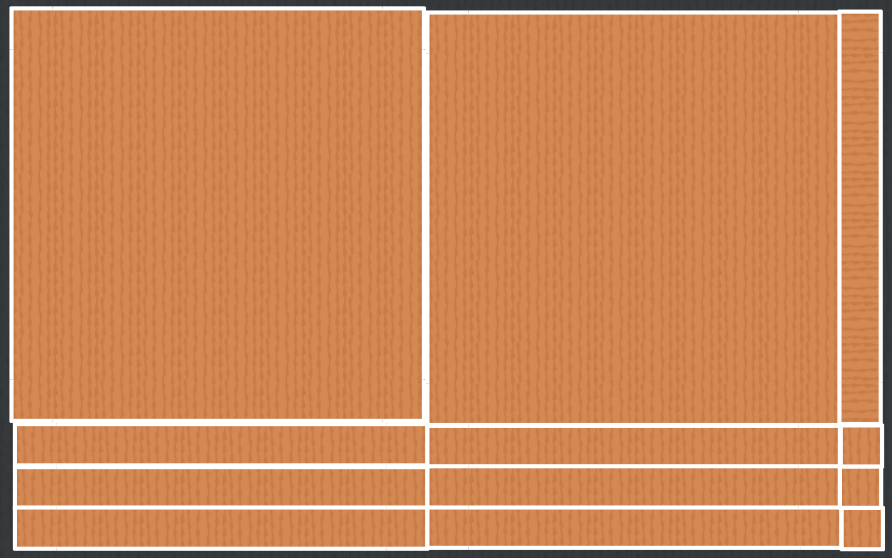


Place Value

- Whole numbers use base 10
- Polynomials have same structure - base *x*
- What is that structure?
- What about fractions?

273

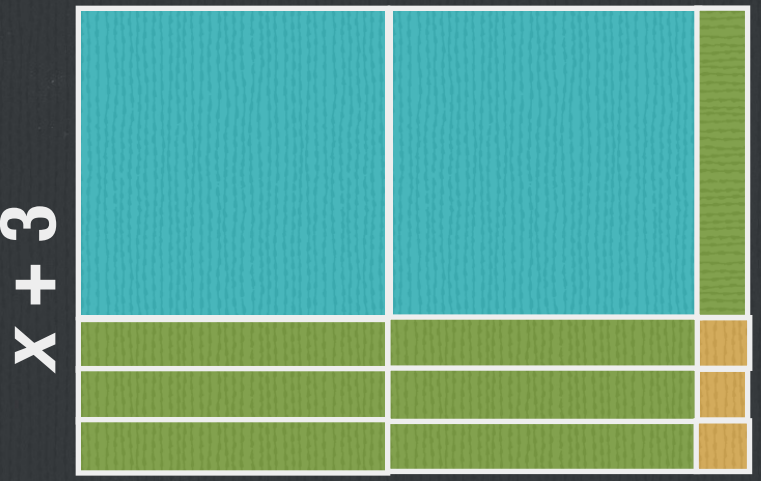
21



= 21 x 13

$2x^2 + 7x + 3$

2x + 1



$= (2x+1)(x+3)$

Algebra to Number?

Factor 651.

Determine 2 numbers with a sum of 50 and a product of 600: 20 and 30

$$651 = 600 + 50 + 1 = 600 + 20 + 30 + 1$$

$$= 20(30+1) + 1(30+1)$$

$$= (30+1)(20+1)$$

$$= (31)(21) = 3 \times 7 \times 31$$

Helpful?
Meaningful?

Other "Interesting" Connections?

- Predict the numerical equivalent…
  - $6x^2+11x+3=(2x+3)(3x+1)$

    $600+110+3=713=23\times 31$
  - $6x^2-7x+2=(3x-2)(2x-1)$

$600-70+2=532=(30-2)\times(20-1)=28\times 19$
  - $9x^2-4=(3x+2)(3x-2)$

    $900-4=896=(30+2)\times(30-2)=32\times 28$

Ten Frame examples 8+9

8 + 9 =

17 – know it cold

10 + 7 – add 1 to 9, subtract 1 from 8

7 + 1 + 9 – decompose the 8 into 7 and 1

18 – 1 – add 10 and adjust or double - 1

16 + 1 – double plus 1

20 – 3 – round up and adjust

Who's right? Does it matter?

Addition

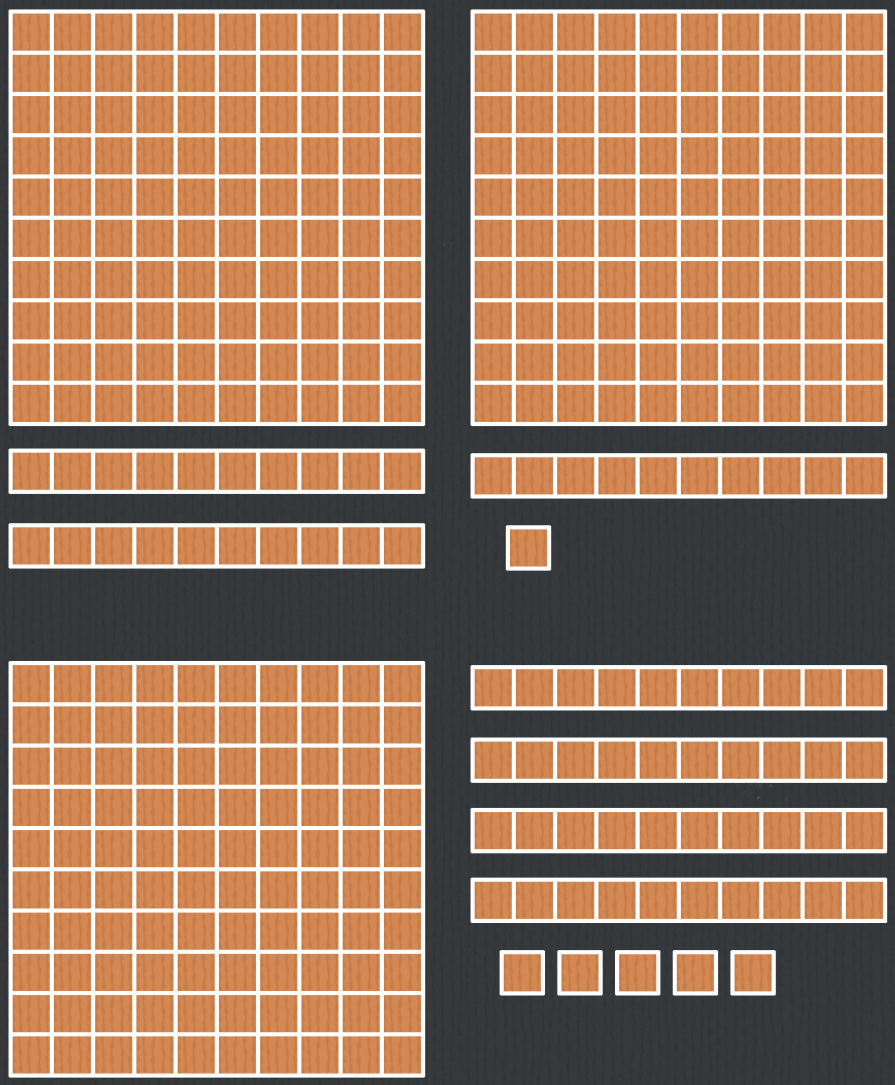
- Focus on the meanings of addition.

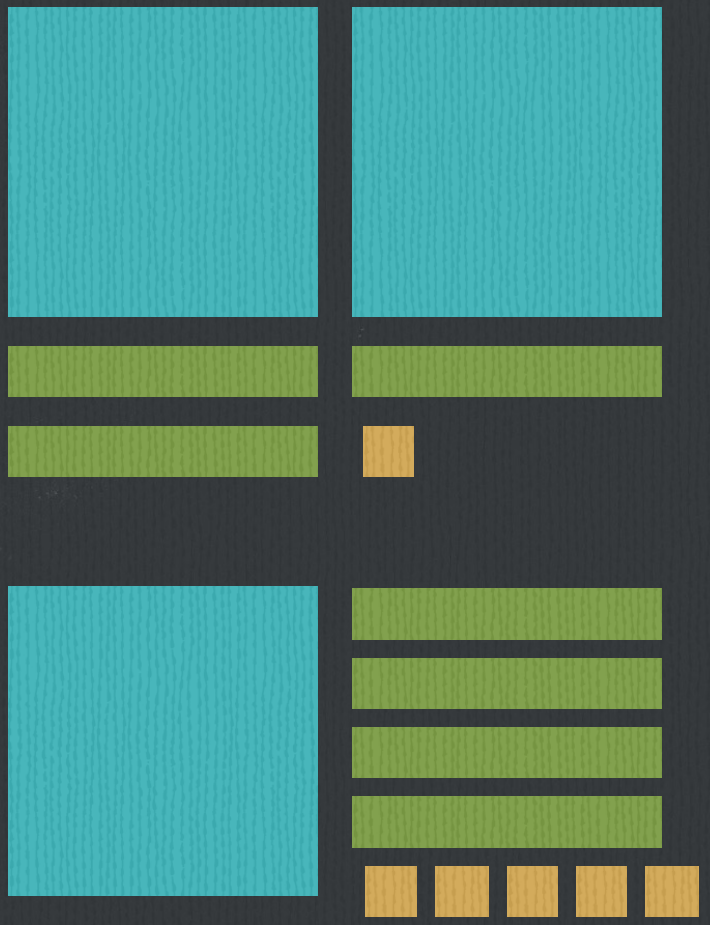
231 + 145

$$\left(2x^2+3x+1\right)+\left(x^2+4x+5\right)$$

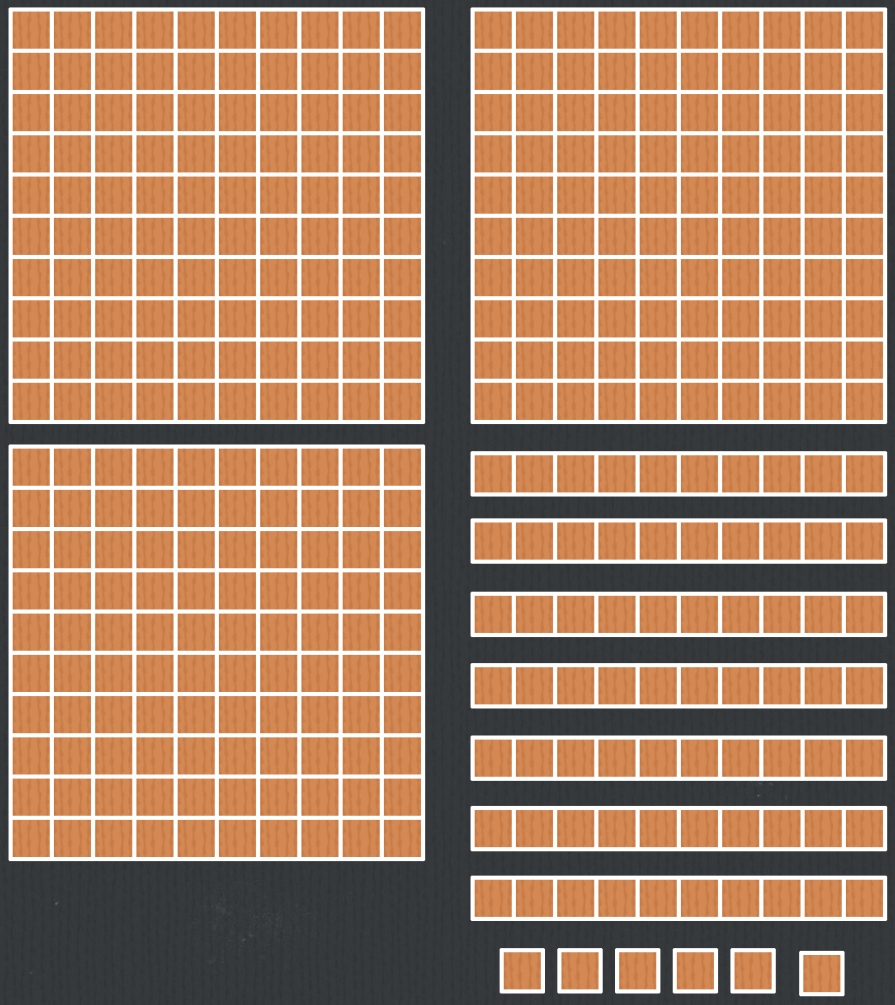
231 + 145

$$\left(2x^2+3x+1\right)+\left(x^2+4x+5\right)$$



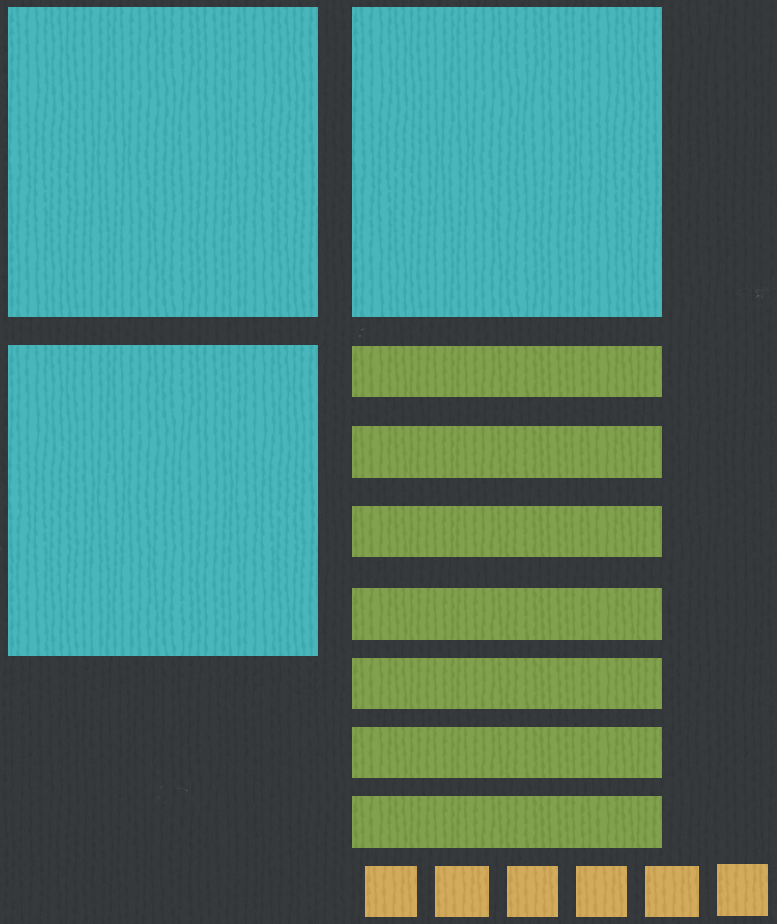


231 + 145



= 376

$$\left(2x^2+3x+1\right)+\left(x^2+4x+5\right)$$



$$= 3x^2+7x+6$$

Subtraction

- Focus on the **meanings** of subtraction.

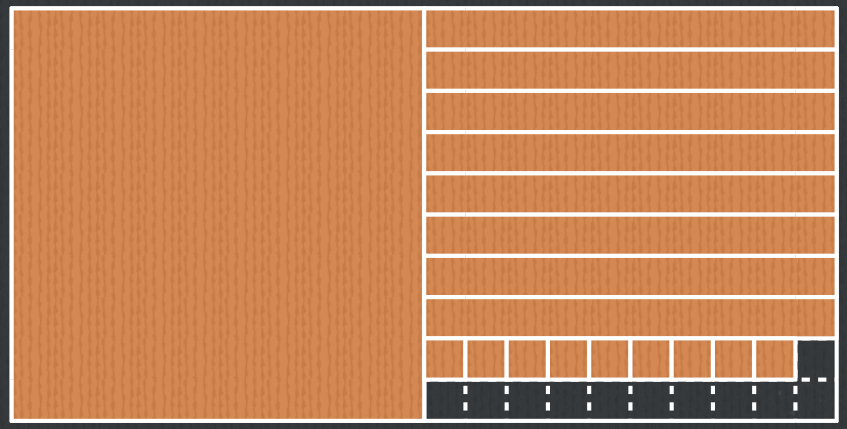
212 – 189

$$\left(2x^2+x+2\right)-\left(x^2+8x+9\right)$$

- We could think **“take away”**.
- Try thinking different, I mean, **“difference”** or **“comparison”**.

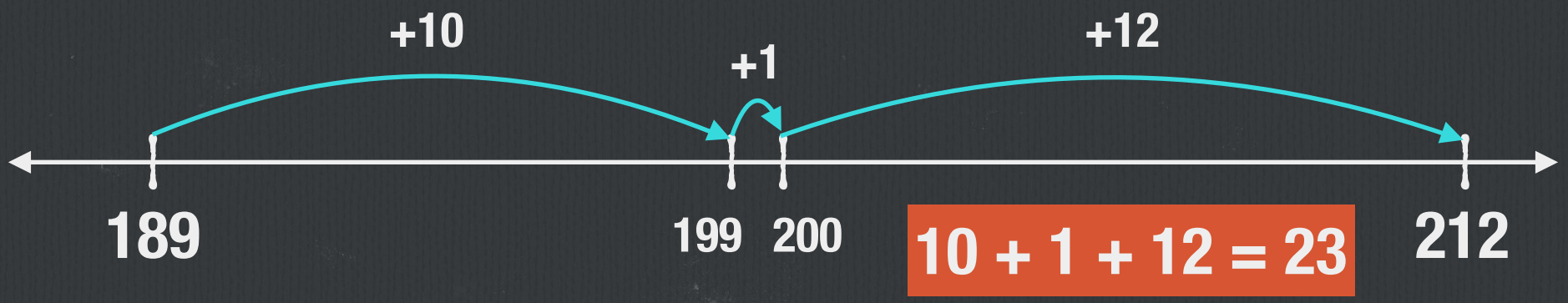
212 – 189

$$\left(2x^2+x+2\right)-\left(x^2+8x+9\right)$$



Compare/contrast with algebraic.

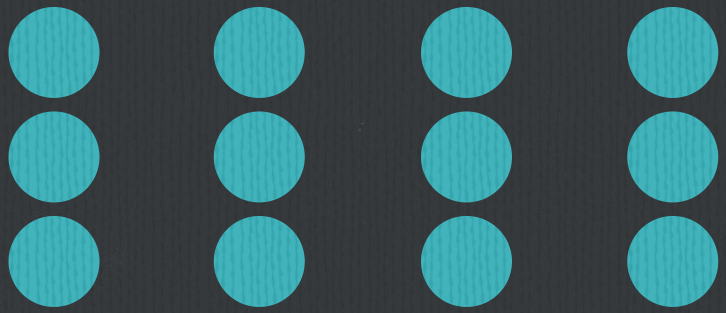
What's missing to make 212?



10 + 1 + 12 = 23

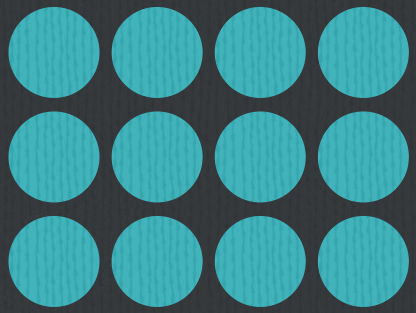
Multiplication

- From “groups of”



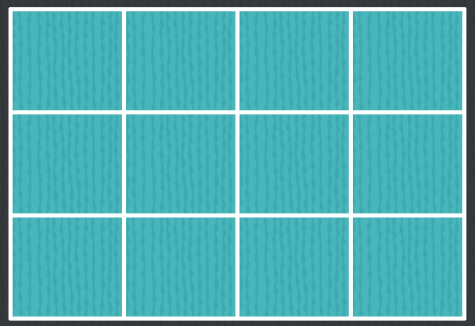
Multiplication

- From “groups of”
- To “arrays!”



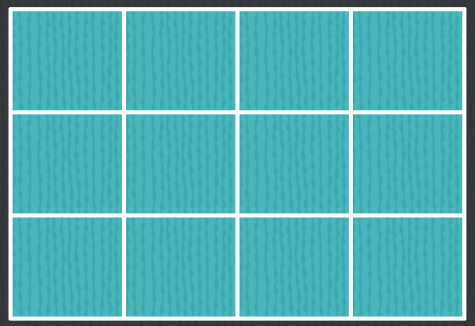
Multiplication

- From “groups of”
- To “arrays!”
- To area model



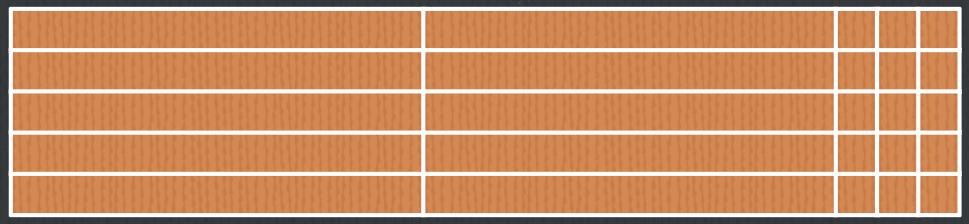
Multiplication

- From “groups of”
- To “arrays!”
- To area model
- Commutivatity
- Other meanings of multiplication.



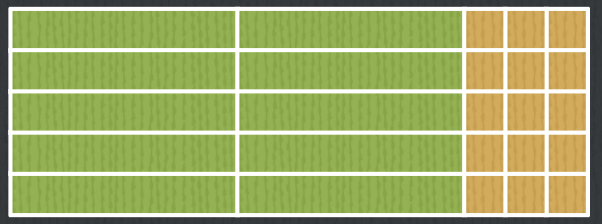
Distribution

5 x 23



= 100 + 15 = 115

$5(2x+3)$



$=10x+15$

5	20	3
	5 x 20 = 100	5x3 =15

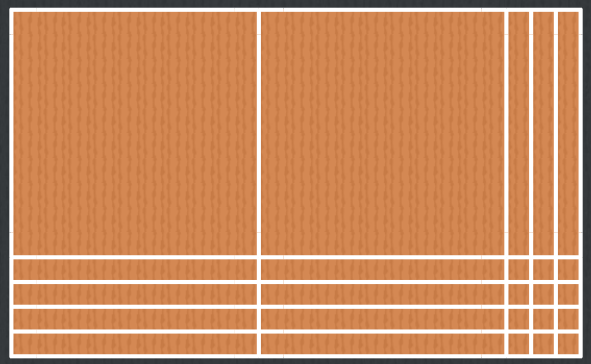
5 x 23

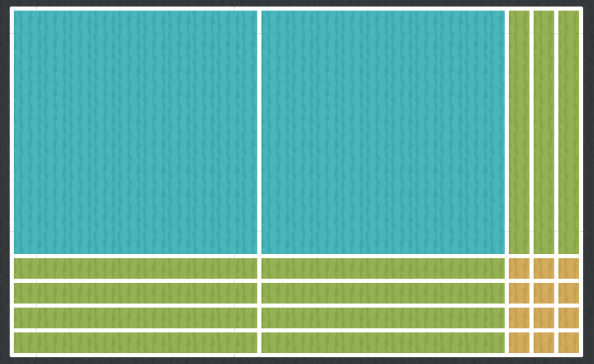
- 20 x 5 = 100
- 3 x 5 = 15

115

23 x 14

$(2x+3)(x+4)$





23 x 14

	20	3
10	20 x 10 = 200	3 x 10 = 30
4	20 x 4 = 80	3 x 4 = 12

200 + 30 + 80 + 12

= 200 + 110 + 12

= 322

$(2x+3)(x+4)$

	$2x$	3
x	$(2x)(x) = 2x^2$	$3x$
4	$4(2x) = 8x$	3 x 4 = 12

$2x^2 + 3x + 8x + 12$

$= 2x^2 + 11x + 12$

Symbolic Connections

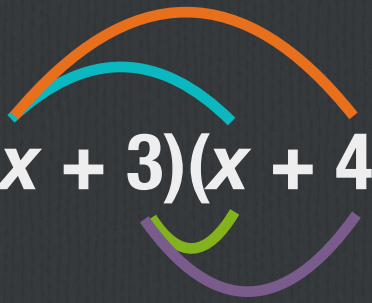
$23 \times 14 = (20 + 3)(10 + 4)$

$= 200 + 80 + 30 + 12$

$= 200 + 110 + 12$

$= 322$

$(2x + 3)(x + 4)$



$= 2x^2 + 8x + 3x + 12$

$= 2x^2 + 11x + 12$

What about that algorithm?

$$\begin{array}{r} {\scriptstyle 1} \\ 23 \\ \times\ 14 \\ \hline 92 \\ 230 \\ \hline 322 \end{array}$$

$$\begin{array}{r} 23 \\ \times\ 14 \\ \hline 12 \\ 80 \\ 30 \\ 200 \\ \hline 322 \end{array}$$

$6 \div 3$

Two meanings of division:

- **Partitive:** If 3 groups, how many in each group?

$6 \div 3$

Quotative is also called Measurement

- How many 3's go into 6?
- Think about which meaning makes sense when dividing algebraic terms.

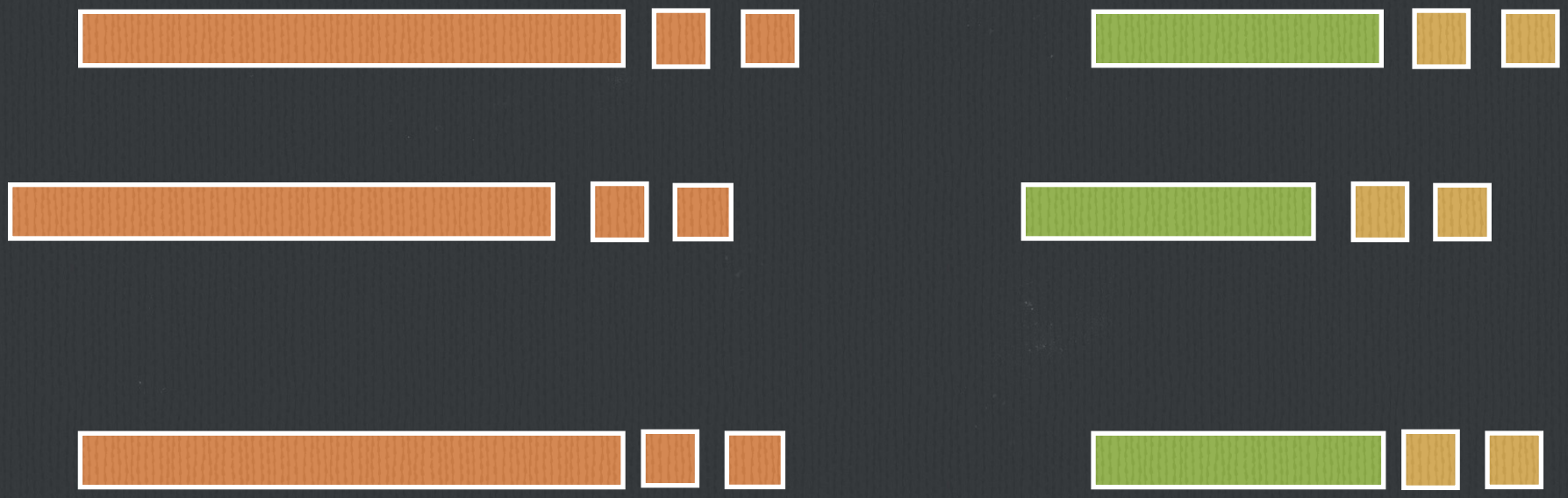
Two meanings of division:

- **Partitive:** If 3 groups, how many in each group?
- **Quotative:** If groups of 3, how many groups?

$36 \div 3$

$(3x + 6) \div 3$

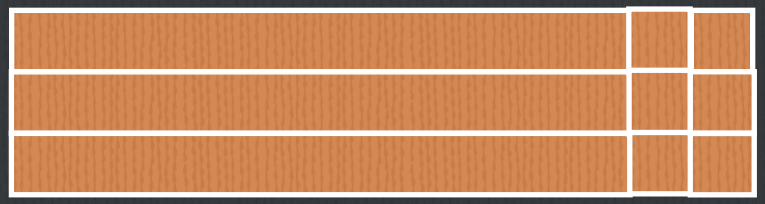
3 groups

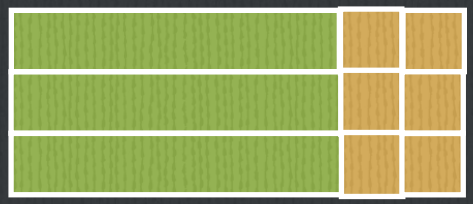


$36 \div 3$

$(3x + 6) \div 3$

3 groups





What about groups of 3?

3972 ÷ 12

$3x^3 + 9x^2 + 7x + 2 \div (x + 2)$

Long division you say?

NOT

3972 ÷ 12

$$3x^3 + 9x^2 + 7x + 2 \div (x + 2)$$

Try thinking quotative/measurement

Live demos

Patterning

4, 7, 10, …

$3x + 1$

Fig. 1

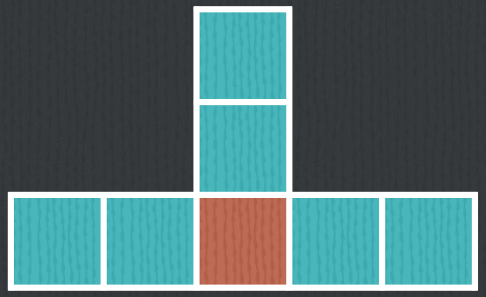


Fig. 2

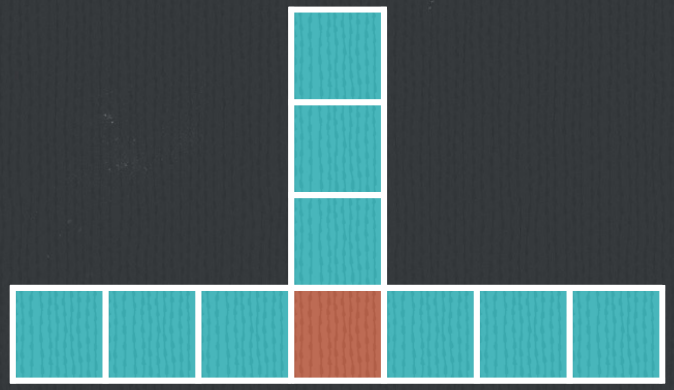


Fig. 3

#MTBoS

Plug: VisualPatterns.org